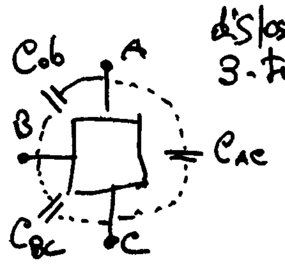
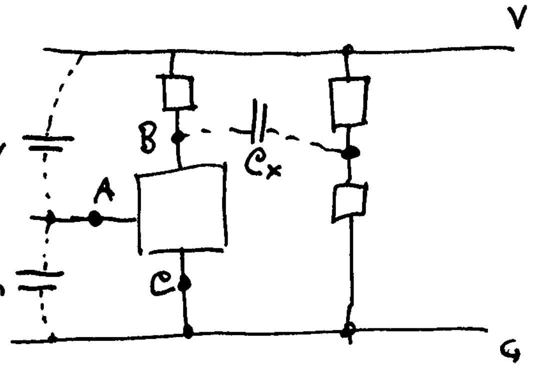
Analisi in frequenza di amplificatori:

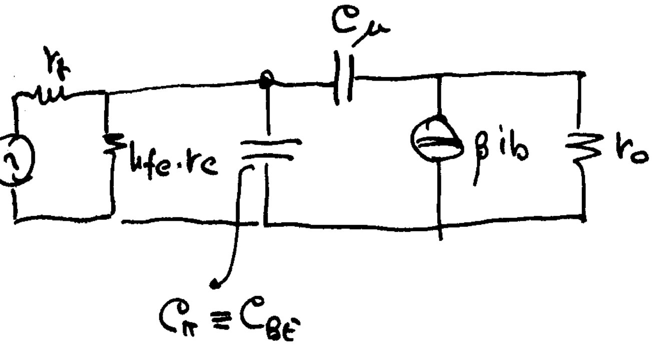
- Dipendenza dalla frequenza generata da elementi reattivi, capacità ed induttanze. Effetto di capacità parassite o capacità di isolamento delle polarizzazioni.

Studio con Plot di Bode.

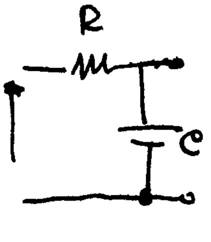




Es: circuito a transistor di base (emettitore comune)



tipici accoppiamenti:



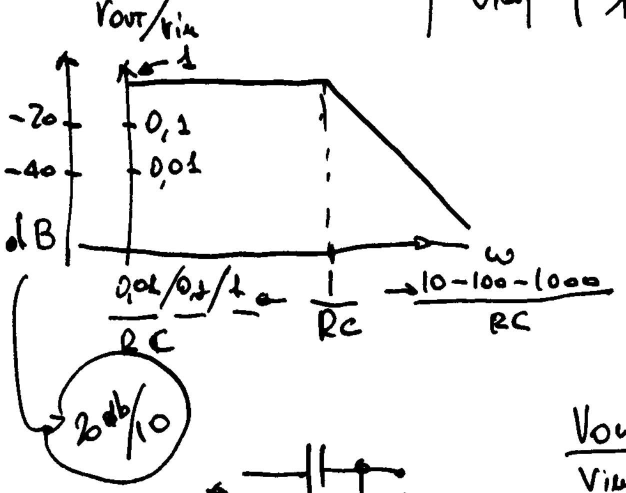
$$\frac{V_{out}}{V_{in}} = \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} = \frac{1}{1 + j\omega RC}$$

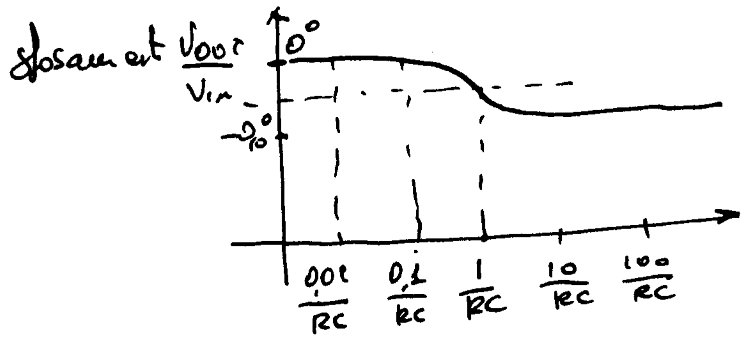
|| - limite di bassa frequenza $\omega \ll \frac{1}{RC}$ $\Rightarrow \frac{V_{out}}{V_{in}} = 1$ sfasamento $= 0$

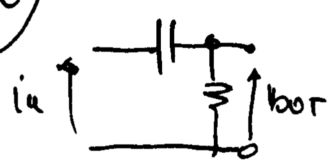
|| - limite alta frequenza $\omega \gg \frac{1}{RC}$ $\Rightarrow \frac{V_{out}}{V_{in}} = \frac{1}{j\omega RC}$ sfasamento $= 90°$

Condizione al contorno: $\omega = \frac{1}{RC}$

$$\left|\frac{V_{out}}{V_{in}}\right| = \left|\frac{1}{1+j}\right| = \frac{1}{\sqrt{2}} = 0{,}707$$ sfasamento $-45°$





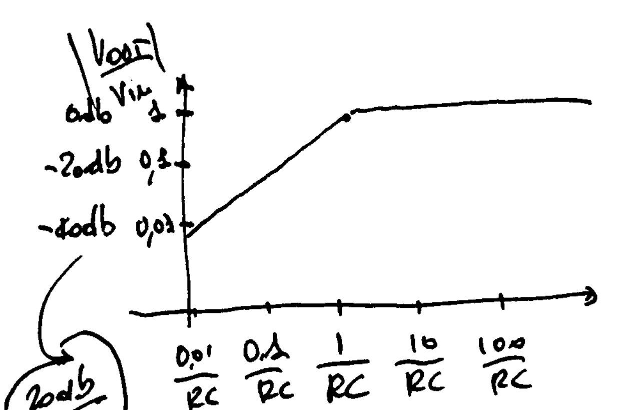


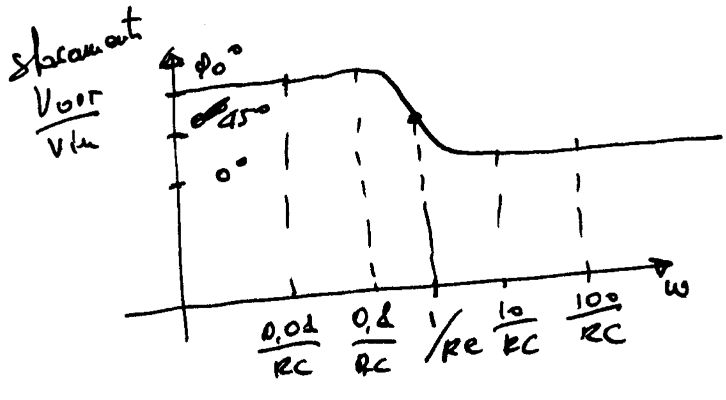
$$\frac{V_{out}}{V_{in}} = \frac{j\omega RC}{1 + j\omega RC}$$

$\omega \ll \frac{1}{RC}$ $\quad \left|\frac{V_{out}}{V_{in}}\right| \to \left|\frac{j\omega RC}{1}\right| = \omega RC$ $\quad$ sfasamento $= 90°$

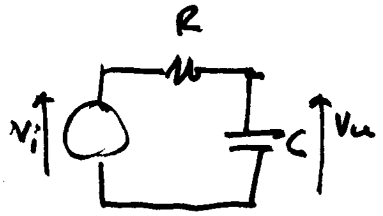
$\omega \gg \frac{1}{RC}$ $\quad \left|\frac{V_{out}}{V_{in}}\right| \to \left|\frac{j\omega RC}{j\omega RC}\right| = 1$ $\quad$ sfasamento $= 0°$

$\omega = \frac{1}{RC}$ $\quad \left|\frac{V_{out}}{V_{in}}\right| \to \left|\frac{1}{1+j}\right| = \frac{1}{\sqrt{2}}$ $\quad$ sfasamento $= 45°$





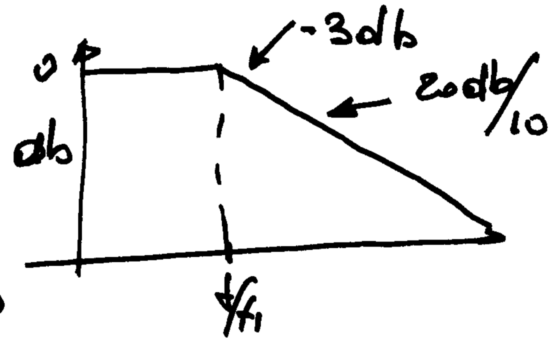
Riprendiamo dai filtri RC e CR.



$$\left|\frac{V_u}{V_i}\right| \text{ in db} = 20\log\left[\frac{1}{1+(f/f_1)^2}\right]^{1/2} =$$

$$= 10\log\frac{1}{1+(f/f_1)^2} = -10\log\left[1+\left(\frac{f}{f_1}\right)^2\right]$$

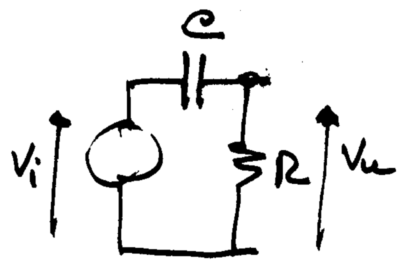
1) Se f/f_1 piccolo $\left|\frac{V_u}{V_i}\right| = 10\log 1 = \underline{\underline{0}}$



2) Se $f/f_1 = 1$ $= 10\log\frac{1}{2} = -10\log 2 = \underline{\underline{-3db}}$

3) Se $f/f_1 \gg 1$ $10\log\frac{1}{(\frac{f}{f_1})^2} = -20\log\frac{f}{f_2}$

↳ Polo:
- diagramma ad un polo
- diagramma di Bode



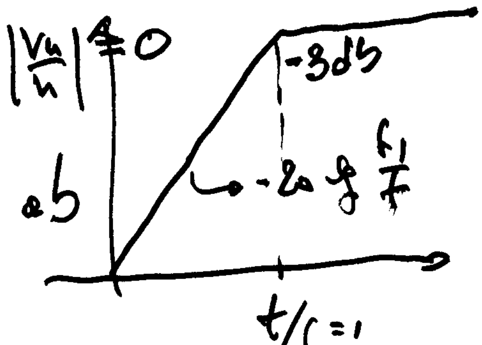
$$\left|\frac{V_u}{V_i}\right| = \frac{1}{\sqrt{1+\left(\frac{f_1}{f}\right)^2}} \qquad \left|\frac{V_u}{V_i}\right|_{db} = 20\log\left[\frac{1}{1+\left(\frac{f_1}{f}\right)^2}\right]^{1/2}$$

$$= -10\log\left[1+\left(\frac{f_1}{f}\right)^2\right]$$

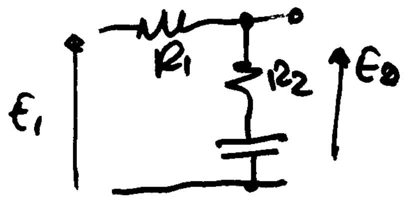
1) se $f \ll f_1$ $= -20\log\frac{f_1}{f}$

2) se $f = f_1$ $= -10\log 2 = 0-3db$

3) se $f \gg f_1$ $= -10\log 1 = 0$



Esempi di Bode Plots.



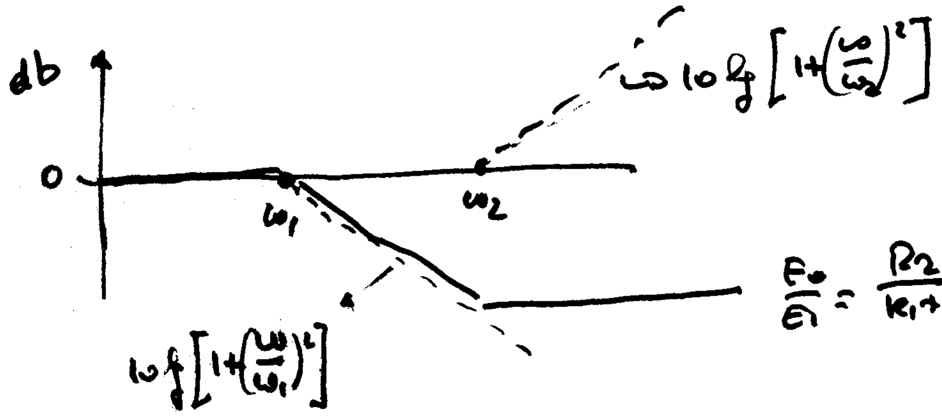
$$\frac{E_0}{E_1} = \frac{1 + j\omega R_2 C}{1 + j\omega C (R_1 + R_2)}$$

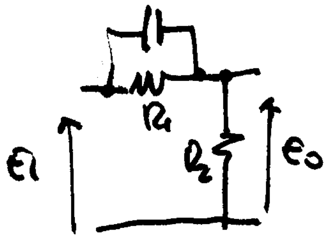
$$\left|\frac{E_0}{E_1}\right| = \left[\frac{1 + \left(\frac{\omega}{\omega_2}\right)^2}{1 + \left(\frac{\omega}{\omega_1}\right)^2}\right]^{1/2}$$

$$\omega_1 = \frac{1}{C(R_1 + R_2)}$$

$$\omega_2 = \frac{1}{R_2 C}$$

$$\left|\frac{E_0}{E_1}\right| db = 10 \lg \left[1 + \left(\frac{\omega}{\omega_2}\right)^2\right] - 10 \lg \left[1 + \left(\frac{\omega}{\omega_1}\right)^2\right]$$





$$\frac{E_0}{E_1} = \underbrace{\frac{R_2}{R_1 + R_2}}_{\alpha} \frac{1 + j\omega C R_1}{1 + j\omega C R_1 R_2 / (R_1 + R_2)}$$

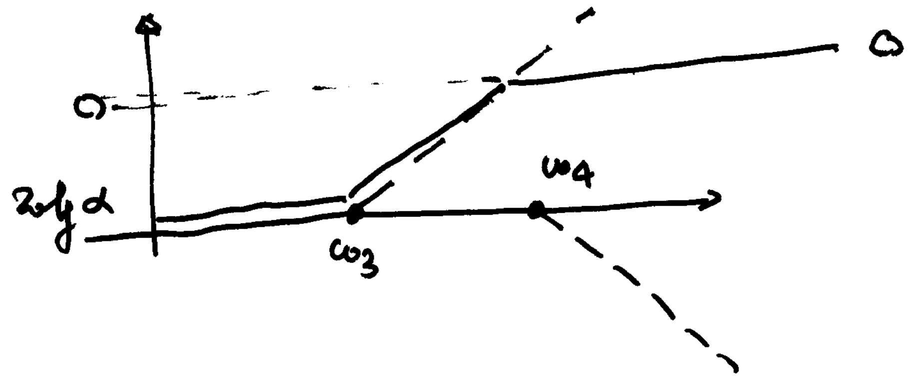
$\frac{E_0}{E_1} = 1$ per $\omega = \infty$ / $\frac{E_0}{E_1} = \frac{R_2}{R_1 + R_2}$ per $\omega \to 0$

$$\left|\frac{E_0}{E_1}\right| = \frac{R_2}{R_1 + R_2} \left[\frac{1 + (\omega/\omega_3)^2}{1 + (\omega/\omega_4)^2}\right]^{1/2}$$

$$\omega_3 = \frac{1}{C R_1}$$

$$\omega_4 = \frac{R_1 + R_2}{C R_1 R_2}$$

$$\left|\frac{E_o}{E_i}\right|_{db} = 20\lg\alpha + 10\lg\left[1+\left(\frac{\omega}{\omega_3}\right)^2\right] - 10\lg\left[1+\left(\frac{\omega}{\omega_4}\right)^2\right]$$



In generale per la cascata ad alte frequenze:

$$\left|\frac{G_H}{G_M}\right| = \left[\frac{1}{1+(f/f_a)^2}\right]^{1/2} \cdot \left[\frac{1}{1+(f/f_b)^2}\right]^{1/2} \cdots$$

modulo

funzione

$\Downarrow$

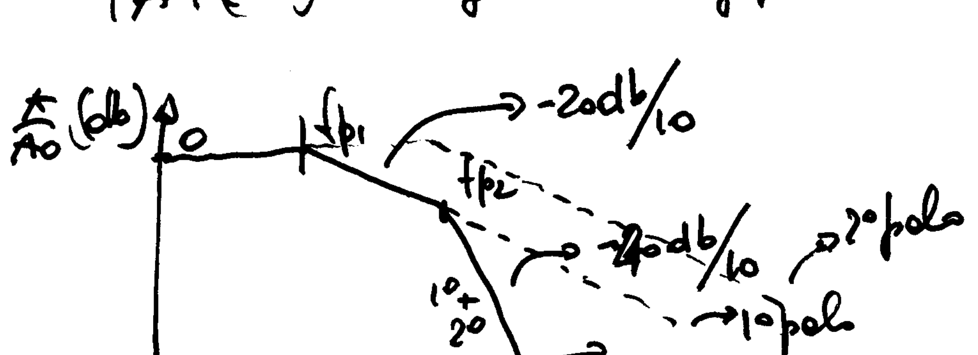
Detta funzione di trasferimento con 2 poli:

$$A(jf) = \frac{A_o}{\left[1+j\left(f/f_{p1}\right)\right]\left[1+j\left(f/f_{p2}\right)\right]}$$

f_{p1} ed f_{p2} "poli"

in db

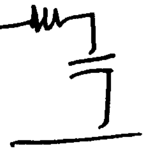
$$|A|(db) = 20\lg A_o - 20\lg\sqrt{1+\left(\frac{f}{f_{p1}}\right)^2} - 20\lg\sqrt{1+\left(\frac{f}{f_{p2}}\right)^2}$$



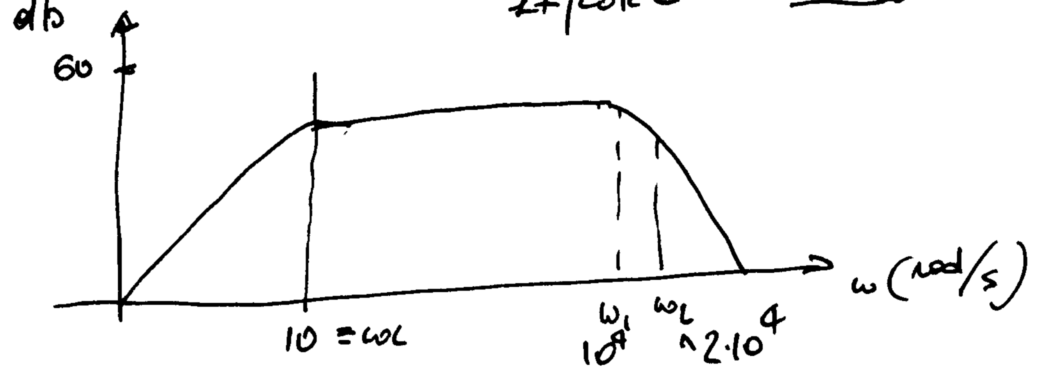
fore $\vartheta = -\cos t_j \frac{t}{T_{p1}} - \sin t_g \frac{t}{T_{p2}}$

Altro Esempio:

$$H(j\omega) = 1000 \frac{j\omega/10}{(1+j\omega/10)(1+j\omega/10^4)\left[1+j\omega/(2\times10^4)\right]}$$

$$C \equiv \frac{j\omega RC}{1+j\omega RC} \equiv$$  $$\frac{1}{j\omega RC} \equiv$$ 

$\equiv j\omega RC$



L'esatto valore di ω_H si calcola da $|H|_{\omega=\omega_H} = \frac{1000}{\sqrt{2}}$ = ampiezza alla frequenza di taglio

Conti eccessivi $|H|$ per ω_L ed ω_H con molti poli ⇒ Allora:

spettro in frequenza = guadagno di medio-banda × f(bassa frequenza) × f(alta frequenza).

$$H(j\omega) = A_0 \cdot H_L \cdot H_H = A_0 \left[\frac{(j\omega/\omega_a)}{(1+j\omega/\omega_a)} \frac{(j\omega/\omega_b)}{(1+j\omega/\omega_b)} \cdots \frac{(j\omega/\omega_m)}{(1+j\omega/\omega_m)}\right]$$

H_L

$$\times \left[\frac{1}{(1+j\omega/\omega_1)(1+j\omega/\omega_2)\cdots(1+j\omega/\omega_n)}\right]$$

H_f

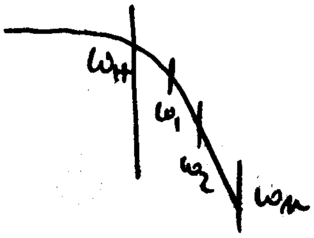
Alle alte frequenze H_L hanno nessun effetto ad $\omega=\omega_H$ il denominatore di H_L va al valore $\left|\frac{j\omega_H}{\omega}\right|$ che cancella il numeratore così che $|H_L| \to 1$

Alte frequenze $\Rightarrow H(j\omega) = A_0 \cdot H_H = \dfrac{A_0}{\underbrace{\left(1+j\frac{\omega}{\omega_1}\right)\left(1+j\frac{\omega}{\omega_2}\right)\cdots\left(1+j\frac{\omega}{\omega_n}\right)}_{\text{denominatore}}}$

il denominatore è:

$$1 + j\omega\left(\frac{1}{\omega_1}+\frac{1}{\omega_2}\cdots\frac{1}{\omega_n}\right) + (j\omega)^2\left(\frac{1}{\omega_1\omega_2}+\frac{1}{\omega_1\omega_n}+\frac{1}{\omega_2\omega_n}\cdots+\frac{1}{\omega_j\omega_n}\right)$$
$$+(j\omega)^3\left(\text{termini del tipo } \frac{1}{\omega_j\omega_k\omega_n}\right) + \cdots + (j\omega)^n\left(\text{termini } \frac{1}{\omega_1\omega_2\cdots\omega_n}\right)$$

$\omega_1, \omega_2, \omega_n > \omega_H$ limite di banda $\Rightarrow$ termini al $(\ )^2$ o al $(\ )^3$ trascurabili.



$$\Rightarrow H(j\omega) = \frac{A_0}{1+j\omega\left(\frac{1}{\omega_1}+\frac{1}{\omega_2}\cdots\frac{1}{\omega_n}\right)}$$

$$\Rightarrow \omega_H = \frac{1}{\left(\frac{1}{\omega_1}+\frac{1}{\omega_2}\cdots+\frac{1}{\omega_n}\right)}$$

ω_H = parallela combinazione $\omega_1 \| \omega_2 \| \cdots \omega_n$

"parallela sovrapposizione"

- il polo di più bassa frequenza domina ω_H

- limite in bassa frequenza!

poli in alta frequenza $\Rightarrow$ poco effetto ogni polo per $\omega < \omega_1, \omega_2 \cdots \omega_n \Rightarrow = 1$ $\left[\frac{1}{(1+j\omega/\omega_1)\cdots(1+j\frac{\omega}{\omega_n})}\right]$

$$H(j\omega) \hat{=} A_0 \cdot H_L = A_0 \frac{1}{(\omega_a/j\omega+1)(\omega_b/j\omega+1)\cdots(\omega_m/j\omega+1)}$$

dividendo per $\frac{j\omega}{\omega_a}$ ……

denominatore: $1 + \frac{1}{j\omega}(\omega_a+\omega_b\cdots+\omega_m)+\frac{1}{(j\omega)^2}(\omega_a\omega_b+\cdots\omega_j\omega_m)$

$$+\left[\frac{1}{j\omega^3}(\omega_j\omega_k\omega_m)\right]+\frac{1}{(j\omega)^m}(\omega_a\omega_b\cdots\omega_m)$$

$\omega_a, \omega_b \cdots \omega_m < \omega_H$ trascuriamo i termini con potenze 2, 3.

$$H(j\omega) = A_0 \frac{1}{1+(1/j\omega)(\omega_a+\omega_b\cdots+\omega_m)}$$

$\times j\omega$ e dividendo per $(\omega_a+\omega_b\cdots\cdots\omega_m)$

$$H(j\omega)=\frac{j\omega/(\omega_a+\omega_b\cdots+\omega_m)}{1+[j\omega/\omega_a+\omega_b\cdots+\omega_m)}$$

$\omega_L = \omega_a+\omega_b\cdots\cdots+\omega_m$

"poli in serie"

un polo più alto sarò dominante

Esercizio precedente:

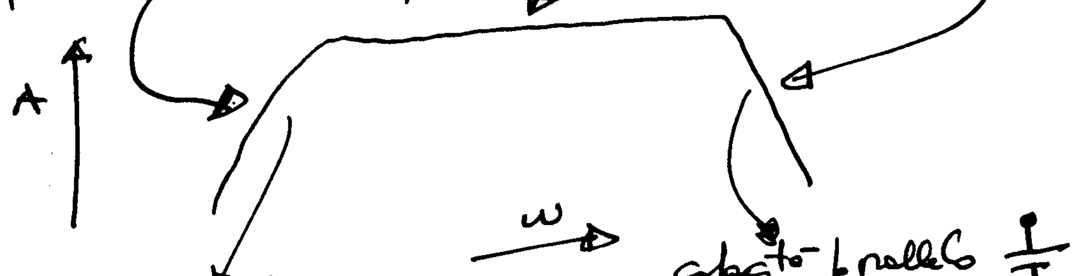
$$\omega_H = \omega_1 \| \omega_2 \qquad \frac{1}{\frac{1}{10^4} + \frac{1}{2\times 10^4}} \sim 0{,}67\times 10^4 \ \frac{rad}{s}$$

$$\omega_L = 10 \ \frac{rad}{sec}$$

⇒ potente tecnica per trovare ω_L e ω_H.

medio-banda ⇒ capacità $f(\omega)$ trascurabili

⇒ capacità a bassa frequenza } e capacità ad alta frequenza



capacità serie

↓

in genere capacità di blocco DC

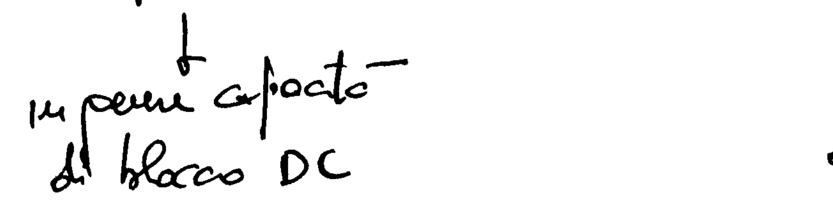
capacità parallelo

↓

in genere capacità parassite

eccezione: C di by-pass però di alta capacità.

Es:



C_L (carico)
C_{CB} ⇒ piccole
C_{BE}

f₁ e f₂ a basse frequenze

$f(C_s \text{ e } C_e)$

$$f_2 = \frac{1}{2\pi C_e (R_L \| R_C)}$$

$$f_1 = \frac{1}{2\pi C_s (h_{fe}\cdot R_E \| (R_1 \| R_2) \| R_s)}$$

alte frequenze:

$$f_3 = \frac{1}{2\pi (R_C \| R_L)\cdot C_L}$$

$$f_4 = \frac{1}{2\pi (h_{fe}\cdot R_E \| (R_1 \| R_2) \| R_s)\cdot C_\pi}$$