# Logic as The Calculus of Computer Science

noneng. S. Baselice (*I have no stamp*) [1]

Ottobre, 2007

[1] Università di Napoli Federico II

# What is a Logic?

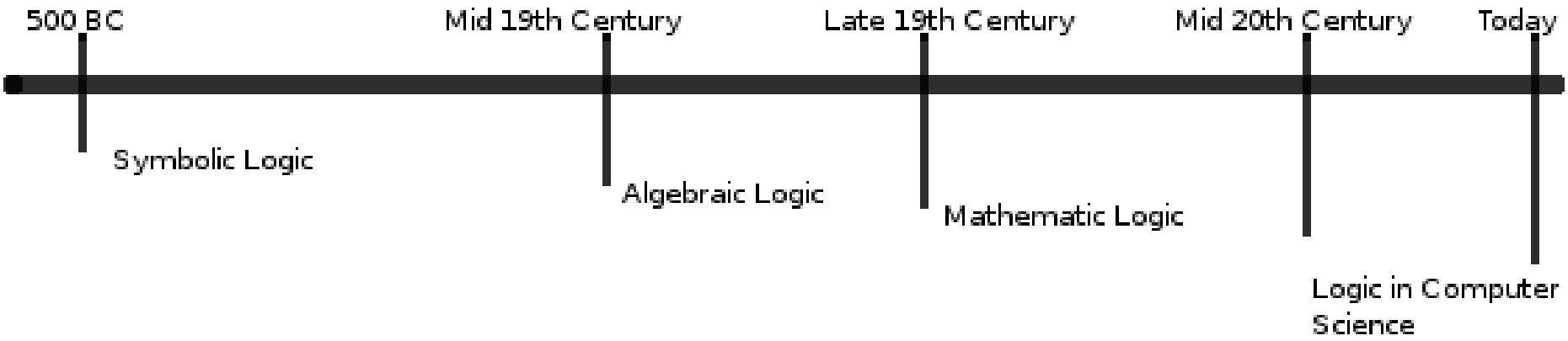
A *Logic* is a formalism with

- a sintax
- a semantics
- an inference mechanism for reasoning

$$((a \vee b) \wedge \neg a) \rightarrow b$$

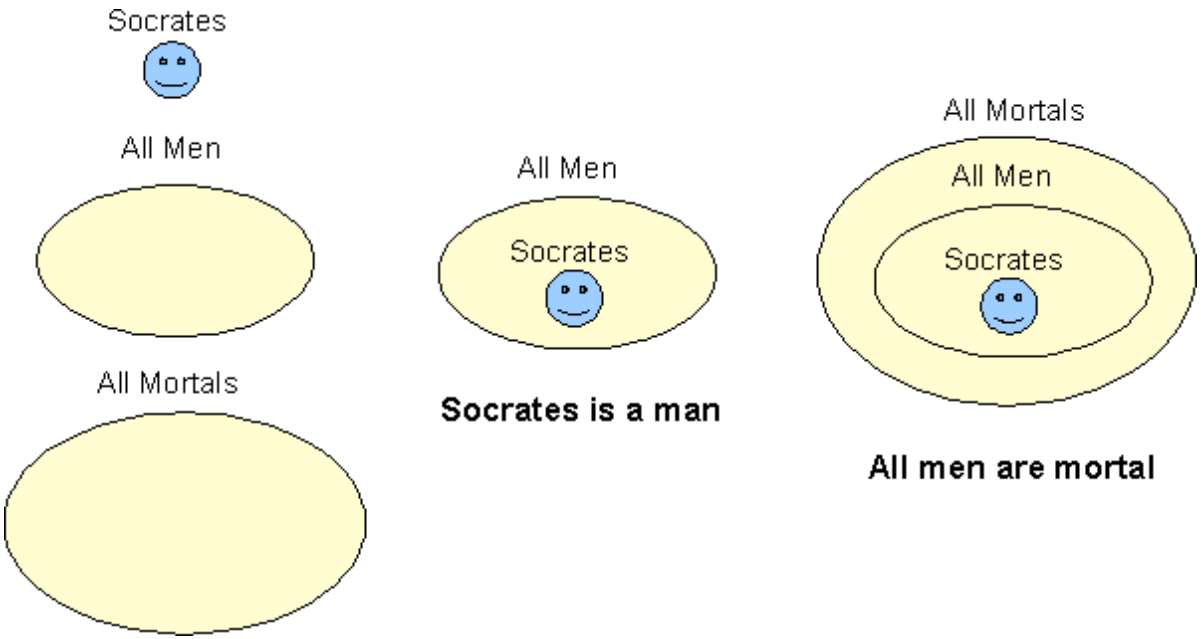
| a | b | a → b |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | 1 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

# Historical Diagram



# The First Age of Logic: Symbolic Logic (500 B.C. - 19th Century)

- Originally, Logic dealt with arguments in the natural languages used by humans.



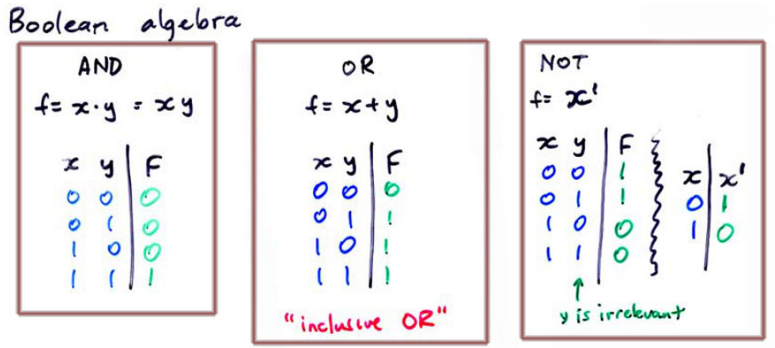
# The First Age of Logic: Symbolic Logic (500 B.C. - 19th Century)

## The Surprise Paradox:

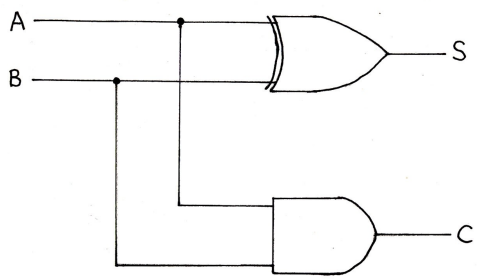

- Natural language is too ambiguous!
- It is needed a *Symbolic Language*.

# The 2nd Age of Logic: Algebraic Logic (Mid to Late 19th Century)

In 1847, George Boole attempted to formulate logic in terms of mathematical language.




A and B are the inputs
S represents the output *sum* and
C represents the output *carry*.



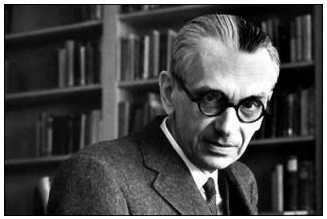

The relevant truth table for this circuit is:

| A | B | C | S |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 0 |

# The 3rd Age of Logic: Mathematical Logic (Late 19th Century to Mid 20th Century)

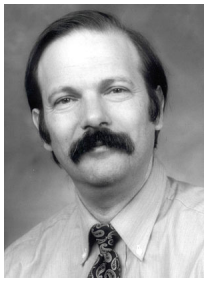
- As mathematical proofs became more sophisticated, *paradoxes* began to show up in them just as they did in natural language.
- Hilbert proposed logic for computing each mathematical problem.

- Gödel's Incompleteness Theorems proved that logic cannot prove every statement and it cannot prove its own consistency.

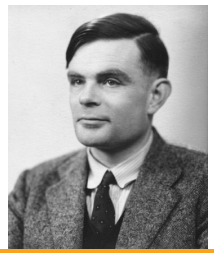


# The 4th Age of Logic: Logic in Computer Science

- ”Logic occupacy a central place in computer science so that logic has been called *the calculus of computer science*. ... Computer science started as Logic.” (*M. Y. Vardi*)



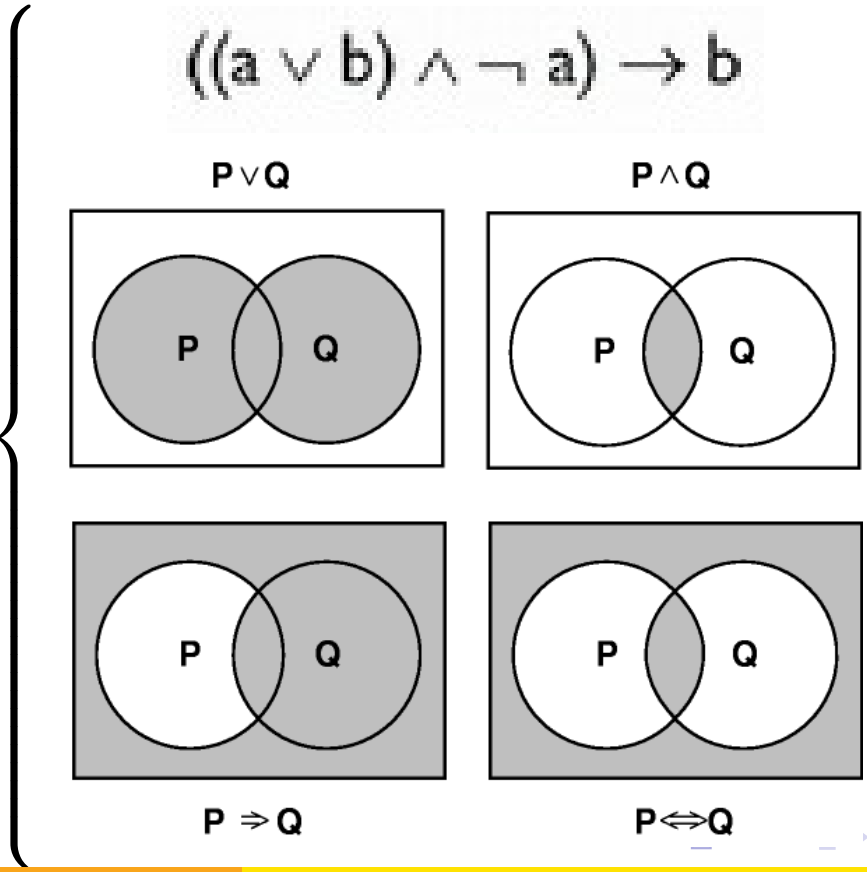
- ”I expected that digital computing machines will eventually stimulate a considerable interest in symbolic logic. ... The language in which one communicates with these machines ... forms a sort of symbolic logic.” (*A. Turing*)



# Propositional Logic

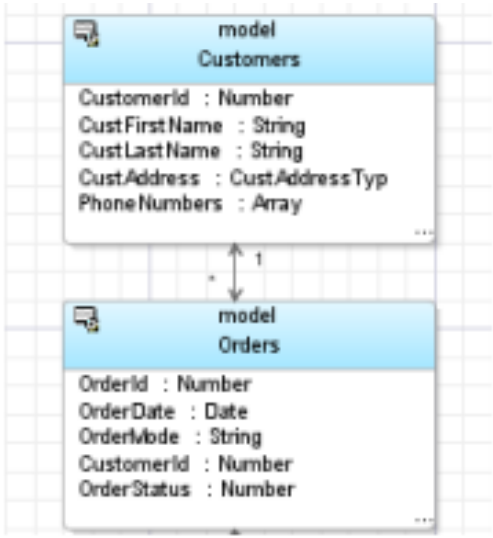
- A formalism for modeling statements that are either *true* or *false*.

*Boolean Logic*



# First Order Logic

- A formalism for specifying properties of mathematical structures, such as *graphs*, *Database System*,...




$customer(Id, Name)$
$order(Id, Date)$
$cust - ord(C, O)$
$Foreign\ Key:$
$(axiom)\ \forall \vec{X}.oder(\vec{X}) \exists! \vec{Y}.(customer(\vec{Y}) \wedge cust - ord(\vec{Y}, \vec{X}))$
$query: SELECT * FROM\ order$
$order(\vec{X})$

# Modal Logic

- A formalism for modeling *necessity* and *possibility*.

Model Checking: $\psi \equiv$"there is a deadlock"

$M\psi$, is it possible there will be a deadlock?

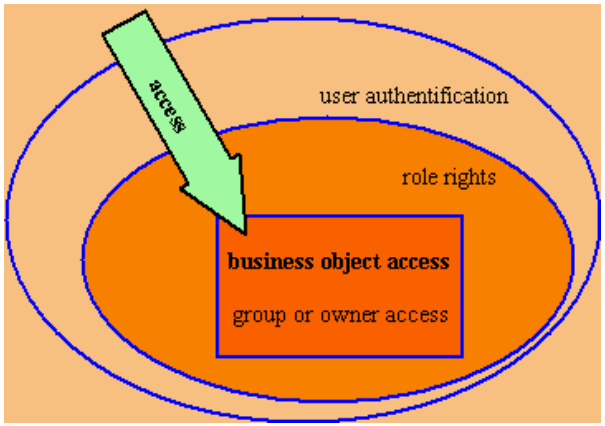
$L\neg\psi$, it is necessary there is no deadlock

# Nonmonotonic Logics

A *Nonmonotonic Logic* $\mathcal{L}$ is s.t. adding a new axiom to a theory $\mathcal{T}$ on $\mathcal{L}$ may cause the lost of a theorem proved in $\mathcal{T}$.

- Access Control Policies:

$$P = \{..., aut(Sogg, Act, Ogg, -) \leftarrow \texttt{not}\ aut(Sogg, Act, Ogg, +), ...\}$$




$$P \models aut(john, read, file, -)$$

$$\Downarrow$$

$$P \cup \{aut(john, read, file, +)\} \not\models aut(john, read, file, -)$$

# Nonmonotonic Logics

A *Nonmonotonic Logic* $\mathcal{L}$ is s.t. adding a new axiom to a theory $\mathcal{T}$ on $\mathcal{L}$ may cause the lost of a theorem proved in $\mathcal{T}$.

- Commonsense Knowledge and Reasoning:

*"My car is in garage."*

*..oops..!*

*"My car is not in garage"*

# Impact of Logic in Computer Science

- Computer-Aided Verification (model checking)
- Database System (SQL, relational algebra)
- Computational Complexity (complexity and expressivity)
- Programming Languages (formal semantics, type theory, rewriting systems,...)
- Computer Security (access control policies, negotiation policies,...)
- Artificial Intelligence (commonsese knowledge and reasoning)
- Distributed Systems (domain description)
- Logic Programming (inference)
- Software Engineering (modeling)

⋮

# References I

C. Baral.

*Knowledge Representation, Reasoning and Declarative Problem Solving*.

Cambridge University Press, Cambridge, 2003.

L. C. Aiello and F. Massacci.

Verifying security protocols as planning in logic programming.

*ACM Trans. Comput. Logic*, 2(4):542–580, 2001.

K. L. Clark.

Negation as failure.

In *Logic and Data Bases*, pages 293–322, 1977.

T. Eiter, W. Faber, and M. Truszczynski, editors.

*Logic Programming and Nonmonotonic Reasoning, 6th International Conference, LPNMR 2001, Vienna, Austria, September 17-19, 2001, Proceedings*, volume 2173 of *Lecture Notes in Computer Science*. Springer, 2001.

# References II

M.H. Van Emden and R.A. Kowalski.

The semantics of predicate logic as a programming language.

*J. ACM*, 23(4):733–742, 1976.

M. Gabbrielli and G. Gupta, editors.

*Logic Programming, 21st International Conference, ICLP 2005, Sitges, Spain, October 2-5, 2005, Proceedings*, volume 3668 of *Lecture Notes in Computer Science*. Springer, 2005.

M. Gelfond and V. Lifschitz.

The stable model semantics for logic programming.

In *Proc. of the 5th ICLP*, pages 1070–1080. MIT Press, 1988.

M. Gelfond and V. Lifschitz.

Classical negation in logic programs and disjunctive databases.

*New Generation Computing*, 9(3-4):365–386, 1991.

# References III


M. L. Ginsberg and D. E. Smith.
Possible worlds and the qualification problem.
In *AAAI*, pages 212–217, 1987.


J. W. Lloyd.
*Foundations of Logic Programming, 1st Edition*.
Springer, 1984.


W. Marek and M. Truszczynski.
Autoepistemic logic.
*J. ACM*, 38(3):587–618, 1991.


J. McCarthy.
Programs with common sense.
In *Proceedings of the Teddington Conference on the Mechanization of Thought Processes*, pages 75–91, London, 1959. Her Majesty's Stationary Office.

# References IV

R. Reiter.

A logic for default reasoning.

*Artificial Intelligence*, 13:81–132, 1980.

J. R. Shoenfield.

*Mathematical Logic*.

Series in Logic. Addison-Wesley, Reading, MA, 1967.

P. Simons, I. Niemelä, and T. Soininen.

Extending and implementing the stable model semantics.

*Artif. Intell.*, 138(1-2):181–234, 2002.

P. J. Stuckey, editor.

*Logic Programming, 18th International Conference, ICLP 2002, Copenhagen, Denmark, July 29 - August 1, 2002, Proceedings*, volume 2401 of *Lecture Notes in Computer Science*. Springer, 2002.

# References V

M.Y. Vardi.

Logic as the calculus of computer science.

Technical report, Rice University.

M.Y. Vardi.

A brief history of logic.

Technical report, Rice University, 2003.

M. De Vos and A. Provetti, editors.

*Answer Set Programming, Advances in Theory and Implementation, Proceedings of the 3rd Intl. ASP'05 Workshop, Bath, UK, September 27-29, 2005*, volume 142 of *CEUR Workshop Proceedings*. CEUR-WS.org, 2005.