15 Beyond the standard model: a brief survey

Grand unification · The SU(5) model · Neutrino masses and oscillations · Grand unification and the Big Bang · Towards a theory of everything?

15.1 Grand unification

In chapter 14 we described an impressive array of experimental tests of the $SU(3)_C \times SU(2)_L \times U(1)_Y$ standard model of the fundamental interactions. To date there is no known discrepancy between the standard model and experiment. Confirmation of the symmetry-breaking mechanism awaits the discovery of the Higgs boson.

The great achievement of the Glashow–Weinberg–Salam model was the 'unification' of the weak and electromagnetic interactions. Strictly speaking, their theory is not a unification in the sense of both couplings arising from a common source. Indeed, the $SU(2) \times U(1)$ gauge group is a product of two disconnected groups of gauge transformations and the coupling strengths g and g' are not related by the theory: their ratio

$$\frac{g'}{g} = \tan\theta_W \qquad (15.1)$$

where θ_W is the Weinberg angle, has to be measured experimentally. Only if an appropriate unifying group G can be found such that

$$G \supset SU(2) \times U(1) \qquad (15.2)$$

(that is to say SU(2) and U(1) are subgroups of the larger group G) will it be possible to predict the relationship between g and g'. Some of the transformations of the new group G will link the previously disconnected groups SU(2) and U(1) thereby relating the coupling strengths g and g'. In fact they will be related by a Clebsch–Gordan coefficient of G.

The standard model is completed by the inclusion of the SU(3) colour group of gauge transformations which describes the strong interactions. Again, this group is disconnected from the electroweak SU(2) × U(1) groups but it is natural to attempt to unify the strong, weak and electromagnetic interactions by searching for a 'grand unifying' group, G, such that

$$\mathrm{G} \supset \mathrm{SU(3)} \times \mathrm{SU(2)} \times \mathrm{U(1)}. \tag{15.3}$$

The basic idea of grand unification is that the symmetry is not broken above some mass scale $\mu = M_X$ where the gauge couplings g_i are related to a single gauge coupling g_G, which evolves with increasing Q^2 in accordance with the β function of G. Below M_X the symmetry is spontaneously broken, presumably by a Higgs mechanism, and the couplings g_i evolve separately in accordance with the β functions of their respective groups until eventually they coincide with their measured values at the mass scale $\mu = M_W$. One possible scenario for this evolution of the coupling constants to the grand unification scale M_X is shown in figure 15.1. The gauge couplings g_i (section 13.8) are related to the

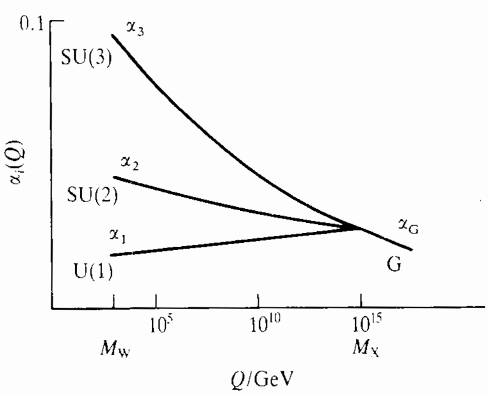

Figure 15.1
Evolution of the gauge couplings $\alpha_i = g_i^2/4\pi$ with Q in a grand unification scheme. Above the grand unification scale, $\mu = M_X \approx 10^{15}$ GeV, all couplings coincide.

couplings of the standard model as follows:

$$\begin{aligned} g_3(Q) &= g_s(Q) && \mathrm{SU(3)} \\ g_2(Q) &= g(Q) && \mathrm{SU(2)} \\ g_1(Q) &= Cg'(Q) && \mathrm{U(1)} \end{aligned} \tag{15.4}$$

where C is a Clebsch–Gordan coefficient of G and the 'fine-structure constants' are given by $\alpha_i = g_i^2/4\pi$. In order that SU(3) × SU(2) × U(1) be embedded in G it is necessary that G possesses at least four commuting generators corresponding to I_3 and Y of weak isospin and I_3^c and Y^c of the SU(3) colour group. Thus G must have rank 4 at least. The simplest group satisfying this requirement is SU(5), originally proposed by Georgi and Glashow.[1] In SU(5), C has the value $\sqrt{(\frac{5}{3})}$.

We can determine an approximate value for the grand unification scale by evolving the couplings from their known values at the W mass through their respective evolution equations to a common intersection $Q = M_X$. For example, the running of the strong coupling was given in equation (13.181) which we rewrite as

$$\alpha_3(Q) = \frac{\alpha_3(\mu)}{1 + 2b_3\alpha_3(\mu)\ln(Q/\mu)}. \tag{15.5}$$

This expression is valid for correction terms at the one-loop level (see section 13.9.2). Simple rearrangement of (15.5) leads to the result

$$\alpha_3^{-1}(\mu) = \alpha_3^{-1}(Q) + 2b_3\ln(\mu/Q). \tag{15.6}$$

Equation (15.6) holds generally for SU(3), SU(2) and U(1), thus,

$$\alpha_i^{-1}(\mu) = \alpha_i^{-1}(Q) + 2b_i\ln(\mu/Q). \tag{15.7}$$

The characteristic values of the b coefficients are

$$b_i = \frac{1}{12\pi}(11n_b - 4n_g) \tag{15.8}$$

where n_b is the number of vector bosons and n_g the number of fermion generations which contribute to the one-loop vacuum polarization diagrams. The values of n_b are 0, 2 and 3 for U(1), SU(2) and SU(3) respectively, and the number of fermion generations is 3. In (15.8) we have neglected numerically unimportant Higgs scalar contributions to b_1 and b_2. Further details can be found in the review by Langacker.[2]

The different approaches of $\alpha_i(Q)$ to the unification scale M_X, depicted in figure 15.1, are governed by the values of b_i given in equation (15.8). The experimental values of the couplings at $Q = M_W$ are

$$\alpha_1^{-1}(M_W) \approx \frac{3}{5}\frac{1-\sin^2\theta_W}{\alpha(M_W)} \approx 59$$

$$\alpha_2^{-1}(M_W) \approx \frac{\sin^2\theta_W}{\alpha(M_W)} \approx 29$$

$$\alpha_3^{-1}(M_W) \approx 8$$

where we have used the values $\sin^2 \theta_W = 0.23$ and $\alpha^{-1}(M_W) = 128$. When evolved from the above values at $Q = M_W$ to the unification scale $\mu = M_X$ the couplings converge approximately to the common value $\alpha_G(M_X) = 0.024$ at $M_X \approx 10^{15}$ GeV: the number of fermion generations does not affect this result (see example 15.1).

One immediate consequence of grand unification is that the weak mixing angle θ_W becomes a prediction of the theory rather than a parameter that has to be determined experimentally. For, in terms of g_1 and g_2, equation (15.1) becomes

$$\frac{g_1(Q)}{Cg_2(Q)} = \tan \theta_W \tag{15.9}$$

and since, for $Q = M_X$, $g_1(M_X) = g_2(M_X)$, θ_W is determined by C. In SU(5), $C = \sqrt{(\frac{5}{3})}$, thus the SU(5) prediction for the weak mixing angle at the unification scale is

$$\sin^2 \theta_W = \tfrac{3}{8}. \tag{15.10}$$

When evolved to $Q = M_W$ (see example 15.1) the predicted value of $\sin^2 \theta_W$ is 0.205, somewhat lower than the measured value[3] of

$$\sin^2 \theta_W = 0.2325 \pm 0.0008. \tag{15.11}$$

Although this result makes it unlikely that SU(5) is the correct grand unifying group, it contains so many important and dramatic features that we will use it as an illustrative model.

15.2 The SU(5) model

Many attempts have been made to formulate 'grand unified theories' or GUTs. In this section we discuss some of the far-reaching predictions of the simplest model – SU(5).

15.2.1 The SU(5) multiplets

In previous chapters we have learnt that there are three distinct generations of fermions: (ν_e, e; u, d), (ν_μ, μ; c, s) and (ν_τ, τ; t, b). Each generation consists of 15 states. In the first generation, for example, there is the electron e^- with two helicity states and the massless neutrino ν with one helicity only. The u and d quarks come in three colours each with two helicity states. By convention, the left-handed helicity states are grouped together. Thus, in the first generation the states under consideration are

$$(\nu_e, e^-, e^+, u_R, u_G, u_B, \bar{u}_R, \bar{u}_G, \bar{u}_B, d_R, d_G, d_B, \bar{d}_R, \bar{d}_G, \bar{d}_B)_L$$

where the subscript L denotes left-handed helicity states. Note that under a CP transformation $e_L^+ \equiv e_R^-$, $\bar{u}_L \equiv u_R$, etc. These 15 states can be accommodated in the $\bar{\mathbf{5}}$ and $\mathbf{10}$ representations of SU(5) which decompose into $(\mathrm{SU(3)_C}, \mathrm{SU(2)_L})$ multiplets as follows:

$$\begin{aligned}\bar{\mathbf{5}} &= (\bar{\mathbf{3}}, \mathbf{1}) + (\mathbf{1}, \mathbf{2}) = (\bar{d}_R, \bar{d}_G, \bar{d}_B)_L + (\nu_e, e^-)_L \\ \mathbf{10} &= (\mathbf{3}, \mathbf{2}) + (\bar{\mathbf{3}}, \mathbf{1}) + (\mathbf{1}, \mathbf{1}) \\ &= (u_R, u_G, u_B, d_R, d_G, d_B)_L + (\bar{u}_R, \bar{u}_G, \bar{u}_B)_L + e_L^+ .\end{aligned}$$

The quintet can be represented as a vector

$$Q = \begin{bmatrix} \bar{d}_R \\ \bar{d}_G \\ \bar{d}_B \\ e^- \\ \nu_e \end{bmatrix}_L . \tag{15.12}$$

The decuplet arises from the antisymmetric part of the product of two fundamental 5-representations (conjugates to the $\bar{5}$-representation)

$$\mathbf{5} \times \mathbf{5} = \mathbf{15} + \mathbf{10}$$

and can be represented as an antisymmetric tensor

$$D = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & \bar{u}_B & -\bar{u}_G & -u_R & -d_R \\ -\bar{u}_B & 0 & \bar{u}_R & -u_G & -d_G \\ \bar{u}_G & -\bar{u}_R & 0 & -u_B & -d_B \\ u_R & u_G & u_B & 0 & -e^+ \\ d_R & d_G & d_B & e^+ & 0 \end{bmatrix}_L . \tag{15.13}$$

The heavier fermion generations belong to multiplets which are replicas of these.

The gauge bosons belong to the 24-dimensional representation of $\mathbf{5} \times \bar{\mathbf{5}} = \mathbf{1} + \mathbf{24}$. The $(\mathrm{SU(3)}, \mathrm{SU(2)_L})$ decomposition of the $\mathbf{24}$ is

$$\mathbf{24} = \underbrace{(\mathbf{8}, \mathbf{1})}_{\text{gluons}} + \underbrace{(\mathbf{1}, \mathbf{3}) + (\mathbf{1}, \mathbf{1})}_{W^\pm, Z, \gamma} + \underbrace{(\mathbf{3}, \mathbf{2}) + (\bar{\mathbf{3}}, \mathbf{2})}_{X, Y \text{ bosons}} \tag{15.14}$$

In addition to the familiar gluons and W, Z and γ bosons there are now superheavy bosons X and Y which form a weak doublet (and antidoublet) and come in three colours giving a total of 12 in all. The X and Y bosons have electric charges $Q = +\frac{4}{3}$ and $+\frac{1}{3}$ respectively: the antiparticles have the opposite sign of charge.

Thus in SU(5) it is necessary to accommodate each generation of fermions in two multiplets, a $\bar{\mathbf{5}}$ and a **10**. In these multiplets quarks and leptons and quarks and antiquarks appear on the same footing and therefore transitions between them can be induced by the appropriate gauge bosons. The colour octet of gluons induces transitions between coloured quarks, the W bosons couple to weak isospin doublets and the Z^0 and γ to fermion–antifermion pairs, transformations which are all beautifully described by the standard model. The new ingredients in SU(5) are the massive gauge bosons X and Y whose existence leads inevitably to dramatic new and far-reaching consequences. These gauge bosons will induce transitions in which baryon number (B) and/or lepton number (L) are no longer conserved. As a result, proton decay would no longer be forbidden: processes such as $p \to \pi^0 e^+$ ($\Delta B \neq 0$, $\Delta L \neq 0$) would be allowed. Neutrinoless double β decay of nuclei should occur and, provided neutrinos have non-zero mass, $\Delta L \neq 0$ transitions will give rise to neutrino oscillations in which transformations between different neutrino species occur. Furthermore, GUTs predict the existence of magnetic monopoles with masses comparable with M_X.

15.2.2 Charge quantization

There are no theoretical constraints in the standard model which demand the quantization of electric charge. Indeed, the electric charge operator is a linear combination of the weak isospin and the weak hypercharge and the latter, being a generator of the Abelian U(1) group, can take on a continuous range of values and can be assigned independently for each representation. The only theoretical constraint is that the charge *difference* between members of a specific doublet is one unit. The charges of the leptons and quarks need not be related by simple factors like 1 or 3. One of the appealing features of the SU(5) model, and others, is that charge quantization occurs naturally, basically because the GUT symmetry fixes the values of I_3 and Y for each member of a multiplet. Since all the fermions of a particular generation appear in the same multiplets of SU(5) their charges are uniquely determined relative to the electron charge.

In general, in any representation of a simple non-Abelian group the generators are traceless. This means that the sum of the eigenvalues of any diagonal generator is zero when taken over all members in a representation. In particular, the electric charge operator is a linear combination of the diagonal generators I_3 and Y, and therefore the sum of the charges of the fermions in any representation must be zero. The fractional charges of the quarks then arise naturally because *the electron is colourless and the quarks come in three colours.* For example, the members of the $\bar{\mathbf{5}}$ are $(\bar{d}_R, \bar{d}_G, \bar{d}_B, e^-, \nu_e)_L$ and therefore the charge of the $\bar{d}$ quark must be $Q_{\bar{d}} = +\frac{1}{3}$ to balance the charge of the electron. The charge of the d quark is then $-\frac{1}{3}$ and because the u_L and d_L quarks form a weak isospin doublet the charge of the u quark is $+\frac{2}{3}$. This charge quantization then guarantees the exact equality between the charges of the electron and proton.

15.2.3 Magnetic monopoles

More than 60 years ago Dirac[4] predicted the existence of magnetic monopoles with magnetic charge e_m given by

$$e_m = n\frac{\hbar c}{2e}$$

where n is an integer. In 1974 't Hooft[5] and Polyakov[6] showed that magnetic monopoles occur naturally in GUTs, thus the discovery of such an object would be a triumph for grand unification schemes. The mass of a magnetic monopole is expected to be in excess of the X boson mass ($\approx 2 \times 10^{14}$ GeV) so that there is no possibility of monopole production in accelerator experiments. However, it is possible that magnetic monopoles could be a remnant of the 'big bang' in which it is envisaged that grand unification held until the temperature dropped below the grand unification mass M_X. Experimental searches for magnetic monopoles are therefore important both for cosmology and particle physics.

The techniques most widely used are based either on ionization or superconducting induction devices. Ionization experiments rely on the fact that a magnetic charge will produce much more ionization than an electric charge with the same velocity. In induction devices the passage of a monopole through the coil will produce a sudden change in the magnetic flux linking the coil and hence a sudden change in the current flowing in the coil. In 1982 Cabrera[7] observed such a flux increase in an experiment at Stanford University but most experiments obtain negative results and isolated candidate events need confirmation.

15.2.4 Proton decay

The most spectacular prediction of the SU(5) grand unification scheme is that protons, which we have hitherto regarded as stable, should decay via the exchange of virtual superheavy gauge bosons X and Y. Some possible mechanisms for the decays are shown in figure 15.2.

In SU(5) the dominant decay mode of the proton is expected to be

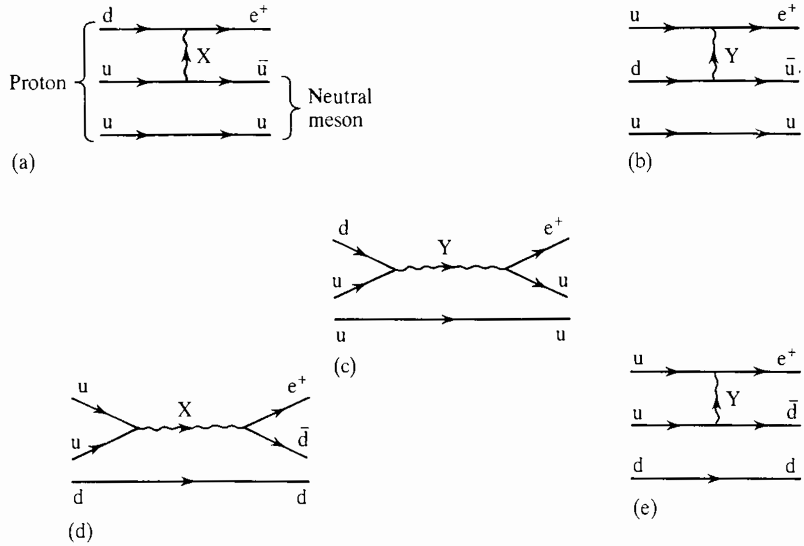

Figure 15.2
Possible mechanisms for proton decay. The quark–antiquark pairs in the final state can combine to produce mesons such as π^0, ρ^0, ω^0, etc.

$p \to e^+\pi^0$. The matrix element for this decay will contain a factor $(q^2 + m_X^2)^{-1}$ for the boson propagator and since the momentum transfer is only of the order of 1 GeV^2, $m_X^2 \gg q^2$ and the decay rate will be proportional to m_X^{-4}. The proton lifetime thus depends crucially on the value of the gauge boson mass. Refined estimates[8] of m_X yield a value of about 2×10^{14} GeV and a proton lifetime

$$\tau_p \approx 2 \times 10^{29 \pm 1.7} \text{ years.} \tag{15.15}$$

The error includes uncertainties arising from the measured value of $\Lambda_{\overline{MS}}$ and other model-dependent factors. Calculations of branching ratios are very model dependent but typical values for exclusive decay modes are given in table 15.1.

The main experimental difficulty in the detection of proton decay arises from the enormous predicted lifetime and the consequent need to shield the detectors from background cosmic radiation. The huge lifetime necessitates the use of massive detectors which are basically of two types. Purified water Cherenkov detectors viewed by thousands of photomultiplier tubes have been designed to detect the Cherenkov radiation arising from electromagnetic showers generated by the decays $p \to e^+\pi^0$. This is the technique used by the IMB (Irvine–Michigan–Brookhaven) and Kamiokande experiments. The experimental difficulties become apparent when one realizes that a 1000 tonne detector contains some 6×10^{32} nucleons so that for a lifetime of 10^{32} years only six nucleons on average will decay in one year. The other type of detector uses sampling calorimeters consisting of iron plates separated by arrays of track detectors such as streamer chambers, proportional wire chambers or drift chambers.

Table 15.1
Typical branching fractions B for exclusive decay modes of the proton

Decay mode	$e^+\pi^0$	$e^+\rho^0$ and $e^+\omega$	$\bar{\nu}_e\pi^+$	μ^+K^0	$\bar{\nu}_\mu K^+$
Branching fraction, %	40	30	16	3	3

To minimize the cosmic ray background the detectors are located either in deep underground mines, in mountain tunnels such as the Mont Blanc and Fréjus tunnels or in purpose-built underground laboratories such as Baksan in the Caucasus.

The present experimental limit[3] for the favoured decay is

$$\tau/B(p \to e^+\pi^0) > 9 \times 10^{32} \text{ years} \tag{15.16}$$

which rules out SU(5) as the grand unification group even allowing for the generous errors quoted in (15.15). Longer proton lifetimes can be accommodated in GUTs based on larger groups than SU(5) but for lifetimes greater than 10^{33} years the background from reactions such as $\bar{\nu}_e p \to e^+\pi^0 n$, induced by cosmic ray antineutrinos, will overwhelm the signal so that positive evidence for proton decay may be difficult to obtain: a positive signal is crucial for the health of grand unification schemes.

15.3 *Neutrino masses and oscillations*

In the standard model neutrinos are assumed to be massless and exist in only one helicity state but there is no fundamental reason why this should be so. In some grand unification schemes neutrino masses appear naturally: a positive observation of a non-zero neutrino mass would help discriminate between the various schemes.

The question of neutrino mass has important implications for cosmology. Various astronomical observations indicate that about 90 per cent of the total gravitational mass of the universe consists of invisible or 'dark matter': a component of this dark matter could be massive neutrinos.

Massive neutrinos could also provide a solution to the so-called solar neutrino problem – the discrepancy between the solar neutrino flux expected from calculations based on the standard solar model (SSM) and the experimentally observed flux of solar neutrinos. The source of energy in the Sun is a series of nuclear reactions which convert hydrogen into helium and produce solar neutrinos with a predicted flux of about $10^{11}\,\text{cm}^{-2}\,\text{s}^{-1}$ at the Earth. The main chain of reactions is initiated by the processes

$$p + p \to {}^2H + e^+ + \nu_e \qquad (E_\nu^{max} = 0.42 \text{ MeV}) \tag{15.17}$$

and, with considerably less probability,

$$p + e^- + p \to {}^2H + \nu_e \qquad (E_\nu = 1.44 \text{ MeV}). \tag{15.18}$$

The resulting deuterons are converted to ^{3}He via the reaction

$$p + {}^2H \to {}^3He + \gamma \tag{15.19}$$

which is followed by

$${}^3He + {}^3He \to {}^4He + 2p. \tag{15.20}$$

As an alternative to (15.20) the ^{3}He can interact with ^{4}He in the Sun to produce energetic neutrinos ($E_\nu^{max} \approx 14$ MeV) via the chain

$$\begin{aligned} &{}^3He + {}^4He \to {}^7Be + \gamma; \qquad {}^7Be + p \to {}^8B + \gamma; \\ &{}^8B \to {}^8Be^* + e^+ + \nu_e; \qquad {}^8Be^* \to 2{}^4He. \end{aligned} \tag{15.21}$$

Only about one in a thousand ^{7}Be nuclei undergo this particular process: the rest are converted to ^{7}Li by electron capture,

$${}^7Be + e^- \to {}^7Li \text{ (or } {}^7Li^*) + \nu_e \qquad (E_\nu = 0.862 \text{ or } 0.383 \text{ MeV}) \tag{15.22}$$

followed by

$${}^7Li + p \to 2{}^4He.$$

In summary, there are three main sources of solar neutrinos:

(a) The so-called p–p neutrinos (equation (15.17)) are the most copious and have a continuous energy spectrum with an endpoint energy of 420 keV.
(b) Reaction (15.22) produces monoenergetic neutrinos with energies of 862 keV (90 per cent) and 383 keV (10 per cent) and an integrated flux about 0.08 times that of the p–p neutrinos.
(c) The ^{8}B decay produces the most energetic neutrinos with an endpoint energy of approximately 14 MeV but the integrated flux is only about 10^{-4} times the p–p neutrino flux and the intensity is less well predicted than for the p–p neutrinos.

There are also contributions to the solar neutrino flux from the p–e–p reaction (15.18) and from ^{13}N, ^{15}O and, to a lesser extent, ^{17}F decays, produced in the carbon–nitrogen–oxygen (CNO) cycle in the Sun. These contributions are much weaker than that from the p–p reaction and much less energetic than that from ^{8}B.

The pioneering chlorine-37 experiment of Davis *et al.*[9,10] has a threshold energy of 0.81 MeV and is therefore sensitive mainly to ^{7}Be and ^{8}B neutrinos. The rate of detection of solar neutrinos through the reaction

$$\nu_e + {}^{37}\mathrm{Cl} \rightarrow {}^{37}\mathrm{Ar} + e^- \qquad (15.23)$$

in the period 1970–85 was approximately one-quarter of that predicted by the SSM. More recent data, accumulated in the same detector in 1987–8, gave a value of about one-half of the SSM prediction. This result is supported by measurements made during the same period in the Kamiokande-II experiment,[11] which is sensitive mainly to ^{8}B solar neutrinos. If neutrinos have mass it is possible that oscillations may take place between neutrino species. The ^{37}Cl detector cannot detect muon-type neutrinos and the Kamiokande-II nucleon decay detector is relatively insensitive to low energy ν_μ so that $\nu_e \rightarrow \nu_\mu$ oscillations could account for the deficiencies in detected flux.

The necessary conditions for such neutrino oscillations to occur *in vacuo* are that at least one of the neutrino species should have non-zero mass and that the neutrino masses be not all degenerate. In addition, there must be a non-conservation of the separate lepton numbers so that the different neutrino types, as defined by the weak charged current, are mixtures of the mass eigenstates. The weak interaction eigenstates ν_e, ν_μ and ν_τ are related to the mass eigenstates ν_1, ν_2 and ν_3 by a unitary mixing matrix similar to the KM matrix describing quark mixing:

$$|\nu_\alpha\rangle = \sum_i U_{\alpha i}|\nu_i\rangle \qquad \alpha = e, \mu, \tau;\ i = 1, 2, 3.$$

In the restricted case of mixing between only two neutrino species the mixing matrix reduces to a 2×2 matrix with only one free parameter, the mixing angle between the neutrino species. For example, the mixing between ν_e and ν_μ is given by

$$\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}. \qquad (15.24)$$

This approach to the problem reveals the important features of neutrino oscillations, has the virtue of simplicity and is often the approach adopted by experimentalists.

If we assume neutrinos are stable and propagation takes place through free space, the mass eigenstates $|\nu_1\rangle$ and $|\nu_2\rangle$ develop in space–time like

$$|\nu_l(x, t)\rangle = |\nu_l(0, 0)\rangle \exp[i(p_l x - E_l t)] \qquad l = 1, 2. \qquad (15.25)$$

If these states are to be spatially coherent we must have $p_1 = p_2 = p$, say.

Then, for $m_l \ll p$, the energies are, to a good approximation,

$$E_l = (p^2 + m_l^2)^{1/2} \approx p + \frac{m_l^2}{2p} \tag{15.26}$$

and, with $c = 1$ and therefore $x = t$,

$$|\nu_l(t)\rangle = |\nu_l(0)\rangle \exp\left(-i\frac{m_l^2 t}{2p}\right). \tag{15.27}$$

Because of the different masses, $|\nu_1(t)\rangle$ and $|\nu_2(t)\rangle$ acquire different phase factors as a function of time. If initially at $x = 0$, $t = 0$, we have a pure ν_e state, as is the case in the interior of the Sun, then it is a simple matter to show that at time t the state is a mixture of ν_e and ν_μ such that

$$P(\nu_e \to \nu_e) = 1 - \sin^2(2\theta)\sin^2\left(\frac{\Delta m^2 t}{4p}\right) \tag{15.28}$$

and

$$P(\nu_e \to \nu_\mu) = \sin^2(2\theta)\sin^2\left(\frac{\Delta m^2 t}{4p}\right) \tag{15.29}$$

where $\Delta m^2 = m_2^2 - m_1^2$ is the difference in mass squared of the mass eigenstates. Equations (15.28) and (15.29) give, respectively, the probability at time t of finding ν_e or ν_μ in an initially pure ν_e state, and show that the intensities of the weak eigenstates oscillate with an amplitude that depends on the mixing angle and a periodicity that depends on the mass difference Δm^2. The characteristic oscillation length in vacuum is

$$\begin{aligned} L_\nu &= 4\pi p/(m_2^2 - m_1^2) \\ &= 2.48[p(\text{MeV}/c)/\Delta m^2((\text{eV}/c^2)^2)] \text{ metres.} \end{aligned}$$

Experimental tests of the neutrino oscillation hypothesis are essentially of two types. In the first class of experiments, 'disappearance' experiments, the flux of neutrinos of one species ν_l is measured at two distances x_1 and x_2 from the point of production. The ratio of the fluxes at the two positions is

$$R = \frac{P_{ll}(x_1/E)}{P_{ll}(x_2/E)}$$

where $P_{ll} \equiv P(\nu_l \to \nu_l)$. In 'appearance' experiments, neutrinos of a specific species, ν_l, travel a distance x to a detector designed to be sensitive to neutrinos of a different species, ν_k. The flux of appearing neutrinos ν_k, relative to the initial ν_l flux is given by $P_{lk}(x/E)$. Several such experiments have been performed at nuclear reactors and particle accelerators and to date no positive evidence for neutrino oscillations has been found. Experimental results are presented as allowed regions on a plot of Δm^2 versus $\sin^2(2\theta)$; for example, experiments at reactors give a limit $\Delta m^2 < 10^{-1}\,(\mathrm{eV}/c^2)^2$ provided $\sin^2(2\theta) > 0.1$.

It must be stressed that the above formalism holds for propagation in vacuum. When propagation through matter is considered account must be taken of the phase factors which arise from coherent forward scattering of neutrinos. In the standard model, in which the neutral current interaction is diagonal and symmetric with respect to neutrino species, neutral-current scattering gives rise to an overall phase shift which has no importance in the present context. Charged-current scattering, however, is not the same for all neutrino species and singles out electron-type neutrinos. As a result, resonant amplification of neutrino oscillations can take place and result in an increased probability that an electron-type neutrino, produced for example in the core of the Sun, arrives at the Earth as a muon-type neutrino. This possibility was first pointed out by Mikheyev and Smirnov[12,13] and is referred to as the Mikheyev–Smirnov–Wolfenstein (MSW) effect.[10,14] The effect is energy dependent and it is possible, for example, to obtain suppression of high energy neutrinos from ^{8}B decay and virtually no suppression of low energy p–p and p–e–p neutrinos. Other scenarios exist in which both low and high energy neutrinos are suppressed.

The GALLEX collaboration has recently measured the rate of ^{71}Ge production from ^{71}Ga by solar neutrinos via the inverse β decay process $^{71}\mathrm{Ga}(\nu_e, e^-)^{71}\mathrm{Ge}$ which has a threshold neutrino energy of 0.236 GeV. The target consists of 30.3 tons of gallium in the form of 8.13 molar aqueous gallium chloride solution (101 tons), shielded by about 3300 metres water equivalent of standard rock in the Gran Sasso Underground Laboratory in Italy. After the first year of operation they reported the first observation of solar p–p neutrinos and obtained an average production rate of ^{71}Ge atoms from solar neutrinos of $(81 \pm 17 \pm 9)$ SNU where the quoted errors are statistical and systematic respectively.* When combined with more recent results[15] the average production rate is $87 \pm 14 \pm 7$ SNU. This experiment is sensitive to solar neutrinos of all energies, particularly p–p and p–e–p neutrinos, and the result is consistent with the observation of the full p–p neutrino flux expected from the SSM together with a reduced flux of ^{8}B and ^{7}Be neutrinos as observed in the chlorine-37 and Kamiokande experiments. The results from all three experiments can be described in terms of the MSW effect with a consistent set of values of the mass and mixing angle parameters, summarized in figure 15.3. The

* 1 SNU = 10^{-36} captures per second per target atom.

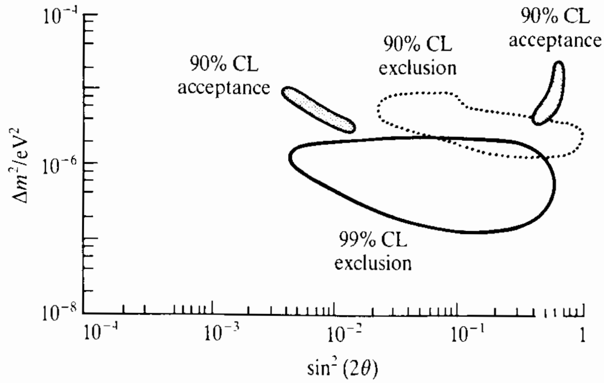

Figure 15.3 Limits on neutrino oscillation parameters. The difference in mass squared of the mass eigenstates is plotted against $\sin^2(2\theta)$, where θ is the mixing angle. In the shaded regions the MSW effect successfully reconciles the chlorine-37 Kamiokande and GALLEX experiments with standard solar models (at the 90 per cent confidence level (CL)). The area inside the dotted line is excluded at the 90 per cent confidence level by the Kamiokande collaboration as a result of a study of day–night effects (Hirata K *et al.* 1991 *Phys Rev Lett* **66** (9)). The area inside the full line is excluded at the 99 per cent confidence level by the GALLEX result.

90 per cent confidence level acceptance regions are shown shaded; at the 99 per cent confidence level the range $2 \times 10^{-7} < \Delta m^2 < 2 \times 10^{-6} (\text{eV}/c^2)^2$ and $6 \times 10^{-3} < \sin^2(2\theta) < 0.6$ is excluded.

At present (1993) the solar neutrino problem is not finally resolved. As we have seen, the GALLEX work, and similar results from a Soviet–American collaboration (SAGE), fall short of the predictions[16] of the SSM (132 ± 7 SNU) and suggest that a mechanism such as the MSW effect must operate. This requires neutrino properties not envisaged by the electroweak theory and not evident in other contexts. The possibility that the SSM does not fully describe conditions in the core of the Sun seems to be ruled out by the observations of Elsworth *et al.*[17] on acoustic waves in the body of the Sun; the frequencies found for the low-order modes which penetrate to the central regions accord well with SSM prediction. The whole question may be clarified when a large heavy-water detector at the Sudbury Neutrino Observatory (SNO) in Canada comes into operation since this will detect not only the flux of electron-type neutrinos but also the total flux of all flavours of neutrino with energy above 2.2 MeV that reach the Earth.

Neutrino oscillation experiments do not measure neutrino masses: they are sensitive to differences of mass squared. Limits on neutrino masses have been obtained in experiments which are sensitive to the kinematics of appropriate decay processes. For example, the detailed shape and endpoint energy of the electron energy spectrum in nuclear β decay is sensitive to the mass of the electron antineutrino, $\bar{\nu}_e$. Greatest sensitivity is achieved if the endpoint energy E_0 is small. The most suitable decay, and one which has been investigated by several groups,[18–23] is tritium decay,

$$^3\mathrm{H} \to {}^3\mathrm{He} + \mathrm{e}^- + \bar{\nu}_\mathrm{e}$$

which has an endpoint energy of 18.6 keV and a half-life of 12.3 years. The current best results[3] are consistent with $m_{\bar{\nu}_e} = 0$ with an upper limit of the order of 7 eV at the 90 per cent confidence level.

A comparable upper limit on the neutrino mass $m_{\bar{\nu}_e}$ was obtained from the observation, in February 1987, of the characteristics of the burst of neutrinos associated with the spectacular collapse of the blue giant star, Sanduleak-69 202, with $M \approx 20 \pm 5\, M_\odot$, which led to the brightest supernova, known as SN 1987a, since Tycho's supernova of 1604. It was a slice of good fortune that scattered around the world were the detectors dedicated to the observation of proton decay. In particular the IMB[24] and Kamiokande[25] experiments observed coincident bursts of neutrinos, lasting for about 10 s, which were well above the normal backgrounds in the detectors, and unambiguously associated with SN 1987a. The neutrino energy distributions were consistent with a thermal spectrum of temperature $T \approx 4$–5 MeV and the pulse length (≈ 10 s) was as expected in conventional models of stellar collapse in which the central core of the star reaches sufficiently high densities that the neutrinos are trapped and diffuse to the surface on this timescale. It is interesting to note that while the neutrinos are emitted essentially directly from the core, electromagnetic radiation has to diffuse out through the supernova atmosphere with the result that the first optical observation of the supernova was some hours after the observation of the neutrino pulse. Massless neutrinos travel with the speed of light so that a burst of such neutrinos would travel through space without dispersion. Massive neutrinos on the other hand would result in a pulse lengthening. The difference δt in time of flight for two neutrinos emitted from the supernova with different energies but the same mass is given by

$$\delta t \approx L\delta v \approx L\delta\gamma/\gamma^3 \approx L(m/E)^3\delta E/m$$

where L is the distance to the supernova, approximately 17×10^4 light years. The absence of any indication of neutrino pulse lengthening resulted in the conservative upper limit*

$$m_{\bar{\nu}_e} < 25\ \mathrm{eV}.$$

The limits on muon-type and tau-type neutrino masses are obtained in a similar fashion to the limits on the mass of electron-type neutrinos

* Since the cross-section $\sigma(\bar{\nu}_e p \to e^+ n)$ is larger than other neutrino cross-sections at low energies it is believed that most of the events observed were induced by electron-type antineutrinos.

from tritium decay. The limit on m_{ν_μ} has been obtained from a study[26] of the muon spectrum in the decay $\pi \to \mu + \nu_\mu$ and that on m_{ν_τ} from a study[27] of the pion spectrum in the decay $\tau \to 5\pi + \nu_\tau$. In summary the current limits on neutrino masses are[3]

$$m_{\nu_e} < 7.3\ \text{eV} \qquad m_{\nu_\mu} < 270\ \text{keV} \qquad m_{\nu_\tau} < 35\ \text{MeV}.$$

Finally, searches for neutrinoless double β decay, although extremely difficult, are important experiments which may help ascertain the exact nature of neutrinos. Double β decay is energetically allowed in only a few nuclei such as, for example,

$$^{76}_{32}\text{Ge} \to {}^{76}_{34}\text{Se} + 2e^-.$$

Such processes violate total lepton number conservation and are forbidden if neutrinos are Dirac particles. They can proceed if the neutrino and its antiparticle are identical (Majorana neutrinos) and have non-zero mass. Simplistically, one can imagine these decays proceeding via the emission and reabsorption of a neutrino,

$$n \to p + e^- + \bar{\nu}_e$$

$$n + \nu_e \to p + e^-$$

with the net result

$$n + n \to p + p + e^- + e^-.$$

These reactions have been written as though neutrinos are Dirac particles and the weak interaction has the familiar V–A structure. Evidently the reaction can proceed only if the neutrino and its antiparticle are identical, lepton number conservation is violated and the weak interactions have a right-handed component.

The observation of neutrinoless β decay, with the full decay energy carried by the electrons, would establish that the electron neutrino is a Majorana particle. To date, neutrinoless double β decay has not been observed. Recent experiments have placed lower limits on the half-lives of such decays, for example $t_{1/2} > 1.1 \times 10^{24}$ years for ^{76}Ge. A recent review of the field has been given by Caldwell.[28] Limits on neutrino mass from double β decay searches are dependent on how the nuclear matrix element is calculated and recent estimates give upper limits to a Majorana neutrino mass in the range 0.5–5 eV (for $t_{1/2} = 10^{24}$ years).

15.4 Grand unification and the big bang

The long-standing cosmological problem of the asymmetry which exists in the universe between matter and antimatter can be understood in terms of the hot big bang model of the origin of the universe and its evolution and the grand unification of the interactions of particle physics. The big bang hypothesis, firmly based on Hubble's discovery of the expansion of the universe, the discovery in 1965 by Penzias and Wilson of the 3 K cosmic background radiation and the abundances of light nuclei, is now the standard model of the origin of the universe.

If the universe were created in a state with the quantum numbers of the vacuum the number of fermions would equal the number of anti-fermions and naively one might expect this symmetry between matter and antimatter to persist as the universe expands and cools. Our very existence and the stability of the world around us is ample evidence of a local asymmetry between matter and antimatter. No plausible mechanism which might lead to a large-scale separation of matter and antimatter has yet been formulated. Although antiprotons have been observed in cosmic ray studies their flux relative to the proton flux is so small that their presence can be attributed to interactions of primary cosmic rays of the matter variety: there is no need to postulate the existence of a source of antimatter. If some distant galaxy indeed consisted of antimatter there would be intense gamma radiation arising from collisons with intergalactic matter. No such radiation has been observed. It is therefore generally believed that the matter–antimatter asymmetry is not just local but universal.

The average density of matter in the universe is estimated to be

$$\rho_m \approx 10^{-31}\ \mathrm{g\,cm^{-3}}. \tag{15.30}$$

This corresponds to a baryon number density

$$n_B \approx 10^{-7}\ \mathrm{cm^{-3}} \tag{15.31}$$

which is considerably less than the density of photons ($\approx 400\ \mathrm{cm^{-3}}$) in the 3 K background radiation. Thus, at the present time the number density ratio of baryons to photons is

$$\frac{n_B}{n_\gamma} \approx 10^{-9\pm 1}. \tag{15.32}$$

In a universe which expands isotropically and adiabatically this ratio should be independent of the time.

In the early universe ($t \lesssim 10^{-6}$ s) when the temperature was greater than the nucleon mass (1 GeV $\equiv 10^{13}$ K) radiation and matter would be in thermal equilibrium through the processes of baryon–antibaryon pair creation and annihilation and the number densities of baryons and photons would be comparable. When the universe expanded and cooled below the threshold for pair creation the annihilation process would continue and cause the baryon number density to fall dramatically, with the result that, at the present time, from an initially matter–antimatter symmetric universe, the ratio of baryons to photons would be

$$\frac{n_{\mathrm{B}}}{n_{\gamma}} \approx 10^{-20}$$

many orders of magnitude less than the observed ratio (equation (15.32)). It therefore appears that there must have been a matter–antimatter asymmetry at temperatures approximately greater than or equal to 1 GeV.

The presently observed ratio (15.32) implies a primordial quark–antiquark asymmetry

$$\frac{n_{\mathrm{B}}}{n_{\gamma}} \approx \frac{n_{\mathrm{q}} - n_{\bar{\mathrm{q}}}}{n_{\mathrm{q}} + n_{\bar{\mathrm{q}}}} \approx 10^{-9}.$$

In a pioneering paper in 1967 Sakharov[29] enunciated the general requirements for the generation of a baryon asymmetry. Clearly, there must be interactions which violate baryon number conservation in order to change an initial state with the quantum numbers of the vacuum ($B = 0$) into one in which $B \neq 0$. Additionally, these B-violating interactions must also violate charge conjugation invariance because if they were C conserving the C transformation, which interchanges quark and antiquark, would leave $n_{\mathrm{q}} = n_{\bar{\mathrm{q}}}$. The interactions must also violate CP because a parity transformation leaves n_{q} and $n_{\bar{\mathrm{q}}}$ unchanged so that a combined CP operation, if exact, would leave $n_{\mathrm{q}} = n_{\bar{\mathrm{q}}}$. The final requirement is that there be a departure from thermal equilibrium at some early stage in the evolution of the universe. The reason for this is that in an equilibrium state we lose sense of the direction of time (the interactions are time-reversal invariant) and therefore, through the CPT theorem, the interactions are CP invariant and leave $n_{\mathrm{q}} = n_{\bar{\mathrm{q}}}$.

GUTs satisfy these requirements although the SU(5) model, outlined in section 15.2, cannot produce an asymmetry large enough to generate the ratio (15.32). Larger groups with more free parameters and hence less predictive power are required. Nevertheless, let us speculatively extrapolate back in time towards the initial singularity to an epoch at $t \lesssim 10^{-36}$ s when $kT \approx 10^{16}$ GeV, i.e. beyond the grand unification mass M_{X}, when matter consisted mainly of quarks and leptons in thermal equilibrium with the superheavy bosons, the vector bosons X or the Higgs boson $\mathrm{H_X}$. In this equilibrium period the abundances of X and $\bar{\mathrm{X}}$, q and $\bar{\mathrm{q}}$, etc. are equal. The dominant decays in simple models such as SU(5) are

$$\left.\begin{matrix}\mathrm{H_X} \\ \mathrm{X}\end{matrix}\right\} \rightarrow \bar{\mathrm{q}} + \bar{\mathrm{l}} \text{ and } \mathrm{q} + \mathrm{q}.$$

Provided that C and CP are violated and provided also that the processes are slow enough relative to the expansion rate of the universe at that time to allow non-equilibrium effects to build up, the universe, as it cools through the transition temperature, can acquire an excess of baryons over antibaryons. The CPT theorem guarantees that the decay rates of the heavy bosons and their antiparticles are identical:

$$\Gamma_{\text{tot}}(\text{X}) = \Gamma(\text{X} \to \bar{\text{q}}\bar{\text{l}}) + \Gamma(\text{X} \to \text{qq}) = \Gamma_{\text{tot}}(\bar{\text{X}}) = \Gamma(\bar{\text{X}} \to \text{ql}) + \Gamma(\bar{\text{X}} \to \bar{\text{q}}\bar{\text{q}}).$$

When the universe expanded and cooled below the grand unification mass, i.e. when the reverse reactions maintaining the abundances of X and $\bar{\text{X}}$ were not sustainable, a baryon-antibaryon asymmetry could occur, leading to a 'freeze-out' of baryon number. This would happen if, through C and CP violation, the partial decay rates of X and $\bar{\text{X}}$ are not equal, i.e. if, for example,

$$\text{B} \equiv \frac{\Gamma(\text{X} \to \text{qq})}{\Gamma_{\text{tot}}(\text{X})} \neq \bar{\text{B}} \equiv \frac{\Gamma(\bar{\text{X}} \to \bar{\text{q}}\bar{\text{q}})}{\Gamma_{\text{tot}}(\bar{\text{X}})}.$$

Detailed calculations show that several grand unification schemes lead to an asymmetry of the order of 10^{-9} and hence to the present ratio of n_{B}/n_{γ}. After this phase transition the superheavy bosons no longer participated in the evolution of the universe. A similar fate was to befall the W and Z bosons when the universe expanded and cooled through the 'Weinberg–Salam' transition at $kT \approx 100$ GeV.

The next important event in the evolution of the universe was the 'freeze-out' of electron neutrinos, an event of crucial importance since it determines the neutron–proton ratio which in turn determines the abundance of light elements, particularly ^{4}He, in the universe. At time $t \approx 10^{-2}$ s after the big bang, the matter in the universe consisted of neutrons, protons, electrons, positrons, electron-type neutrinos and antineutrinos. At this epoch the density of matter in the universe was sufficiently high that even the neutrinos were trapped within the characteristic size of the universe. In such a cosmic fluid the light particles would predominate: there would be about one proton or neutron for every 10^9 photons, electrons or neutrinos. The fluid would be driven to thermal equilibrium by the weak interactions

$$\begin{aligned} \nu_e + \text{n} &\rightleftarrows \text{e}^- + \text{p} \\ \bar{\nu}_e + \text{p} &\rightleftarrows \text{e}^+ + \text{n} \end{aligned} \tag{15.33}$$

which would occur so rapidly that there would be roughly equal numbers of neutrons and protons. At equilibrium the neutron–proton ratio is given by the Boltzmann law

$$\frac{n}{p} = \exp\left(-\frac{\Delta mc^2}{kT}\right)$$

where Δm is the neutron–proton mass difference and $T \approx 10^{11}$ K at this epoch. As the universe expanded and cooled further the n/p ratio steadily dropped until, at $t \approx 1$ s, the rate of the reactions (15.33) became small compared with the expansion rate at this time and the equilibrium could not be maintained: the neutrinos decoupled from the nucleons, electron–positron annihilation began to dominate and the neutron–proton ratio was frozen at about 15 per cent. Some 13 s later the temperature of the universe had dropped sufficiently that e^+e^- pairs could no longer be created and light nuclei, deuterium and helium began to form. When the photon energy dropped further still, so that photodisintegration of the newly formed nuclei could no longer occur, the abundances of ^{4}He became frozen at ^{4}He/p $\approx$ 25 per cent. At this stage, $t \approx 35$ minutes, the temperature was still too high for neutral atoms to be formed: this only happened some 10^5 years later. With electrons locked in neutral atoms the universe became transparent to the electromagnetic radiation which continued to cool with the expansion of the universe to the present temperature (2.7 K) of the background radiation.

Detailed calculations, although fraught with difficulty, yield a value for the primordial ^{4}He abundance of 24 ± 2 per cent in good agreement with the observed abundance. This primordial ^{4}He abundance depends sensitively on the rate of cooling of the universe just prior to the epoch of nucleosynthesis and this in turn depends on the number of fundamental fermion species produced earlier, with each additional neutrino species contributing about 1 per cent to the helium mass fraction. The observed helium abundance constrains the number of light neutrino species to be less than four. This observation was strikingly confirmed by recent measurements of the width of the Z^0 at LEP which limit the number of light neutrino species to three.

The important events in the evolution of the universe since the big bang are summarized in figure 15.4.

15.5 Towards a theory of everything?

Although it seems inevitable that the strong, electromagnetic and weak interactions are unified at some extremely high energy, there are some features of GUTs which are unsatisfactory. Perhaps the most unsatisfactory feature is that gravity is not included in the unification schemes. The main difficulty in developing a theory which unifies gravity with the

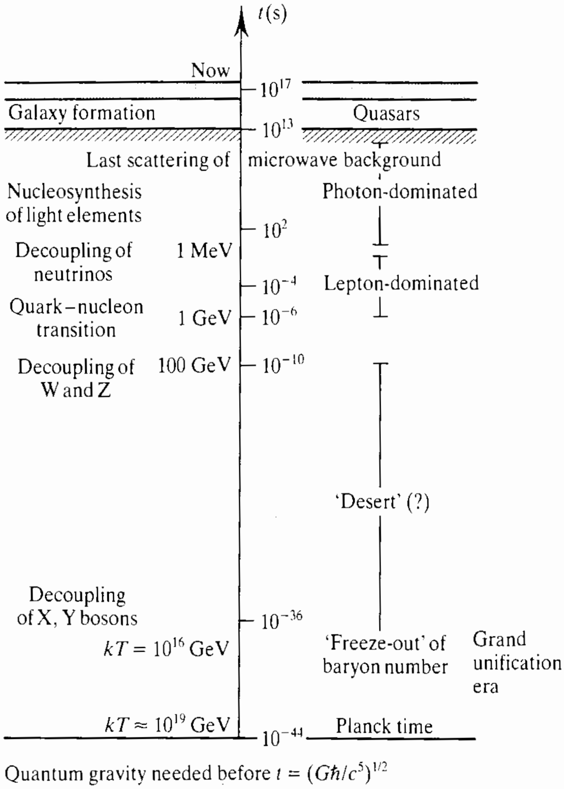

Figure 15.4
The main stages in the evolution of the universe from the Planck time ($\approx 10^{-44}$ s) to the present.

other forces is that general relativity, although a gauge theory, does not incorporate quantum effects, while the standard model of elementary particle physics depends on quantum mechanics in an essential way.

Another problem, which at first sight is unrelated to attempts to find a 'theory of everything' (TOE), is the so-called hierarchy problem which concerns the enormous difference between the unification scales of the electromagnetic and weak interactions ($M_W \approx 10^2$ GeV) and the grand unification scale ($M_X \approx 10^{15}$ GeV). Presumably, a Higgs mechanism analogous to that associated with electroweak symmetry breaking is also responsible for the GUT symmetry breaking. It is expected that these Higgs bosons, H_W and H_X, have masses of the order of M_W and M_X respectively. It is conceivable that the parameters of the scalar potential could be 'fine-tuned' to give this hierarchy but not in a 'natural' way. One of the problems encountered in this approach concerns the mass of the light Higgs boson. There will be contributions to the Higgs mass from radiative corrections involving gauge boson, scalar boson and fermion loops. To obtain a well-defined Higgs boson mass, $M_H \approx M_W$, requires fine tuning of the parameters of the theory so that cancellations at a precision of $(M_W/M_X)^2 \approx 10^{-26}$ take place in each order of perturbation theory. This 'naturalness' problem can be solved by introducing a

Table 15.2
Spectrum of SUSY particles

Particle	Spin	Sparticle	Spin
Quark q	$\frac{1}{2}$	Squark $\tilde{q}$	0
Lepton l	$\frac{1}{2}$	Slepton $\tilde{l}$	0
Neutrino ν	$\frac{1}{2}$	Sneutrino $\tilde{\nu}$	0
Photon γ	1	Photino $\tilde{\gamma}$	$\frac{1}{2}$
Gluon g	1	Gluino $\tilde{g}$	$\frac{1}{2}$
W boson	1	Wino $\tilde{W}$	$\frac{1}{2}$
Z boson	1	Zino $\tilde{Z}$	$\frac{1}{2}$
Higgs boson H	0	Shiggs $\tilde{H}$	$\frac{1}{2}$

symmetry between bosons and fermions known as supersymmetry or SUSY. Each point-like particle is postulated to have a SUSY partner, known as a 'sparticle', with a spin which differs from that of the particle by half a unit. If the fermion–boson pairs have identical couplings their contributions to the radiative corrections have opposite sign and exactly cancel. Divergences in the mass renormalization problem are controlled, or, to put it another way, the naturalness (fine-tuning) problem is solved, provided that the masses of the fermion–boson pairs satisfy the condition $|m_B^2 - m_F^2| \lesssim 1\ \text{TeV}^2$.

If the above ideas are correct there should be a doubling of the spectrum of known particles. These, together with their supersymmetric partners, are shown in table 15.2. With the exception of spin, the supersymmetric particles have the same quantum numbers as their normal partners. For example, like the gluon, the gluino is a colour octet, flavour singlet with $C = -1$. The selectron, like the electron, carries a conserved electron number. In supersymmetric theories particles are assigned a new *multiplicative* quantum number known as R-parity. All ordinary particles have an R-parity of $+1$ while the supersymmetric partners have $R = -1$. Formally, the R-parity of any particle (sparticle) with spin j, baryon number B and lepton number L is defined as

$$R = (-1)^{2j+3B+L}. \tag{15.34}$$

R-parity is not necessarily conserved but is *imposed* as a discrete symmetry with the consequence that SUSY particles are always produced in pairs. Furthermore, the lightest supersymmetric particle must be stable because, through R-conservation, it cannot decay into ordinary particles. It is generally assumed that the photino ($\tilde{\gamma}$) is the lightest SUSY particle. If, on the other hand, the scalar neutrino (or sneutrino) were the lightest SUSY particle the photino would decay, $\tilde{\gamma} \to \nu\tilde{\nu}$.

Several experimental searches for supersymmetric particles have been made in the last decade or so but to date there is no evidence for such particles. The charged scalar leptons, for example, interact electromagnetically and should be produced in pairs in e^+e^- annihilations.

Recent searches for sleptons produced in the reactions

$$
\begin{aligned}
e^+e^- \rightarrow Z^0 \rightarrow \tilde{l}^+\tilde{l}^- & \\
& \hookrightarrow l^-\tilde{\gamma} \\
& \hookrightarrow l^+\tilde{\gamma}
\end{aligned}
\tag{15.35}
$$

have been performed at LEP. (Note that the photino is a neutral Majorana fermion so that $\bar{\tilde{\gamma}} \equiv \tilde{\gamma}$.) If photinos are light the sleptons will decay very rapidly, with approximately 100 per cent branching ratio, into an ordinary lepton and a photino. Since they interact extremely weakly the photinos will escape detection and give a very characteristic signature for the processes (15.35): the sleptons could have masses of about 1 TeV so the final state will contain an unlike-sign lepton pair with a large momentum imbalance. Searches of this kind have placed lower limits on the masses of sleptons of the order of 40 GeV. Current lower limits on the masses of other SUSY particles are $m_{\tilde{\gamma}} > 5$ GeV, $m_{\tilde{q}} > 100$ GeV and $m_{\tilde{g}} > 100$ GeV. If supersymmetry is indeed a symmetry of nature, and provides a solution to the naturalness problem, SUSY particles should have masses less than 1 TeV. This hypothesis could be tested definitively at the proposed high-luminosity supercolliders, LHC and SSC, in which beams of protons will collide with total centre-of-mass energies of 16 and 40 TeV respectively.

Supersymmetry impinges on the question of the stability of the proton: supersymmetric models predict the predominance of $K^0\mu^+$, $K^+\bar{\nu}_\mu$ and $K^+\bar{\nu}_\tau$ final states in contrast to the predictions of SU(5) (see table 15.1), which favours the decay $p \rightarrow \pi^0 e^+$ and in which second generation fermions are suppressed. A possible identification of decay modes involving kaons would signal the presence of supersymmetry and explain a longer proton lifetime than that predicted by SU(5).

A compelling and attractive feature of supersymmetry is that locally supersymmetric theories relate the generators of supersymmetry to the generators of space–time transformations so that there is an inevitable connection with general relativity which may lead to the ultimate unification of gravity with the strong and electroweak forces.

In order to appreciate this connection let us recall (chapter 8) that the momentum operators are the generators of translations in space. Correspondingly, the four-momentum operators p^μ generate space–time translations. The generators of rotations in space are the angular momentum operators. Lorentz transformations may be regarded as rotations in space–time and the rotation group is in fact a subgroup of the Lorentz group of transformations. The laws of physics are invariant under this group of space–time transformations, collectively known as the Poincaré group. In addition to these space–time symmetries we have met various internal symmetries: the generators T_a of a non-Abelian internal symmetry form a Lie algebra

$$[T_a, T_b] = \mathrm{i} f_{abc} T_c$$

where the f_{abc} are the structure constants of the group. The generators T_a commute with the Hamiltonian and therefore with the generators, p^μ and $M^{\mu\nu}$ of the Poincaré group:

$$[T_a, H] = [T_a, p^\mu] = [T_a, M^{\mu\nu}] = 0.$$

A supersymmetric transformation, connecting fermion fields ψ and boson fields φ, and changing the total angular momentum by half a unit, is effected by a spin $\frac{1}{2}$ Majorana generator Q_α:

$$Q_\alpha \psi = \varphi$$

where $\alpha = 1, 2, 3, 4$ is a spinor index. With the introduction of supersymmetry the algebra is modified and now includes anticommutators as well as commutators:

$$[Q_\alpha, p^\mu] = 0 \tag{15.36}$$

$$[Q_\alpha, M^{\mu\nu}] = \tfrac{1}{2}(\sigma^{\mu\nu} Q)_\alpha \tag{15.37}$$

$$\{Q_\alpha, \bar{Q}_\beta\} = -2(\gamma_\mu)_{\alpha\beta} p^\mu \tag{15.38}$$

where γ^μ are the Dirac matrices, $\sigma^{\mu\nu} = (\mathrm{i}/2)[\gamma^\mu, \gamma^\nu]$ and $\bar{Q}_\beta = Q_\beta^{\mathrm{T}} \gamma^0$, where the superscript T signifies the transpose. Equation (15.37) expresses the fact that Q_α transforms as a spinor, while equation (15.36) shows that the spinor charges are conserved. The anticommutation relation (15.38) shows that *two successive supersymmetry transformations generate a translation in space–time*, and herein lies the hope of achieving the ultimate unification of all the known forces in nature.

In special relativity the line element

$$(\delta s)^2 = \sum_{\mu,\nu=0}^{3} g_{\mu\nu} \delta x^\mu \delta x^\nu \tag{15.39}$$

is invariant, i.e. is the same in all inertial frames of reference. In (15.39) $g_{\mu\nu}$ is the Minkowski metric whose only non-vanishing components are

$$g_{00} = 1 \qquad g_{11} = g_{22} = g_{33} = -1.$$

The central postulate of general relativity is that the gravitational field, arising from the presence of matter, can be described by replacing the Minkowski 'flat space' metric by a more general metric which depends on the space–time coordinates

$$(\delta s)^2 = \sum_{\mu,\nu=0}^{3} g_{\mu\nu}(x)\delta x^\mu \delta x^\nu. \qquad (15.40)$$

The curvature of the metric is determined by solving the Einstein field equations, a set of ten non-linear, second-order, hyperbolic partial differential equations, for the ten components ($g_{\mu\nu}(x) = g_{\nu\mu}(x)$) of the metric tensor. The gravitational force can be viewed as arising from deviations, or fluctuations, of the curved space from the flat Minkowski space

$$g_{\mu\nu}(x) = g_{\mu\nu} + h_{\mu\nu}(x) \qquad (15.41)$$

where the $h_{\mu\nu}(x)$ measure the size of the fluctuations. For sufficiently small fluctuations the Einstein equations become a set of ten *linear* equations in $h_{\mu\nu}(x)$.

From the particle physics viewpoint, forces are conveyed via the exchange of field quanta. Particles with half-integer spin (fermions) cannot give rise to static forces between interacting particles, so that the field quanta must be bosons. Furthermore, detailed arguments show that static forces can arise only if the boson spin is less than or equal to 2. The exchange of a spin 1 boson (the photon for example) gives rise to a repulsive force between identical particles so that the quantum of the gravitational field, the *graviton*, must have spin 0 or spin 2. Only a spin 2 graviton has enough degrees of freedom to correspond to the ten fields $h_{\mu\nu}(x)$.

If supersymmetry is made locally gauge invariant new fields are introduced whose quanta are spin $\frac{3}{2}$ *gravitinos*, the supersymmetric partners of the graviton, and *gauginos*, the spin $\frac{1}{2}$ partners of the vector gauge bosons. A local supersymmetry is called supergravity. There are in fact eight supergravity theories corresponding to the supersymmetry generators Q_α^N ($N = 1, 2, \ldots, 8$), where N gives the number of gravitinos in the theory. Note that, because gravitinos are fermions, they will not themselves give rise to static forces so that the predictions of general relativity are protected in supergravity theories. Unfortunately, these attempts to unify the fundamental forces of nature seem doomed to failure because they are plagued with the usual infinities arising from radiative corrections: it appears that the theories are not renormalizable.

The notion that space–time is a continuum of space–time 'points' would seem to be singularly inappropriate in a quantum theory that purports to unify gravitation with the other fundamental forces: localization of a particle at a 'point' would require an infinite amount of energy. It has been conjectured that the difficulties which arise in attempts to create renormalizable theories of supergravity are unavoidable when this basic inconsistency is ignored. Quantum gravity, with its natural scale given by the Planck length $L_P = (G\hbar/c^3)^{1/2} \approx 10^{-35}$ m, where G is the gravitational constant, raises the possibility that the notion of a space–time continuum may not be valid at distances less than L_P and that a different model of space–time may be needed. In *supersymmetric string theories* the structureless 'point-like' particles of conventional quantum field theories are replaced by one-dimensional string-like objects. These theories produce finite results but for internal consistency require the space–time in which these distributed objects move to be ten-dimensional! The six extra spatial dimensions are visualized as being compactified, or 'rolled up' into closed loops on a scale of 10^{-35} m or less. These ideas are currently causing considerable excitement but much work remains to be done before the physicists' dream of a theory of everything becomes a reality.

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