Surface Resistivity and Surface Resistance Measurements
Using a Concentric Ring Probe Technique

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Abstract The relationship between surface resistivity and surface resistance is established and explained.

1 Introduction

Concepts of surface resistance and surface resistivity can be sometimes confusing. Definitions of both terms can be found in many books and standards [1–4]. Surface resistance, R_s, is defined in all of the aforementioned literature sources as the ratio of a DC voltage U to the current, I_s flowing between two electrodes of specified configuration that are in contact with the same side of a material under test (Figure 1).

$$R_s = \frac{U}{I_s} \qquad (1)$$

Surface resistivity ρ_s, on the other hand, is determined by the ratio of DC voltage U drop per unit length L to the surface current I_s per unit width D.

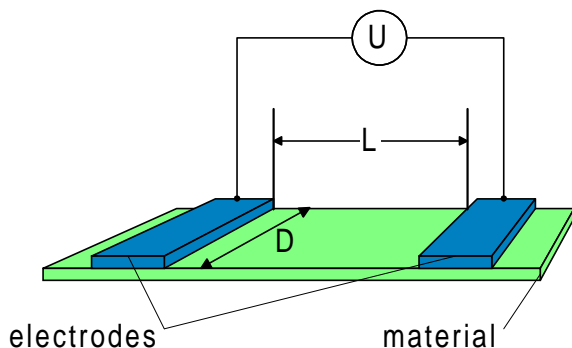$$\rho_s = \frac{\frac{U}{L}}{\frac{I_s}{D}} \qquad (2)$$

Figure 1: Basic setup for surface resistance and surface resistivity measurement.

Surface resistivity is a property of a material. Theoretically it should remain constant regardless of the method and configuration of the electrodes used for the surface resistivity measurement. A result of the surface resistance measurement depends on both the material and the geometry of the electrodes used in the measurement. The physical unit of surface resistivity is Ohm (Ω). The legitimate unit of the surface resistance is also Ohm. Because of that surface resistivity and the surface resistance are often mixed up. In order to differentiate between the two, surface resistivity is often expressed also in Ohm/square (Ω/sq.) which is <u>not</u> a valid unit from the dimensional analysis point of view.

2 Surface resistivity and surface resistance

2.1 Current density and surface current density

It is possible to establish a relationship between the surface resistance and surface resistivity for any electrode configuration. An idea of the current density is very helpful in understanding of that relationship. Consider two samples of a material as shown in Figure 2. With a constant voltage U and both samples made of the same material the amount of current flowing through the material will be different. The thicker bar (sample #1) conducts "more easily" than the thin bar (sample #2). One may use a water pipe analogy - given a constant water pressure, there will be more water per unit time coming through the pipe with a larger diameter. The flow density, be it water or electric current, is the amount of flow passing through a unit area of the pipe or the sample of the material. The surface area is perpendicular to the direction of the flowing current (or water).

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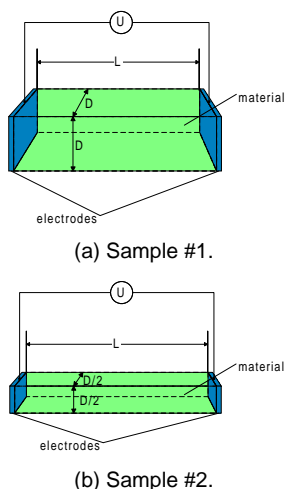## Using a Concentric Ring Probe Technique

(a) Sample #1.

(b) Sample #2.

Figure 2: Current density.

When the voltage **U** is kept constant, the current density for the thin and the thick bar is the same. The electric current density is often expressed by:

$$J = \frac{I}{S}$$

where **I** is the current and **S** is the surface area, and is measured in [A/m^2]. Surface current density is the next concept helpful in understanding the relationship between the surface resistance and surface resistivity. Consider Figure 1, where both electrodes are on the same side of the material. It is assumed that electric current flows on the surface of the material only. In reality, this is not exactly true. There is always a portion of that current flowing through the bulk of the material. However, in order to make it possible to compare surface properties of various materials, it had been presumed that the surface current flows through infinitesimally thin surface layer. This layer is so thin, that the thickness of it can be neglected. Surface current density **J$_s$** is therefore defined as:

$$J_s = \frac{I}{D},$$

where D is a width of the electrode.

2.2 Concentric ring electrodes configuration

The relationship between surface resistivity and the surface resistance for a concentric ring probe geometry can be found by defining a surface current density in the area between rings. Knowing the surface current density, it is possible to find an electric field intensity between the electrode rings (Figure 3).
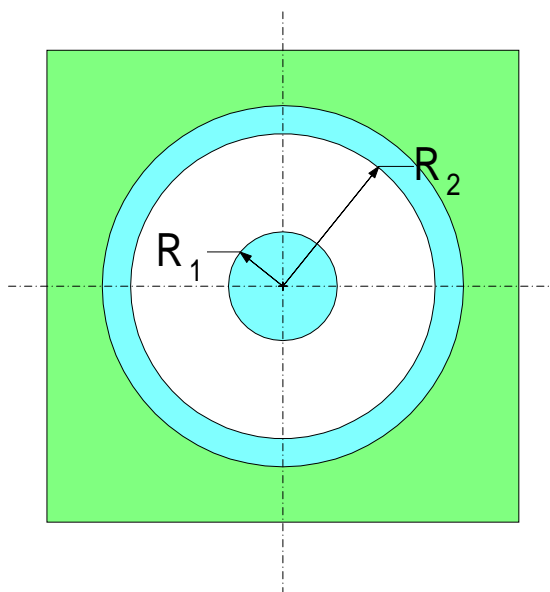

Figure 3: Surface resistance and surface resistivity measurement configuration for concentric ring electrodes.

Define:

- **R$_1$** outer radius of the center electrode,

- **R$_2$** inner radius of the outer ring electrode,

as it is shown in Figure 3 (see also Figure 5). The surface current density **J$_s$** for a concentric rings configuration is determined as:

$$J_s = \frac{I_s}{2\pi \cdot r}, \tag{3}$$

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where the radius r varies from R_1 to R_2. It is important to remember that when testing the surface resistivity (or resistance) of any material, it is assumed that all the currents flow between electrodes along the surface and do not penetrate into the bulk of the material. In order to ensure that the surface currents are measured properly, some more advanced techniques for surface resistivity measurements have been developed [1, 2, 4]. The Ohm's law describes relationship between a current density J and an electric field intensity E. It is also valid for the surface currents:

$$J_s = \frac{E}{\rho_s}, \qquad (4)$$

Therefore, it is possible to find electric field between the concentric rings by solving the following dependency (using equation 3 and 4):

$$E = \frac{\rho_s I_s}{2\pi \cdot r}, \qquad (5)$$

The voltage between electrodes can be found by integrating the electric field E from R_1 to R_2:

$$
\begin{aligned}
U_{R_1,R_2} &= \int_{R_1}^{R_2} E \, dr = \\
&= \int_{R_1}^{R_2} \frac{\rho_s I_s}{2\pi \cdot r} \, dr \\
&= \frac{\rho_s I_s}{2\pi} \int_{R_1}^{R_2} \frac{1}{r} \, dr \\
&= \frac{\rho_s I_s}{2\pi} \ln\left(\frac{R_2}{R_1}\right) \qquad (6)
\end{aligned}
$$

Substituting $R_s = \frac{U}{I_s}$:

$$R_s = \frac{\rho_s}{2\pi} \ln\left(\frac{R_2}{R_1}\right) \qquad (7)$$

After rearrangements, the surface resistivity is related to the surface resistance by a constant that depends on the geometry of the electrodes only:

$$\rho_s = R_s \frac{2\pi}{\ln\left(\frac{R_2}{R_1}\right)} = R_s \cdot k \qquad (8)$$

Where k is frequently called a geometry coefficient.

Figure 4: Surface resistivity measurement setup.

Figure 5: Concentric ring probe.

Figure 4 presents a typical surface resistivity measurement setup using a resistivity meter and a concentric ring probe (Figure 5). The resistivity meter is capable of measuring surface resistivities directly, utilizing various configuration of

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electrodes. Meter provides a constant voltage *U* and measures the current *I* flowing between electrodes. Resistance R_s is then easily calculated and the value of resistivity is equal to the value of resistance multiplied by the geometry coefficient. Usually electrodes are especially constructed to simplify calculations of surface resistivity and the geometry coefficient is equal to a simple integer. Most of industrial standards use this simplified approach [1, 2, 4].

Example 1: Consider the following measurement configuration:

R_1 = 15.3 [mm],

R_2 = 28.6 [mm],

U = 10 [V].

The current measured during the test was equal to I_s = 1·10⁻⁶ [A]. The surface resistivity of the material under test can be calculated from the equation 8:

$$
\begin{aligned}
\rho_s &= R_s \frac{2\pi}{\ln\left(\frac{R_2}{R_1}\right)} = \\
&= \frac{U}{I_s} \frac{2\pi}{\ln\left(\frac{R_2}{R_1}\right)} = \\
&= \frac{10\,[V]}{10^{-6}\,[A]} \frac{2\pi}{\ln\left(\frac{28.6\,[mm]}{15.3\,[mm]}\right)} = \\
&= 10^7\,[\Omega] \cdot \frac{2\pi}{\ln(1.869)} = \\
&= 10^7\,[\Omega] \cdot 10 = \\
&= 10^8\,[\Omega]
\end{aligned}
$$

3　Additional remarks

While conducting the surface resistivity and the surface resistance tests, it is important to consider some additional components affecting the test results. The electric resistivity of any dielectric material depends on many environmental factors. It can change with humidity, temperature, etc. For this reason it is recommended to condition the test sample before the measurement. Another important aspect is to ensure a proper contact between electrodes and the tested material. Electrode systems can be made of various materials and may come in various shapes and configurations. The way of electrode contacts the material under test has a very significant influence on the result of a measurement. The DC voltage level used for testing is also an important issue. Usually resistivity of the material depends on the value of the applied voltage and the time span during which the sample was energized. All these contributing factors and criteria are described in appropriate guides and standards [1–4].

References

[1] ASTM Standard D 257-99. Standard test methods for D-C resistance or conductance of insulating materials, 1999.

[2] ESD STM 11.11-2001 Standard. Surface resistance measurement of static dissipative planar materials, 2001.

[3] Michael B. Heaney. *The Measurement, Instrumentation and Sensors Handbook*, chapter Electrical Conductivity and Resistivity. CRC Press, 1999.

[4] IEC 61340-5-1 Standard. Electrostatics - part 5-1: Protection of electronic devices from electrostatic phenomena - general requirements, 1998.