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Supernova neutrino energy spectra and resonant transition effects

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MSW and NSFP Resonant transition effects for three flavour Majorana neutrinos in supernova are considered and the deformed neutrino distributions are obtained for different choices of the electron-tau mixing angle.

1. Introduction

The mechanisms proposed to explain the deficit in the observed number of solar neutrinos with respect to the theoretical predictions of the Standard Solar Model (SSM) are basically three. The pure oscillation in vacuum is strongly disfavoured by the experimental evidence for a quite strong energy dependence of the flux depletion mechanism. Certainly, the most promising solution is provided by the resonant oscillation mechanism, the Mikheyev-Smirnov-Wolfenstein effect (MSW) [1], which has the correct energy dependence. Nevertheless, one can imagine the superposition of a Neutrino Spin-Flavour Precession (NSFP) mechanism too [2]. This effect is relevant when a strong transverse magnetic field is present and if the magnetic dipole moments of neutrinos are large enough.

In this paper we will consider MSW and NSFP processes for three-flavour Majorana in supernova [3], [4], where the conditions of large density and strong magnetic field are likely to be obtained.

2. Resonant transitions of Majorana neutrinos

For three flavour Majorana neutrinos the independent degrees of freedom, which are coupled with weak interaction, can be denoted by ν_{eL}, $\nu_{\mu L}$, $\nu_{\tau L}$ and for antineutrinos by $\overline{\nu}_{eR}$, $\overline{\nu}_{\mu R}$, $\overline{\nu}_{\tau R}$. For simplicity, hereafter we will omit the indication of chirality being clear that it is left-handed for neutrinos and right-handed for antineutrinos.

In presence of a transverse magnetic dipole moment a neutrino can flip its spin and thus change its chirality. However, CPT invariance forbids these transitions for Majorana neutrinos, unless they change flavour at the same time, namely $\nu_{\alpha L} \to \overline{\nu}_{\beta R}$, with $\alpha, \beta = e, \mu, \tau$ and $\alpha \neq \beta$.
This kind of transitions are called Neutrino Spin-Flavour Precessions (NSFP), and their probability can be large if the amplitudes of the transverse magnetic field and of the off-diagonal dipole magnetic moment are large enough. As neutrino oscillations in matter (MSW), even the NSFP can receive a resonant enhancement from the presence of a dense medium. This is for example the case of stellar matter or the extreme condition of a supernova.

Let us denote with ν the vector in the flavour space $\nu \equiv (\nu_e, \nu_\mu, \nu_\tau)$, and analogously $\overline{\nu} \equiv (\overline{\nu}_e, \overline{\nu}_\mu, \overline{\nu}_\tau)$. The evolution equation for neutrino wave functions travelling along the radial coordinate $r \simeq ct$ is

$$i\frac{d}{dr}\begin{pmatrix} \nu \\ \overline{\nu} \end{pmatrix} = \begin{pmatrix} H_0 & B_\perp M \\ -B_\perp M & \overline{H}_0 \end{pmatrix}\begin{pmatrix} \nu \\ \overline{\nu} \end{pmatrix} \quad (1)$$

where the symmetric matrix H_0 is the 3×3 hermitian effective Hamiltonian ruling the resonant flavour transition in the flavour basis

$$H_0 = \frac{1}{2E} U \mathcal{M} U^\dagger + \begin{pmatrix} N_1(r) & 0 & 0 \\ 0 & N_2(r) & 0 \\ 0 & 0 & N_2(r) \end{pmatrix} (2)$$

where U is the mixing matrix, $\mathcal{M}$ the diagonal squared mass matrix, $\mathcal{M} = diag(m_1^2, m_2^2, m_3^2)$, $N_1(r) = \sqrt{2} G_F (N_e - N_n/2)$, $N_2(r) = -G_F N_n/\sqrt{2}$, N_e and N_n being the electron and neutron number density, respectively. The corresponding Hamiltonian for antineutrinos, denoted with $\overline{H}_0$, is obtained from (2) by replacing

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$N_1(r),\ N_2(r) \to -N_1(r),\ -N_2(r)$. Finally, M is the matrix of magnetic dipole moments

$$M = \begin{pmatrix} 0 & \mu_{e\mu} & \mu_{e\tau} \\ -\mu_{e\mu} & 0 & \mu_{\mu\tau} \\ -\mu_{e\tau} & -\mu_{\mu\tau} & 0 \end{pmatrix} \tag{3}$$

The resonant conditions for transformations $\nu_e \leftrightarrow \nu_\mu$, ν_τ and for $\nu_e \leftrightarrow \overline{\nu}_\mu$, $\overline{\nu}_\tau$ can be obtained by requiring the approaching of two different eigenvalues of H in Eq.(1). For small mixing angles this essentially coincides with the condition of having coincident diagonal elements.

In the following we will assume, as in [5], that $\nu_e \leftrightarrow \overline{\nu}_\mu$ transitions account for the solar neutrino problem, though it should be mentioned that in this case the predictions strongly depend on the magnetic field configurations. Typical values of neutrino parameters able to reproduce the data are $\Delta m^2_{e\mu} \simeq \left(10^{-8} \div 10^{-7}\right)\ eV^2$ and $\sin\left(2\theta_{e\mu}\right) \lesssim 0.2 \div 0.3$.

3. Effects on thermal neutrino spectra

To get predictions on supernova neutrino spectra it is necessary to specify density and magnetic field profiles. In a supernova the mass density ranges from $\sim 10^{-5} g/cm^3$ in the external envelope up to $\sim 10^{15} g/cm^3$ in the dense core. We will assume, hereafter, the following density profile [6]

$$\rho \simeq \rho_0 \left(\frac{R_0}{r}\right)^3 \tag{4}$$

with $\rho_0 \simeq 3.5 \times 10^{10} g/cm^3$, and $R_0 \simeq 1.02 \times 10^7 cm$. The quantity R_0 denotes the radius of the so-called *neutrinosphere*, which represents the bounding surface of the region in which neutrinos of a given flavour are in thermal equilibrium. Note that, the electron fraction number Y_e outside the neutrinosphere can be assumed almost constant and fixed at the value $Y_e = 0.42$. For the transverse magnetic field, one can instead consider the simple expression [7]

$$B_\perp(r) \simeq B_0 \left(\frac{R_B}{r}\right)^2 \tag{5}$$

where r denotes the radial coordinate, and the constant $B_0 R_B^2 \simeq 10^{24}$Gauss cm^2.

We will assume that inside the neutrinosphere the resonance conditions are not satisfied, as suggested by predictions on neutrino mass spectrum of a wide range of unified gauge models. In first approximation neutrinos are therefore emitted from this surface with a Fermi-Dirac distribution

$$n^0_{\nu_\alpha}(E) \simeq \frac{0.5546}{T_\alpha^3} E^2 \left[1 + \exp\left(\frac{E}{T_\alpha}\right)\right]^{-1} \tag{6}$$

with the different flavours equally populated. The index α in Eq.(6) denotes the particular neutrino species. Since the production and scattering cross-sections for electron-neutrinos are larger than for the other flavours, ν_e are produced a bit more copiously with respect to the other ones. Thus, their neutrinosphere is larger than that for ν_μ , ν_τ . This implies that the temperature T_α in (6) for the ν_e -sphere is lower than the one of ν_μ , ν_τ . Furthermore, since ν_μ and ν_τ are produced and scatter on the surrounding matter only through neutral currents, they have identical spectra. Obviously, since ν_e , ν_μ , ν_τ and $\overline{\nu}_e$, $\overline{\nu}_\mu$, $\overline{\nu}_\tau$ are produced in pairs, the magnitude of neutrino and antineutrino distributions are equal for each flavour. For the temperature of ν_e and ν_μ, ν_τ neutrinosphere we adopt the typical values $T_e \simeq 3\ MeV$ and $T_\mu = T_\tau \simeq 6\ MeV$. As far as the $\overline{\nu}_e$ distribution is concerned, it is characterized by $T_{\bar{e}} \simeq 4\ MeV$, whereas for $\overline{\nu}_\mu$ and $\overline{\nu}_\tau$ we have $T_{\bar{\mu}} = T_{\bar{\tau}} \simeq 6\ MeV$.

It can be easily seen that in a supernova (for $Y_e = 0.42$ outside the neutrinosphere) four resonance conditions can be fulfilled [3]. With decreasing density, and for τ neutrinos more massive than μ ones, we first encounter the region for e–τ resonance transitions, and then that for e–μ ones. In order we have: $\overline{\nu}_e \leftrightarrow \nu_\tau$, $\nu_e \leftrightarrow \nu_\tau$, $\overline{\nu}_e \leftrightarrow \nu_\mu$ and $\nu_e \leftrightarrow \nu_\mu$.

To deduce the total transition probabilities, it is important to establish if the different resonance regions overlap, namely if the resonance widths are larger than their separation in r. In many grandunified models it is natural to expect $m_{\nu_\mu} << m_{\nu_\tau}$. In this paper we make this assumption, so that the resonance regions involving e–τ flavours and those involving e–μ flavours are well separated between them. For example, for 10 MeV neutrinos with $m_{\nu_\mu} \simeq 10^{-3}\ eV$ and $m_{\nu_\tau} \simeq 10\ eV$ we have the e–τ resonances around the region of density $\approx 10^8\ g/cm^3$ (deep in the supernova); on the contrary, the resonances e–μ is in the external envelope of supernova ($\approx 1\ g/cm^3$). Further, the MSW-NSFP resonance region non-overlapping condition is given by

$$L_\rho \left|\tan 2\theta_{\alpha\beta}\right| + \left|\frac{2\mu_{\alpha\beta} B_\perp(r_1)}{(\Delta m^2_{\alpha\beta}/2E)}\right| L_\rho \lesssim r_2 - r_1 \qquad (7)$$

where r_1 is the radial position of the NSFP resonance, while r_2 is that of the MSW one, and $L_\rho \equiv -[(1/\rho)d\rho/dr]^{-1}$ is assumed to vary slowly between r_1 and r_2. In Eq.(7) α and β label the flavours involved in the resonance ($\alpha, \beta = e, \mu, \tau$), $\theta_{\alpha\beta}$ denote the mixing angles, and $\Delta m^2_{\alpha\beta}$ is the squared mass differences of the relevant mass eigenstates. For supernova neutrinos ($E \approx 0 \div 50\ MeV$), assuming (4) and (5), it is easy to see that for all cases considered below, the non-overlapping condition (7) is well satisfied for both $\nu_e, \overline{\nu}_e \leftrightarrow \nu_\tau$ and $\nu_e, \overline{\nu}_e \leftrightarrow \nu_\mu$ transitions.

For the MSW and NSFP resonances we have a simple semi-analytical formula for the survival probability

$$P_{\alpha\beta}(\nu_\alpha \to \nu_\alpha) = \frac{1}{2} + \left(\frac{1}{2} - \exp\left\{-\frac{\pi}{2}\gamma_{\alpha\beta} F_{\alpha\beta}\right\}\right) \cdot$$
$$\cdot \cos 2\theta_{\alpha\beta} \cos 2\theta^m_{\alpha\beta} \qquad (8)$$

where α, β refer to the neutrino or antineutrino states involved in the transition, $\gamma_{\alpha\beta}$ are the adiabaticity parameters, $\theta^m_{\alpha\beta}$ is the effective mixing angle in matter and $F_{\alpha\beta}$ is the non adiabatic correction factor. For the explicit expressions of all these parameters see for example [3].

The outcoming neutrino distributions can be simply written in terms of initial distribution and probabilities (8) as follows

$$n_{\nu_e} = P(\nu_e \to \nu_e)\, n^0_{\nu_e} +$$
$$[1 - P(\nu_e \to \nu_e)]\, n^0_{\nu_x} + P(\overline{\nu}_e \to \nu_e)\, n^0_{\overline{\nu}_e} \qquad (9)$$

$$n_{\nu_\mu} + n_{\nu_\tau} = [1 - P(\nu_e \to \nu_e)]\, n^0_{\nu_e}$$
$$+ [1 - P(\overline{\nu}_e \to \overline{\nu}_e) - P(\overline{\nu}_e \to \nu_e)]\, n^0_{\overline{\nu}_e}$$
$$+ [P(\nu_e \to \nu_e) + P(\overline{\nu}_e \to \overline{\nu}_e)]\, n^0_{\nu_x} \qquad (10)$$

$$n_{\overline{\nu}_e} = P(\overline{\nu}_e \to \overline{\nu}_e)\, n^0_{\overline{\nu}_e} +$$
$$[1 - P(\overline{\nu}_e \to \overline{\nu}_e)]\, n^0_{\nu_x} \qquad (11)$$

where $n^0_{\nu_\mu} = n^0_{\nu_\tau} = n^0_{\nu_x}$. Note that to obtain the above expression we have only used the unitarity and the observation that $P(\nu_e \to \overline{\nu}_e)$ is vanishing (up to the first order in the mixing angle).

Finally, the survival probabilities $P(\nu_e \to \nu_e)$ and $P(\overline{\nu}_e \to \overline{\nu}_e)$ can be written as products of the single survival probabilities at the resonances, namely

$$P(\nu_e \to \nu_e) \simeq P_{e\tau}(\nu_e \to \nu_e)\, P_{e\mu}(\nu_e \to \nu_e) \quad (12)$$

$$P(\overline{\nu}_e \to \overline{\nu}_e) \simeq P_{\bar{e}\tau}(\overline{\nu}_e \to \overline{\nu}_e)\, P_{\bar{e}\mu}(\overline{\nu}_e \to \overline{\nu}_e) \quad (13)$$

while the transition probability $P(\overline{\nu}_e \to \nu_e)$ takes the expression

$$P(\overline{\nu}_e \to \nu_e) \simeq P_{\bar{e}\tau}(\overline{\nu}_e \to \overline{\nu}_e) \cdot$$
$$[1 - P_{\bar{e}\mu}(\overline{\nu}_e \to \overline{\nu}_e)]\, [1 - P_{e\mu}(\nu_e \to \nu_e)] +$$
$$P_{e\mu}(\nu_e \to \nu_e)\, [1 - P_{\bar{e}\tau}(\overline{\nu}_e \to \overline{\nu}_e)] \cdot$$
$$[1 - P_{e\tau}(\nu_e \to \nu_e)] \qquad (14)$$

All the expressions (12)–(14) rely on the fact that all single resonances are well separated.

From Eq.s(9)–(11), the deformed thermal neutrino spectra are obtained once the neutrino parameters are fixed. For the electron–muon sector, the relevant parameters can be fixed according to the explanation assumed for the solar neutrino problem. In this paper, since we are interested in a possible scenario in which the neutrino electromagnetic properties play the essential role we will choose the NSFP explanation [5]. and $\Delta m^2_{e\mu} \simeq 10^{-8}\ eV^2$ and $\sin(2\theta_{e\mu}) \simeq 0.2$ and $\mu_{e\mu} \simeq 10^{-11}\mu_B$.

Concerning the parameters for the electron–tau sector, they are less constrained. However, we can fix $\Delta m^2_{e\tau} \simeq m^2_{\nu_\tau} \simeq 25\ eV^2$ in order to be able to identify ν_τ has the hot component of dark matter, whereas, for the transition magnetic moment we can assume $\mu_{e\tau}$ to be of the same order of $\mu_{e\mu}$, since, typically, the enhancement to the electromagnetic properties is due to physics beyond the electroweak interactions, which hardly distinguishes between τ–leptons and μ–leptons. Hence,

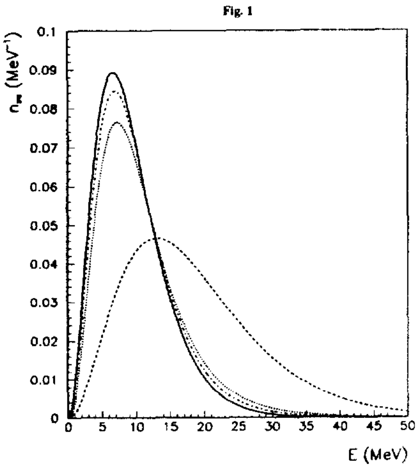

Figure 1. The energy spectra for ν_e. The solid line represents the initial distribution. Dashed, dotted and dashed-dotted lines corresponds to $\theta_{e\tau} = 10^{-1}, 10^{-4}, 10^{-8}$ respectively.

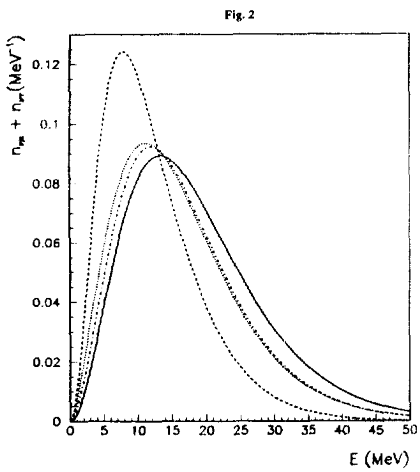

Figure 2. The energy spectra for muon and tau neutrinos, with the same notation of Figure 1.

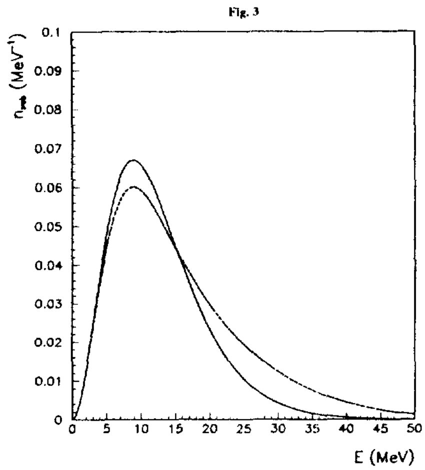

Figure 3. The energy spectra for $\overline{\nu}_e$, with the same notation of Figure 1.

the only remaining parameters is $\theta_{e\tau}$, for which we will choose three indicative values, namely, 10^{-1}, 10^{-4} and 10^{-8}.

In Figures 1–3, we show the deformed neutrino distributions for ν_e, $\nu_\mu + \nu_\tau$, and $\overline{\nu}_e$, respectively, versus their initial distributions.

REFERENCES

1. L. Wolfenstein, Phys. Rev. D17 (1978) 236; S.P. Mikheyev and A.Yu. Smirnov, Nuovo Cim. C9 (1986) 17; Sov. J. Nucl. Phys. 42 (1986) 913; Sov. Phys. Usp. 30 (1987) 759.
2. E.Kh. Akhmedov, Sov. Phys. JETP 68 (1989) 690.
3. S. Esposito, V. Fiorentino, G. Mangano and G. Miele, hep-ph/9704374.
4. H. Athar, J. T Peltoniemi and A.Yu. Smirnov, Phys. Rev. D51 (1995) 6647.
5. E.Kh. Akhmedov, A. Lanza and S.T. Petcov, Phys. Lett. B348 (1995) 124.
6. T. Janka and W. Hillebrandt, Astron. Astroph. Suppl. 78 (1989) 375.
7. S. Sahu, V.B. Semikoz and J.W.F. Valle, hep-th/9512390.