Control of the laser frequency of the Virgo gravitational wave interferometer with an in-loop relative frequency stability of 1.0×10^{-21} on a 100 ms time scale

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***Abstract*— The measurement of the space-time structure variations induced by strong cosmic events (supernovae, coalescing binaries of neutron stars, etc.) requires an oscillator with a relative stability of 10^{-21} on time scales typically ≈100 ms. We demonstrate that the Virgo interferometer with a wavelength of 1.064 μm has a laser frequency with an in-loop stability of 1.0×10^{-21} on a 100 ms time scale, and an in-loop frequency noise of 2×10^{-7} $\mathrm{Hz}/\sqrt{\mathrm{Hz}}$ at 10 Hz. We show that this fits the specifications. Two references successively stabilize the laser frequency. The first one is a 144 m long suspended cavity; the second one is the common mode of two perpendicular 3 km long Fabry-Perot cavities. The differential mode of the relative length variations of these two optical cavities is the port where we expect the signal for the gravitational waves; this out-of-loop measurement, less sensitive to laser frequency noise, does not show up correlations with the in-loop error signal. This is the best ever performance of short term laser frequency stabilization reported.**

Fig. 1. The Virgo interferometer

I. INTRODUCTION

A. An interferometer as a transducer for gravitational waves

The general relativity theory predicts vibrations of the structure of the space-time. Violent events emit these gravitational waves: supernovae, final spiraling phases before coalescence of binary neutron stars or of black holes, etc. These waves appear in the metric of a flat space time as relative deviations of the space elements, in the transverse traceless gauge. For a wave propagating along the z axis, there are two polarizations h_+ and h_x:

$$\mathrm{d}s^2 = c^2\mathrm{d}t^2-(1+h_+(t))\,\mathrm{d}x^2-(1-h_+(t))\,\mathrm{d}y^2+2\,h_\mathrm{x}(t)\,\mathrm{d}x\,\mathrm{d}y. \tag{1}$$

They travel at the speed of light c, they affect the space in the transverse dimensions of their propagation vector, and are tensorial. Let's consider the case of a h_+ gravitational wave traveling through a Michelson interferometer, with free-falling mirrors, whose arms are aligned along the x and y axes, and such that the gravitational wave vector is perpendicular to the plane of the interferometer. Assume that the gravitational wave wavelength is much longer than the interferometer arm length. One arm has a relative stretch of a factor $h_+/2$, while the other is relatively shorter by $-h_+/2$: the comparison of the phases of the light circulating in the interferometer measures a quantity proportional to h_+. The output port of the Michelson interferometer can measure this phase difference.

B. The Virgo instrument

The Virgo interferometer [1], seated near Pisa, Italy, is based on a Michelson interferometer structure, see fig.1. The arms are made of 3 km long Fabry Perot cavities of finesse 50, with suspended mirrors, to enhance the phase change of the light caused by gravitational waves. A 10W Nd:YAG laser illuminates the interferometer. An electro-optic crystal modulates the laser phase at $f_1 = 6.25$ MHz. The interferometer output is tuned on the dark fringe; once demodulated, this port is analyzed to look for gravitational waves. The tuning to the dark fringe gives the best signal to noise ratio for a modulation/demodulation sensing technique. Most of the laser light is reflected back to the laser; an additional mirror, the power recycling mirror, provides an additional resonant cavity with suspended mirrors to improve by a factor 30 the amount of light on the beam-splitter, thus minimizing the resolution once limited by the photon shot noise. A 144 m long triangular resonant cavity with suspended mirrors, the "input mode cleaner", filters the input laser beam angular and transversal displacements. The mirrors are suspended to a chain of five pendulums [2] to filter out the seismic motion from the 10 Hz – 10 kHz detection band. Above the 0.6 Hz resonant frequency of the last pendulum, the mirror response along the optical axis is equivalent to the one of free falling test masses.

II. LASER FREQUENCY CONTROL IN THE VIRGO INSTRUMENT

A. Requirements for the laser frequency stabilization

The laser wavelength is the intermediary ruler to compare the relative variations of the two 3 km Fabry Perot cavity lengths. We show that when stabilized, the contribution of the laser frequency noise to the dark fringe noise budget should be negligible. From the study of the coupling paths of the laser frequency noise to the dark fringe, we give the specifications for the in-loop laser frequency noise.

1) Passive filtering: The first isolation of the dark fringe, measuring the differential mode of the two cavities, from the laser frequency noise comes from a passive filtering; the second effect is the isolation from the symmetric response of the two Fabry-Perot cavities.

A resonant optical cavity is a low pass filter for laser frequency noise with a pole equal to the half of the cavity line-width. The input mode cleaner provides a passive filtering with a pole at 500 Hz; the power recycling cavity, coupled to the long arms, provides a passive filtering with a pole at 8 Hz.

The symmetry of the optical properties of the resonant cavities provides additional isolation of the dark fringe from the laser frequency noise. The first asymmetry is the finesse asymmetry, equivalent to an optical path length difference. A spurious Fabry-Perot cavity in the input couplers, between the anti-reflective coating on the outer face and the T=12% of the inner face, modulates the input coupler reflectivity, and thus the finesse. We expect the finesse asymmetry to vary from ±4%, depending on the temperature-dependent thickness of the input substrates. This etalon effect is designed for a possible tuning of the finesse asymmetry. A second asymmetry is the reflectivity asymmetry, contributing to a contrast defect. The non-perfect interference gives a contribution from photons that are sensitive to only one of the cavities, thus very sensitive to the laser frequency noise. A detailed calculation of the transfer function from the laser frequency noise, after input mode cleaner, to the demodulated dark fringe, leads to:

$$\frac{\tilde{h}}{\tilde{\nu}/\nu_0} = \frac{\Delta F}{F}\frac{1}{1+if/f_{\mathrm{rec}}} + \sqrt{\frac{1-C}{2}}\,\frac{f_{\mathrm{p}}}{f_{\mathrm{rec}}}\,\frac{1+if/f_{\mathrm{p}}}{1+if/f_{\mathrm{rec}}} \qquad (2)$$

where $\nu_0 = 288$ THz is the laser frequency, F is the average finesse and ΔF the finesse difference, $f_{\mathrm{rec}} = 8$ Hz is the measured pole of the power recycling cavity coupled to the long arms, $f_{\mathrm{p}} = 500$ Hz is the pole of the long arms, $1-C = 9\times10^{-7}$ is the measured contrast defect.

The target for the spectral density of resolution for the Virgo interferometer is $\tilde{h} = 9\times10^{-23}/\sqrt{\mathrm{Hz}}$ at 50 Hz and $\tilde{h} = 6\times10^{-22}/\sqrt{\mathrm{Hz}}$ at 10kHz. When taking into account the asymmetry function, the specification for the laser frequency noise, after the input mode cleaner cavity, is then 4×10^{-6} $\mathrm{Hz}/\sqrt{\mathrm{Hz}}$ at a frequency of 50 Hz, and 3×10^{-4} $\mathrm{Hz}/\sqrt{\mathrm{Hz}}$ at a frequency of 10 kHz.

Our free-running laser frequency noise is $\tilde{\nu}(f) = 10$ kHz/f; filtered by the 500 Hz pole of the input mode cleaner, this gives 32 $\mathrm{Hz}/\sqrt{\mathrm{Hz}}$ at 50 Hz and $8\times10^{-4}\mathrm{Hz}/\sqrt{\mathrm{Hz}}$ at 10 kHz. we see that a passive isolation is not sufficient: an active isolation, using very stable references, is required.

2) Active stabilization in two stages: We need very stable phase references to be able to meet the specifications for the laser frequency noise. An analysis of possible feedback loops shows that two loops are necessary.

The best reference at our disposal is the common mode variations of the two long arm cavities $\delta L_1 + \delta L_2$, whereas the dark fringe looks for gravitational waves giving a signal proportional to $\delta L_1 - \delta L_2$. We thus stabilize the laser frequency on this reference, using a Pound-Drever-Hall like technique. The unity gain frequency is actually limited to a fraction of the 50 kHz free spectral range of the long arms. We found out that the demodulated signal from the light reflected on the anti-reflective coating of the beam-splitter gives the best signal to noise ratio as well as best unity gain frequency. We optimized the filter loop response and achieved a unity gain frequency of 23 kHz. The loop gain at 10 kHz is 2.3 so that the target for negligible contribution of the laser frequency noise would not achieved at 10 kHz, and required loop gain of $\approx 10^7$ at 50 Hz difficult.

We thus pre-stabilize our laser frequency, using the input mode cleaner cavity as a reference. The error signals of the two loops are added electronically. That feedback loop has a unity gain of 300 kHz, one third of the free spectral range of that cavity. Below 30 kHz the loop shape is $1/f^4$, so that the loop gain at 50 Hz is 10^{12}.

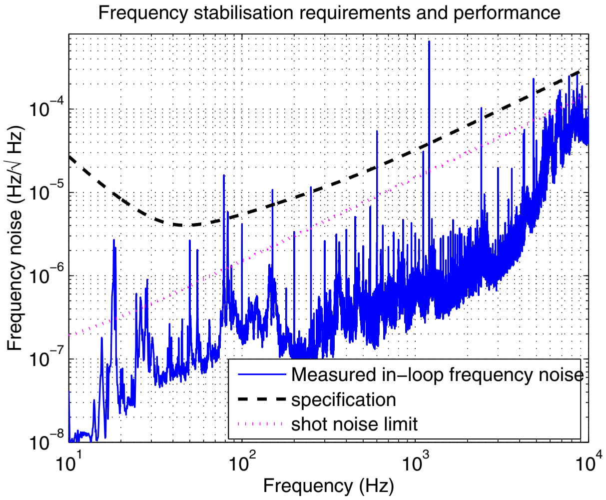

Fig. 2. Spectral density of the error signal of the lock of the laser frequency on the interferometer common mode.

Using two feedback loops, we have designed a system able to meet the requirement of negligible laser frequency noise in the channel where the gravitational waves are searched.

B. Experimental results

We show that the in-loop measured frequency noise meets the specifications, taking into account the passive filtering capacities of the interferometer. We have an in-loop laser frequency noise of $1.9\times10^{-7}\mathrm{Hz}/\sqrt{\mathrm{Hz}}$, limited by the in-loop shot noise that is reported out-of-loop. The coherence of the laser frequency stabilization error signal with the dark fringe channel does not show significant coherence, except on a few lines.

1) In loop measurements: The demodulated signal from the light caught on the anti-reflective coating of the beam-splitter is the error signal for the laser frequency stabilization on the common mode of the two Fabry-Perot cavities. The fit of the open loop transfer function with the various loop elements gives a sensitivity of the error signal of 12 V/Hz. The fit, around the free spectral range, from the transfer function of a perturbation added to the loop, to the error signal, shows that the recycling cavity coupled to the long arms has a pole at 8 Hz. With the sensitivity and pole information, the error signal is properly calibrated. The result appears in fig. 2. Except for some lines, the measured curved fits the specifications.

The in-loop error signal measurement noise due to shot noise infers an out-of-loop limit on the laser frequency stabilization. This level is estimated with the 43 mW on the photodiode; it is actually a limit that would be measured on a perfect out-of-loop measurement.

The spectral density can be converted into a relative Allan standard deviation [3]. For this calculation, to avoid a divergence in the data, the error signal is not corrected from the cavity pole at 8 Hz. The result is presented in fig. 3.

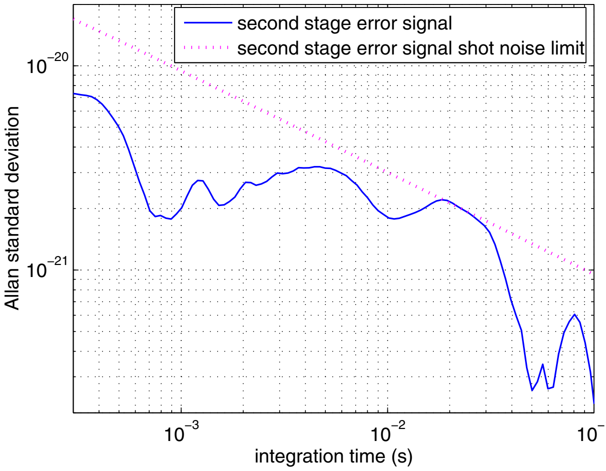

Fig. 3. Relative Allan standard deviation corresponding to the data shown in fig. 2, not corrected from the cavity pole

The specification on the spectral density of the in-loop error signal is met. An in-loop relative stability of 1.0×10^{-21} on a 100 ms timescale is measured.

2) Out of loop laser frequency noise: The interferometer and its laser frequency control is designed so that the most sensitive channel, the dark fringe, is not sensitive to the laser frequency noise. A proper out-of-loop measurement is thus not possible. We show that the laser frequency noise does not seem to be the main contributor to the noise budget of the dark fringe, as required.

A first information is the measurement of the coherence between the common mode (laser frequency noise) error signal, and the differential mode (demodulated dark fringe). The coherence is obtained as the spectral density of the cross correlation, calibrated with the spectral densities of the two signals. The result is shown in fig. 4. Except on some lines, the coherence is very low.

The coherence is not an information on how much laser frequency noise is present on the dark fringe. We can forecast how much laser frequency noise is present on the dark fringe, using the measured asymmetry function described in equation 2. This gives the data displayed in fig. 5. The out-of-loop frequency noise due to the re-injected shot noise actually contributes to 30% of the dark fringe for frequencies above ≈ 500 Hz; but this does not appear on the coherence as the in-loop signal is not shot noise limited.

If the error signal of the laser frequency stabilization signal would be shot noise limited, it might be possible to feed-forward this contribution and reduce the out-of-loop laser frequency noise contribution further.

III. CONCLUSION

We have shown that the laser of the Virgo interferometer is controlled with an in-loop error of $2 \times 10^{-7}\mathrm{Hz}/\sqrt{\mathrm{Hz}}$, shot noise limited, at 10 Hz. According to our analysis, the laser frequency noise does not seem to be the main contributor to the interferometer dark fringe, as required by the laser frequency control design.

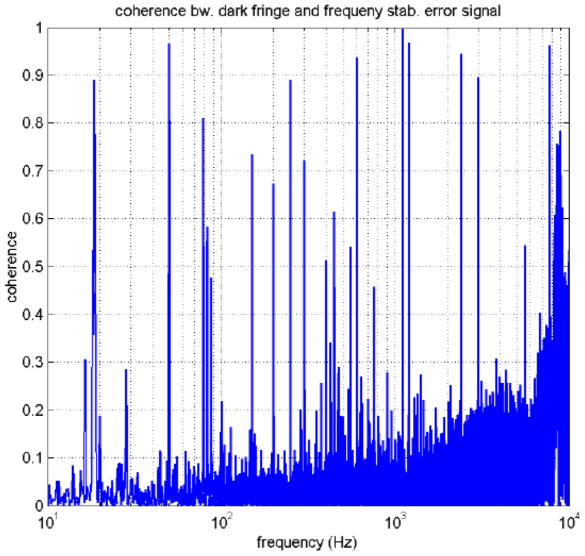

Fig. 4. Coherence between in-loop laser frequency noise and the interferometer dark fringe port

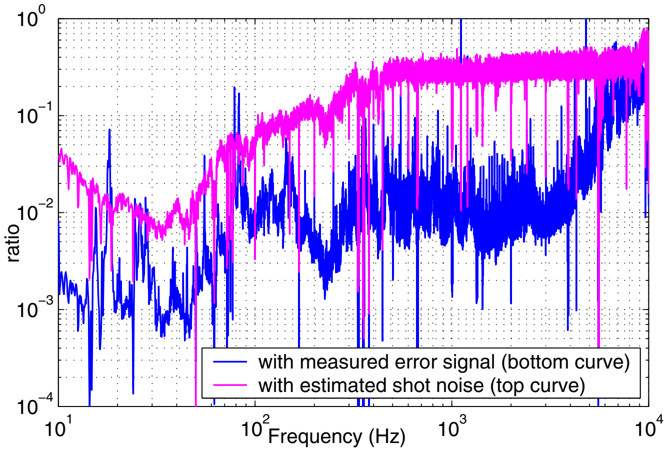

Fig. 5. Expected contribution of the laser frequency noise in the dark fringe

REFERENCES

[1] F. Acernese, M. Alshourbagy, P. Amico, F. Antonucci, S. Aoudia, K. Arun, P. Astone, S. Avino, L. Baggio, and G. B. etal., "Virgo status," *Class. Quantum Grav.*, vol. 25, no. 18, p. 184001, 2008.

[2] A. Bozzi, C. Bourgoin, S. Cortese, A. Errico, D. Enard, S. Mataguez, A. Paoli, A. Pasqualetti, P. Popolizio, and F. R. etal., "Last stage control and mechanical transfer function measurement of the virgo suspensions," *Rev. Sci. Instrum.*, vol. 73, no. 5, pp. 2143–1249, 2002.

[3] F. Hartmann and F. Stoeckel, "Stabilité de fréquence et pureté spectrale des lasers," *J. Phys. Colloques (Paris)*, vol. 39, pp. C1–32–C1–36, 1978.