Enriched Modal Logics

Aniello Murano

Università degli Studi di Napoli "Federico II"

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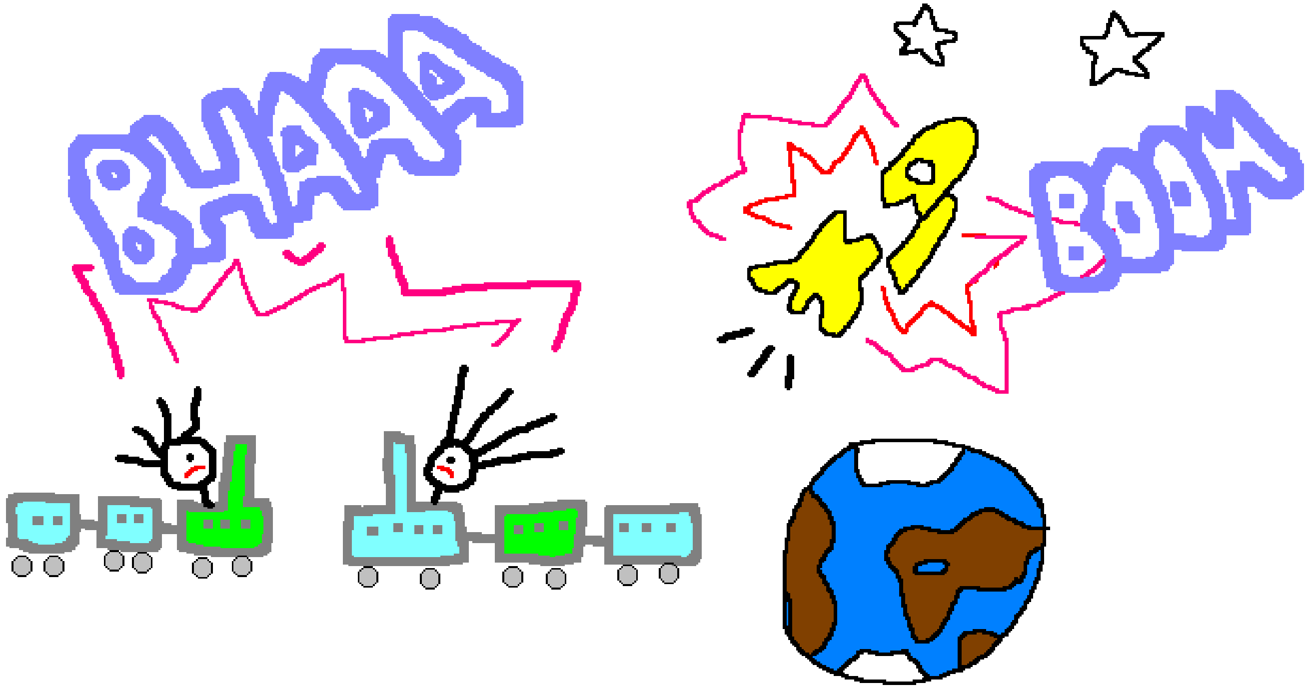
Motivations



Is the system correct?

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Formal Verification:

- System → A mathematical model M
- Desired Behavior → A formal specification ψ
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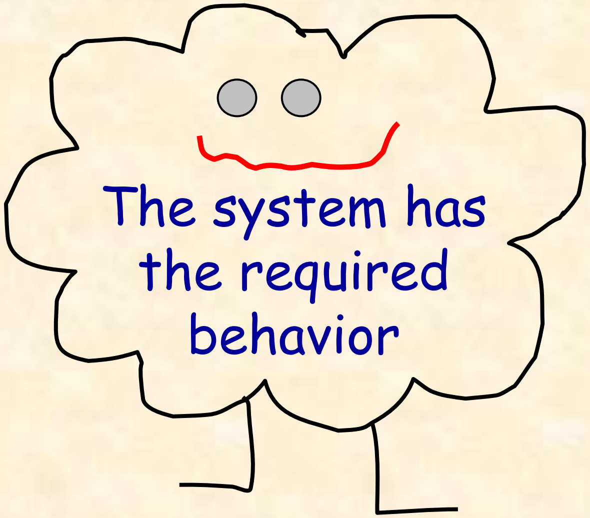

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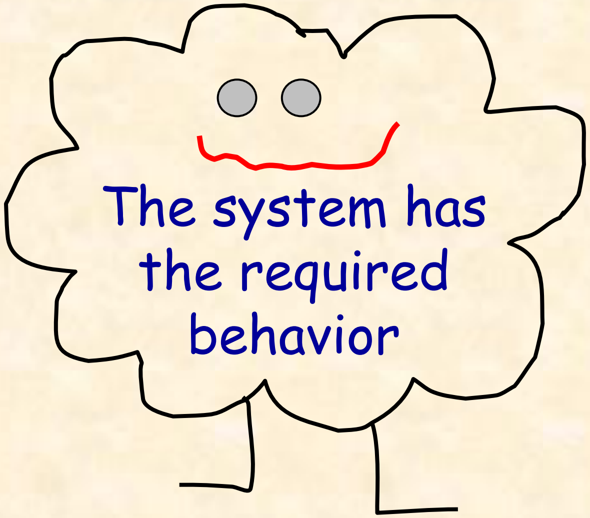

- Model Checking: Does M satisfies ψ ?
- Satisfiability: Is there M for ψ ?

A Basic Model: Kripke Structure

- A system can be represented as a **Kripke Structure**: a labeled-state transition graph

$M = (AP, S, S_0, R, Lab)$

  - AP is a set of atomic propositions.
  - S is a finite set of states.
  - $S_0 \subseteq S$ is the set of initial states.
  - $R \subseteq S \times S$ is a transition relation, total: $\forall s \in S, \exists s' . R(s, s')$.
  - $Lab : S \rightarrow 2^{AP}$ labels states with propositions true in that states.

- A path is a system run!

System Specification

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- Modal and Temporal logic allow description of the temporal ordering of events
- Two main families of logics:
- Linear-Time Logics (**LTL**)
  - Each moment in time has a unique possible future.
  - LTL expresses path properties based on the paths state labels.
  - Useful for hardware specification.
- Branching-Time Logics (**CTL**, **CTL***, and μ-**CALCULUS**)
  - Each moment in time may split into various possible future.
  - CTL* expresses state properties from which LTL-like properties are satisfied in an existential or universal way .
  - Useful for software specification.

μ-calculus is a very expressive logic

- Can express several practical properties.
- Corresponds to alternating parity tree automata
- Important connections with MSO
- Strictly subsumes classical logics such as CTL, LTL, CTL*, …
- Identifies powerful classes of Description Logics

- Decision problems:
  - Model checking: UP ∩ co-UP
  - Satisfiability: ExpTime-complete

μ-calculus limitations

- Several important constructs cannot be easily translated to the μ-calculus:
  - Inverse Programs to travel relations in backward
  - Graded modalities to enable statements on a number of successors
  - Nominals as propositional variables true exactly in one state

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- Extensions of the μ-calculus with these abilities induces families of enriched μ-calculi.
- Similarly, we can define families of enriched temporal logics.

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I part

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- ❑ Satisfiability of fully enriched μ-calculus: Undecidable
- ❑ Satisfiability of the other families we consider: ExpTime-complete
  - ◆ Upper bound via Fully Enriched Automata (FEA).
  - ◆ The upper bound holds also in case numbers are coded in binary

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- Graded Computation Tree Logic (GCTL)
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- Graded Computation Tree Logic (GCTL)

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- Open questions on GCTL and its extensions:
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- Some achievements in open system verification.

I part: Enriched µ-calculi

Some known results

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- Satisfiability for Fully enriched μ-calculus is undecidable [Bonatti, Peron 2004]
- ExpTime-completeness of satisfiability for enriched μ-calculi:
  - μ-calculus with inverse programs [Vardi'98]
  - μ-calculus with graded modalities [Kupferman,Sattler,Vardi'02]
  - full hybrid logic [Sattler,Vardi'01]
  - full graded logic in unary coding [Calvanese, De Giacomo, Lenzerini'01]

The fully enriched μ-calculus

- The μ-calculus is a propositional modal logic with least(μ) and greatest (ν) fixpoint operators [Kozen 1983].

- The fully enriched μ-calculus extends the μ-calculus with
  - graded modalities: $\langle$**n**,$\alpha\rangle$ (atleast formulas) and **[n**,α**]** (allbut formulas)
  - nominals propositions: Nominal set **Nom**
  - inverse programs: Use of both program sets **Prog** and **Prog-**

The fully enriched μ-calculus (Syntax)

- Let AP, Var, Prog, and Nom be sets of atomic proposition, propositional variables, atomic, programs and nominals
- Syntax:

  $\varphi := true \mid false \mid p \mid \neg p \mid y \mid \varphi \vee \varphi \mid \varphi \wedge \varphi \mid \langle n,\alpha\rangle\varphi \mid [n,\alpha]\varphi \mid \mu y.\ \varphi(y) \mid \nu y.\ \varphi(y)$

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- Fragments of the fully enriched μ-calculus:
  - full graded μ-calculus (without nominals)
  - hybrid graded μ-calculus (without inverse programs)
  - full hybrid μ-calculus (without graded modalities)

Semantics: The enriched model

- The semantics of the fully enriched μ-calculus is given with respect to enriched Kripke structures

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  - $R : Prog \rightarrow 2^{W \times W}$ assigns to programs transitions relation over S
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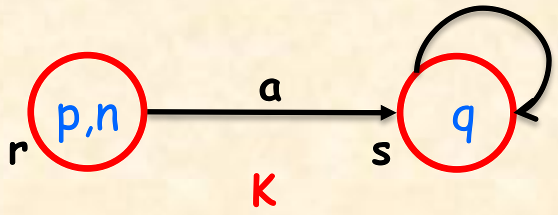

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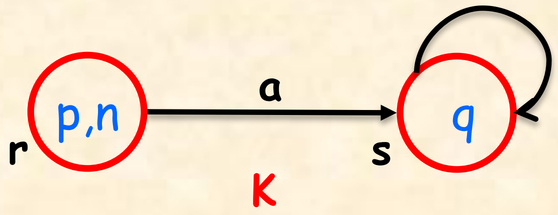
- Given a Kripke structure, atomic propositions and boolean connectivities are interpreted as usual:
  - K satisfies the nominal n at the starting state r, since Lab(n)={s}
  - K does not satisfy q at r, but at s.



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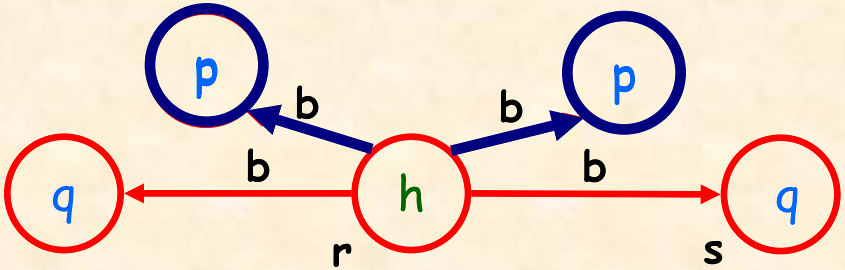

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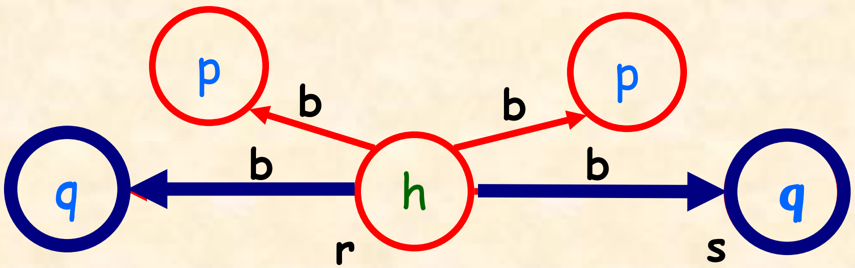

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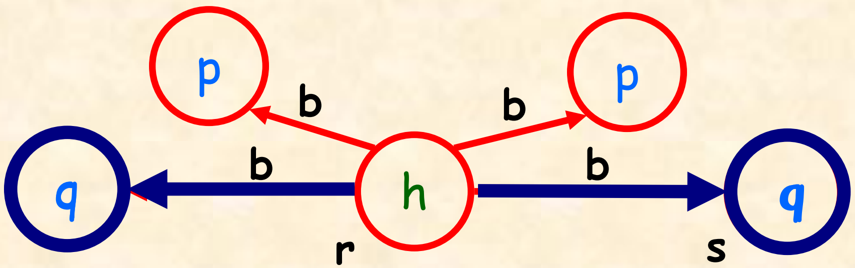

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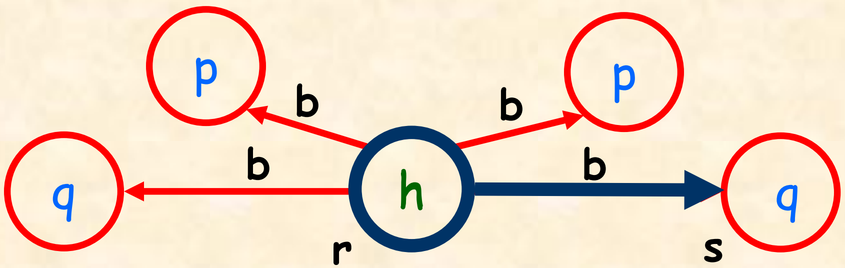

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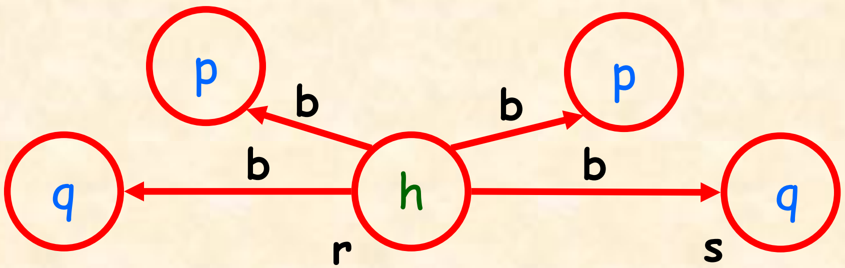

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- ν and μ are useful to express liveness and safety:
  - AGp: p always true along all a-paths is $\nu X.\ p \wedge [0,a]X$
  - EFp: there exists an a-path where p eventually holds is $\mu X.\ p \vee \langle 0,a\rangle X$

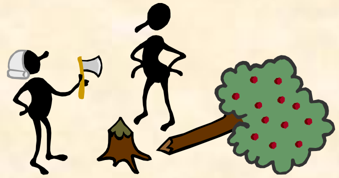
- Note that $\langle 0,\alpha\rangle\varphi$ is $\langle\alpha\rangle\varphi$ and $[0,\alpha]\varphi$ is $[\alpha]\varphi$

Structure properties

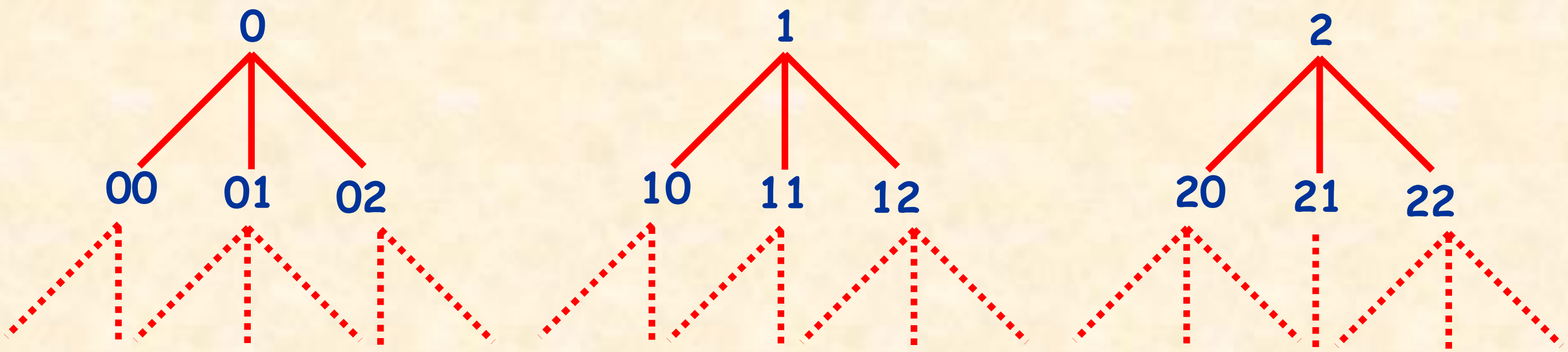
- In branching-time temporal logic, important model features to symplify decisions reasonings are:
- Finite-model property:
  - Is there a finite model satisfying the formula
  - It is possible to use exhaustive (brute-force) methods!
- Tree-model property:
  - Is there a tree-model shape satisfying the formula
  - It is possible to use tree automata !

- In enriched μ-calculus we need **forest structures** as models

Forest structures



- A forest $F \subseteq N^+$ is a collection of trees:



- The elements of F are nodes, the degree of F is the maximum number of node's successors, and 0, 1, and 2 are roots of F.
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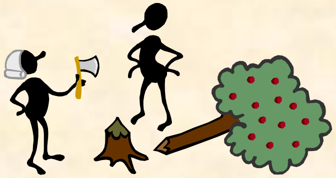

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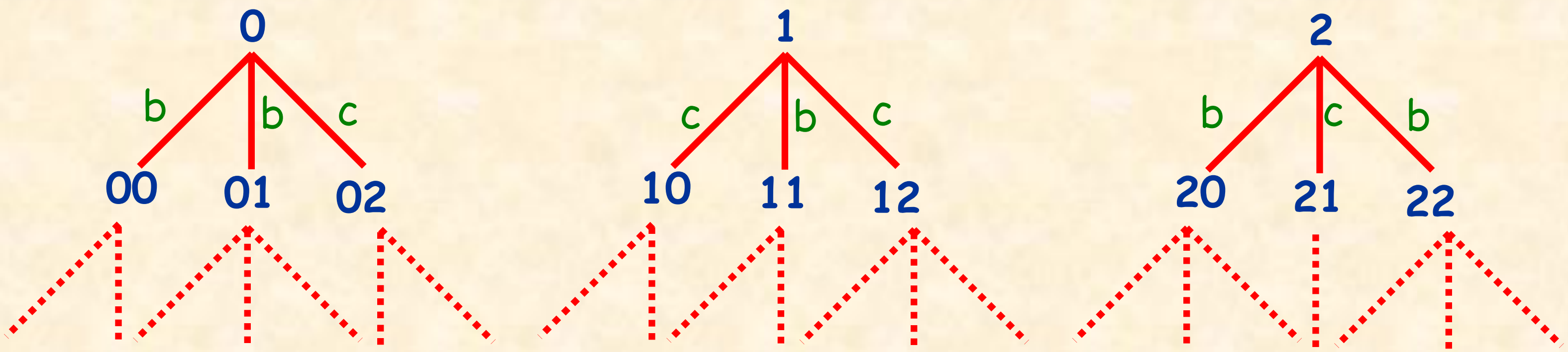

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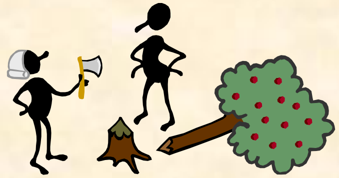

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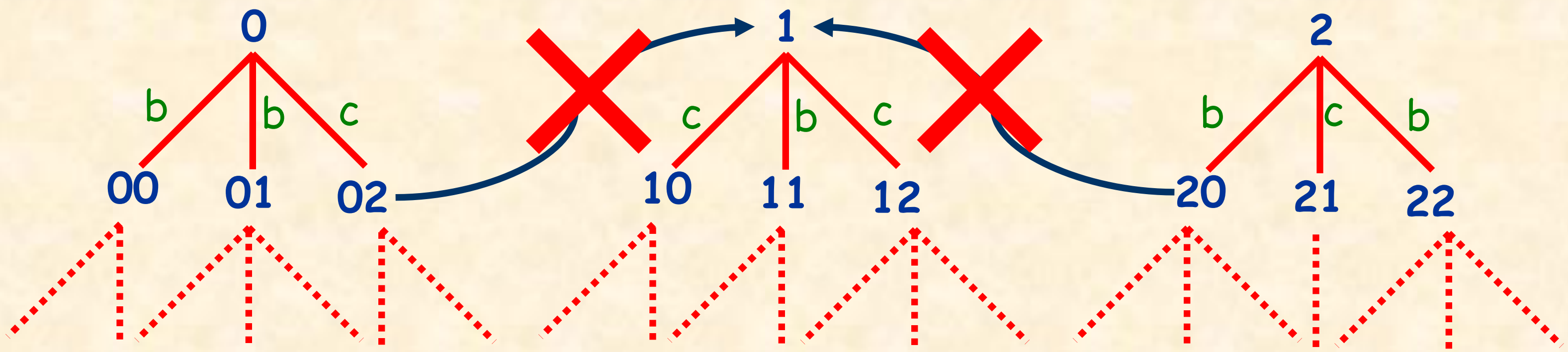

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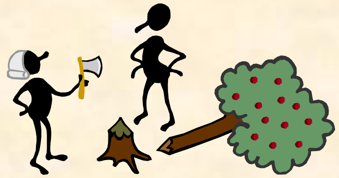

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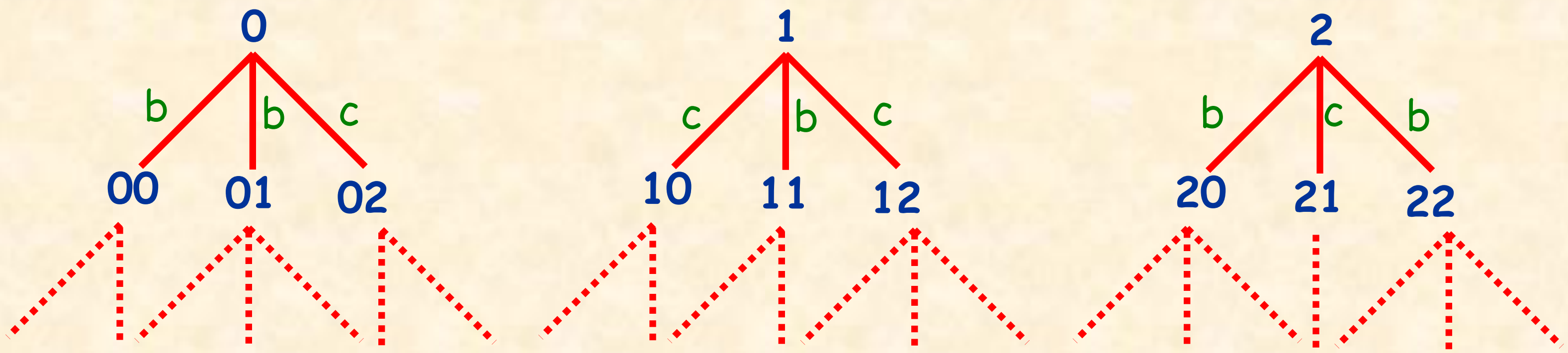

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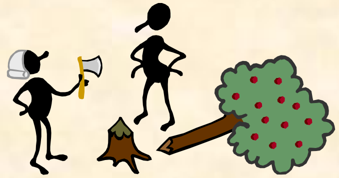

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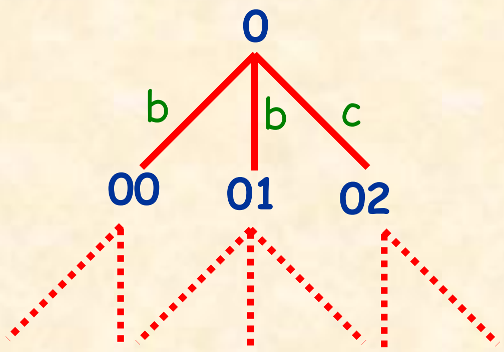

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Forest and tree model property

- Given a sentence φ of the full graded μ-calculus with m atleast subsentences and counting up to b

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φ is satisfiable

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- The hybrid graded μ-calculus does not enjoy the tree model property.

- Given a sentence φ of the hybrid graded μ-calculus with k nominals, m atleast subsentences and counting up to b

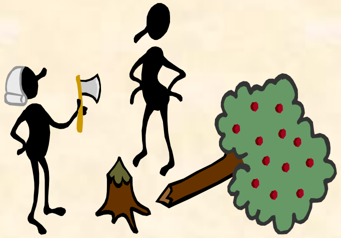
φ is satisfiable

⬇

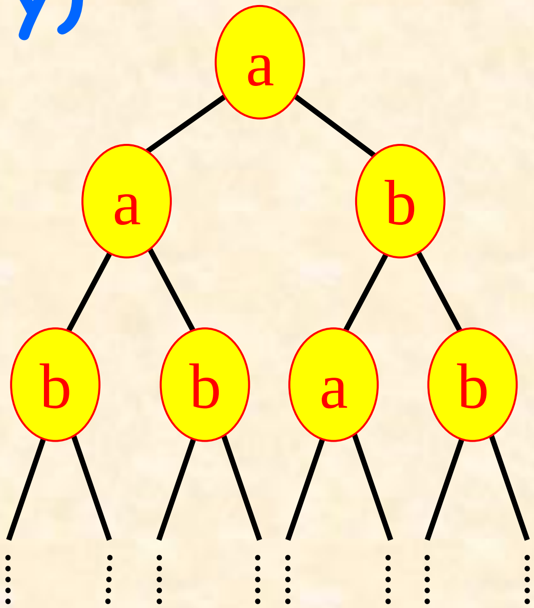
φ has a quasi forest model
whose degree is at most $\max\{k+1, m \cdot (b+1)\}$

Solving enriched mu-calculi

- We use an automata-theoretic approach.
- In modal μ-calulus, we translate a formula to an alternating parity tree automaton anch check for its emptiness.
  - The translation is polynomial
  - Checking for emptiness can be done in ExpTime
  - Satisfiability of μ-calculus is solvable in ExpTime.
- For the enriched μ-calculi, we need an enriched version of parity tree automata.
- Let us first recall alternating automata on infinite tree…



Nondeterministic (binary) tree automata: NTA

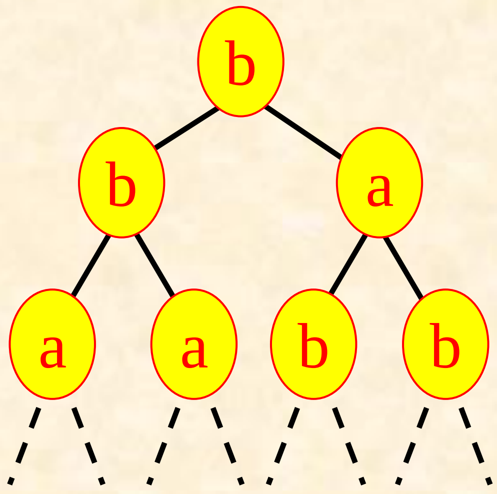


a (Σ-labeled) tree t

- A infinite (binary) tree is **$t : \{0,1\}^* \rightarrow \Sigma$**
- A **path** is an **infinite** sequence of nodes starting at the root
- An **NTA** is a tuple **A** = < Q, Σ, δ, Q_0, **F** >
  - **δ** : $Q \times \Sigma \rightarrow 2^{Q \times Q}$ is a tree transition relation
  - Runs are binary trees labeled with states accordingly to **δ**
  - **F** is an acceptance condition satisfied on each path of a run

Runs

- A **run** r : $\{0,1\}^* \rightarrow Q$ is built in accordance with δ and $r(\varepsilon) \in Q_0$. Thus, runs are Q-labeled trees.
- Let $(q,q) \in \delta(p,a)$ and q_0 initial state



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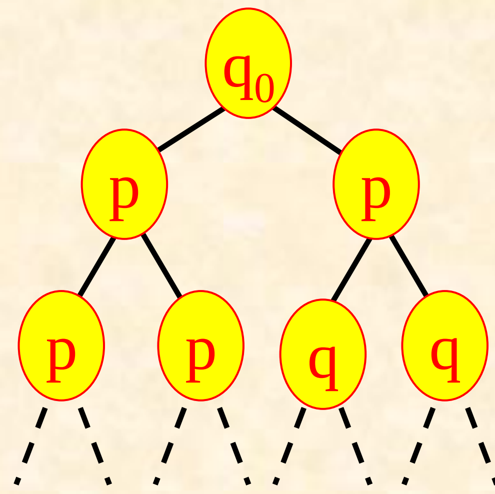

the corresponding run r

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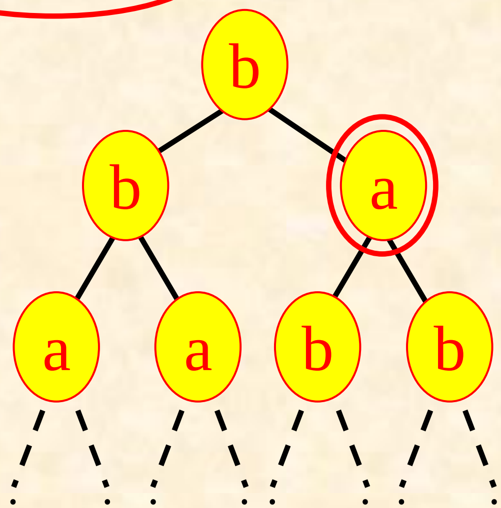

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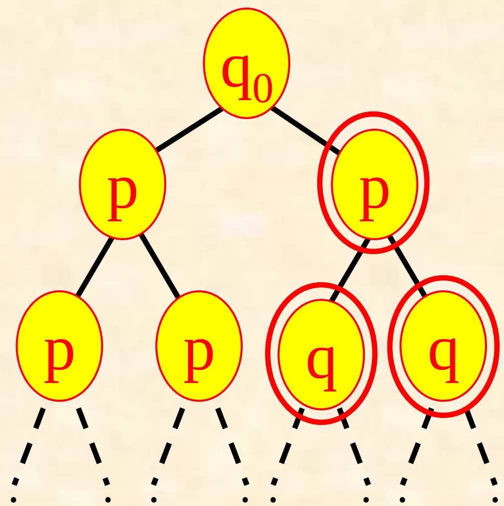

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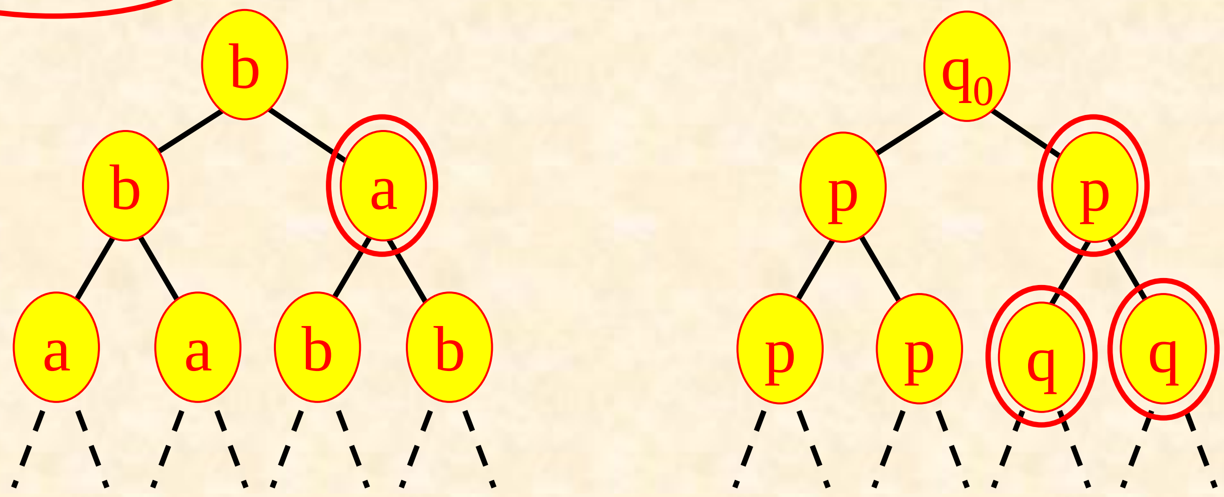
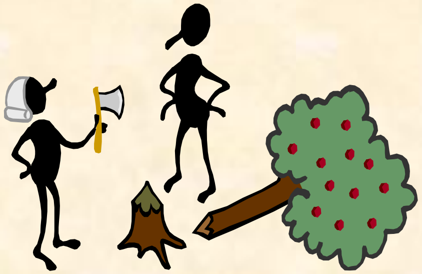


a tree t the corresponding run r

- A run is **accepting** if the acceptance condition is satisfied on every path



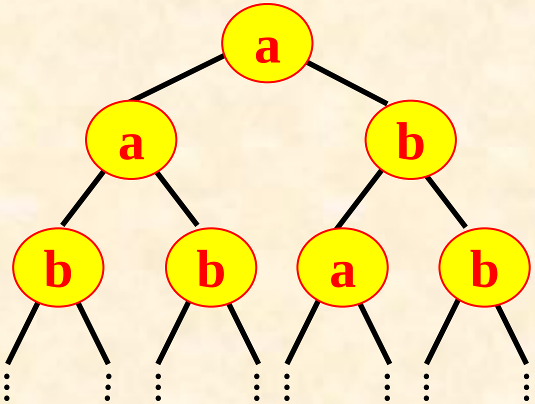
Alternating automata on infinite trees

- An alternating (finite-state) automaton on infinite Σ-labeled D-trees is a tuple

$$A = \langle Q, \Sigma, \delta, q_0, F \rangle$$

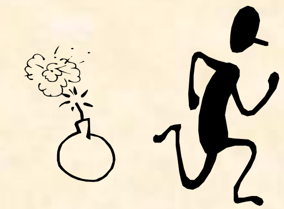
  - $\delta : (Q \times \Sigma) \rightarrow B^+(D \times Q)$
  - positive Boolean formulas of pairs of directions and states

For example
$\delta(p,a)=$
$(1,p) \wedge (1,q)$

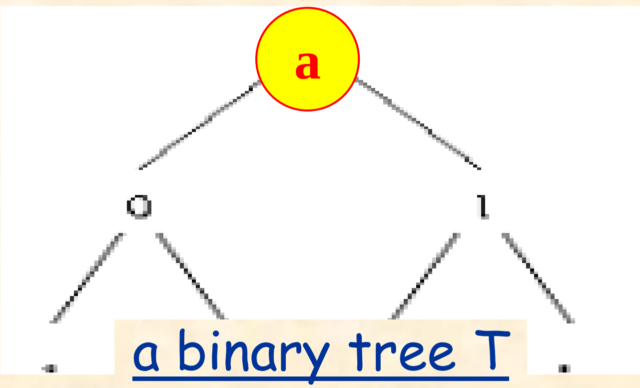


Σ-labeled binary tree

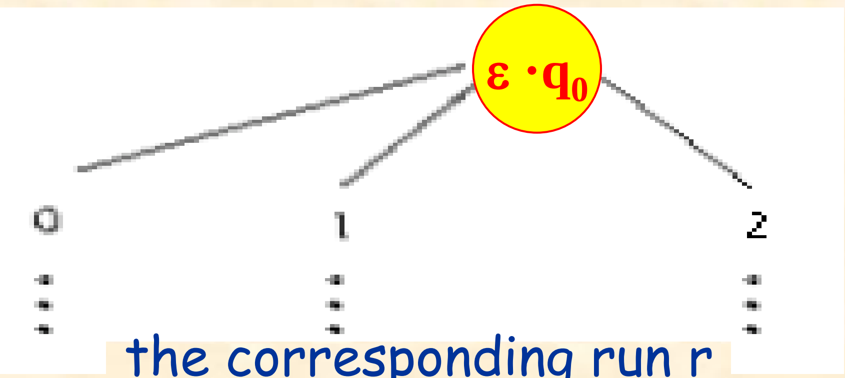
Runs



- A run on a Σ-labeled D-trees is a $(D^* \times Q)$-labeled tree. The root is labeled with (ε, q_0) and labels of each node and its successors must satisfy the δ

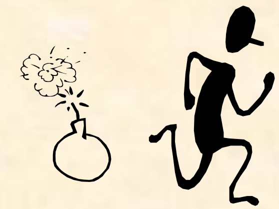


a binary tree T



the corresponding run r

Runs



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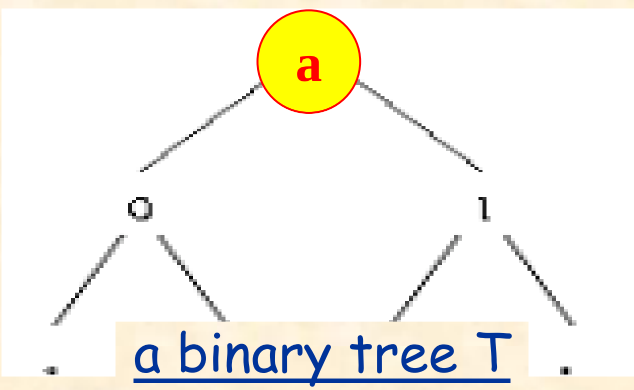

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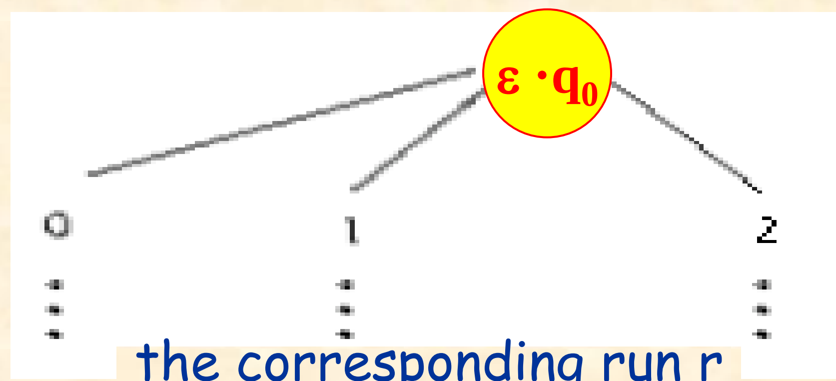

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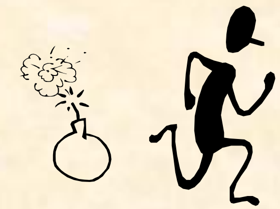

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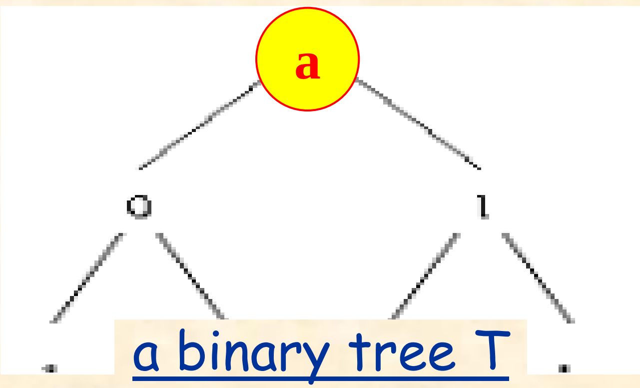

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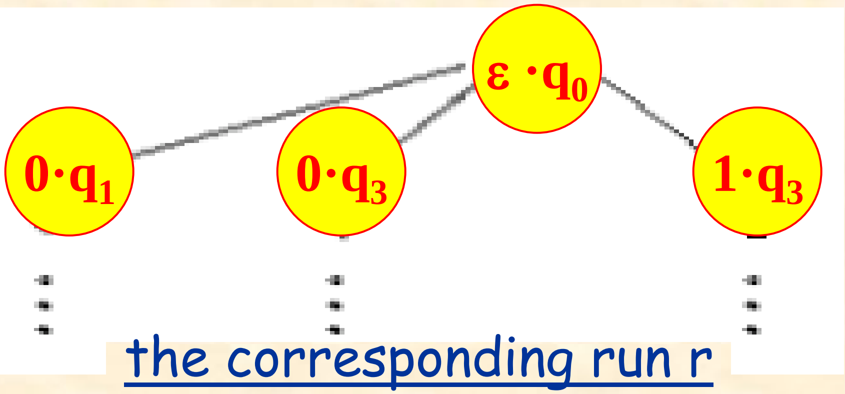

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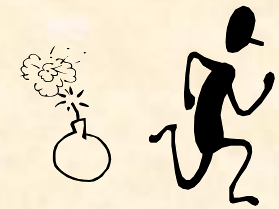

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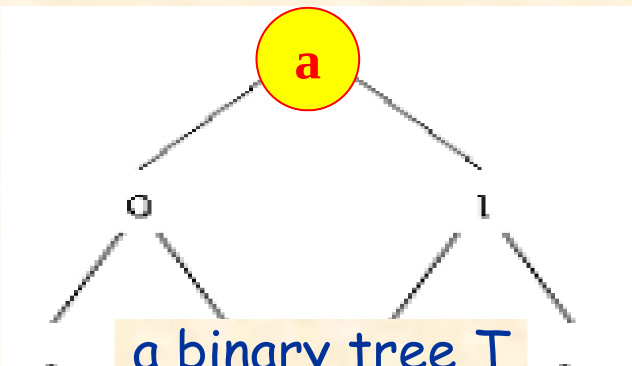

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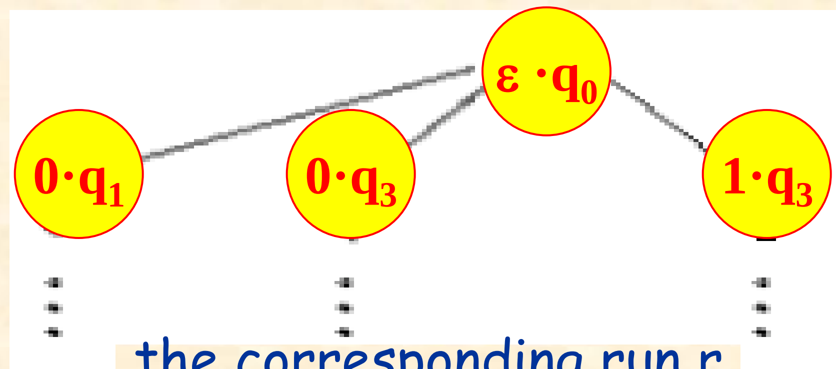
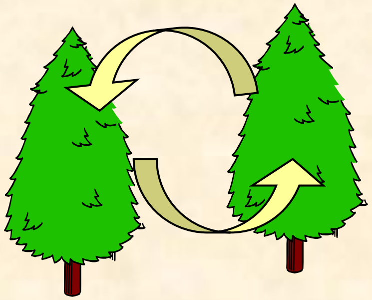


the corresponding run r

- There is no one-to-one correspondence between nodes of T and r
- As in nondeterministic automata, a run is accepting if the acceptance condition is satisfied on every path.

Fully Enriched Automata

- Fully enriched automata (FEA) run on infinite labeled forests $\langle T, V \rangle$.
- FEA generalize alternating automata on infinite trees as the fully enriched μ-calculus extends the standard μ-calculus:



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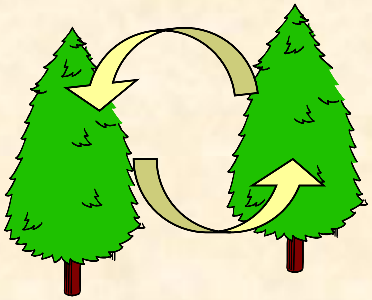

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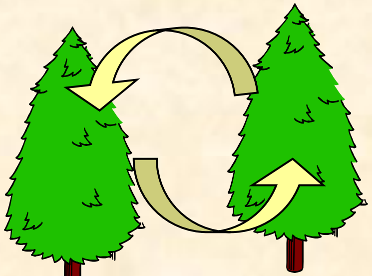

- $\delta(q,\sigma)$ is a positive boolean combination of pairs of directions and states.
- Formally,
  - $\delta : Q \times \Sigma \rightarrow B^+(D_b \times Q)$, where D_b can be -1, ε, $\langle root \rangle$, [root], $\langle n \rangle$, or [n], with $0 \leq n \leq b$.
  - (-1, q) and (ε, q) send a copy to the predecessor and to the current node.
  - ($\langle$root$\rangle$, q) and ([root], q) send a copy to some or all roots of the forest.
  - ($\langle$n$\rangle$, q) and ([n], q) send a copy in state q to n+1 and all but n successors of the current node, respectively.

Runs for FEA

- For a FEA A with a transition δ: $Q \times \Sigma \rightarrow B^{+}(D_b \times Q)$
- A run over a forest $\langle F,V \rangle$ is a $(F \times Q)$-labeled tree, built in accordance with δ and $r(\varepsilon) = (c, q_0)$, for a root c of F.

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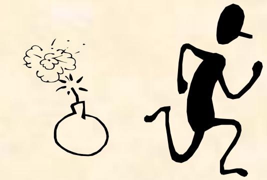

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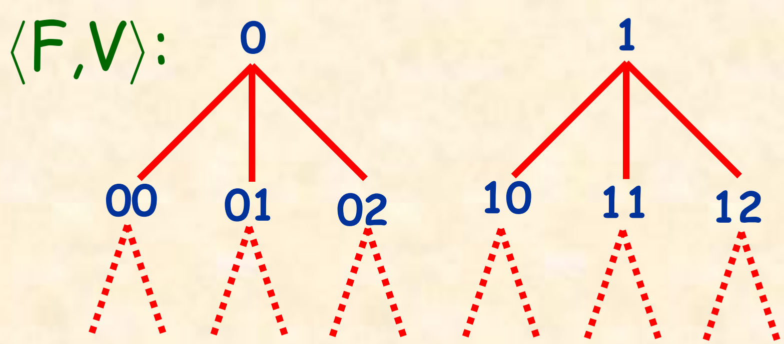

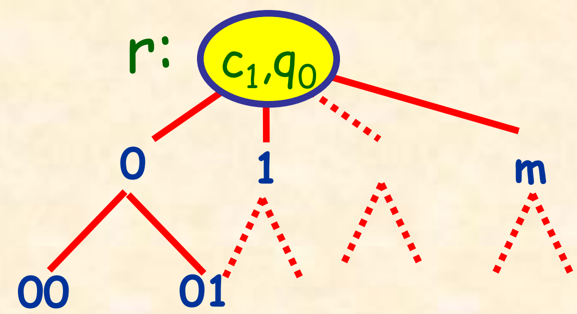

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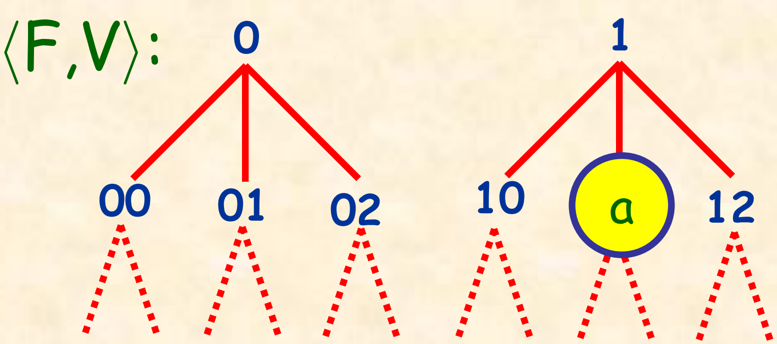

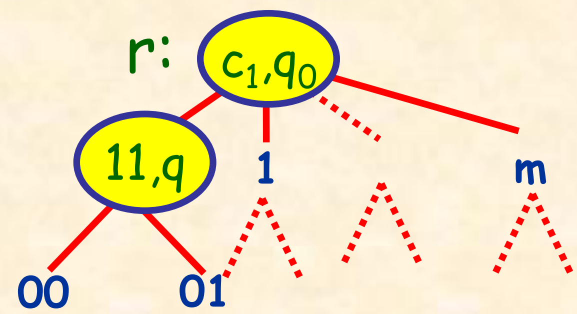

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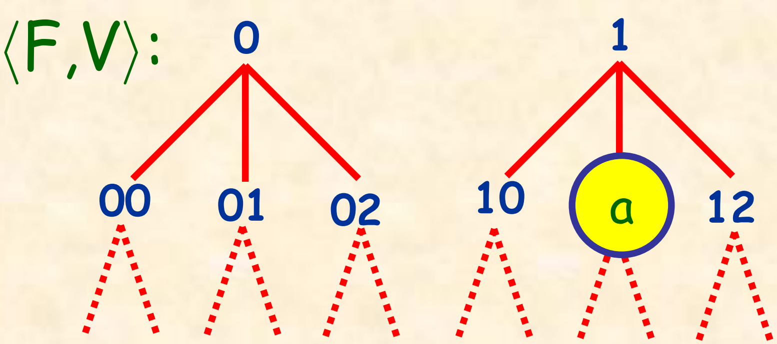

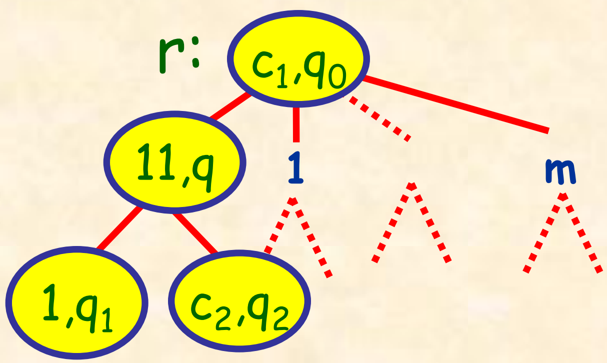

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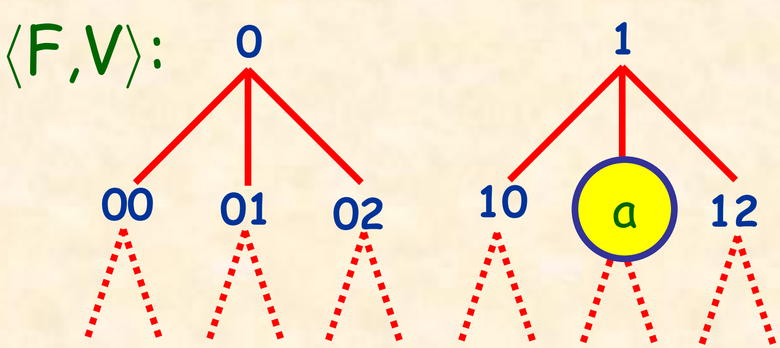

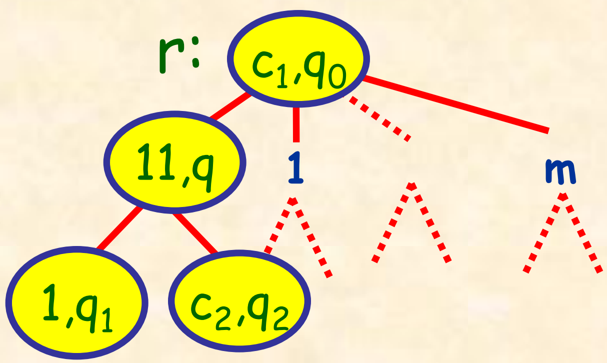

- We use a parity condition.

Acceptance conditions

- Büchi condition: $F \subseteq Q$. A run r is accepting iff for every path, there exists a final state appearing infinitely often
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- Emptiness:
  - Nondeterministic Buchi Tree Automata (NBT) : PTime-Complete
  - Alternating Buchi Tree Automata (ABT) : ExpTime-Complete
  - Nondeterministic Parity Tree Automata (NPT) : UP ∩ Co-UP
  - Alternating Parity Tree Automata (APT) : ExpTime-Complete

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- In both cases, φ is satisfiable if $L(A_\varphi) \neq \emptyset$

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- The result follows from the blow-up involved in building the GNPT and from the complexity for checking its emptiness.

A strategy tree with detour

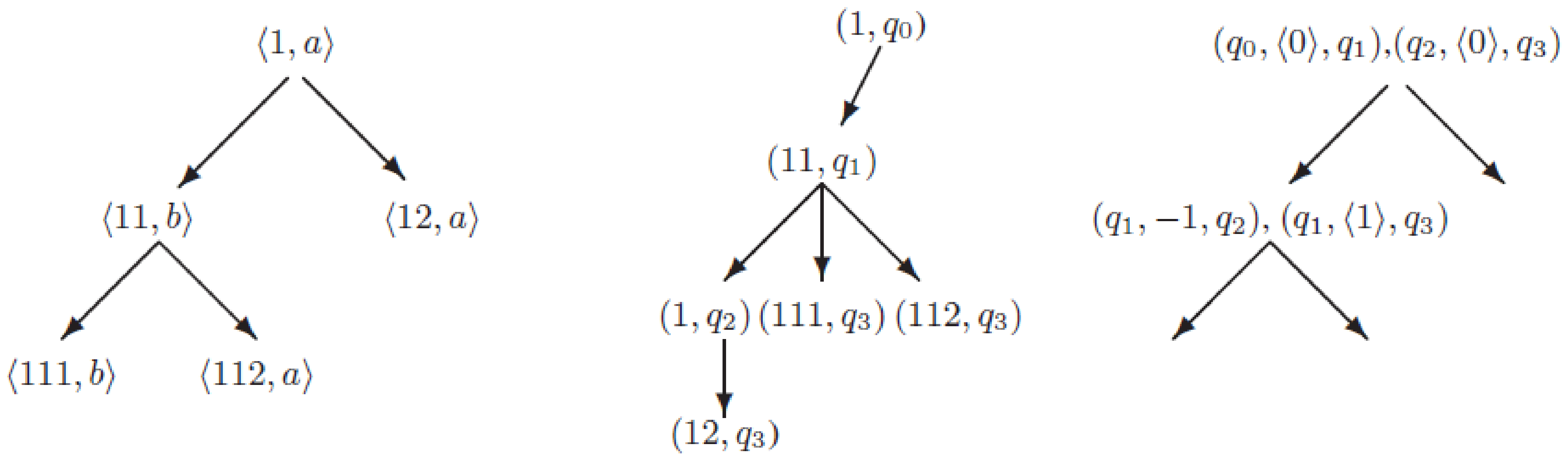

Figure 2: A fragment of an input tree, a corresponding run, and its strategy tree.

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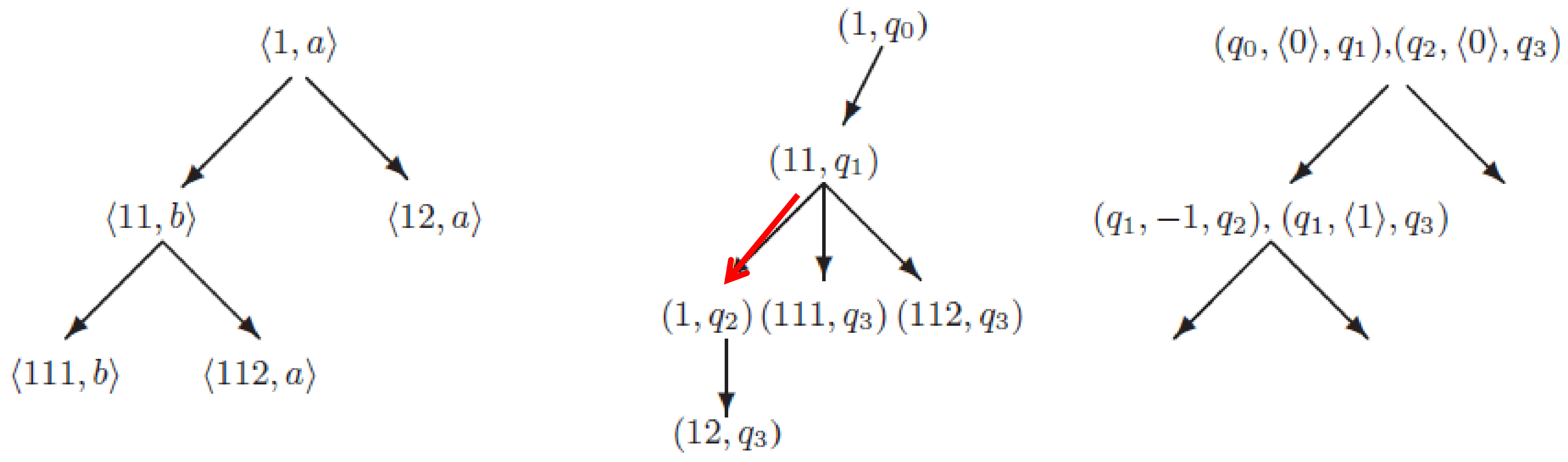

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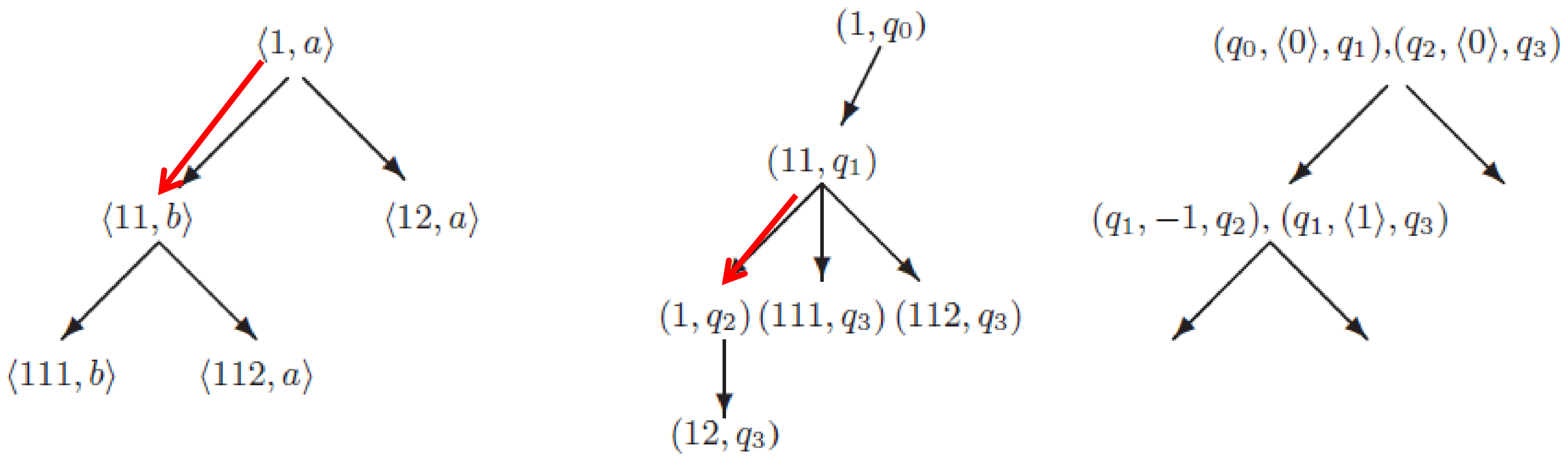

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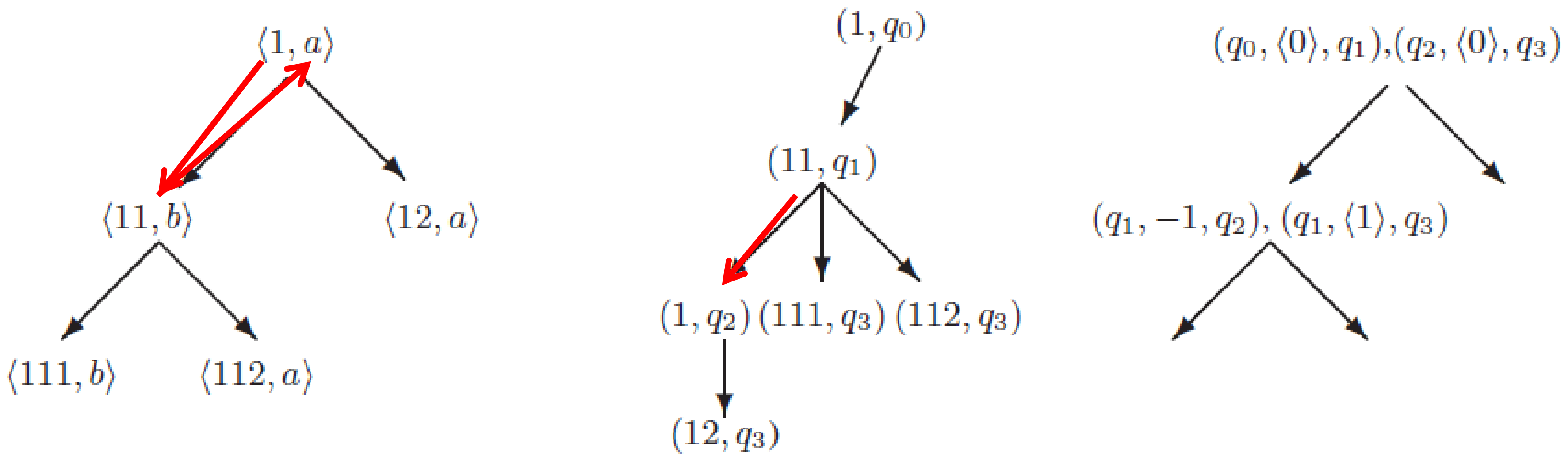

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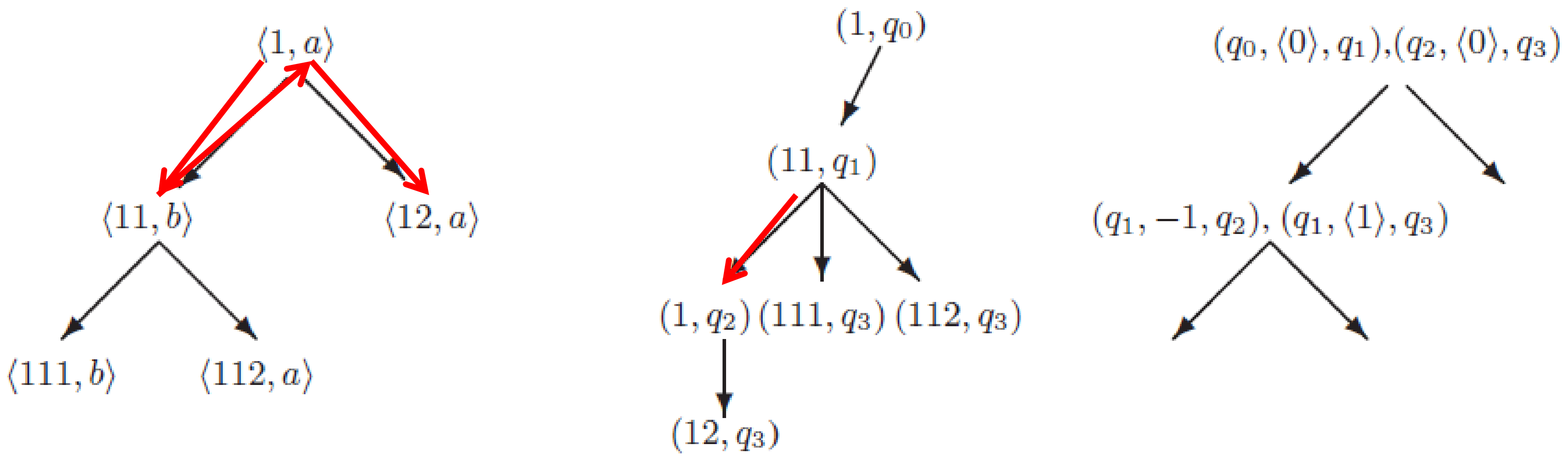

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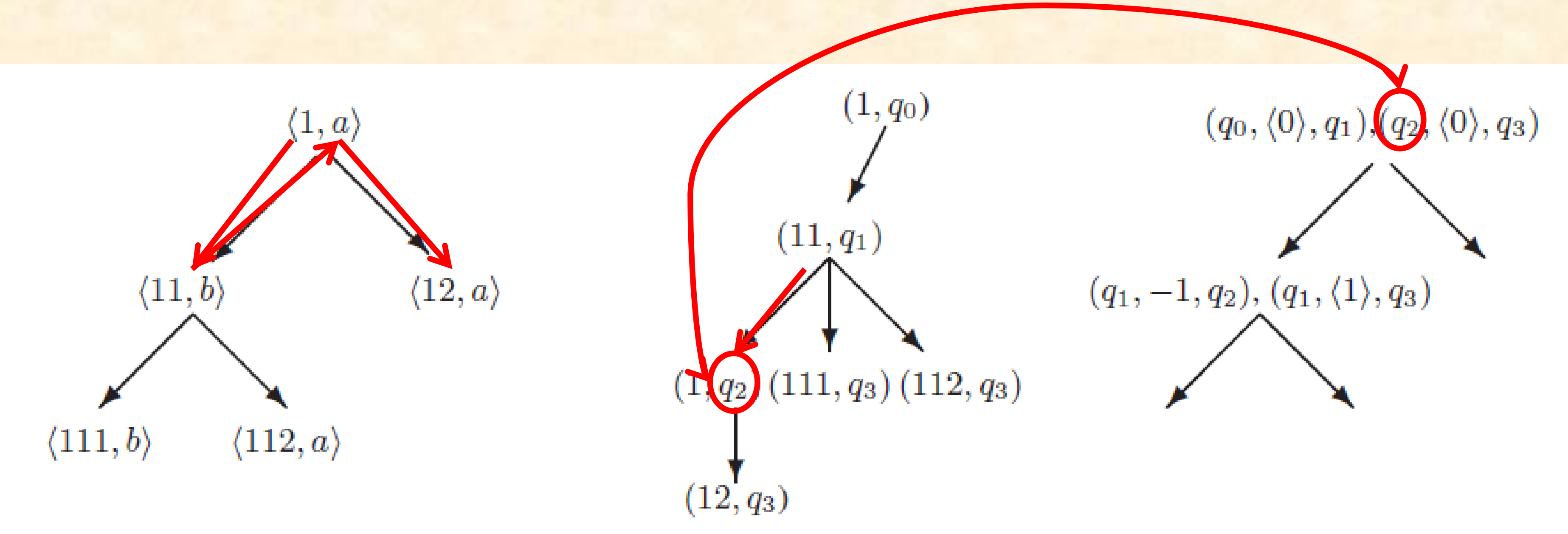

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A Summary for Enriched μ-calculi

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	Inverse programs	Graded modalities	Nominals	Complexity
fully enriched	×	×	×	Undecidable[1]
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hybrid graded		×	×	
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- Moving from μ-calculus to CTL with graded modalities, we need to move from graded world modalities to graded path modalities!

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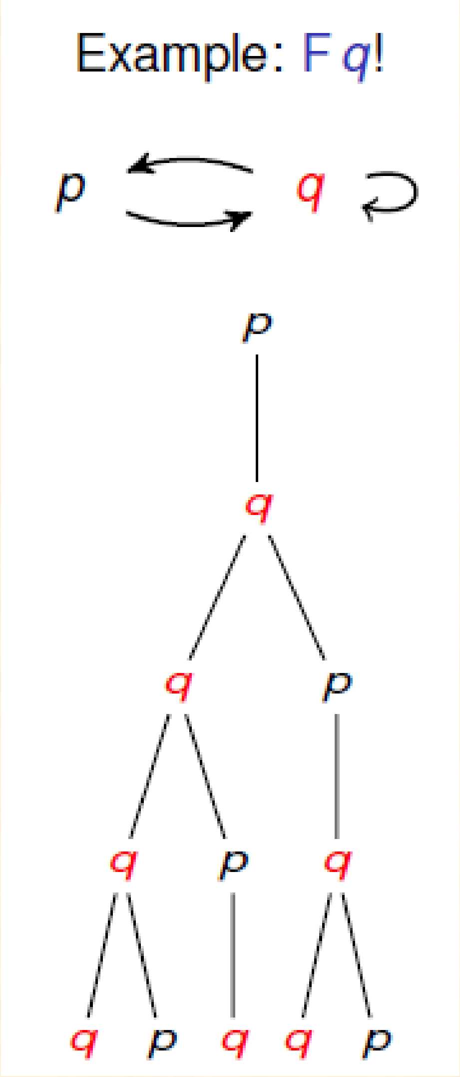
- CTL* uses state and path formulas built inductively as follows:
- State-formulas:
  - $\varphi := p \mid \neg\varphi \mid \varphi \wedge \varphi \mid \varphi \vee \varphi \mid E^{\geq n}\, \psi \mid A^{<n}\, \psi$
  - where $p \in AP$ and ψ is a path-formula
- path-formulas (LTL):
  - $\psi := \varphi \mid \psi \wedge \psi \mid \neg\psi \mid X\psi \mid \psi\, U\, \psi$
  - where φ is a state-formula, and ψ a path-formula
- GCTL formulas are obtained by forcing each temporal operator to be coupled with a path quantifier

Counting paths

- What does counting paths mean?
  - A property ensured by a common prefix may be satisfied on an infinite number of paths.



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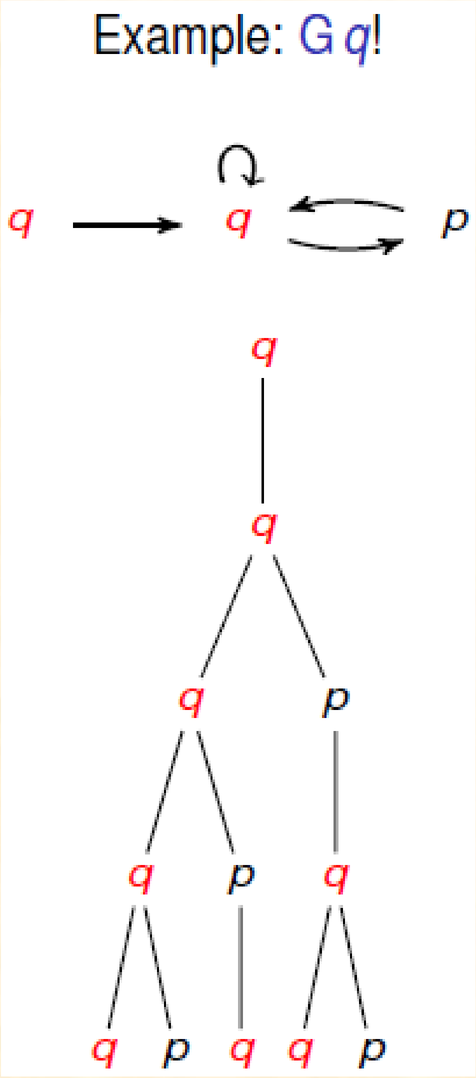

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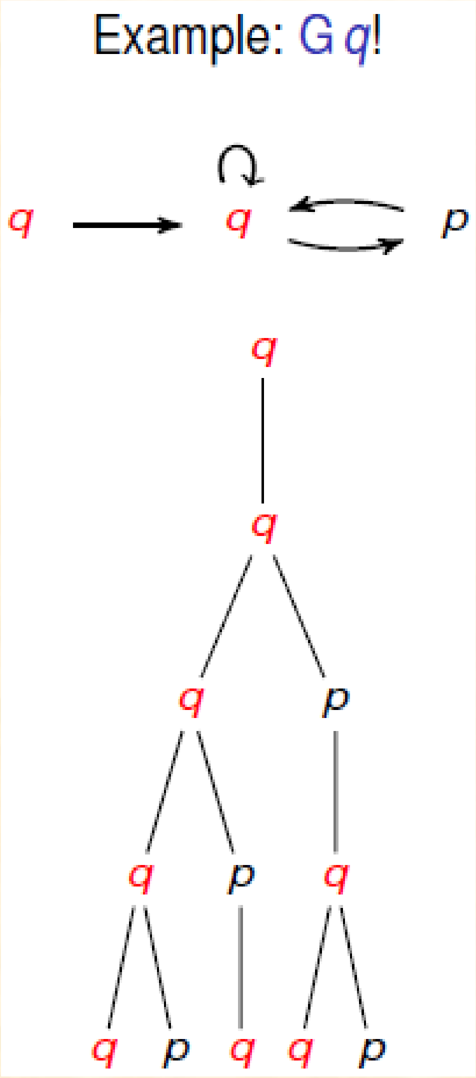
- We restrict to minimal and conservative paths
- Two paths are equivalent if
  - their common prefix satisfy the formula.
  - no matter how these prefixes are extended in the structure, the paths satisfy the formula.



Semantics of GCTL*

- For a Kripke structure K, a world w, and a GCTL* path formula ψ,
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- Let P(K, w, ψ) be the set of minimal and conservative paths of K starting in w and satisfying ψ
  - $K, w \models E^{\geq n} \psi$ iff $|P(K, w, \psi)| \geq n$
  - $K, w \models A^{<n} \psi$ iff $|P(K, w, \neg\psi)| < n$
- For n=1, we write Eψ and Aψ instead of $E^{\geq 1} \psi$ e $A^{<1} \psi$

Solving GCTL in unary coding

- Let ψ be a GCTL formula with grades coded in unary.
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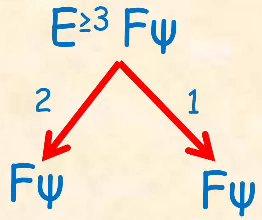

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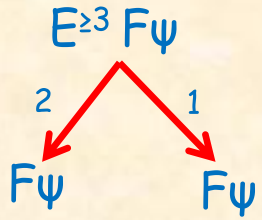

- By means of an opportune extension of the Myhano-Hayashi tecnique, we translate in Exponential Time P_ψ in an NBT B_ψ
- Since the emptiness of $L(B_\psi)$ can be checked in polynomial time, we get that the satisfiability problem for GCTL is in ExpTime.
- ExpTime hardness comes from the satisfiability problem for CTL

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  - The tree encoding turns each level of the tree in a binary tree, i.e., brothers of a node become its successors.
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  - The satellite is an (exponential) NBT and ensures that each tree model satisfies some structural properties along its paths.
- As the satellite automaton is already an NBT, this avoids to inject an extra exponent when moving both automata to a unique NBT.
- Thus, also in the binary coding, the satisfiability question for GCTL is ExpTime-complete

What about GCTL*

- Solving graded CTL* is even more appealing.
- There are several question to investigate.
- Is GCTL* more succinct than Graded mu-calculus?
- What about the satisfiability?
  - Using a slight variation of the previous reasoning used for GCTL, we get a 3ExpTime upper bound.
  - As CTL* satisfiability is 2ExpTime-complete, it is an open question to decide the exact complexity of the problem for GCTL*

Further directions about GCTL and GCTL*

- What about GCTL/ GCTL* plus backwords modalities?
- CTL and CTL* have been investigated with respect to (linear and branching) Past modalities.
- PCTL (PCTL*) is (2)ExpTime-complete.
- What about GCTL/GCTL over more enriched structures: Hierachical, pushown, weighted etc…

Enriched modalities vs. open systems

- Enriched mu-calculi has been investigated in the setting of module checking.
- Same results as in the satisfiability case:
  - Undecidable if we consider the fully enriched mu-calculus.
  - ExpTime-complete for every fragment.

Conclusion

Results on the satisfiability problem for Enriched μ-calculi				
	Inverse programs	Graded modalities	Nominals	Complexity
fully enriched	x	x	x	Undecidable[1]
full hybrid	x		x	ExpTime[2]
full graded	x	x		ExpTime 2ary (1ary[4])
hybrid graded		**x**	x	**ExpTime 1ary/2ary**
graded		**x**		ExpTime 1ary/2ary[3]
full	x			ExpTime[5]
Results on the satisfiability problem for GCTL				
GCTL		**x**		**ExpTime 1ary/2ary**
Past CTL	x			ExpTime[6]

1. [Bonatti, Peron 2004]
2. [Sattler, Vardi 2001]
3. [Vardi 1998]
4. [Calvanese, De Giacomo, Lenzerini, 2001]
5. [Kupferman, Sattler, Vardi, 2002]
6. [Kupferman, Pnueli 1995]

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Thank you for your attention!