Supernova Neutrinos and Resonant Transitions [1]

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Abstract

The resonant transition effects MSW and NSFP for three flavour Majorana neutrinos in a supernova are considered. In this scenario, the deformed thermal neutrino distributions are obtained for different choices of the electron-tau mixing angle.

Among the mechanisms proposed to explain the solar neutrino problem, certainly, the most promising solution is provided by the resonant oscillation mechanism, the Mikheyev-Smirnov-Wolfenstein effect (MSW) [1], which is the most natural and seems to have the correct energy dependence. Nevertheless, one can imagine the superposition of a Neutrino Spin-Flavour Precession (NSFP) mechanism too [2, 3]. This effect is relevant when a strong transverse magnetic field is present and if the magnetic dipole moments of neutrinos are large enough. Such conditions can occur in a supernova and for these reasons, recently, the resonant neutrino oscillations in a supernova, for three-flavour Majorana neutrinos, have been considered [4].

For three flavour Majorana neutrinos the light degrees of freedom weakly interacting can be denoted by ν_{eL} , $\nu_{\mu L}$, $\nu_{\tau L}$ ($\overline{\nu}_{eR}$, $\overline{\nu}_{\mu R}$, $\overline{\nu}_{\tau R}$ for antineutrinos).[2]

In presence of a transverse magnetic dipole moment a neutrino can flip its spin and thus change its chirality. This kind of transitions are called Neutrino Spin–Flavour Precessions (NSFP) [2], and their probability can be relevant in presence of huge transverse magnetic field and for large off-diagonal dipole magnetic moment.

In analogy with MSW, even the NSFP can receive a resonant enhancement from the presence of a dense medium [2], and this is for example the case of stellar matter or the extreme condition of a supernova.

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[2] For simplicity, hereafter we will omit the indication of chirality being clear that it is left-handed for neutrinos and right-handed for antineutrinos.

Denoting with $\nu \equiv (\nu_e\ , \nu_\mu\ , \nu_\tau\ )$, and analogously $\overline{\nu} \equiv (\overline{\nu}_e\ , \overline{\nu}_\mu\ , \overline{\nu}_\tau\ )$, the unitary mixing matrix for ν and $\overline{\nu}$ can be written as [5]

$$U = \begin{pmatrix} C_\varphi C_\omega & C_\varphi S_\omega & S_\varphi \\ -C_\psi S_\omega - S_\psi S_\varphi C_\omega & C_\psi C_\omega - S_\psi S_\varphi S_\omega & S_\psi C_\varphi \\ S_\psi S_\omega - C_\psi S_\varphi C_\omega & -S_\psi C_\omega - C_\psi S_\varphi S_\omega & C_\psi C_\varphi \end{pmatrix} \quad , \tag{1}$$

where for simplicity CP violation terms have been neglected.

The evolution equation for neutrino wave functions travelling along the radial coordinate $r \simeq ct$ is

$$i\frac{d}{dr}\begin{pmatrix} \nu \\ \overline{\nu} \end{pmatrix} = H \begin{pmatrix} \nu \\ \overline{\nu} \end{pmatrix} = \begin{pmatrix} H_0 & B_\perp M \\ -B_\perp M & \overline{H}_0 \end{pmatrix}\begin{pmatrix} \nu \\ \overline{\nu} \end{pmatrix} \quad . \tag{2}$$

The symmetric matrix H_0 is the 3×3 hermitian effective Hamiltonian ruling the resonant flavour transition in the flavour basis. It reads

$$H_0 = \frac{1}{2E} U \begin{pmatrix} m_1^2 & 0 & 0 \\ 0 & m_2^2 & 0 \\ 0 & 0 & m_3^2 \end{pmatrix} U^\dagger + \begin{pmatrix} N_1(r) & 0 & 0 \\ 0 & N_2(r) & 0 \\ 0 & 0 & N_2(r) \end{pmatrix} \quad , \tag{3}$$

where $N_1(r) = \sqrt{2} G_F (N_e - N_n/2)$, $N_2(r) = -G_F N_n/\sqrt{2}$, and N_e, N_n being the electron and neutron number density, respectively. The Hamiltonian for antineutrinos, $\overline{H}_0$, is obtained from H_0 by replacing $N_1(r),\ N_2(r) \rightarrow -N_1(r),\ -N_2(r)$. The quantity M is the matrix of magnetic dipole moments

$$M = \begin{pmatrix} 0 & \mu_{e\mu} & \mu_{e\tau} \\ -\mu_{e\mu} & 0 & \mu_{\mu\tau} \\ -\mu_{e\tau} & -\mu_{\mu\tau} & 0 \end{pmatrix} \quad . \tag{4}$$

The resonant conditions for transformations $\nu_e \leftrightarrow \nu_\mu\ ,\ \nu_\tau$ and for $\nu_e \leftrightarrow \overline{\nu}_\mu\ ,\ \overline{\nu}_\tau$ can be obtained by requiring the approaching of two different eigenvalues of H in Eq.(2). For small mixing angles this essentially coincides with the condition of having coincident diagonal elements, namely, neglecting second order terms in S_ω,S_ψ and S_ϕ

$$N_1(r_s) - N_2(r_s) = 2\Delta \qquad (\nu_e \leftrightarrow \nu_\mu) \quad , \tag{5}$$

$$N_1(r_s) - N_2(r_s) = \Delta - \Lambda \qquad (\nu_e \leftrightarrow \nu_\tau) \quad , \tag{6}$$

$$N_1(r_s) + N_2(r_s) = 2\Delta \qquad (\nu_e \leftrightarrow \overline{\nu}_\mu) \quad , \tag{7}$$

$$N_1(r_s) + N_2(r_s) = \Delta - \Lambda \qquad (\nu_e \leftrightarrow \overline{\nu}_\tau) \quad , \tag{8}$$

where $\Delta = (m_2^2 - m_1^2)/4E$ and $\Lambda = (m_1^2 + m_2^2 - 2m_3^2)/4E$. In the same way, the resonant conditions for $\overline{\nu}_e \leftrightarrow \overline{\nu}_\mu\ ,\ \overline{\nu}_\tau$ and for $\overline{\nu}_e \leftrightarrow \nu_\mu\ ,\ \nu_\tau$ are obtained from the charge conjugate transitions (5)-(8) by changing sign in the corresponding r.h.s. . Note that r_s denotes the value of coordinate r for which each resonant condition is verified.

Once that neutrino parameters, namely ψ, φ, ω, Δ and Λ, are fixed one can study the occurrence of resonant conditions (5)–(8) and charged conjugate ones, by using the particular density profile for electrons and neutrons, contained in $N_1(r)$ and $N_2(r)$.

In Ref.[6] the above description for neutrino dynamics in presence of matter has been used in order to explain the solar neutrino problem. The authors assume there for simplicity that flavour mixing occurs in the e–μ sector only. As a result of their analysis, it is shown that for a small value of the mixing angle $\theta_{e\mu} = \omega$, $\nu_e \leftrightarrow \overline{\nu}_\mu$ transitions are sufficient to account for the solar neutrino problem. In this case the predictions strongly depend on the magnetic field configurations even if a best fit is achieved for the so-called LIN2 [6] solar magnetic field parametrization. Typical values of neutrino parameters able to reproduce the data are $\Delta m^2_{e\mu} \simeq \left(10^{-8} \div 10^{-7}\right)\ eV^2$ and $\sin\left(2\theta_{e\mu}\right) \lesssim 0.2 \div 0.3$ [6].

The neutrino flux dynamics contained in Eq.(2) can be also applied to the case of a supernova. In this case, the mass density ranges from $\sim 10^{-5} g/cm^3$ in the external envelope up to $\sim 10^{15} g/cm^3$ in the dense core, with the following density profile

$$\rho \simeq \rho_0 \left(\frac{R_0}{r}\right)^3 \quad , \tag{9}$$

with $\rho_0 \simeq 3.5{\times}10^{10} g/cm^3$, and $R_0 \simeq 1.02{\times}10^7 cm$ denoting the radius of the neutrinosphere. Note that, the electron fraction number Y_e outside the neutrinosphere can be assumed almost constant and fixed at the value $Y_e = 0.42$.

As far as the transverse magnetic field is concerned, one can safely assume the simple expression

$$B_\perp(r) \simeq B_0 \left(\frac{R_B}{r}\right)^2 \quad , \tag{10}$$

where r denotes the radial coordinate, and the constant $B_0 R_B^2 \simeq 10^{24}$Gauss cm^2.

In first approximation neutrinos are emitted from neutrinosphere with a Fermi-Dirac distribution

$$n^0_{\nu_\alpha}(E) \simeq \frac{0.5546}{T^3_\alpha}\, E^2 \left[1 + \exp\left(\frac{E}{T_\alpha}\right)\right]^{-1} \quad , \tag{11}$$

with the different flavours equally populated. The index α in Eq.(11) denotes the particular neutrino species. Due to their different interactions, the temperature of ν_e and ν_μ , ν_τ neutrinosphere result $T_e \simeq 3\ MeV$ and $T_\mu = T_\tau \simeq 6\ MeV$. As far as the $\overline{\nu}_e$ distribution is concerned, it is characterized by $T_{\bar{e}} \simeq 4\ MeV$, whereas for $\overline{\nu}_\mu$ and $\overline{\nu}_\tau$ we have $T_{\bar{\mu}} = T_{\bar{\tau}} \simeq 6\ MeV$.

Assuming that the solar neutrino problem is solved in terms of $\nu_e \rightarrow \overline{\nu}_\mu$ NSFP, as explained in Ref.[6], from Eq.s(5)–(8) we see that in a supernova (for $Y_e = 0.42$ outside the neutrinosphere) four resonance conditions can be fulfilled. With decreasing density, and for τ neutrinos more massive than μ ones, we first encounter the region for e–τ resonance transitions, and then that for e–μ ones. In order we have: $\overline{\nu}_e \leftrightarrow \nu_\tau$, $\nu_e \leftrightarrow \nu_\tau$, $\overline{\nu}_e \leftrightarrow \nu_\mu$ and $\nu_e \leftrightarrow \nu_\mu$.

To deduce the total transition probabilities, it is important to establish if the different resonance regions overlap, namely if the resonance widths are larger than their separation in r. In many GUT models it is natural to expect $m_{\nu_\mu} << m_{\nu_\tau}$, thus in this paper we make this assumption. As a consequence, the resonance regions involving e–τ flavours and those involving e–μ flavours are well separated between them. For example, for 10 MeV neutrinos with $m_{\nu_\mu} \simeq 10^{-3}\ eV$ and $m_{\nu_\tau} \simeq 10\ eV$ we have the e–τ resonances around the region of density $\approx 10^8\ g/cm^3$ (deep in the supernova). On the contrary, the resonances e–μ is in the external envelope of supernova ($\approx 1\ g/cm^3$). For supernova neutrinos ($E \approx 0 \div 50\ MeV$), assuming (9) and (10), it is easy to see that for a wide range of models there is non overlapping. Thus, each resonance can be considered independently from the others.

For the MSW resonance the survival probability reads [5, 7]

$$P_{\alpha\beta}\left(\nu_\alpha \rightarrow \nu_\alpha\right) = \frac{1}{2} + \left(\frac{1}{2} - \exp\left\{-\frac{\pi}{2}\gamma_{\alpha\beta}F_{\alpha\beta}\right\}\right)\cos 2\theta_{\alpha\beta}\cos 2\theta^m_{\alpha\beta} \quad , \tag{12}$$

where the adiabaticity parameter $\gamma_{\alpha\beta}$ and $\theta^m_{\alpha\beta}$ are given by

$$\gamma_{\alpha\beta} = \left.\frac{\Delta m^2_{\alpha\beta} L_\rho \sin^2 2\theta_{\alpha\beta}}{2E\cos 2\theta_{\alpha\beta}}\right|_{\rm res} \quad , \qquad \tan 2\theta^m_{\alpha\beta} = \frac{2\Delta m^2_{\alpha\beta}\sin 2\theta_{\alpha\beta}}{2\sqrt{2}EG_F N_e - \Delta m^2_{\alpha\beta}\cos 2\theta_{\alpha\beta}} \quad . \tag{13}$$

For NSFP transitions one has [7]

$$P_{\bar\alpha\beta}\left(\overline{\nu}_\alpha \rightarrow \overline{\nu}_\alpha\right) = \frac{1}{2} + \left(\frac{1}{2} - \exp\left\{-\frac{\pi}{2}\gamma_{\bar\alpha\beta}F_{\alpha\beta}\right\}\right)\cos 2\theta_{\alpha\beta}\cos 2\theta^m_{\bar\alpha\beta} \quad , \tag{14}$$

$$\gamma_{\bar\alpha\beta} = \left.\frac{8E\mu^2_{\alpha\beta}B^2_\perp L_\rho}{\Delta m^2_{\alpha\beta}}\right|_{\rm res} \quad , \quad \tan 2\theta^m_{\bar\alpha\beta} = \frac{4E\mu_{\alpha\beta}B_\perp}{2\sqrt{2}EG_F\left(N_e - N_n\right) - \Delta m^2_{\alpha\beta}\cos 2\theta_{\alpha\beta}} \quad . \tag{15}$$

Note that for both the adiabatic parameters $\gamma_{\alpha\beta}$ and $\gamma_{\bar\alpha\beta}$ of Eq.(13) and (15), the adiabatic condition reads: $\gamma_{\alpha\beta}, \gamma_{\bar\alpha\beta} >> 1$.

The non adiabatic correction factor $F_{\alpha\beta}$ of Eq.s(12) and (14), neglecting non adiabatic effects induced by the magnetic field with respect to those due to density, is given by [5]

$$F_{\alpha\beta} \simeq \left(1 - \tan^2(\theta_{\alpha\beta})\right)\left\{1 + \frac{1}{3}\left[\log\left(1 - \tan^2(\theta_{\alpha\beta})\right)\right.\right.$$

$$+ \quad 1 - \frac{1+\tan^2(\theta_{\alpha\beta})}{\tan^2(\theta_{\alpha\beta})} \log\left(1+\tan^2(\theta_{\alpha\beta})\right)\Bigg]\Bigg\} \quad . \tag{16}$$

In terms of survival or transition probabilities the outcoming neutrino distributions result

$$\begin{aligned}
n_{\nu_e} \quad &= \quad P(\nu_e \to \nu_e)\, n^0_{\nu_e} + [1 - P(\nu_e \to \nu_e) - P(\overline{\nu}_e \to \nu_e)]\, n^0_{\nu_x} \\
&+ \quad P(\overline{\nu}_e \to \nu_e)\, n^0_{\overline{\nu}_e} \quad , \\
n_{\nu_\mu} \quad &+ \quad n_{\nu_\tau} = [1 - P(\nu_e \to \nu_e)]\, n^0_{\nu_e} + [1 - P(\overline{\nu}_e \to \overline{\nu}_e) - P(\overline{\nu}_e \to \nu_e)]\, n^0_{\overline{\nu}_e} \\
&+ \quad [P(\nu_e \to \nu_e) + P(\overline{\nu}_e \to \overline{\nu}_e) + P(\overline{\nu}_e \to \nu_e)]\, n^0_{\nu_x} \quad , \\
n_{\overline{\nu}_e} \quad &= \quad P(\overline{\nu}_e \to \overline{\nu}_e)\, n^0_{\overline{\nu}_e} + [1 - P(\overline{\nu}_e \to \overline{\nu}_e)]\, n^0_{\nu_x} \quad ,
\end{aligned} \tag{17}$$

where $n^0_{\nu_\mu} = n^0_{\nu_\tau} = n^0_{\nu_x}$. Note that to obtain the above expression we have only used the unitarity and the observation that $P(\nu_e \to \overline{\nu}_e)$ is vanishing (up to the first order in the mixing angle).

The survival probabilities $P(\nu_e \to \nu_e)$ and $P(\overline{\nu}_e \to \overline{\nu}_e)$ can be written as products of the single survival probabilities at the resonances, namely

$$P(\nu_e \to \nu_e) \quad \simeq \quad P_{e\tau}(\nu_e \to \nu_e)\, P_{e\mu}(\nu_e \to \nu_e) \quad , \tag{18}$$

$$P(\overline{\nu}_e \to \overline{\nu}_e) \quad \simeq \quad P_{\bar{e}\tau}(\overline{\nu}_e \to \overline{\nu}_e)\, P_{\bar{e}\mu}(\overline{\nu}_e \to \overline{\nu}_e) \quad . \tag{19}$$

while the transition probability $P(\overline{\nu}_e \to \nu_e)$ takes the expression

$$\begin{aligned}
P(\overline{\nu}_e \to \nu_e) \quad \simeq \quad & P_{\bar{e}\tau}(\overline{\nu}_e \to \overline{\nu}_e)\, [1 - P_{\bar{e}\mu}(\overline{\nu}_e \to \overline{\nu}_e)]\, [1 - P_{e\mu}(\nu_e \to \nu_e)] \\
+ \quad & P_{e\mu}(\nu_e \to \nu_e)\, [1 - P_{\bar{e}\tau}(\overline{\nu}_e \to \overline{\nu}_e)]\, [1 - P_{e\tau}(\nu_e \to \nu_e)]
\end{aligned} \tag{20}$$

According to the above results (17), the deformed thermal neutrino spectra are obtained once that neutrino parameters are fixed.

For the electron–muon sector, since we are interested in the situation in which e.m. properties of neutrinos play an essential role, the relevant parameters can be fixed assuming the NSFP explanation [6] for solar neutrino problem. In this case, the deficit in the solar neutrino flux is mainly due to the conversion $\nu_e \to \overline{\nu}_\mu$, being assumed Majorana neutrinos, which is the natural choice occurring in GUT theories where a see-saw mechanism is at work.

In the NSFP framework, the values for neutrino parameters able to reproduce the data are $\Delta m^2_{e\mu} \simeq 10^{-8}\ eV^2$ and $\sin(2\theta_{e\mu}) \simeq 0.2$ and $\mu_{e\mu} \simeq 10^{-11}\mu_B$ [6].

Concerning the parameters for the electron–tau sector, they are less constrained. However, we can fix $\Delta m^2_{e\tau} \simeq m^2_{\nu_\tau} \simeq 25\ eV^2$ in order to be able to identify ν_τ has the hot component of dark matter [8], whereas, for the transition magnetic moment we can assume $\mu_{e\tau}$ to be of the same order of $\mu_{e\mu}$, since, typically, the enhancement to the electromagnetic properties is due to physics beyond the electroweak interactions, which hardly distinguishes between τ–leptons and μ–leptons. Hence, the only remaining parameters is $\theta_{e\tau}$, for which we will choose three indicative values, namely, 10^{-1}, 10^{-4} and 10^{-8}. Values in this range are for example predicted by SUSY GUT theories [9], where one could expect in principle an enhancement of the neutrino electromagnetic properties. In ref. [10] the bound $\theta_{e\tau} \leq 10^{-3}$ for $\Delta m^2_{e\tau} = 1 \div 100 eV^2$ has been obtained from an analysis of heavy elements nucleosynthesis in supernovae, assuming adiabaticity of mixing processes. They also consider a particular site for rapid neutron capture processes. We have nevertheless also considered the case of a large mixing angle $\theta_{e\tau} \sim 10^{-1}$ in view of the uncertainties which still affect the description of supernova explosion dynamics.

In Figures 1–3, we show the deformed neutrino distributions for ν_e, the sum of ν_μ and ν_τ , and $\overline{\nu}_e$ distributions, respectively, versus their initial distributions. In all figures, the solid line represents the initial distribution and the predictions for the outcoming neutrino distributions are obtained for the above three values of $\theta_{e\tau}$ [4].

The MSW $\nu_e \leftrightarrow \nu_\tau$ conversion becomes less and less efficient as $\theta_{e\tau}$ decreases, as can be seen by the other two lines in Figures 1 and 2. For the other values of $\theta_{e\tau}$, in fact, $\gamma_{e\tau} << 1$. However, as one can see from the dashed-dotted line of Figure 1, corresponding to the extremely small value $\theta_{e\tau} = 10^{-8}$, a conversion of ν_e in other kind of neutrinos still remains. In fact, in this case the only conversions remaining are the NSFP as one can see by observing the antineutrinos spectra of Figure 3. Note that since we have fixed $\mu_{e\mu} \sim \mu_{e\tau} \sim 10^{-11}\ \mu_B$, all the deformed antineutrino distributions are almost coincident, because, in this case, they are almost independent of the on neutrino mixing angles.

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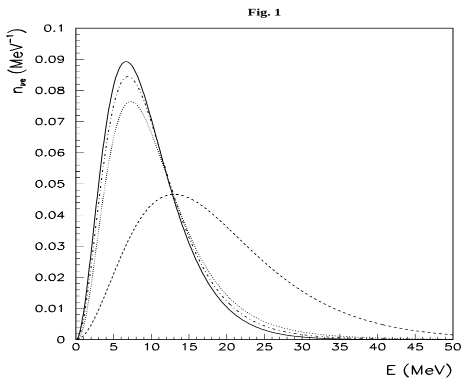

Figure 1: The energy spectra for ν_e versus energy are reported. The solid line represents the initial distribution, whereas the dashed line corresponds to the distorted spectra for $\theta_{e\tau} = 10^{-1}$. The dotted line and dashed–dotted correspond to $\theta_{e\tau} = 10^{-4}$ and 10^{-8}, respectively [4].

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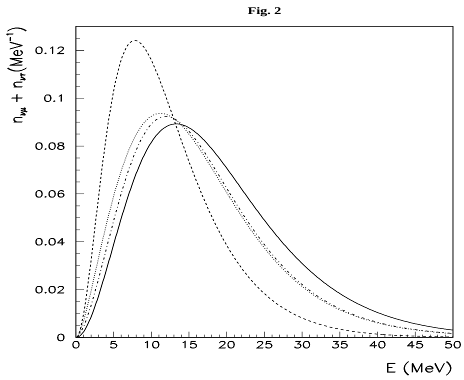

Figure 2: The energy spectra for $n_{\nu_\mu} + n_{\nu_\tau}$, with the same notation of Figure 1 [4].

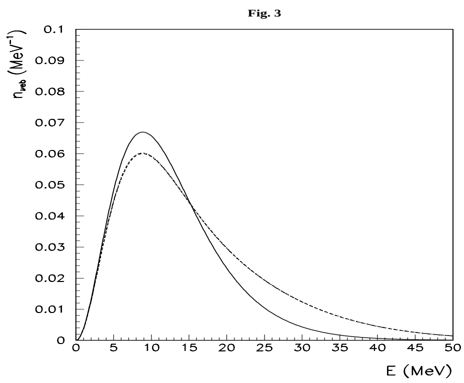

Figure 3: The energy spectra for $n_{\overline{\nu}_e}$, with the same notation of Figure 1 [4].