THE "VOLUMETTI" BY ETTORE MAJORANA

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Abstract. We report on a careful analysis of the five notebooks ("Volumetti") left unpublished by Ettore Majorana.

Ettore Majorana published only few papers (9 different articles, one of which posthumous) on frontier physics topics, discussed in the years going from the end of the second and the beginning of the third decade of the XX century. His scientific activity, however, was not limited to those papers, and many handwritten notes exist, which record his work on different areas of physics (and applied mathematics). Most of the unpublished manuscripts are deposited at the *Domus Galilaeana* in Pisa (Italy) and have been catalogued [1]; they consist essentially in:

- the thesis for his "laurea" degree;
- 12 folders with first-hand notes;
- 18 booklets (the "Quaderni"),
- 5 notebooks (the "Volumetti").

In the present work, we have analyzed Majorana's notebooks, known as the "Volumetti" [2]. These are 5 small, orderly notebooks written in Rome between 1927 and 1932. Each of them is composed of about 100-150 sequentially numerated pages (making, all together, 587 pages) of approximate size 11cm × 18cm. Each notebook was written during a period of time of about one year, starting from the years (1927 - early 1828) in which Ettore Majorana was completing his studies at the University of Rome. Therefore, the content of these notebooks ranges from typical topics covered in academic courses to frontier research problems, as one can immediately sees by looking at the table of contents reproduced below in the appendix. Note that, in the original manuscripts, the author himself indicated the table of contents for each notebook in its first page. Probably, Majorana used such notebooks as helpers in his studies and researches.

Although some points still remain to be clarified, a careful analysis of the manuscripts reveals in any case the original method adopted by the author in writing the notebooks.

The key observation is that, in some cases, *numerated* blank pages appear between the end of a section and the beginning of the following one. Most likely, this points out that Majorana used to approach the issues treated in the notebooks by following well defined mental schemes. However, we can possibly exclude that the author made an *ab initio* plan of his notebooks (as, for example, for writing a book) since some topics are studied in different (not adjacent) sections. The table of contents for each notebook was written by the author as soon as a particular section was completed. It is remarkable to observe that a date, written by Majorana on the initial blank page of each notebook, records when the notebook was finished. Indeed, the last Volumetto, which is the smallest one, was probably left unfinished and does not contain this explicit information. Nevertheless, we can fix the "closing" date of

the Volumetti to be in the year 1932. Actually, in this year the word *neutron* was coined to denote the neutral nucleon, and this word has been used by Majorana in the second section of the last notebook, thus establishing a lower limit for that date. Furthermore, an upper limit can be set by looking at Section 8 of the same book, which contains material used by the author in his seventh article published in 1932.

A quick look at the table of contents of the Volumetti shows that a great variety of topics was tackled by Majorana, which range from classical electromagnetism to statistical mechanics, from Earth physics to atomic, molecular and nuclear physics, from relativity to group theory. Despite such a mixing between different arguments (which can be detected by looking at different notebooks, as well as at each single book), the method with which a topic is treated is never obvious.
As an example, one can refer here to the study of the shift of the melting point for a given substance when it is placed in a magnetic field region (see Section 28 of the first notebook) or, more interestingly, to the study of heat propagation in a one-dimensional medium (considered in Section 38 of the same notebook). In the latter case it is particularly interesting the first method used by the author to study the problem of heat propagation along a rectilinear, homogenous and isotropic bar of finite, or infinite, length. He considered the two end points of the bar as heat sources of different but fixed temperature, and then developed a "method of sources" analogous to the method of images used in electrostatics.
Also noticeable are the contemporary physics topics faced by Majorana in an original and very clear way, like the Fermi explanation of the electromagnetic mass of the electron (Section 41 of Volumetto I), the Dirac equation with its applications, and the Lorentz group (in Volumetti III and IV), which are an indication, in some cases, of the preferred literature.
As far as frontier research arguments are concerned, we quote here two illuminating examples only: the study of quasi-stationary states in Section 28 of Volumetto IV, anticipating the Fano theory of about 20 years [3], and the Fermi theory of atoms in Section 7 of Volumetto II. In the latter Section, the author set forth (for the first time) analytic solutions to the Thomas-Fermi equation, with boundary conditions appropriate for atoms,in terms of simple quadratures; to our best knowledge, such solutions remain today the only known semi-analytical solutions to the Thomas-Fermi equation. Notice that the interest for these pages, however, is not limited to the exhibition of some particular analytical solutions, but lies also in the very original approach used to obtain them (see Ref. [4]).

The methods used by Majorana in his studies, as they emerge1 from the Volumetti, are those typical of a high-level physicist. He got started from a well-defined physical problem, which is then faced by using all the possible tools. Let us add that even many tables and/or compilations of known mathematical results, as well as several tables of experimental data, appear in the Volumetti. Note also that the strictly mathematical topics present in the notebooks are, almost exclusively, treated by Majorana in view of their applicability to physical problems. Moreover, when the author suggests a new model or obtains new results from well established theories, they have been always compared with the existing experimental results.

As a conclusion, the personality of Ettore Majorana which arises from the analysis of his notebooks can be represented in the form envisaged by Fermi [1]: "... Because, you see, in the world there are various categories of scientists; people of

a secondary or tertiary standing, who do their best but do not go very far. There are also those of high standing, who come to discoveries of great importance, fundamental for the development of science. But, then, there are geniuses like Galileo or Newton. Well, Ettore was one of them. Majorana had what no one else in the world had ...".

Appendix A. Table of Contents for the "Volumetti"

In the following we give the table of contents for the five notebooks by Ettore Majorana considered in this paper.

VOLUMETTO I

VOLUMETTO II

26. Definite integrals
27. Series expansion
28. Radiation theory (Part 5): free electron scattering
29. De Broglie waves
30. $e^2 \simeq hc$?
31. The equation $y'' + Py = 0$
32. Indetermination of the vector and of the scalar potential
33. On the spontaneous ionization of an hydrogen atom placed in a high potential region
34. Scattering of an α particle with a radioactive nucleus
35. Retarded potential
36. The equation $y'' = xy$
37. Resonance degeneracy for many-electron atoms
38. Various formulas
    - 38.1 Schwarz formula
    - 38.2 Maximum value of random variables
    - 38.3 Binomial coefficients
    - 38.4 Expansion of $1/(1-x)^n$
    - 38.5 Relations between the binomial coefficients
    - 38.6 Mean values of r^n between concentric spherical surfaces

VOLUMETTO III

1. Evaluation of some series
2. The equation $\Box H = r$
3. Equilibrium of a rotating heterogeneous liquid body (Clairaut Problem)
4. Determination of a function from its moments
5. Probability curves
6. Evaluation of the integral $\int_0^{\pi/2} \frac{\sin kx}{\sin x}\, dx$
7. Infinite products
8. Bernoulli numbers and polynomials
9. Poisson brackets
10. Elementary physical quantities
11. "Chasing the dog"
12. Statistical potential in molecules
13. The group of proper unitary transformations in two variables
14. Exchange relations for infinitesimal transformations in the representations of continuous groups
15. Empirical relations for a two-electron atom
16. The Group of Rotations $O(3)$
17. The Lorentz Group
18. Dirac matrices and the Lorentz Group
19. The spinning electron
20. Characters of D_j and reduction of $D_j \times D'_j$
21. Intensity and selection rules for a central field
22. The anomalous Zeeman effect (according to the Dirac theory)
23. Complete sets of first order differential equations

VOLUMETTO IV

1. Connection between the susceptibility and the electric moment of an atom in its ground state
2. Ionization probability for a hydrogen atom in an electric field
3. Legendre polynomials expansion in the interval $-1 \leq x \leq 1$
4. Multiplication rules for Legendre polynomials
5. Green functions for the differential equation $y'' + \left(\frac{2}{x} - 1\right) y + \phi(x) = 0$
6. On the series expansion of the integral logarithm function
7. Fundamental characters of the group of permutations of f objects
8. Expansion in spherical harmonics of the plane wave
9. The Rutherford formula deduced from Classical Mechanics
10. The Rutherford formula deduced as first approximation of the Born method
11. The Laplace equation
12. Polarization forces between hydrogen atoms
13. Integral representation of the Bessel functions
14. Cubic symmetry
15. Formulae
16. Plane waves in the Dirac theory
17. Improper operators
18. Integral representation of the hydrogen eigenfunctions
19. Deflection of an α ray induced by a heavy nucleus (Classical Mechanics)
20. Scattering from a potential of the form $\frac{a}{r} - \frac{b}{r^2}$
21. The set of orthogonal functions defined by the equation $y_a'' = (x - a)y_a$
22. Fourier integral expansions
23. Circular integrals
24. Oscillation frequencies of ammonia
25. Spherical functions with spin (s=1)
26. Scattering of fast electrons (relativistic Born method)
27. Frequently used atomic quantities
28. Quasi-stationary states
29. Spherical functions with spin (II)

VOLUMETTO V

1. Representations of the Lorentz group
2. Proton − neutron scattering
3. Zeros of half order Bessel functions
4. Statistics and thermodynamics
   - 4.1 Entropy of a system in equilibrium
   - 4.2 Perfect gases
   - 4.3 Mono-atomic gas
   - 4.4 Bi-atomic gas
   - 4.5 Numerical expressions for the entropy of a gas
   - 4.6 Free energy of bi-atomic gases
5. Frequently used polynomials

References

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