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Tunable mechanical monolithic horizontal sensor with high Q for low frequency seismic noise measurement

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Abstract. A tunable mechanical horizontal monolithic seismometer/accelerometer, developed for applications in the fields of geophysics and interferometric detection of gravitational waves of second and third generation, is described. The large measurement band ($10^{-3} \div 10\,Hz$) with sensitivities of $\approx 10^{-12}\,m/\sqrt{Hz}$, as seismometer, and better than $10^{-11}\,m/s^2/\sqrt{Hz}$, as accelerometer, have been obtained with an optimised mechanical design and the introduction of a very sensitive laser interferometric optical readout, the latter aimed also to ensure a very good immunity to environmental noises. Prototypes of seismometers are operational in selected sites both to acquire seismic data for scientific analysis of seismic noise and to collect all the useful information to understand their performances in the very low frequency band ($10^{-6} \div 10^{-3}\,Hz$).

1. Introduction

The structure of the tunable mechanical horizontal monolithic sensor is based on the Folded Pendulum (hereafter FP), called also *Watt-linkage*, a suspension system developed in 1962 [1], recently applied as ultra-low frequency pendulum resonator for vibration isolation in interferometric detectors of gravitational waves [2]. Based on the FP basic scheme, single-axis monolithic accelerometers were developed as sensors in the control system for advanced seismic attenuators [3]. Recently, tunable monolithic seismometer/accelerometers of small size have been developed at the University of Salerno with extremely soft flexures at the pendulum's hinges coupled to the innovative application of laser optics readout techniques, that have largely improved the sensor sensitivity in the low frequency band and increased its immunity to environmental noises [4, 5]. Special tuning procedures have been also developed to decrease the sensor natural frequency to values as low as $\approx 70\,mHz$, allowing its use both as seismometer and accelerometer of small dimensions for seafloors or boreholes [4]. On the basis of these good scientific results, we have developed a low-noise high-resolution horizontal monolithic FP sensor for geophysical application aimed to explore the low frequency band of the seismic spectrum, both as a stand-alone sensor or as part of large and geographically distributed seismic networks.

2. Theoretical Model

The FP dynamics is theoretically well described by the simplified Lagrangian model developed by J.Liu et al. [2], very useful to understand and discuss the FP global dynamics together its main characteristics and expected performances. The FP mechanical scheme, shown in Figure 1, consists of two vertical beams, a pendulum of length l_p and mass m_p and an inverted pendulum of length l_{ip} and mass m_{ip}, linked to a central rigid mass, m_c. The distance between the FP central mass pivot points is fixed and equal to l_d. In the model of Liu et al. [2] the two vertical beams are modeled with equivalent concentrated masses positioned in their geometrical centre, that is $l_{b_1} = l_p/2$ for the pendulum and $l_{b_2} = l_{ip}/2$ for the inverted pendulum, approximations well justified by the mechanical design of our FP prototypes. The central mass, m_c, is modeled, instead, with two equivalent concentrated masses, m_{c_1} and m_{c_2} ($m_c = m_{c_1} + m_{c_2}$), positioned at the pivot points of the central mass at distances l_{c_1} and l_{c_2}, measured with respect to the pivot points of the pendulum arm and of the inverted pendulum, respectively. The addition of a tuning mass, m_t, at a distance l_t from the pendulum-central-mass pivot point, changes the values of the equivalent masses m_{c_1} and m_{c_2}, increased by fractions of the tuning mass, function of its position, l_t, according to the relations

$$m_{c1_{new}} = m_{c1_{old}} + m_t\left(1 - \frac{l_t}{l_d}\right) \qquad m_{c2_{new}} = m_{c2_{old}} + m_t\left(\frac{l_t}{l_d}\right) \tag{1}$$

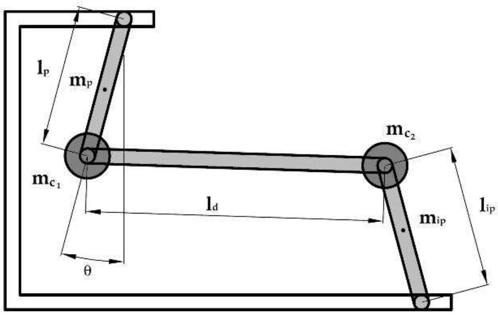

Figure 1. FP mechanical scheme.

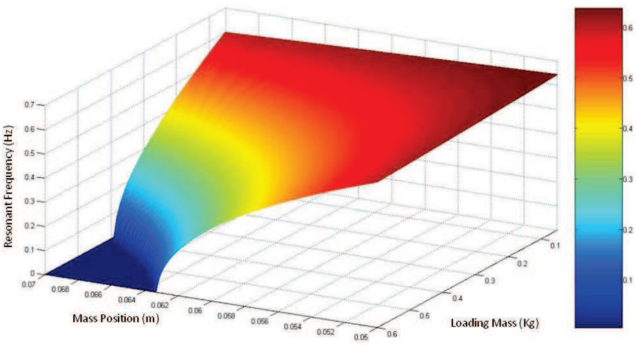

Figure 2. f_o vs. the tuning mass value, m_t, and the position, l_t, for the FP prototype.

Assuming equal lenght of the two vertical beams, $l_b = l_{b_1} = l_{b_2}$, and equal distance of the pivot points of the central mass measured $l_c = l_{c_1} = l_{c_2}$, hypothesis satisfied by our FP monolithic prototypes, the FP resonance frequency, f_o, is expressed by [4, 5]

$$f_o = \frac{\omega_o}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{(m_{b_1} - m_{b_2})\frac{gl_b}{2l_c^2} + (m_{c_1} - m_{c_2})\frac{g}{l_c} + \frac{k_\theta}{l_c^2}}{(m_{b_1} + m_{b_2})\frac{l_b^2}{3l_c^2} + (m_{c_1} + m_{c_2})}} = \frac{1}{2\pi}\sqrt{\frac{K_{eq}}{M_{eq}}} \tag{2}$$

where ω_o is the resonance angular frequency, K_{eq} is the equivalent stiffness constant, sum of the equivalent gravitational linear stiffness constant, $K_{g_{eq}}$, and of the equivalent elastic constant, $K_{e_{eq}}$, given by

$$K_{g_{eq}} = (m_{b_1} - m_{b_2})\frac{gl_b}{2l_c^2} + (m_{c_1} - m_{c_2})\frac{g}{l_c} \qquad K_{e_{eq}} = \frac{k_\theta}{l_c^2} \tag{3}$$

while M_{eq} is the equivalent mass

$$M_{eq} = (m_{b_1} + m_{b_2})\frac{l_b^2}{3l_c^2} + (m_{c_1} + m_{c_2}) \tag{4}$$

Another important FP design parameter is the tuning sensitivity, S_{f_o}, relevant to guarantee a comfortable and stable tuning of f_o. An analytic expression for S_{f_o} is easily obtained deriving Equation 2 with respect to the tuning mass position, l_t, that is

$$S_{f_o} = \frac{df_o}{dl_t} = \frac{g}{2\pi l_c l_d} \frac{m_t}{\sqrt{M_{eq}(m_t) K_{eq}}} \tag{5}$$

Equation 5 shows that the FP sensitivity is function of the value of the tuning mass, m_t. The heavier is the tuning mass, the smaller are its displacements necessary to tune f_o. This relation must be taken into full account during the FP design phase. In Figure 2, the FP resonance frequency is plotted as function of the value and position of the tuning mass for the FP prototype. The frequency plateau shows the FP instability region.

The FP theoretical transfer functions, describing its dynamics as seismometer and as accelerometer are easily obtained using the simplified Lagrangian model developed by J.Liu et al. [2], but do not include the dissipation effects, that are measurable and are the real limitation to the FP performances. To improve the FP model we introduced a global dissipation term in the lagrangian in the form of the global quality factor $Q(\omega_o)$ [4, 5]. The dependance of the quality factor on the FP resonance frequency, ω_o, has been experimentally demonstrated and will be discussed in the following section. Therefore, defining the coordinates of the pendulum frame (fixed to the ground) as x_g and the coordinate of the FP central mass as x_c (see Figure 1), then the FP transfer function as seismometer is [4, 5]

$$\frac{x_c(\omega) - x_g(\omega)}{x_g(\omega)} = \frac{(1 - A_c)\omega^2}{-\omega^2 + i\frac{\omega_o}{Q(\omega_o)}\omega + \omega_o^2} \tag{6}$$

A FP accelerometer is, instead, implemented with a force feed-back control, with a generated feed-back force proportional to the ground acceleration, experimentally obtained when the current used to drive the actuator coil is proportional to the ground acceleration. Thus defining the ground acceleration as $a_g(\omega) = \omega^2 x_g(\omega)$, then, with the same reasonment done before, the FP transfer function as accelerometer is [4, 5]

$$\frac{x_c(\omega) - x_g(\omega)}{a_g(\omega)} = \frac{(1 - A_c)}{-\omega^2 + i\frac{\omega_o}{Q(\omega_o)}\omega + \omega_o^2} \tag{7}$$

where

$$A_c = \frac{\left(\frac{l_b}{3l_c} - \frac{1}{2}\right)(m_{b_1} - m_{b_2})}{M_{eq}} \tag{8}$$

is the parameter related to the centre of percussion effects [2].

3. Test of the monolithic prototype

All the tests were performed on an Aluminium prototype (mod. 08F_100_AL1), shown in Figure 3, shaped with precision machining and electric discharge machining (EDM) from a $134 \times 134 \times 40\,mm$ block of metal (Alloy 7075-T6). The four flex joints, the sensor most critical parts, have an elliptical profile with $100\,\mu m$ minimum thickness and ellipticity ratio of $\epsilon = 16/5$. This shape ensures robustness and long-term durability to the mechanics [4, 5]. The pendula arms ($71.5\,mm$ length and spaced by $102\,mm$) minimize the mass and the moment of inertia, without reducing rigidity and symmetry. The values of the masses of the pendulum arm, of the inverted pendulum arm and of the central mass are $m_p \approx 40\,g$, $m_{ip} \approx 50\,g$ and $m_c \approx 600\,g$, respectively. The FP frequency tunability was obtained as described in [4], machining a large recess in the central mass to hosting suitable shaped and positioned tuning masses.

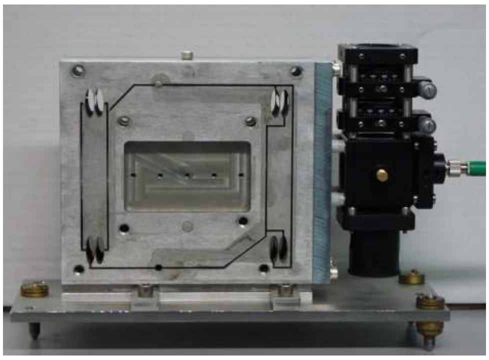

Figure 3. Mechanical monolithic Folded Pendulum with interferometric optical reaodut.

The gaps between the central mass-arms and arms-frame are $1\,mm$ large, much larger than the ones of previous versions of the FP sensor [4], for two reasons: to increase the FP dynamics and the quality factor for applications in air.

The width of the gaps is an important design parameter for a FP seismometer, because it determines its mechanical dynamics. In fact, according to the simplified lagrangian model [2] the lower is the FP natural frequency, the lower is the restoring force to external perturbations and the amplitude of the seismic signal that saturates the FP. Therefore, the width of the gaps are directly evaluted as function of the chosen FP natural resonance frequency and of the expected seismic signal maximum amplitude. The width of the gap is not a problem for a FP accelerometer, being the central mass forced in its rest position by a force feed-back control.

The performances of the monolithic FP strongly depend on the quality factor, Q, largely determined by the damping effect of the air in the gaps, and, therefore, function of their width and of the air pressure. It is also well known that Q decreases together with the decrease of the FP natural frequency, f_o, so that the knowledge of this function, even if based only on experimental data, is necessary to predict the FP behaviour and to understand its limits in the low frequency band. On the other hand, it was demonstrated in [4] that the same FP resonance frequency can be obtained with tuning masses of different weight suitably positioned. Therefore, we expected to find a dependance of Q on the value and position of the tuning mass, m_t, so that $Q = Q(f_o, m_t)$. Although this relationship is largely dependent on the design, structure and material of FP sensor, nevertheless an empirical knowledge would be very useful to predict the global behaviour of Q at different resonance frequencies.

For this task we performed a set of measurements in air, using four different tuning masses of values $m_t = 100\,g, 240\,g, 350\,g, 500\,g$. We performed the tuning procedure with each tuning mass, changing the resonance frequency and measuring the quality factor. We then fitted the measured points for each value of the tuning mass, m_t. In Figure 4 all the measurements and the fitting curves are shown. This figure shows that the heavier is the value of the tuning mass, the higher is the value of Q. Moreover, the dependance of the Q on the frequency, f_o, is linear, with a slope increasing at the increasing of the tuning mass.

We performed linear fittings (Q vs. f_o) (Figure 4) to understand the FP real limits at low frequency, although we were aware that at very low frequencies the function is parabolic ($Q = a \cdot f_o^2$). These fittings demonstrate that the prototyype cannot be used in air, already at $f_o = 10\,mHz$. We then performed three sets of measurements, tuning the FP at three different resonance frequencies, that are $300\,mHz$, $370\,mHz$, $450\,mHz$, measuring the FP Q at different values of pressure, p. The measurements, reported in Figure 5, show that Q largely increases in the pressure range $0.1\,mBar \div 1\,mBar$, suggesting the need of a deeper study in vacuum, to get rid of the gas damping, extending the Q measurements towards the low frequencies, down

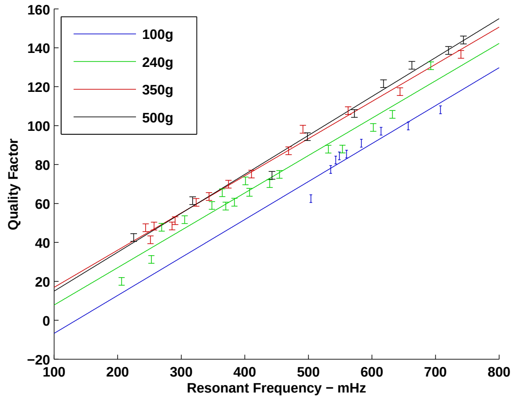

Figure 4. Q vs. f_o in air for $m_t = 100\,g,\ 240\,g,\ 350\,g,\ 500\,g$.

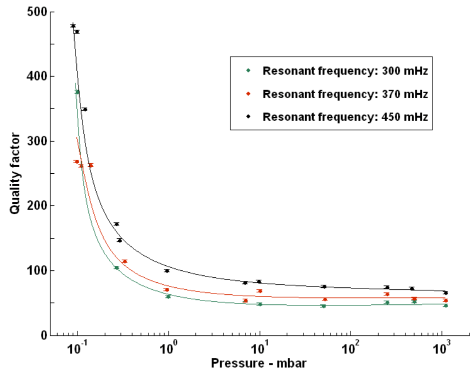

Figure 5. Q vs. p for $m_t = 120\,g$.

to tents of mHz resonance frequencies, in order to understand how long the expected square dependence of Q vs frequency lasts or new dissipation mechanisms come into play.

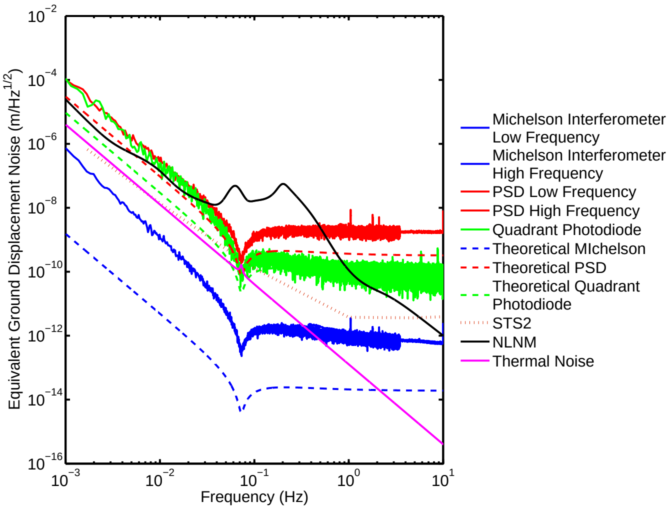

Figure 6. Theoretical and experimental sensitivity curves of the monolithic FP seismometer with optical lever and laser interferometric readouts at a natural frequency $f_o = 70\,mHz$ and temperature $T = 300\,K$.

Figure 6 shows the sensitivity curves of the FP seismometer, obtained with optical levers (PSD and quadrant photodiodes) and interferometer readouts. The theoretical sensitivity curves, evaluated according to the model described in [4] are reported for comparison, together with the sensitivity of the STS-2 by Streckeisen [6] and the Peterson New Low Noise Model (NLNM), that is the Minimum Earth Noise [7]. Note the sensitivity of $\approx 10^{-12}\,m/\sqrt{Hz}$ in the band $10^{-3} \div 10\,Hz$. At the present, two monolithic horizontal sensors are located in a blind-ended (side) tunnel 2000 ft deep in the Homestake (South Dakota, USA) mine hosting the Deep Underground Science and Engineering Laboratory (DUSEL) for the seismic characterization of the Homestake site in the frequency band $10^{-4} \div 30\,Hz$.

Figure 7 shows the sensitivity curves of the FP accelerometer, obtained with optical levers (PSD and quadrant photodiodes) and interferometer readouts, together with the sensitivity of the STS-2 by Streckeisen. Note the sensitivity better than $10^{-11}\, m/s^2/\sqrt{Hz}$ in the band $10^{-1} \div 10\, Hz$, obtained with the interferometric readout.

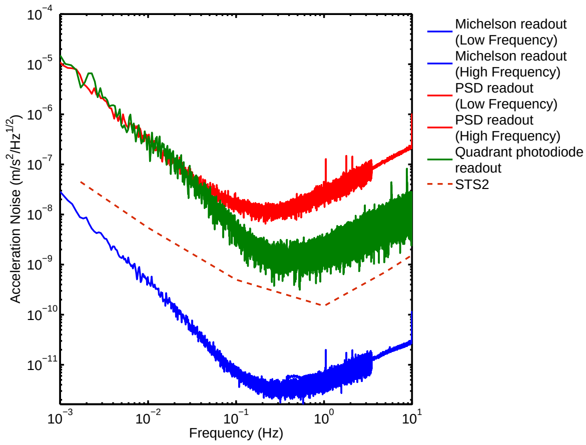

Figure 7. Sensitivity curves of the monolithic FP accelerometer with optical levers and laser interferometric readouts.

4. Conclusions and future developments

We described a mechanical horizontal monolithic sensor developed at the University of Salerno. The instrument is a monolithic tunable folded pendulum, very sensitive in the low-frequency seismic noise band ($10^{-3} \div 10\, Hz$), with very good immunity to environmental noises. The tunability of the resonance frequency and the integrated laser optical readout are its main characteristics. The interferometric readout has largely improved the sensitivity of FP seismometer that is now $\approx 10^{-12}\, m/\sqrt{Hz}$ in the band $10^{-1} \div 10\, Hz$. The sensitivity of the FP accelerometer is instead already better than $10^{-11}\, m/s^2/\sqrt{Hz}$ in the band $10^{-1} \div 10\, Hz$. Prototypes of monolithic seismometers are operational in selected sites both to acquire seismic data for scientific analysis of seismic noise and to collect all the useful information to understand their performances in the very low frequency band ($10^{-6} \div 10^{-3}\, Hz$).

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