A Logical Analysis of Majorana's Papers of Theoretical Physics

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Abstract: We study two celebrated Majorana's papers through a method of investigation which relies upon the recently recognised distinction between classical logic and *grosso modo* the several kinds of non-classical logics, i.e. the failure of the double negation law. This law fails when a double negated sentence is not equivalent to the corresponding positive sentence, owing to the lack of scientific evidence of the latter one. All recognised double negated sentences inside the text of each paper are listed; the mere sequence of such sentences gives the logical thread of Majorana's arguing. This one is recognised to be of the Lagrangian kind, which mixes logical arguing and mathematical calculation; i.e. the author puts a fundamental problem which is solved by anticipating the mathematical hypothesis which is capable of solving it, and then by drawing from this hypothesis the mathematical consequences in order to obtain the desired result. Furthermore Majorana's rhetoric of presentation results to be a juridical one, owing to his style of presenting the laws to which an ideal theoretical physicist has to conform himself in order to solve the problem at issue.

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1. Introduction

Recently the interest for the human and scientific figure of Ettore Majorana acquired vigour. Apart several biographies and books (about his life and in particular about the unsolved problem of his disappearance [1]), the following books have been edited: the last lectures at Naples University in Italian language [2]; and, in English language, his "Volumetti" [3], where Majorana was accustomed to solve for his own interest scientific

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problems of various kinds; it seems that his accuracy and precision was the higher one with respect to any other scientist's notebook. This element supports what Fermi spoke about him:

"...in the world there are various categories of scientists: people of a secondary or tertiary standing, who do their best but do not go very far. There are also those of high standing, who come to discoveries of great importance, fundamental for the development of science' (and here I had the impression that he placed himself in that category). 'But then there are geniuses like Galileo and Newton. Well, Ettore was one of them. Majorana had what no one else in the world had ..." [4].

Maybe such an evaluation appears somewhat apologetic; but Fermi saw some other elements. In the following we want to analyse a characteristic feature of Majorana papers - i.e. the kind of logic in his arguments - which was missed in previous analysis.

2. Majorana as A Theoretical Physicist: Papers and Publications

The scientific production by E. Majorana includes nine printed publications [5].

In last years some studies wanted to put such papers inside a historical background, then to interpret them and possibly to develop them for achieving new results.

Some decades ago E. Amaldi wrote a review on Majorana's scientific production [6]; he first offered a detailed valuations of them; it constitutes a first characterisation of Majorana's work in theoretical physics.

Furthermore in last years some improvements have been considered of the ideas that Majorana started to develop and then have been forgotten - though independently recuperated by others -; in particular, his solutions of those differential equations which are useful in many physical problems [7].A first valuation for Majorana's work on symmetries has been given; along with Giovanni Gentile jr, for the first time he introduced this mathematical tool in the Italian theoretical physics scenario [8]. It is also interesting to note that he was almost the unique (though remote) Weyl's follower in Weyl's attitude of founding quantum mechanics and the field theory through group theory only [9].

The problem is open whether it is possible to reconstruct Majorana's scientific production according to a line of development which represents a conception of theoretical physics as a whole, for instance a conception similar to H. Weyl's.

The present paper is aimed to improve the comprehension of Majorana's papers by means of a technique of mathematical logic. Among his papers we have chosen two of the most important papers in theoretical physics: *Teoria relativistica di particelle con momento intrinseco arbitrario* (Relativistic theory of particles with arbitrary intrinsic momentum) of 1932 and *Teoria simmetrica dell'elettrone e del positrone* (Symmetric theory of electron and positron) of 1937 [10]. The historical background of these papers is offered by the above mentioned Amaldi's review.

3. Analysis of A Scientific Writing by Means of Non-Classical Logic

In the above-mentioned text Majorana illustrates a theory which is based upon a very general problem, i.e. the relativistic generalisation of Dirac theory of the electron for particles of arbitrary spin. For solving this problem, a deductive method of solution does not *a priori* exist. The final result is not deductively drawn from some a priori principle; rather, the arguments proceed in a heuristic way, without absolute certainty.

Elsewhere, one of us (A. D.) suggested the model of theoretical organisation aimed to search a new scientific method capable to solve a given universal problem [11]. Inside this kind of organisation, called a problem-based one, several double negated sentences (DNS) play a crucial role, each one being not equivalent to the corresponding positive sentence, owing to the lack of (either experimental or theoretical) scientific evidence of the latter one. For instance, in the historical development of thermodynamics very often the following sentence occurs: "It is impossible the perpetual (= without an end) motion" (we underlined the negations for evidentiating them to the reader; the same will be done in the following DNSs); the corresponding positive statement is "Each motion has an end"; in order to state it we have to know either the instant or the place where the given motion will end; this knowledge is impossible, owing to our ignorance of both the friction function which affects the motion and the length of the path along which friction is exerted upon the body in motion. Hence, the positive sentence is not supported by scientific facts.

In 20^{th} Century the studies upon the foundations of mathematical logic resulted in establishing more precisely the borderline between classical logic and *grosso modo* non-classical logics; this bordeline is the failure of rather than the law of excluded middle (either A or *not*-A), the double negation law (*not*-*not*-A $=A$) [12]. This change in the attention is very relevant for, since inside a scientific writing a scholar easily recognises a sentence *not*-*not*-A and then easily he tests whether in this sentence the double negation logical law holds true or not, by testing whether the corresponding sentence A is supported by scientific evidence or not.

According to what occurs in the problem-based theories, in Majorana's paper too occur DNSs; hence, Majorana was surely out of a full deductive theory in classical logic. Hence, it is useful to analyse his writings under this logical aspect. To this aim it is necessary first to list all DNSs included by Majorana's paper.

4. Inspection of All DNSs in *Teoria Relativistica* (1932)

Let us analyse the paper by recognising the DNSs inside it. In order to make easier the reader to recognise the DNS nature of each sentence listed in the following, the corresponding positive sentence is written inside square brackets, after a sign $\neq$.

There exists several kinds of DNSs. Beyond the plain DNSs (either a sentence or a word including two negations say, in-variant, etc.) there are the "weak" DNSs (in the following: wDNS), that is those including a modality, as "may", "allow", "want", "must " (in the following each of these words is written in Italic and the order number of its DNS is marked by an asterisk*). The appearance of these words is quite improper in a scientific text, meant in a traditional sense, i.e. as a deductive exposition; because this exposition has to be based upon either experimental or deductive certainties; if instead such modalities occur, they show that the arguing is heuristic in nature; in such a case they have to be expressed by means of DNSs; for ex. "*may* = it is not true that it is not ".

Furthermore, by taking into account that Majorana ignored at all non-classical logic, we will consider as a DNS also sentences which are DNS in a more general sense, for example the words "equivalent", "analogous", similar, each one to be meant as "not unequal".

1*. (p. 335) [...] A relativistic generalisation of the previous theory *must* [= it is not true that is not] satisfy [≠ satisfies] to the following conditions when its degree of accuracy is growing up:

2*. (*a*) The theory *allows* the study of [≠ studies] particles having velocity almost determined in magnitude and direction,...

3. ... giving results which are equivalent [= that are not different from] [≠ equal] to the non relativistic theory,...

4. (p. 336) without however the necessity of choosing [= differentiate] a particular reference system [≠ for all systems].

5*. (*b*) The theory moreover *allows* to study [≠ studies] processes where the velocities of the particles vary slowly, but in an extended range, owing to weak external fields.

6. (*c*) The theory is valid in general, whatever be undetermined [≠ for an arbitrary determination of] the velocity of the particles.

7. [...] On the contrary, it is likely [= it is not true that it is not] [≠ is true] that...

8. ... a theory satisfying (*b*) and only in part (*c*) does not hurt in subtantial difficulties [≠ is valid].. .

9*. ... its physical content *can* be [= it is not true that it is not] [≠ is] essentially the same justifying the Schroedinger equation.

10. The most remarkable example of such generalisations is just offered by DIRAC's theory, but since this applies only to particles with intrinsic momentum s=1/2, I looked for *analogous* [= not different] [≠ equal] in form equations to DIRAC's, though somewhat more cumbersome,...

11*. ... which *allow* to consider [≠ consider] particles with arbitrary momentum, also for a vanishing value of it

12*. [...] Equations of this kind [that is Dirac-like] present a difficulty of principle. The operator -1 indeed *must* transform [≠transforms] as the temporal component of a 4-vector...

13*. ... and so β *cannot be* [≠ is] merely a multiple of the unity matrix,...

14*. ... but *must* have [≠ has] at least two different eigenvalues, say β_1 and β_2 ;...

15*. ... but from this fact follows that the rest-energy of the particles, which is obtained from the (1) by putting $p = 0$, *must have* [$\neq$ has] at least two different values, $\beta_1 mc^2$ and $\beta_2 mc^2$.

16*. According to the DIRAC equation the possible values of the rest mass are, as it is well-known, $+m$ and -m, from which, due to the relativistic invariance follows that the energy *can have* [$\neq$ has] two different values owing to the sign of each value of p : $W = \pm\sqrt{m^2c^4 + c^2p^2}$.

17. The indeterminacy in the sign of energy can be overcame [= cancelled] [$\neq$is determined], by using equations of the fundamental kind (1),...

18. (p. 337) ... only if the wave function has infinite components which *not* [Italic in the text] allow to be broken [= not-reducible] [$\neq$ elementary] in finite tensors or finite spinors.

19*. 1. The equation (1) *can be* [$\neq$ is] deduced from the variational principle [a formula follows].

20. One of the conditions of relativistic in-variance [$\neq$ constancy]...

21. ... has of course to be in-variant [$\neq$ constant] the form [a formula follows].

22*. If now we want that the rest-energy result [$\neq$ when the rest-energy is] always positive,...

23*. ... the eigenvalues of β *must be* [$\neq$ are] all positive ones and the form [a formula follows] will be positive definite.

24*. Then it is *possible* to reduce [$\neq$ reduces] through a non-unitary transformation $\psi \rightarrow \varphi$ such a form to the unity form: [a formula follows].

25. By substituting in (2) for ψ his expression by means of φ one will have: [a formula follows] from which the equations equivalent [$\neq$ egual] to (1) follow: [a formula follow].

26*. We now *must* determine the law of transformation of φ according to a LORENTZ rotation and moreover expression of matrices...

27. ... in such a way that the relativistic in-variance [$\neq$ constancy] of the variational principle be satisfied (4)...

28. ... and hence the function to be integrated be invariant [$\neq$ constant].

29*. Let us start by stating the law of transformation of φ by observing first of all that the invariance of [a formula follow] means that we *must consider* [$\neq$ we consider] unitary transformations only.

30*. (p. 338) The operators a e b *must be* [$\neq$ are] Hermitian in a unitary representation and *viceversa*;...

31*. ... furthermore, in order to the infinitesimal transformations to be integrable *must satisfy* [$\neq$ satisfy] some commutation relations which are deduced from both (6) and (7): (formulas follow) and the orther ones which are obtained by permutation of x, y and z.

32. (p. 339) The denomination " zero-index " means that is zero the in-variant [= the constant operator]: [a formula follows].

33*. [...] 2. We now *must* determine [$\neq$ we determine] the operators [a formula follows]...

34. ... in such a way that (4) be invariant [$\neq$ constant].

35*. Because we consider unitary transformations only, the above-mentioned operators transform as the Hermitian forms related to them, and hence owing to the invariance [≠ constancy] of the integrand function in (4)...

36. ... it is *necessary* that they constitute [≠ constitute] a covariant vector [a formula follows].

37*. [...] The operators γ *must satisfy* [≠ satisfy] the exchange relations [formulas follow] and some others obtained by cyclical permutation of x, y, z.

38. (p. 342) [...] Without difficulties [≠ easily] it is verified that...

39*. [...] These states *can be regarded* [≠ are] as belonging to the imaginary value ik of the mass.

40*. We now *want to consider* [≠ consider] quickly the introduction of the electromagnetic field in equation (16).

41. (p. 343) For instance we may add invariant [≠ constant] terms,...

42. ... analogous [≠ equal] to those introduced by Pauli in the theory of the magnetic neutron, which considers the field strength instead of the electromagnetic potentials,...

43. ... so that to do not unsettle [≠ maintain as a constant]...

44. ... the invariance [≠ the constancy] of the equations according to the indeterminacy of the potentials.

In total, in 10 pages of this paper we recognised 21 DNSs (among them 6 are due to the word "invariant") and 23 wDNSs.

5. Analysis of the Arguing in *Teoria Relativistica* (1932)

The same kind of analysis when applied to scientific texts written by some other authors, gives each time a list of DNSs, whose sequence is sufficient to manifest the logical thread of the presentation at issue. This is the case for instance of the discursive part of S. Carnot's booklet, i.e. the part originating most of modern thermodynamics. His theory is a problem-based theory since it puts a universal problem: whether a maximum efficiency in converting heat into work there exists.

Its sequence of DNSs can be broken into some units of arguing; each unit starts from a DNS setting a problem, then some DNSs establish methodological principles leading to construct a result which, in a rigorous way, is obtained by means of an *ad absurdum* argument, such as it is the celebrated S. Carnot's theorem in thermodynamics [13]. But this is not the case of Majorana's paper, because the DNSs are intermixed with mathematical derivations; where each mathematical formula includes an exact equality; hence, the negation of the negation of the formula gives again the same formula, in agreement with classical logic and contrarily to the case of a DNS.

However, a more sophisticated case than S. Carnot's thermodynamics is given by Lagrange's mechanics. This theory too wanted to solve a great problem, which Lagrange himself stated, i.e., how to develop a theory of mechanical phenomena including constraints. Also inside Lagrange's book (*Mécanique analytique*) we recognise DNSs; it is even possible to point out some units of arguing; each one of them starting from a new problem; which then is solved. In this case, however, the solving argument belongs to classical logic, being developed by means of mathematical derivations. Hence, the cycle of arguing links the non-classical logic of DNSs with the classical logic of mathematical formulas. In which way are all they linked together inside a consistent logical development?

Lagrange suggests a formalism constituted by a set of differential equations from which one achieves the wanted solution; which is derived in such an easy way ("a regular and uniform march", claims Lagrange in p. *vi* of his book) to give us the feeling of making use of a "magic wand" [14]. Let us analyse the deep logical structure of this formalism.

Lagrange declares that his theory develops a general principle, capable to govern the whole theoretical physics. Then Lagrange presents his celebrated differential equations. Although he tried to justify them by means of pseudo-derivations, one recognises that these equations are no less than a generalisation of the principle of virtual works [15]. Hence, these differential equations are put *a priori* by him. Which is their theoretical import?

Actually, the Principle of virtual works results from a problem-based theory, aimed to solve the constraints problem. Its core actually is a DNS, i.e. "No work from nothing "; or, in more specific terms "It is impossible that the not real forces of the constraints furnish positive work". Here we recognise the last step of an *ad absurdum* argument. Its mathematical formula, translating an essentially DNS, includes an inequality (likely as the mathematical formula for the variations of entropy in thermodynamics). It is a methodological principle, addressing to obtain specific solutions. But, the long practice of its effectiveness led theorists to rise up it to an *a priori* principle; from a philosophical viewpoint, owing to its rejection of an absurdity, it constitutes a general reality principle. This idealisation of a methodological principle in a so general *a priori* principle opens the door to a more idealistic operation.

Let us consider the same argument leading to the principle of virtual works from a classical logic viewpoint. A logical scheme of natural deduction may help the reader [16].

Cycle of arguing in discursive terms	***Translation of the cycle in mathematical terms***
Common Knowledge: (X), (Y), (Z),...	General axiom-principle (= $\neg\bot$)
$\neg$ T	Specific principles: X, Y, Z,...
Methodological Principles	Conditions (= PM mathematised)
AA	Mathematical derivations
--------------	--------------
$\bot \rightarrow \neg\neg$ T	T

Legenda: (X, Y, Z) = physical conditions; X, Y, Z = the corresponding mathematised hypotheses. MP = Methodological Principle; AA = *Ad absurdum* Argument; $\bot$ =

Absurd. To draw consequences is represented by the vertical line.

Notice that whereas the first column shows the (non-classical) implication ¬ T → ⊥, the second column shows the classical converse of this implication (let us recall that if M → N, the classical converse (which yet is rejected by non-classical logic) is ¬ N → ¬ M), i.e. ¬⊥ → ¬¬ T → T ; just what classical logic allows, i.e. to reverse through a conversion laws an *ad absurdum* argument in an affirmative argument, however the conclusion may be idealistic in nature.

Let us reverse the path of the previous argument leading to the methodological principle of the virtual works, so to start from its final step; then the negated *absurdum* is equivalent a positive sentence. Which by a mathematical step, can be referred no more to the constraints reactions, but to the resulting active forces; then it, by concerning these forces which can be *a posteriori* experimented, seems a full physical principle. Hence, its philosophical nature of a reality principle together with the (semi-)physical nature of its mathematical formula qualify it as an instance of theoretical omniscience ("all the necessary equations for the solution of each problem", p. 12).

When it is applied no more to forces, but, through the Lagrange's celebrated step $\Sigma(f_i\text{-}ma_{i)}=0$, to velocities it becomes an omniscient principle on dynamical equilibria. In total, Lagrange generalises the negated *absurdum* to a very general mathematical formula, playing the role of an omniscient principle.

However, the nature of the theory to be a problem-based one, is not lost. The last step, the solution of the problem at issue, is obtained not in forces causing effects (trajectories), but in in-variants (about energy, momentum, etc.), i.e. the same results originated by each theory based upon a general problem.

Majorana's paper, though including DNSs, does not conform to Carnot's problematic model but to Lagrange's; indeed, *ad absurdum* arguments are absent. In order to solve the problem at issue he introduces a sophisticated formalism (variational calculus applied upon the action principle, theory of group representations), which in fact is the Lagrangian formalism. Then, he develops the arguments in a full mathematical way; once he recognises the suitable hypotheses (which actually suggest the restrictions under which the problem is solvable) he puts them as mathematical premises; then the subsequent development merely verify that they effectively solve the problem at issue. We conclude that his arguing is similar to Lagrange's.

6. The Scheme of Arguing in *Teoria Relativistica* (1932)

From both the direct reading of the paper (without taking in account DNSs) and by the knowledge of the scientific subject dealt with, one recognises the following thread of the speech.

[*General problem*] (p. 335) To generalise Schroedinger's mechanics to the relativistic framework

The generalisation has to satisfy three conditions: (a), (b) e (c) [see DNSs no.s 2, 5 and 6]

The third condition is not easy to be satisfied

Dirac equation has already solved the problem under the conditions (b) and in part (c)

But Dirac equation holds only for spin $s = 1/2$.

[*Specific problem*] (p. 336) A generalisation is required for arbitrary spin

In Dirac equation the mass operator assumes two eigenvalues, a positive and a negative one

[*Methodological principle*: it seems an unnatural fact the negative mass or, equivalently (in the theory of relativity), negative energy]

[*Methodological principle*] (p. 336) Looking for an equation which is similar to Dirac's, but dealing with only positive mass.

[*Anticipation of the solution in a mathematical form*] (pp. 336-337) The equation can be obtained by introducing a condition upon the wave function: this has an infinite number of components forming an irreducible representation of the Lorentz group; Majorana thus makes use of unitary infinite-dimensional representations of the Lorentz group.

[*Having apperceived by intuition the possible final solution, the development comes back to the problem and a mathematical step is performed*] (p. 337) In order to go from the wave function pertaining to Dirac equation to that pertaining to the novel equation to be recognised, a non-unitary transformation is required.

[*Position of the principle of mathematical omnisolution from which to draw the desired solution*] (p. 337) Variational principle on the action function.

From this principle Dirac equation can be obtained.

In Dirac equation the operator $\beta\ m\ c\hat{}2$ [$=\beta\ m\ c^2$] is the only one including the mass; it is an invariant.

[*Anticipation of the mathematical solution*] (p. 337) In order to obtain only positive mass values, it is necessary that the matrix β has only positive eigenvalues.

[*Methodological principle*] (p. 337) This can be achieved by changing the β operator by means of a non-unitary transformation.

[*Result*] (p. 337) The general form of the Majorana equations are obtained, as a modification of the Dirac's one.

[*Subordinate Problem*] The subsequent task is to establish the invariance of the wave function under the Lorentz group, being the matrices γ occurring in the Majorana equation undefined.

[*Methodological principle*] (p. 337) An infinitesimal Lorentz transformation does it. A Hermitian infinite-dimensional representation of the above group is considered.

[*Result*] (p. 339) Infinite spinors and tensors.

[*Subordinate Problem*] (p. 339) Majorana looks for the invariance of γ matrices [*Methodological principle*] under the Lorentz group; he obtains an explicit expression for them.

[*General result*] (p. 342) Hence, the obtained infinite-dimensional representations of the γ's gives the complete determination of his equations.

[*Further problem*] (p. 342) Generalisation in order to include electromagnetic interaction.

[*Methodological principle, chosen among various other methodological principles*] (p. 343) A transformation on both W and p introducing electric charge, scalar and potential vectors; the new equations are obtained and discussed.

Let us now come back to the list of DNS.

The interpretative analysis of the paper in units of non-classical logic arguing, expressed by DNSs, is as follows:

1 ° unit: The DNSs 1-11 describe the theoretical background; the general problem is put by (DNS 1,10). This unit will be closed by means of the unit 4 °.

2 ° unit: The specific problem of eliminating negative values of mass is expressed (DNSs 12-16); some methodological principles for solving it follow, i.e. the use of Dirac-like equation, variational principle (DNSs 17-19) and Lorentz invariance of the equation (DNSs 20-21); the problem is then reiterated (DNS 22), the condition for the invariance of the coefficients is introduced in a mathematised form (DNS 24) and the result (Majorana equations) is obtained (DNS 25).

3 ° unit: The problem of the transformation law of the wave function φ is put (DNSs 26-28) and after the methodological principle of unitarian representations (DNS 29), the result is obtained (DNS 30-32).

4 ° unit: After both the problem of determining the operators for the Lorentz invariance of the equations (DNSs 33-34), and the methodological principle (DNS 35-36), the result is obtained and discussed (DNS 37-38)

5 ° unit: After the problem (DNS 40), he chooses among many others a methodological principle for introducing electromagnetic fields in analogy to Pauli theory in the magnetic neutron (DNSs 41-44).

In total, we have five units of arguing; but they are not cycles of arguing, because, by relying the arguing upon mathematical deduction, it never ends by means of an *ad absurdum* theorem; i.e. that kind of arguing which has to close a correct arguing in non-classical logic. It is manifest to the reader that this lack is due to the introduction of the mathematical hypotheses which anticipate the final point of the arguing and, owing to the mathematical formalism, have to be verified as (even partial) solutions only of the problem set at the beginnings.

7. Analysis of *Teoria Simmetrica* (1937)

An accurate reading of this paper suggests that the deductive thread of thinking in Majorana can be organised according to the following steps:

- Variational principle using real variables
- Method of quantization
- Dirac equation in terms of real variables
- Relativistic invariance according to the Jordan-Wigner method

Instead, his exposition is a complex one:

- *Problem*: Replacement of the Dirac equation

- *Problem*: generalization of variational principle
- *Methodological principle*: the method of quantization;

from which he obtains an undetermined formula for the equation of motion.

- These equations can be represented by an Hamiltonian if a certain relationships on his

magnitudes hold true

- Then he shows that Dirac equations (written in terms of real variables) can be obtained from the variational principle (using real variables)
- By means of the mentioned method, these equations may be quantized
- They are relativistically invariant because they reiterate a well-known scheme
- Applications

The reader can see that again the exposition is not organised in a deductive one; rather Majorana puts a problem and then a more general problem; after the solution of the latter one, he can solve his starting problem. This method of resolution starts from a variational principle in the Lagrangian formalism and then he puts the Hamilton equations as the closing step of the presentation.

Here we could reiterate the same interpretative analysis as above. First, to list the DNSs in the text and then to group them according to units of arguing. But for not boring the reader, we leave out the list of DNSs, whose number however is almost the double of the previous one: 82 DNSs in 14 pages of the paper; 43 of which are normal DNSs and 41 wDNSs.

The set of cycles of arguing shows one time more that his theory is a problem-based theory. The analysis through cycles of arguing is already apparent from the previous scheme: a first cycle sets the problem and then the more general problem is stated. A second cycle applies the method of quantization in order to obtain the formula of the equations of motion, which are then compared with the Hamiltonian and thus declared valid (actually this argument can be considered an implicit *ad absurdum* arguing). After this step, he closes the first cycle of arguing by deriving the solution of his problem from the solution obtained for the general problem (variational principle in real variables). The remaining part is devoted to verifications and logical consequences.

8. Global Considerations on the Two Papers

Let us consider the distributions of the DNSs and the wDNSs. Notice that there in both paper the recourse to DNSs is frequent (a mean which varies from 2 per page to 3 per page). Moreover, there is not a substantial difference between the two distributions of DNSs and wDNSs; hence, by summing up the two kinds of DNS, weak or not, we obtain a double number of DNS (around 5) per page.

Summing up, the two papers illustrate a consistent way of arguing in the particular organisation of a physical theory, the problem-based one. The examination of the DNSs, including the wDNSs, shows that by making use of non-classical logic Majorana well recognises and formalises all the problems he faces to. On the other hand he states

TAB. THE DISTRIBUTIONS OF DNSs INSIDE THE TWO MAJORANA PAPERS

	DNS	**wDNS**				DNS per page
		necessity	permission	possibility	want	
Teoria relativistica	21	10	3	8	1	2, 1 DNSs and 2,3 wDNS per page (in 10 pages)
Teoria simmetrica	43	14	5	20	-	3 DNS and 2,9 wDNS per page (in 14 pages)
Total	64	24	8	28	1	2,6 DNS and 2,5 wDNS per page (in 24 pages)

the action principle, from which he derives all mathematical consequences useful for his scopes. In this way he fashions his theory as a kind of Lagrangian theory, just the principle to which he makes appeal.

9. The Weak DNS and the Rhetoric of the Scientific Presentation by E. Majorana

The following kinds of rhetoric in presenting a scientific novelty seem to come out:

1) Historical report of the way the scientific discovery happened in the mind of the researcher or in the laboratory; hence, the set of the data are described by listing them in the way they occurred along the time of the discovery.

2) Exposition of formal laws which hold true for both the obtained data and the calculations (hence, the set of data and calculations: must, obey, result, etc.).

3) The list of obligations and potentialities that the " ideal solver" wants in order to reiterate the experience to solve the problem (hence the solver: must, it is necessary, see, it is possible to him, he is allowed, etc.).

4) A programmatic arguing which then leaves the developments as the executive part.

5) The deductive or axiomatic exposition of the novelty, obtained as a consequence of a known theory (hence the verbs are: is, holds true, follows, etc.).

In the paper at issue (but also inside the "Volumetti") Ettore Majorana's rhetoric is apparently the third one; he presents the laws holding true for the ideal scientist; hence, it is the "juridical" way of speaking to a scientist. About this point, one can show that for instance in the second paper the DNSs can be grouped in different sets according to which they refer to Nature laws N, to calculation laws C, to the attitude asked to the theoretical physicist F: the maximum number is for those of kind F: 10, plus 34 wDNSs. We mean that Majorana imposes these laws to each scientist; in this sense his presentation is more than a "juridical" one; it can be considered a "royal" one.

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[4] Reported in the letter from G. Cocconi to E. Amaldi (18 July, 1965). The letter is quoted in E. Recami: *op. cit.* p. 23.

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