Jets, Kinematics, and Other Variables

A Tutorial for Physics With p-p (LHC/Cern) and p-$\bar{p}$ (Tevatron/FNAL) Experiments

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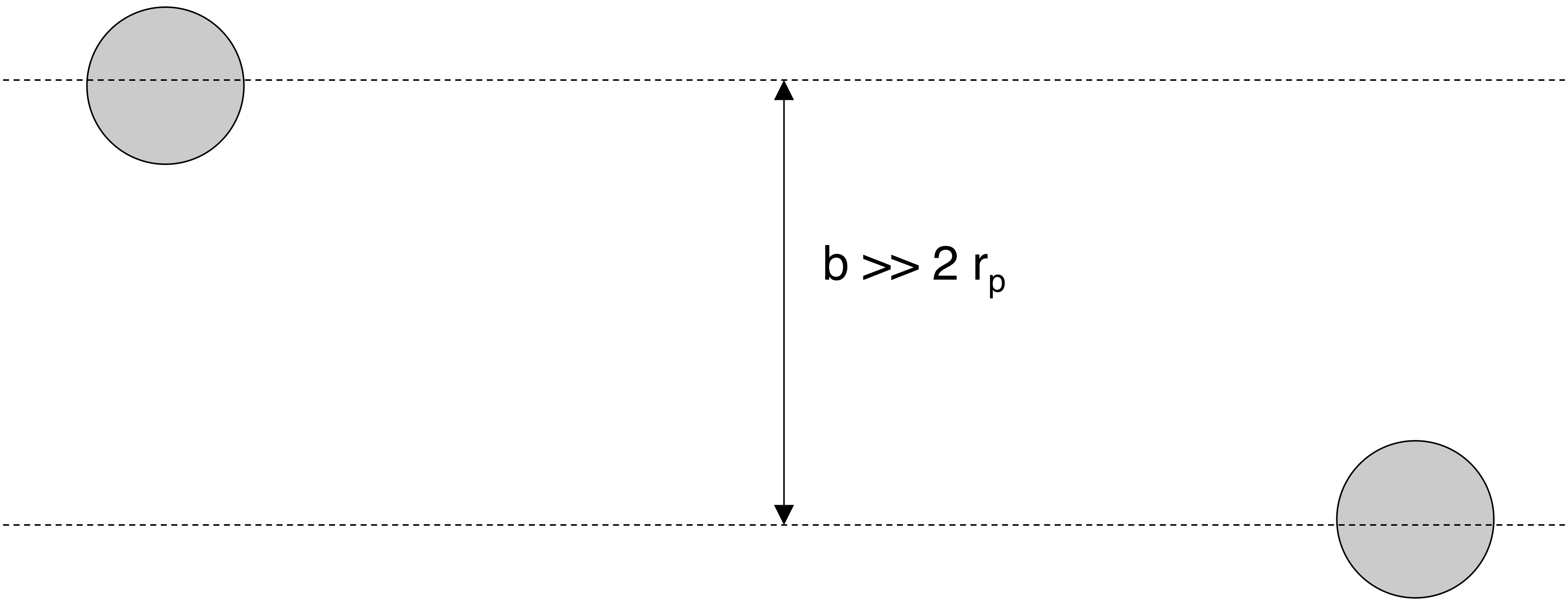
Nucleon-nucleon Scattering

Elastic scattering

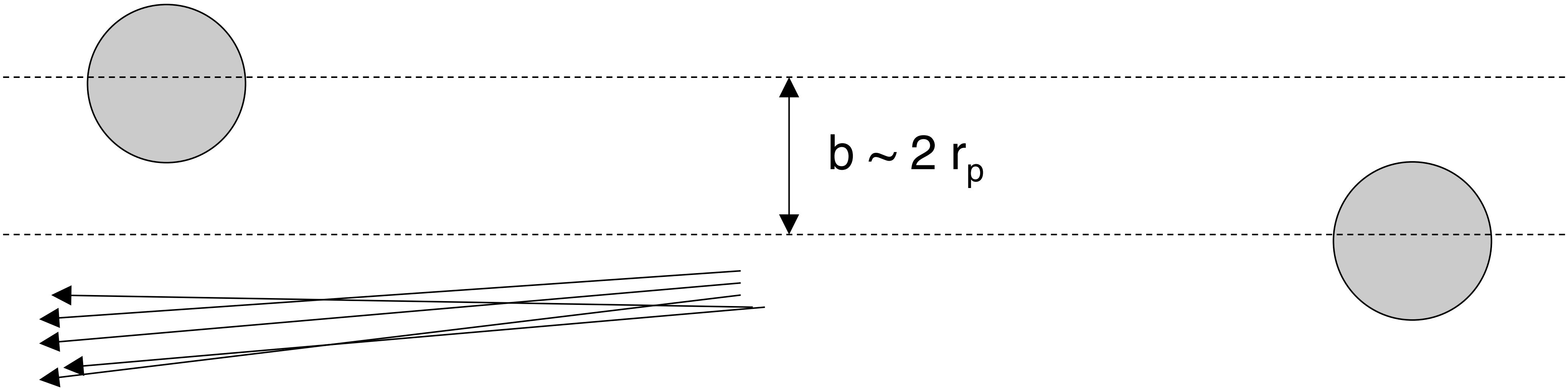
Forward-forward scattering, no disassociation (protons stay protons)



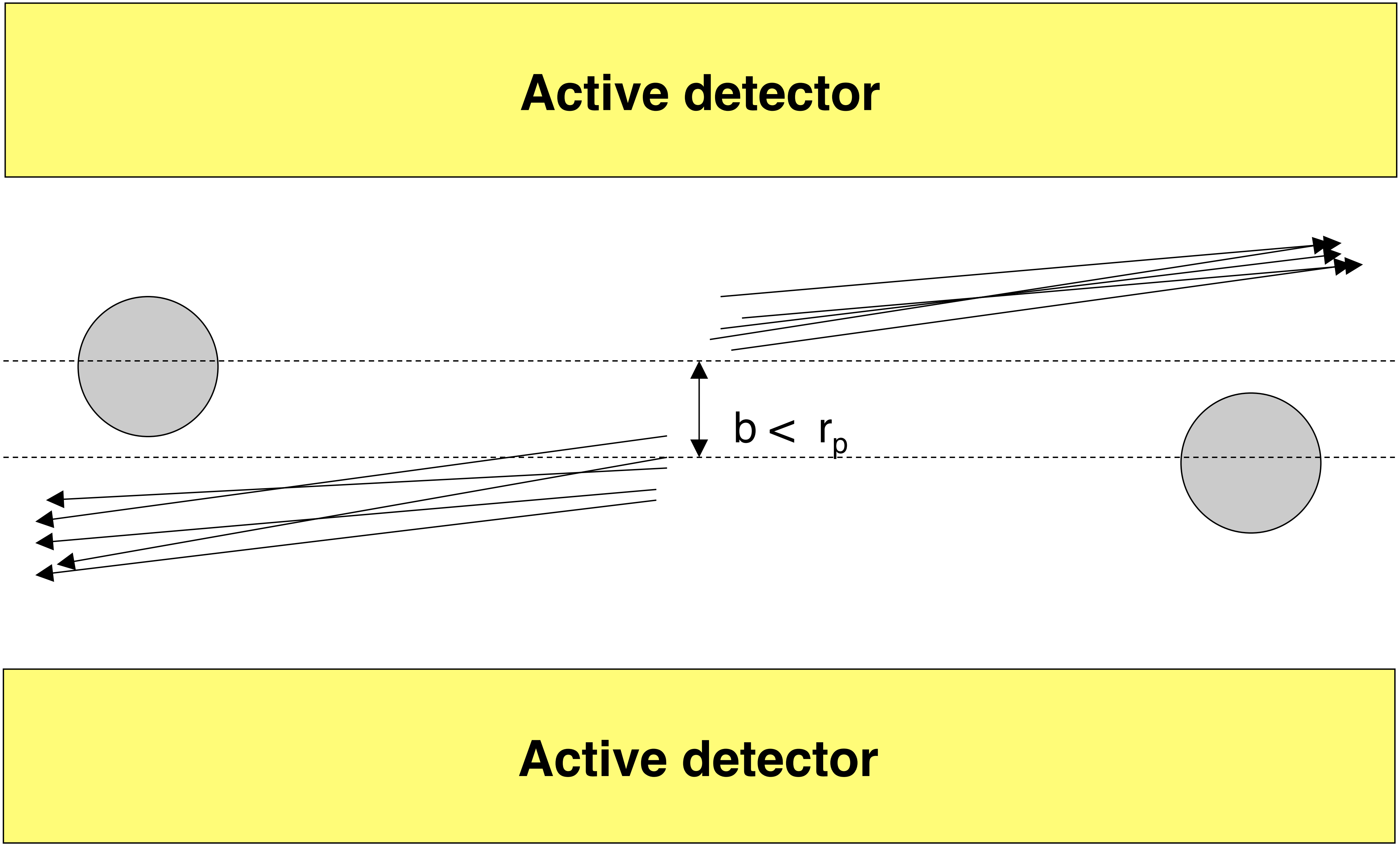
"Single-diffractive" scattering

One of the 2 nucleons disassociates into a spray of particles

- Mostly $\pi^{\pm}$ and π^{0} particles
- Mostly in the forward direction following the parent nucleon's momenum

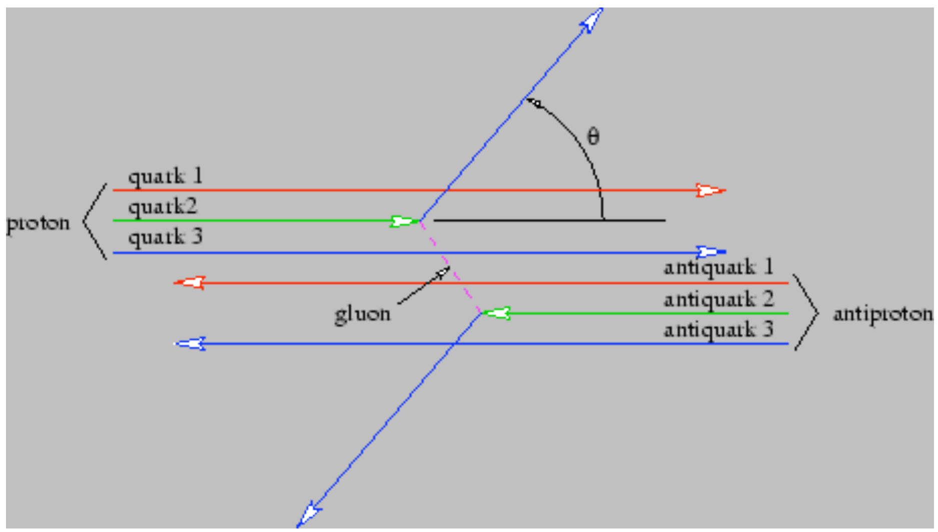


“Double-diffractive” scattering



Proton-(anti)Proton Collisions

- At "high" energies we are probing the nucleon structure
  - "High" means Compton wavelength $\lambda_{beam} \equiv hc/E_{beam} << r_{proton} \sim hc/\text{"1GeV"} \sim 1\text{fm}$
    - E_{beam}=1TeV@FNAL 5-7 TeV@LHC
  - We are really doing *parton–parton* scattering (*parton* = quark, gluon)
- Look for scatterings with large momentum transfer, ends up in detector "central region" (large angles wrt beam direction)
  - Each parton has a momentum distribution –
    - CM of hard scattering is not fixed as in e^+e^- will be move along z-axis with a boost
    - This motivates studying boosts along z
  - What's "left over" from the other partons is called the "underlying event"
- If no hard scattering happens, can still have disassociation
  - An "underlying event" with no hard scattering is called "minimum bias"



"Total Cross-section"

- By far most of the processes in nucleon-nucleon scattering are described by:
  - σ(Total) ~ σ(scattering) + σ(single diffractive) + σ(double diffractive)
    - *"elastic"*: σ(scattering)
    - *"inelastic"*: σ(single diffractive) + σ(double diffractive)
- This can be naively estimated….
  - hard sphere scattering, partial wave analysis:
  - $\sigma \sim 4\text{xArea}_{proton} = 4\pi r_p^2 = 4\pi \times (1\text{fm})^2 \sim 125\text{mb}$
- But! total cross-section stuff is NOT the reason we do these experiments!
- Examples of "interesting" physics @ Tevatron
  - W production and decay via lepton
    - $\sigma \cdot Br(W \to e\nu) \sim 2\text{nb}$, 1 in $50\text{x}10^6$ collisions
  - Z production and decay to lepton pairs
    - About 1/10 that of W to leptons
  - Top quark production
    - σ(total) ~ 5pb, 1 in $20\text{x}10^9$ collisions
- Rates for similar things at LHC will be ~10x higher

arXiv.org > hep-ph > arXiv:0709.0395

High Energy Physics – Phenomenology

The total cross section at the LHC

P. V. Landshoff

(Submitted on 4 Sep 2007)

We do not have the ability to perform precise calculations of long-range strong interaction effects, because the effective QCD coupling is not small and so we cannot use perturbation theory. Nevertheless, I show that we know a lot, though not nearly enough. As a measure of our lack of knowledge, the best prediction for the total cross section at LHC energy is 125 +/- 25 mb.

Comments: Lectures at School on QCD, Calabria, July 2007
Subjects: **High Energy Physics – Phenomenology (hep-ph)**; High Energy Physics – Experiment (hep-ex)
Report number: DAMTP-2007-82
Cite as: arXiv:0709.0395v1 [hep-ph]

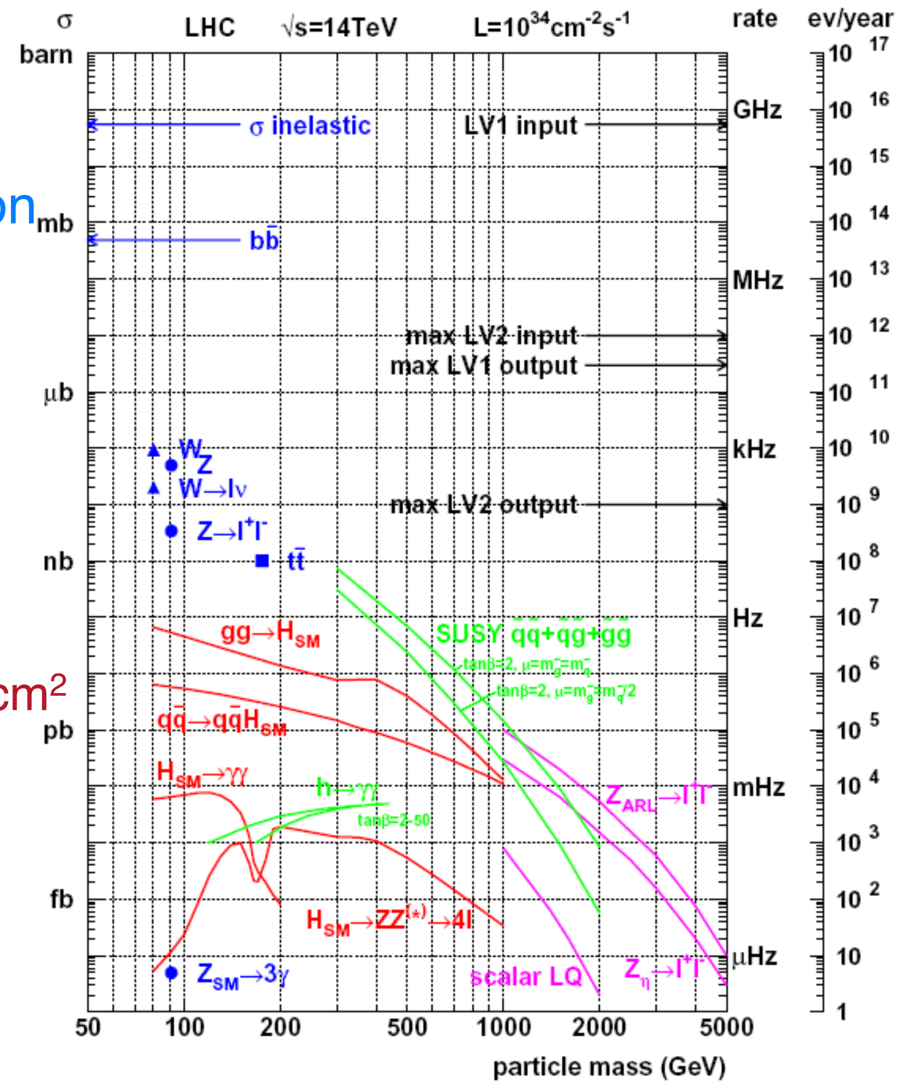
Needles in Haystacks

- What determines number of detected events N(X) for process “X”?
  - Or the rate: R(X)=N(X)/sec?
- N(X) per unit cross-section should be a function of the brightness of the beams
  - And should be constant for any process:

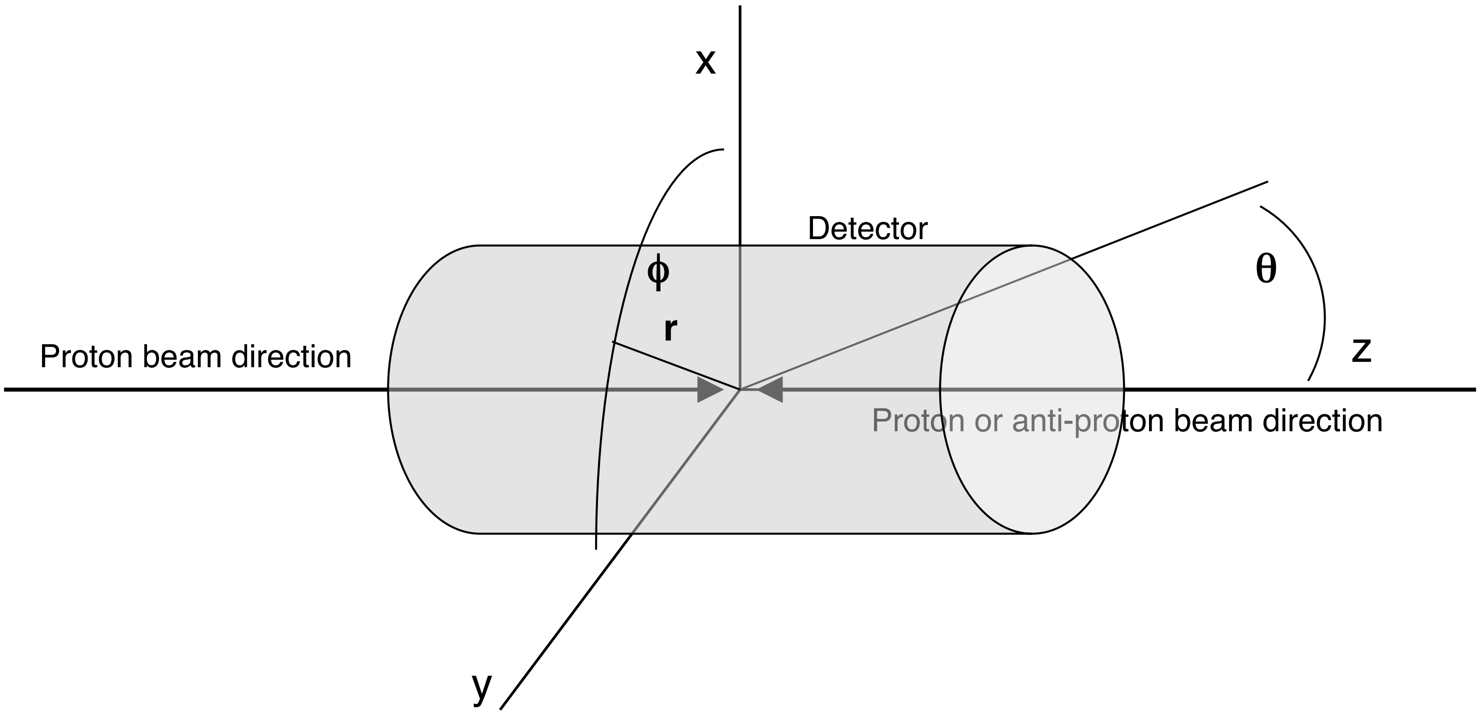
  $N(X)/\sigma(X)$ = constant==L (luminosity)

  $R(X)/\sigma(X) = \mathcal{L}$ (instantaneous luminosity)
- Units of luminosity:
  - “Number of events per barn”
  - Note: 1nb = 10^{-9} barns = 10^{-9}x10^{-24}cm^2 = 10^{-33} cm^2
  - LHC instantaneous design luminosity

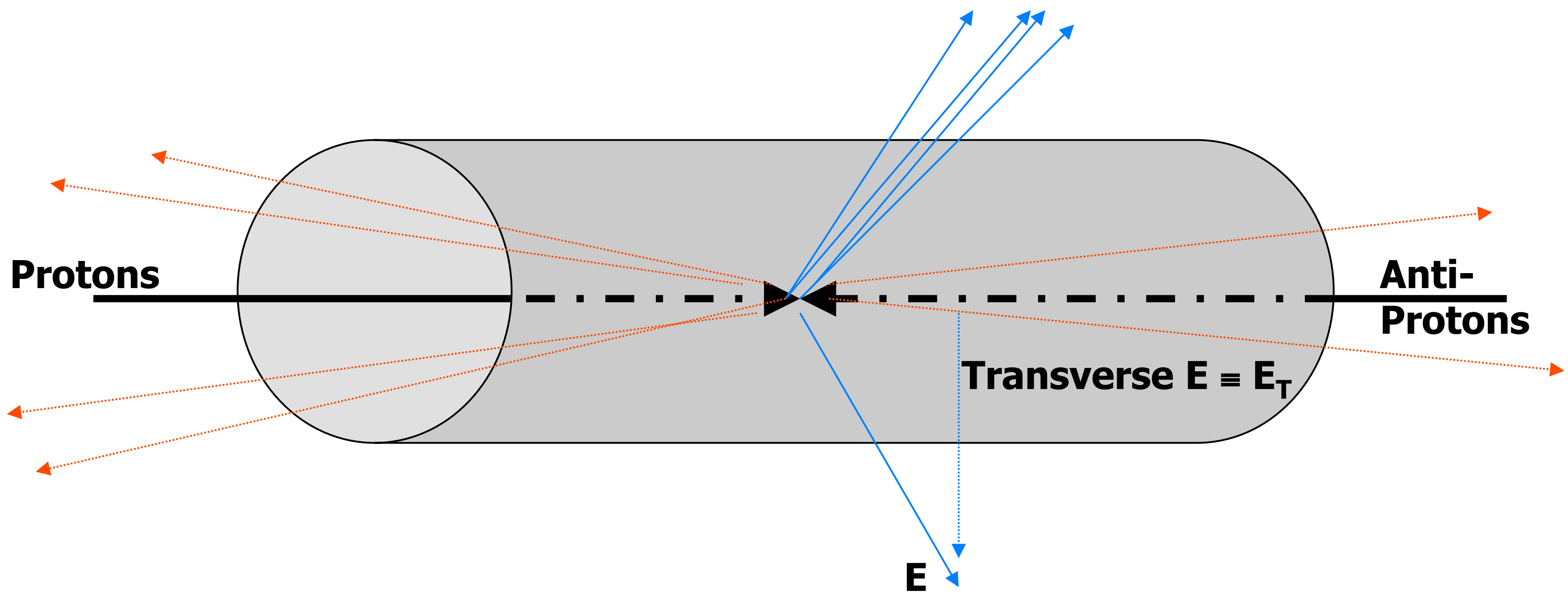
    10^{34} cm^{-2} s^{-1} = 10 nb^{-1}/s, or 10 events per nb cross-section per second, or “10 inverse nanobarns per second”
    - e.g. 10 t-tbar events per second



Coordinates



Detect the “hard scattering”



Phase Space

- Relativistic invariant phase-space element:

$$d\tau = \frac{d^3 p}{E} = \frac{dp_x dp_y dp_z}{E}$$

  - Define $p\bar{p}$ or pp collision axis along *z-axis*:
  - Coordinates $\mathbf{p^\mu = (E, p_x, p_y, p_z)}$ – Invariance with respect to boosts along z?
    - 2 longitudinal components: E & p_z (and dp_z/E) NOT invariant
    - 2 transverse components: p_x p_y, (and dp_x, dp_y) ARE invariant

- Boosts along z-axis
  - For convenience: define $\mathbf{p^\mu}$ where only 1 component is not Lorentz invariant
  - Choose $\mathbf{p_T}$**, m,** ϕ as the "transverse" (invariant) coordinates
    - $\mathbf{p_T \equiv p\,}$***sin***(θ) and ϕ is the azimuthal angle
  - For 4th coordinate define "rapidity" (y) $y \equiv \frac{1}{2}\ln\frac{E + p_z}{E - p_z}$ or $p_z = E\tanh y$
    - …How does it transform?

Boosts Along beam-axis

- Form a boost of velocity β along z axis
  - $p_z \Rightarrow \gamma(p_z + \beta E)$
  - $E \Rightarrow \gamma(E + \beta p_z)$
  - Transform rapidity:

$$y = \frac{1}{2}\ln\frac{E+p_z}{E-p_z} \Rightarrow \frac{1}{2}\ln\frac{\gamma(E+\beta p_z)+\gamma(p_z+\beta E)}{\gamma(E+\beta p_z)-\gamma(p_z+\beta E)}$$

$$= \frac{1}{2}\ln\frac{(E+p_z)(1+\beta)}{(E-p_z)(1-\beta)} = y + \ln\gamma(1+\beta)$$

$$y \Rightarrow y + y_b$$

- Boosts along the beam axis with $v=\beta c$ will change y by a constant y_b
  - $(p_T, y, \phi, m) \Rightarrow (p_T, y+y_b, \phi, m)$ with $y \Rightarrow y + y_b$, $y_b \equiv \ln\gamma(1+\beta)$ simple additive to rapidity
  - Relationship between y, β, and θ can be seen using $p_z = p\cos(\theta)$ and $p = \beta E$

$$y = \frac{1}{2}\ln\frac{1+\beta\cos\theta}{1-\beta\cos\theta}$$ or $$\tanh y = \beta\cos\theta$$ where β is the CM boost

$$d\tau \equiv \frac{d^3 p}{E} = \frac{dp_x dp_y dp_z}{E}$$

Phase Space (cont)

- Transform phase space element dτ from **(E,p_x,p_y,p_z)** to **(p_t, y, φ, m)**

$$dp_x dp_y = \frac{1}{2} dp_T^2 d\phi$$

&

$$dy = dp_z\left(\frac{\partial y}{\partial p_z} + \frac{\partial y}{\partial E}\frac{\partial E}{\partial p_z}\right)$$

$$= dp_z\left(\frac{E}{E^2 - p_z^2} - \frac{p_z}{E^2 - p_z^2}\frac{p_z}{E}\right)$$

$$= \frac{dp_z}{E}$$

using

$$y \equiv \frac{1}{2}\ln\frac{E + p_z}{E - p_z}$$

- Gives: $$d\tau = \frac{1}{2} dp_T^2 d\phi dy$$

- Basic quantum mechanics: dσ = lM l²dτ
  - If lM l² varies slowly with respect to rapidity, dσ/dy will be ~constant in y
  - Origin of the "rapidity plateau" for the min bias and underlying event structure
  - Apply to jet fragmentation - particles should be uniform in rapidity wrt jet axis:
    - We expect jet fragmentation to be function of momentum perpendicular to jet axis
    - This is tested in detectors that have a magnetic field used to measure tracks

Transverse Energy and Momentum Definitions

- Transverse Momentum: momentum perpendicular to beam direction:

$$p_T^2 = p_x^2 + p_y^2 \quad \text{or} \quad \boxed{p_T = p\sin\theta}$$

- Transverse Energy defined as the energy if p_z was identically 0: $E_T \equiv E(p_z=0)$

$$\boxed{E_T^2 = p_x^2 + p_y^2 + m^2 = p_T^2 + m^2 = E^2 - p_z^2}$$

- How does E and p_z change with the boost along beam direction?
  - Using $\tanh y = \beta\cos\theta$ and $p_z = p\cos\theta$ gives $p_z = E\tanh y$

then $$E_T^2 = E^2 - p_z^2 = E^2 - E^2\tanh^2 y = E^2\text{sech}^2 y$$

or $\boxed{E = E_T\cosh y}$ which also means $\boxed{p_z = E_T\sinh y}$

  - (remember boosts cause $y \rightarrow y + y_b$)
  - Note that the sometimes used formula $E_T = E\sin\theta$ is not (strictly) correct!
  - But it's close – more later….

Invariant Mass $M_{1,2}$ of 2 particles p_1, p_2

- Well defined: $M_{1,2}^2 = (p_1 + p_2)^2 = m_1^2 + m_2^2 + 2\left(E_1 E_2 - \vec{p_1} \cdot \vec{p_2}\right)$

- Switch to $p^\mu = (p_T, y, \phi, m)$ (and do some algebra…)

$$\vec{p_1} \cdot \vec{p_1} = p_{x_1} p_{x_2} + p_{y_1} p_{y_2} + p_{z_1} p_{z_2} = E_{T_1} E_{T_2} \left(\beta_{T_1} \beta_{T_2} \cos\Delta\phi + \sinh y_1 \sinh y_2\right)$$

with $E = E_T \cosh y$ and $\beta_T \equiv p_T / E_T$

- This gives $$M_{1,2}^2 = m_1^2 + m_2^2 + 2 E_{T_1} E_{T_2} \left(\cosh\Delta y - \beta_{T_1} \beta_{T_2} \cos\Delta\phi\right)$$
  - With $\beta_T \equiv p_T/E_T$
  - Note:
    - For $\Delta y \rightarrow 0$ and $\Delta\phi \rightarrow 0$, high momentum limit: $M \rightarrow 0$: angles "generate" mass
    - For $\beta \rightarrow 1$ ($m/p \rightarrow 0$) $$M_{1,2}^2 = 2 E_{T_1} E_{T_2} \left(\cosh\Delta y - \cos\Delta\phi\right)$$

This is a useful formula when analyzing data…

Invariant Mass, multi particles

- Extend to more than 2 particles:

$$M_{1,2,3}^2 = (p_1 + p_2 + p_3)^2 = (p_1 + p_2)^2 + 2(p_1 + p_2)p_3 + m_3^2$$

$$= M_{1,2}^2 + [2p_1p_3] + [2p_2p_3] + m_3^2$$

$$= M_{1,2}^2 + [p_1^2 + 2p_1p_3 + p_3^2] - m_1^2 - m_3^2 + [p_2^2 + 2p_2p_3 + p_3^2] - m_2^2 - m_3^2 + m_3^2$$

$$= M_{1,2}^2 + M_{1,3}^2 + M_{2,3}^2 - m_1^2 - m_2^2 - m_3^2$$

- In the high energy limit as m/p → 0 for each particle:

$$\boxed{M_{1,2,3}^2 = M_{1,2}^2 + M_{2,3}^2 + M_{1,3}^2}$$

⇒ Multi-particle invariant masses where each mass is negligible – no need to id

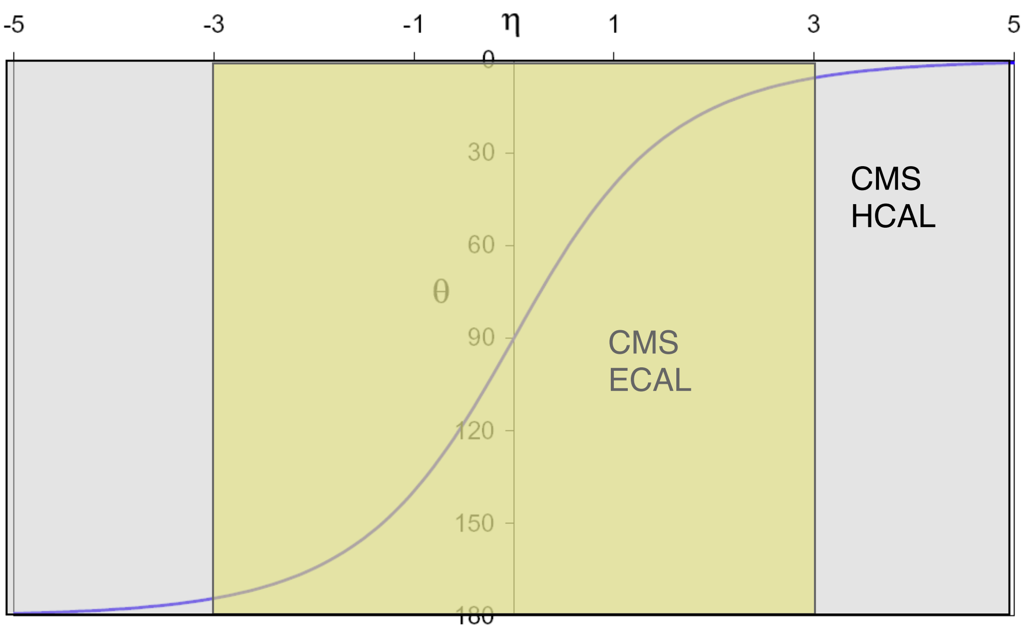
- ⇒ Example: t →Wb and W →jet+jet
- – Find M(jet,jet,b) by just adding the 3 2-body invariant masses in quadriture
- – Doesn't matter which one you call the b-jet and which the "other" jets as long as you are in the high energy limit

Pseudo-rapidity

"Pseudo" rapidity and "Real" rapidity

- Definition of y: $\tanh(y) = \beta \cos(\theta)$
  - Can almost (but not quite) associate position in the detector (θ) with rapidity (y)
- But…at Tevatron and LHC, most particles in the detector (>90%) are π's with $\beta \approx 1$
- Define "pseudo-rapidity" defined as $\eta \equiv y(\theta,\beta=1)$, or $\tanh(\eta) = \cos(\theta)$ or

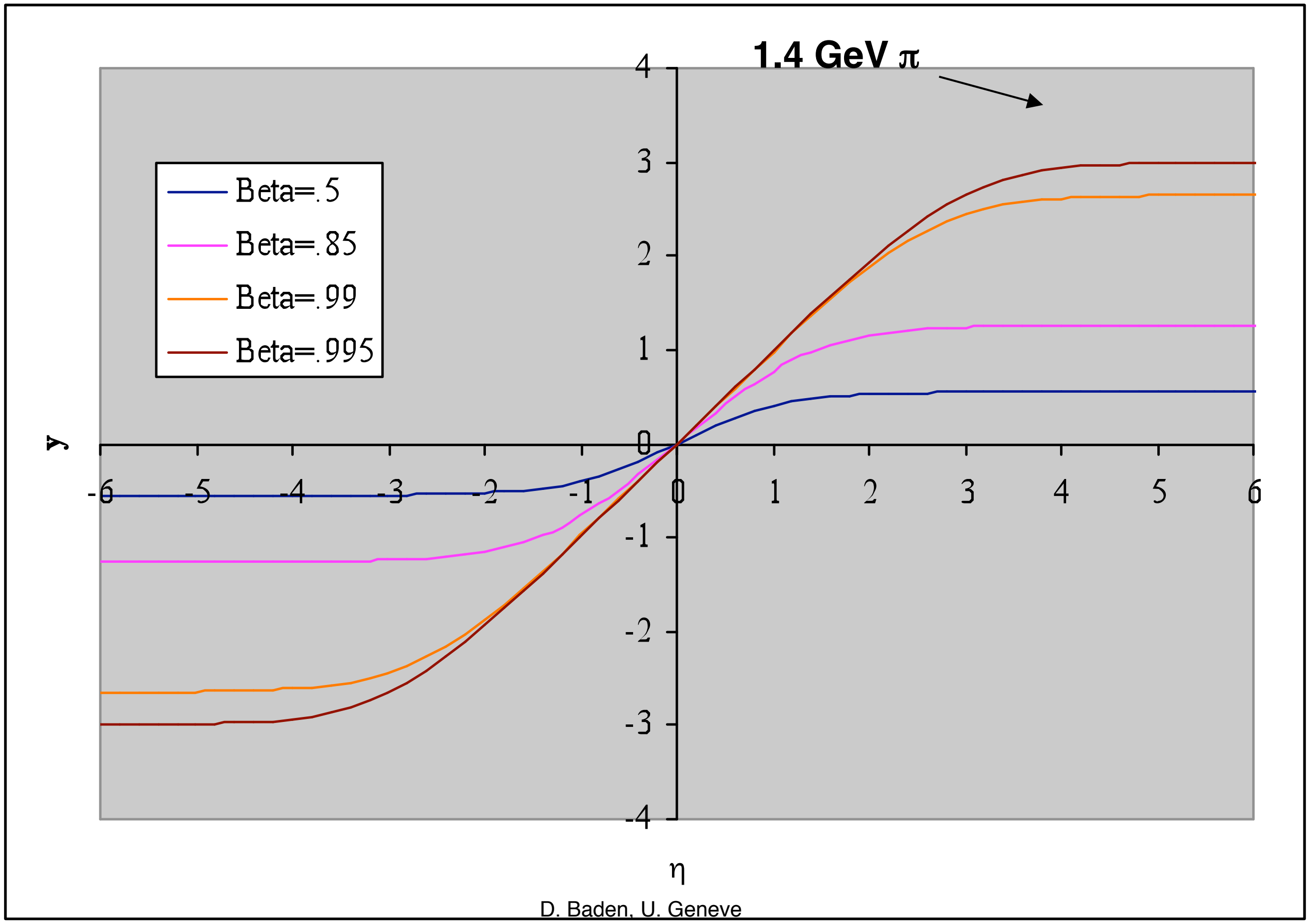
$$\eta = \frac{1}{2}\ln\frac{1+\cos\theta}{1-\cos\theta} = \ln\frac{\cos\theta/2}{\sin\theta/2} = -\ln(\tan\theta/2)$$



(η=5, θ=0.77°)

Rapidity (y) vs "Pseudo-rapidity" (η)

- From $tanh(\eta) = \cos(\theta) = tanh(y)/\beta$
  - We see that $|\eta| \geq |y|$
  - Processes "flat" in rapidity **y** will not be "flat" in pseudo-rapidity **η**
    - (y distributions will be "pushed out" in pseudo-rapidity)



$|\eta| - |y|$ and p_T – Calorimeter Cells

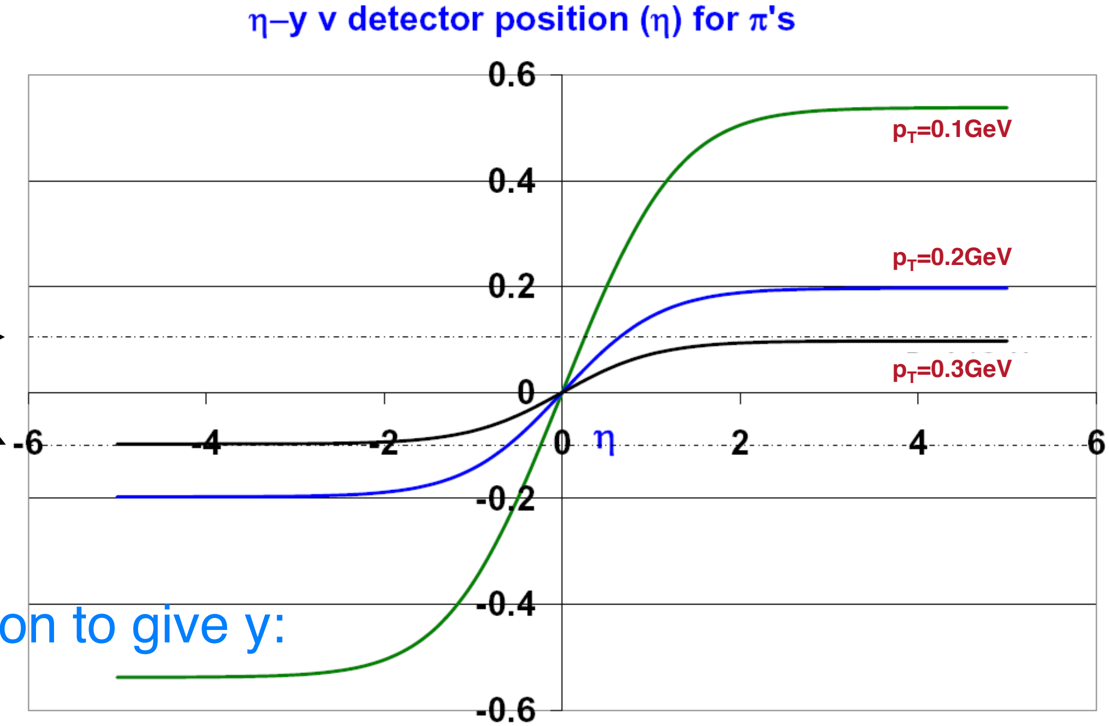
- At colliders, Center-of-Mass can be moving with respect to detector frame
- Lots of longitudinal momentum can escape down beam pipe
  - But transverse momentum p_T is conserved in the detector
- Plot $\eta - y$ for constant m_π, $p_T \Rightarrow \beta(\theta)$

DØ calorimeter cell width

$\Delta\eta=0.1$

CMS HCAL cell width 0.08

CMS ECAL cell width 0.005



- For all η in DØ/CDF, can use η position to give y:
  - Pions: $|\eta|-|y| < 0.1$ for $p_T > 0.1$GeV
  - Protons: $|\eta|-|y| < 0.1$ for $p_T > 2.0$GeV
  - As $\beta \to 1$, $y \to \eta$ (so much for "pseudo")

Rapidity "plateau"

- Constant p_t, rapidity plateau means $d\sigma/dy \sim k$
  - How does that translate into $d\sigma/d\eta$?

$$\frac{d\sigma}{d\eta} = \frac{d\sigma}{dy}\frac{dy}{d\eta} = k\frac{dy}{d\eta}$$

  - Calculate $dy/d\eta$ keeping m, and p_t constant
  - After much algebra… $dy/d\eta = \beta(\eta)$

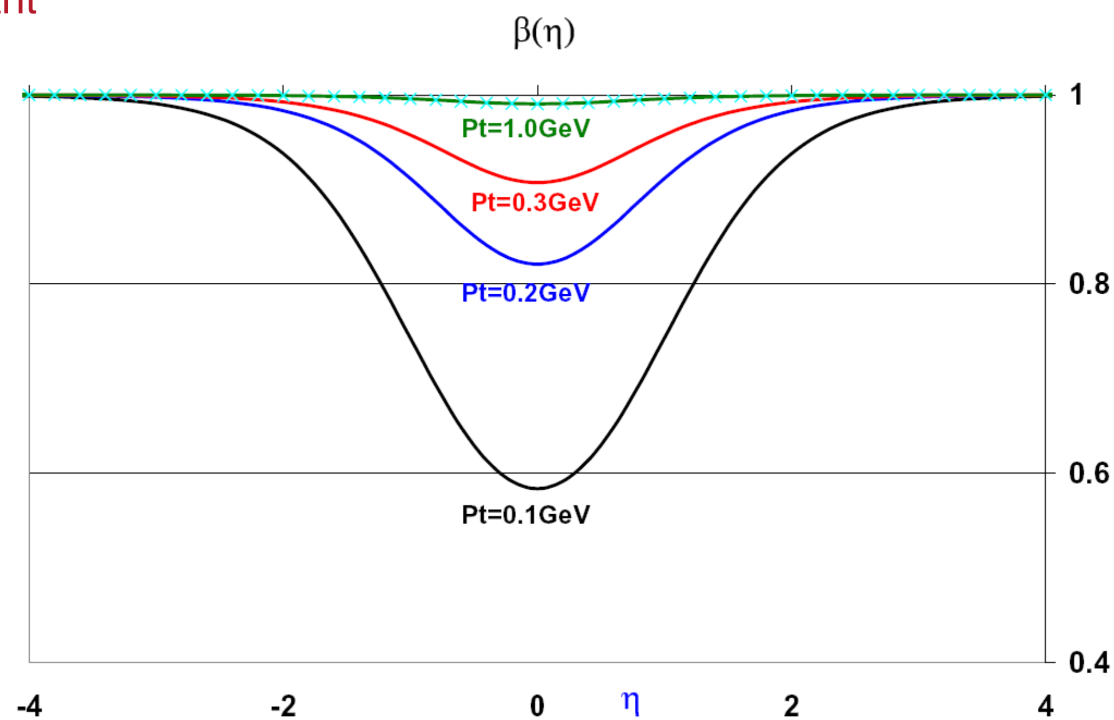
$$\frac{d\sigma}{d\eta} = \frac{d\sigma}{dy}\frac{dy}{d\eta} = k\frac{dy}{d\eta} = k\beta(\eta)$$

  - "pseudo-rapidity" plateau…only for $\beta \to 1$

…some useful formulae…

$$\tanh(y) = \beta(\eta)\tanh(\eta)$$

$$\beta(\eta) = \frac{p}{E} = \sqrt{\frac{p_T^2 + p_Z^2}{p_T^2 + p_Z^2 + m^2}} = \frac{\cosh(\eta)}{\sqrt{m^2/p_T^2 + \cosh^2\eta}}$$



Transverse Mass

Measured momentum conservation

- Momentum conservation: $$\sum_{particles} p_Z = P_{CM} \quad \text{and} \quad \sum_{particles} \vec{p}_T = 0$$
- What we measure using the calorimeter: $$\sum_{cells} p_Z = P_{CM} \quad \text{and} \quad \sum_{cells} \vec{p}_T = 0$$
- For processes with high energy neutrinos in the final state: $$\sum_{cells} \vec{p}_T + \vec{p}_{T\nu} = 0$$
- We "measure" p_ν by "missing p_T" method: $$\vec{\not{p}}_T = \vec{p}_\nu \equiv -\sum_{cells} \vec{E}_T$$
  - e.g. $W \rightarrow e\nu$ or $\mu\nu$
- Longitudinal momentum of neutrino cannot be reliably estimated
  - "Missing" measured longitudinal momentum also due to CM energy going down beam pipe due to the other (underlying) particles in the event
  - This gets a lot worse at LHC where there are multiple pp interactions per crossing
    - Most of the interactions don't involve hard scattering so it looks like a busier underlying event

Transverse Mass

- Since we don't measure p_z of neutrino, cannot construct invariant mass of W
- What measurements/constraints do we have?
  - Electron 4-vector
  - Neutrino 2-d momentum (p_T) and m=0
- So construct "transverse mass" M_T by:
  1. Form "transverse" 4-momentum by ignoring p_z (or set p_z=0) $p_T^\mu \equiv \left(E_T, \overrightarrow{p_T}, 0\right)$
  2. Form "transverse mass" from these 4-vectors:

$$M_{T1,2}^2 \equiv \left(p_{T_1} + p_{T_2}\right)^\mu \left(p_{T_1} + p_{T_2}\right)_\mu$$

- This is equivalent to setting $\eta_1 = \eta_2 = 0$
- For e/μ and ν, set $m_e = m_\mu = m_\nu = 0$ to get:

$$M_{T1,2}^2 = 2E_{T_1} E_{T_2} \left(1 - \cos\Delta\phi\right) = 4E_{T_1} E_{T_2} \sin^2(\Delta\phi/2)$$

  - This is another way to see that the opening angle "generates" the mass

Transverse Mass Kinematics for Ws

- Transverse mass distribution?
- Start with $M_W^2 = M_{e,\nu}^2 = 2E_{T_e}E_{T_\nu}\left(\cosh\Delta\eta - \cos\Delta\phi\right)$
- Constrain to M_W=80GeV and $p_T(W)$=0
  - $\cos\Delta\phi = -1$
  - $E_{Te} = E_{T\nu}$
  - This gives you $E_{Te}E_{T\nu}$ versus $\Delta\eta$

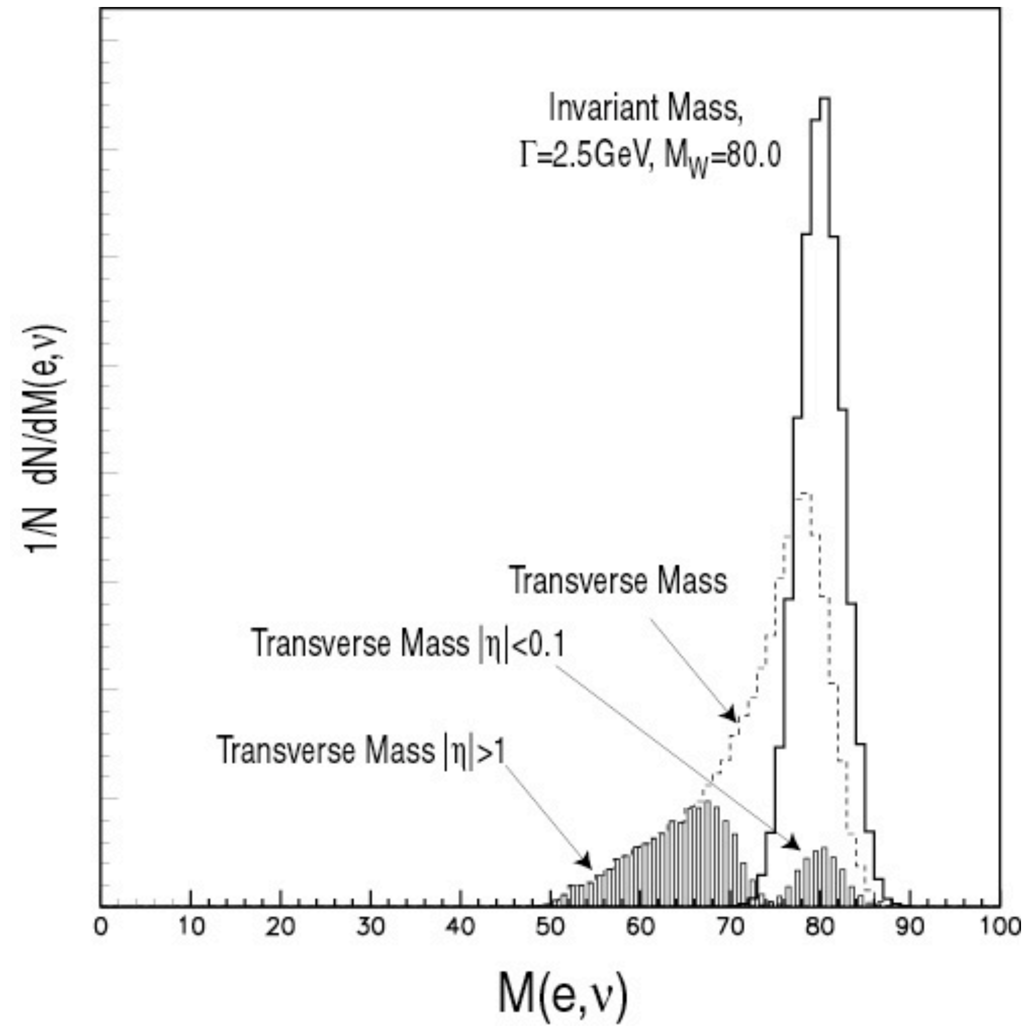
$$E_{Te}E_{T\nu} = \frac{80^2}{2(\cosh\Delta\eta + 1)}$$

- Now construct transverse mass

$$M_{Te,\nu}^2 = 2E_{Te}E_{T\nu}\left(1-\cos\Delta\phi\right) = 2\frac{80^2}{\cosh\Delta\eta+1}$$

$\Delta\phi=\pi$

  - Cleary $M_T=M_W$ when $\eta_e=\eta_\nu=0$



Neutrino Rapidity

- Can you constrain M(e,ν) to determine the pseudo-rapidity of the ν?
  - Would be nice, then you could veto on θ_ν in "crack" regions

- Use M(e,ν) = 80GeV and $M_W^2 = 80^2 = 2E_{Te}E_{T\nu}\left(\cosh\Delta\eta - \cos\Delta\phi\right)$

to get $$\cosh\Delta\eta = \frac{80^2}{2E_{Te}E_{T\nu}} + \cos\Delta\phi$$

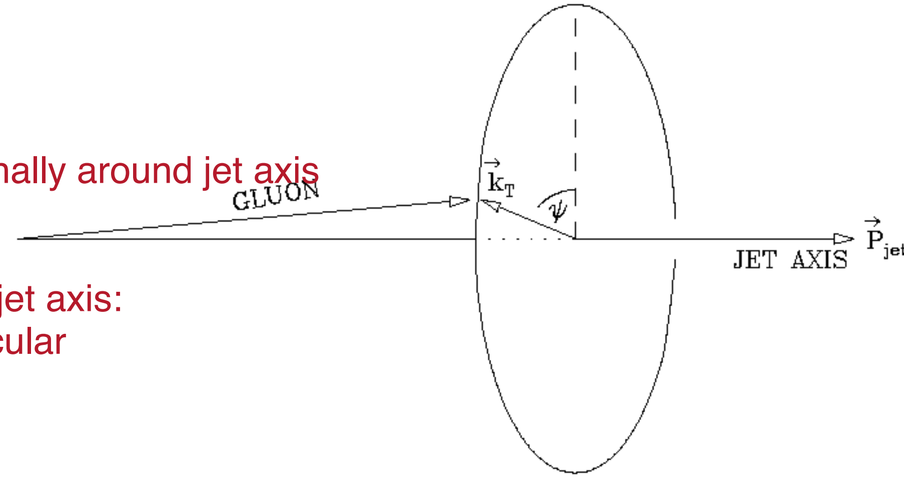
and solve for Δη: $$\Delta\eta = \ln\frac{\cosh\Delta\eta + \sqrt{\cosh^2\Delta\eta + 1}}{2}$$

- Since we know η_e, we know that $\eta_\nu = \eta_e \pm \Delta\eta$
  - Two solutions. Neutrino can be either higher or lower in rapidity than electron
  - Why? Because invariant mass involves the opening angle between particles.
  - Perhaps this can be used for neutrino's (or other sources of missing energy?)

Jets

Jet Definition

- How to define a "jet" using calorimeter towers so that we can use it for invariant mass calculations
  - And for inclusive QCD measurements (e.g. $d\sigma/dE_T$)



- QCD motivated:
  - Leading parton radiates gluons uniformly distributed azimuthally around jet axis
  - Assume zero-mass particles using calorimeter towers
    - 1 particle per tower
  - Each "particle" will have an energy $\vec{k}_T$ perpendicular to the jet axis:
  - From energy conservation we expect total energy perpendicular to the jet axis to be zero on average: $$\sum_{particles} \vec{k}_T = 0$$
  - Find jet axis that minimizes k_T relative to that axis
  - Use this to define jet 4-vector from calorimeter towers
  - Since calorimeter towers measure total energy, make a basic assumption:
    - Energy of tower E_i is from a single particle with that energy
    - Assume zero mass particle (assume it's a pion and you will be right >90%!)
    - Momentum of the particle is then given by

$$\vec{p}_i = E_i \hat{n}_i \text{ and } \hat{n}_i \text{ points to tower } i \textit{ with energy } E_i$$

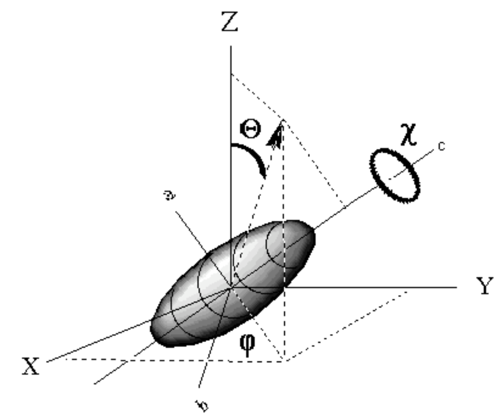
  - Note: m_i=0 does NOT mean M_{jet}=0
    - Mass of jet is determined by opening angle between all contributors
    - Can see this in case of 2 "massless" particles, or energy in only 2 towers:

$$M^2_{12} = 2E_1E_2(1-\cos\theta_{12}) = 4E_1E_2\sin^2\frac{\theta_{12}}{2}$$

    - Mass is "generated" by opening angles.
    - A rule of thumb: Zero mass parents of decay have θ_{12}=0 always

Quasi-analytical approach

- Transform each calorimeter tower to frame of jet and minimize k_T
  - 2-d Euler rotation (in picture, $\phi=\phi_{jet}$, $\theta=\theta_{jet}$, set $\chi=0$)



$$M(\phi_{jet},\theta_{jet}) = \begin{pmatrix} -\sin\phi_{jet} & \cos\phi_{jet} & 0 \\ -\cos\theta_{jet}\cos\phi_{jet} & -\cos\theta_{jet}\sin\phi_{jet} & \sin\theta_{jet} \\ \sin\theta_{jet}\cos\phi_{jet} & \sin\theta_{jet}\sin\phi_{jet} & \cos\theta_{jet} \end{pmatrix}$$

  - Tower in jet momentum frame: $\vec{E}'_i = M(\theta_{jet},\phi_{jet}) \times \vec{E}_i$ and apply $\sum_{particles} \vec{k}_T = 0$

$$E'_{xi} = -E_{xi}\sin\phi_{jet} + E_{yi}\cos\phi_{jet}$$

$$E'_{yi} = -E_{xi}\cos\theta_{jet}\cos\phi_{jet} - E_{yi}\cos\theta_{jet}\sin\phi_{jet} + E_{zi}\sin\theta_{jet}$$

$$E'_{zi} = E_{xi}\sin\theta_{jet}\cos\phi_{jet} + E_{yi}\sin\theta_{jet}\sin\phi_{jet} + E_{zi}\cos\theta_{jet}$$

  - Check: for 1 tower, $\phi_{tower}=\phi_{jet}$, should get $E'_{xi} = E'_{yi} = 0$ and $E'_{zi} = E_{jet}$
    - It does, after some algebra…

Minimize k_T to Find Jet Axis

- The equation $\sum_{particles} \vec{k_T} = 0$ is equivalent to $\sum_i E'_{xi} = \sum_i E'_{yi} = 0$ so…

$$\sum E'_{xi} = -\sin\phi_{jet} \sum E_{xi} + \cos\phi_{jet} \sum E_{yi} = 0 \quad \Rightarrow \quad \tan\phi_{jet} = \frac{\sum E_{yi}}{\sum E_{xi}}$$

$$\sum E'_{yi} = -\cos\theta_{jet}\left(\cos\phi_{jet} \sum E_{xi} - \sin\phi_{jet} \sum E_{yi}\right) + \sin\theta_{jet} \sum E_{zi} = 0$$

$$\tan\theta_{jet} = \frac{\sqrt{\left(\sum E_{xi}\right)^2 + \left(\sum E_{yi}\right)^2}}{\sum E_{zi}}$$

- Momentum of the jet is such that:

$$\tan\phi_{jet} = \frac{p_{y,jet}}{p_{x,jet}} \qquad \tan\theta_{jet} = \frac{p_{T,jet}}{p_{z,jet}}$$

$$p_{x,jet} = \sum E_{xi}$$
$$p_{y,jet} = \sum E_{yi}$$

$$p_{T,jet} = \sqrt{\left(\sum E_{xi}\right)^2 + \left(\sum E_{yi}\right)^2}$$
$$p_{z,jet} = \sum E_{zi}$$

Jet 4-momentum summary

- Jet Energy: $$E_{jet} = \sum_{towers} E_i$$
- Jet Momentum: $$\vec{p}_{jet} = \sum_{towers} E_i \hat{n}_i$$
- Jet Mass: $$M_{jet}^2 = E_{jet}^2 - p_{jet}^2$$
- Jet 4-vector: $$p_\mu^{jet} = \left(E_{jet}, \vec{p}_{jet}\right) = \left(\sum_{cells} E_i, \sum_{cells} E_i \hat{n}_i\right)$$
- Jet is an object now! So how do we define E_T?

E_T of a Jet

- For any *object*, E_T is well defined:

$$E_{T,jet} \equiv \sqrt{E_{jet}^2 - p_{z,jet}^2} = \sqrt{p_{T,jet}^2 + m_{jet}^2}$$

correct

- There are 2 more ways you could imagine using to define E_T of a jet but neither are technically correct:

Alternative 1

$$E_{T,jet} = E_{jet} \sin\theta_{jet}$$

or

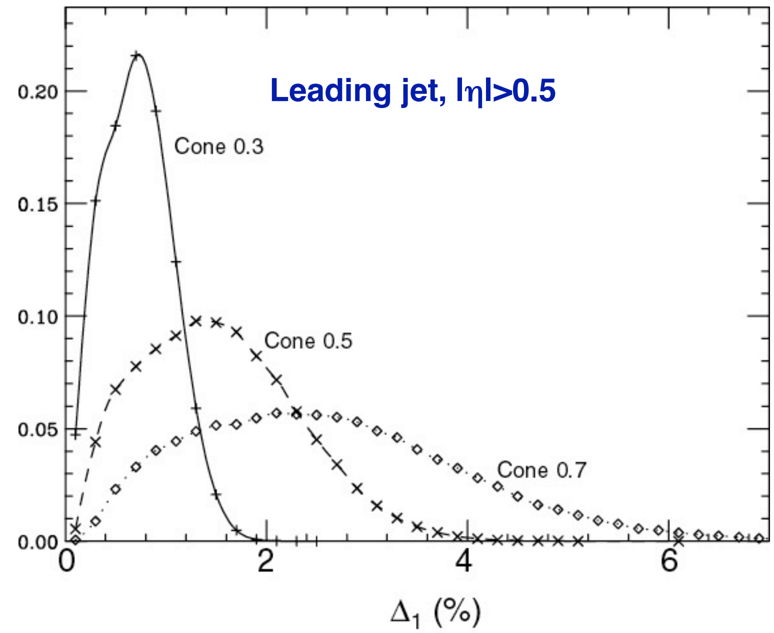
Alternative 2

$$E_{T,jet} = \sum_{towers} E_{T,i}$$

  - How do they compare?
  - Is there any E_T or η dependence?

True E_T vs Alternative 1

- True: $E_{T,jet} = \sqrt{p_{T,jet}^2 + m_{jet}^2}$
- Alternative 1: $E_{T,jet} = E_{jet}\sin\theta_{jet} = \sqrt{p_{jet}^2 + m_{jet}^2}\sin\theta_{jet} = \sqrt{p_{T,jet}^2 + m_{jet}^2\sin^2\theta_{jet}}$
- Define $\Delta_1 \equiv \frac{E_{T,jet} - E_{jet}\sin\theta_{jet}}{E_{T,jet}} = 1 - \frac{\sqrt{p_{T,jet}^2 + m_{jet}^2\sin^2\theta_{jet}}}{\sqrt{p_{T,jet}^2 + m_{jet}^2}}$ which is always >0
  - Expand in powers of $\frac{m_{jet}^2}{p_{T,jet}^2}$: $\Delta_1 \to \frac{m_{jet}^2\tanh^2\eta_{jet}}{2p_{T,jet}^2}$
  - For small η, tanhη → η so either way is fine
    - Alternative 1 is the equivalent to true def central jets
      - Agree at few% level for |η|<0.5
  - For η ~ 0.5 or greater....cone dependent
    - Or “mass” dependent....same thing



True E_T vs Alternative 2

Alternative 2: towers $E_{T,jet} = \sum_{towers} E_{T,i}$ harder to see analytically…imagine a jet w/2

- TRUE:

$$E^2_{T,jet} = E^2_{jet} - p^2_{z,jet} = (E_1 + E_2)^2 - (p_{z1} + p_{z2})^2$$
$$= E_1^2 + 2E_1E_2 + E_1^2 - p_{z1}^2 + 2p_{z1}p_{z2} + p_1^2$$
$$= E_{T1}^2 + E_{T2}^2 + 2E_1E_2(1 - \cos\theta_1\cos\theta_2)$$

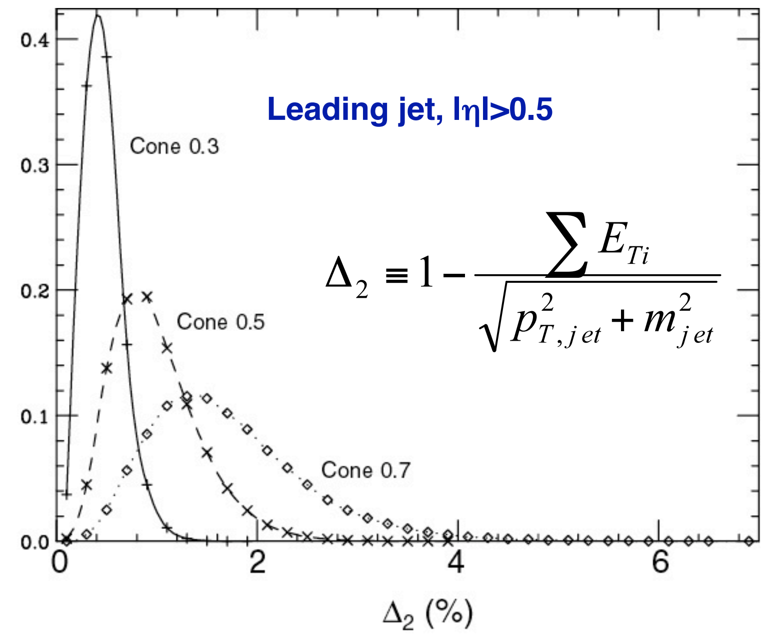
- Alternative 2:

$$(E_{T1} + E_{T1})^2 = E_{T1}^2 + E_{T2}^2 + 2E_{T1}E_{T2}$$
$$= E_{T1}^2 + E_{T2}^2 + 2E_1E_2\sin\theta_1\sin\theta_2$$

- Take difference:

$$E^2_{T,jet} - (E_{T1} + E_{T2})^2 = 2E_1E_2(1 - \cos\theta_1\cos\theta_2 - \sin\theta_1\sin\theta_2)$$
$$= 2E_1E_2(1 - \cos\delta\theta) = E_1E_2\sin^2\delta\theta/2$$

Always > 0!



- So this method also underestimates "true" E_T
  - But not as much as Alternative 1

Jet Shape

- Jets are defined by $\sum_{particles} \vec{k}_{T,i} = 0$ but the "shape" is determined by

$$\sum_{particles} k_{T,i}^2 = \sum_{particles} E'^2_{x,i} + E'^2_{y,i} \geq 0$$

- From Euler:

$$E'_{xi} = -E_{xi}\sin\phi_{jet} + E_{yi}\cos\phi_{jet} = E_{Ti}\sin\delta\phi_i$$

$$E'_{yi} = -E_{xi}\cos\theta_{jet}\cos\phi_{jet} - E_{yi}\cos\theta_{jet}\sin\phi_{jet} + E_{zi}\sin\theta_{jet}$$

$$= -E_{Ti}\cos\delta\phi_i\cos\theta_{jet} + E_{zi}\sin\theta_{jet}$$

$$\delta\phi \equiv \phi_i - \phi_{jet}$$

$$\delta\theta \equiv \theta_i - \theta_{jet}$$

- Now form $\sum_{particles} k_{T,i}^2$ for those towers close to the jet axis: $\delta\theta \to 0$ and $\delta\phi \to 0$

$$E'_{xi} \to E_{Ti}\delta\phi_i$$

$$E'_{yi} \to -E_{Ti}\cos\theta_{jet} + E_{zi}\sin\theta_{jet} = -E_i\sin\theta_i\cos\theta_{jet} + E_i\cos\theta_i\sin\theta_{jet} = E_i\sin\delta\theta_i \sim E_i\delta\theta_i$$

- From $\tanh\eta = \cos\theta$ we get $d\theta = -\sin\theta d\eta$ which means

$$E'_{xi} \to E_{Ti}\delta\phi_i$$

$$E'_{yi} \to E_i\delta\theta_i = -E_i\sin\theta_i\delta\eta_i \to -E_{Ti}\delta\eta_i$$

So…

$$k_{T,i}^2 = E'^{\,2}_{xi} + E'^{\,2}_{yi} = E_{T,i}^2\left(\delta\phi_i^2 + \delta\eta_i^2\right)$$

and…

$$\sum_{particles} k_{T,i}^2 = \sum_{particles} E'^2_{x,i} + E'^2_{y,i} = \sum_{particles} E_{T,i}^2\left(\delta\phi_i^2 + \delta\eta_i^2\right)$$

Jet Shape – E_T Weighted

- Define $\delta R_i^2 \equiv \delta\phi_i^2 + \delta\eta_i^2$ and $\delta R_i = \sqrt{\delta R_i^2} = \sqrt{\delta\phi_i^2 + \delta\eta_i^2}$
  - This gives: $\sum_{particles} k_{T,i}^2 = \sum_{particles} E_{T,i}^2 \delta R_i^2$ and equivalently, $k_{Ti} = E_{Ti}\delta R_i$
  - Momentum of each “cell” perpendicular to jet momentum is from
    - E_{ti} of particle in the detector, and
    - Distance from jet in $\eta\phi$ plane
  - This also suggests jet shape should be roughly circular in $\eta\phi$ plane
    - Providing above approximations are indicative overall….
- Shape defined:
  - Use energy weighting to calculate true 2nd moment in $\eta\phi$ plane

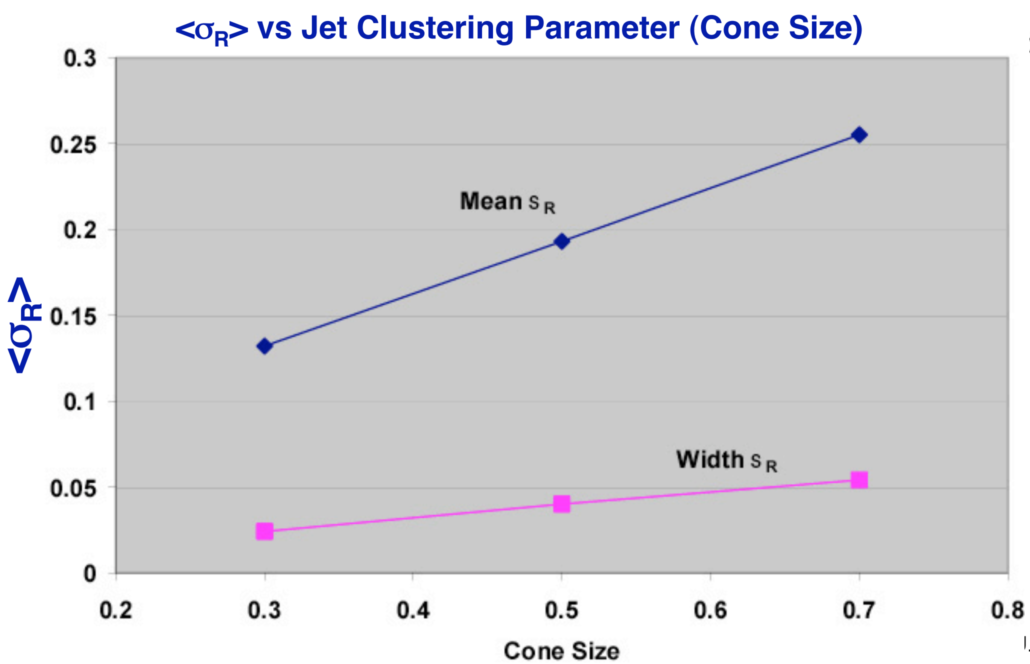
$$\sigma_R^2 \equiv \frac{\sum_{particles} k_{T,i}^2}{\sum_{particles} E_{T,i}^2} = \frac{\sum_{particles} E_{T,i}^2 \delta R_i^2}{\sum_{particles} E_{T,i}^2} = \sigma_{\eta\eta} + \sigma_{\phi\phi} \quad \text{with} \quad \sigma_{\eta\eta} \equiv \frac{\sum_{particles} E_{T,i}^2 \delta\eta_i^2}{\sum_{particles} E_{T,i}^2} \qquad \sigma_{\phi\phi} \equiv \frac{\sum_{particles} E_{T,i}^2 \delta\phi_i^2}{\sum_{particles} E_{T,i}^2}$$

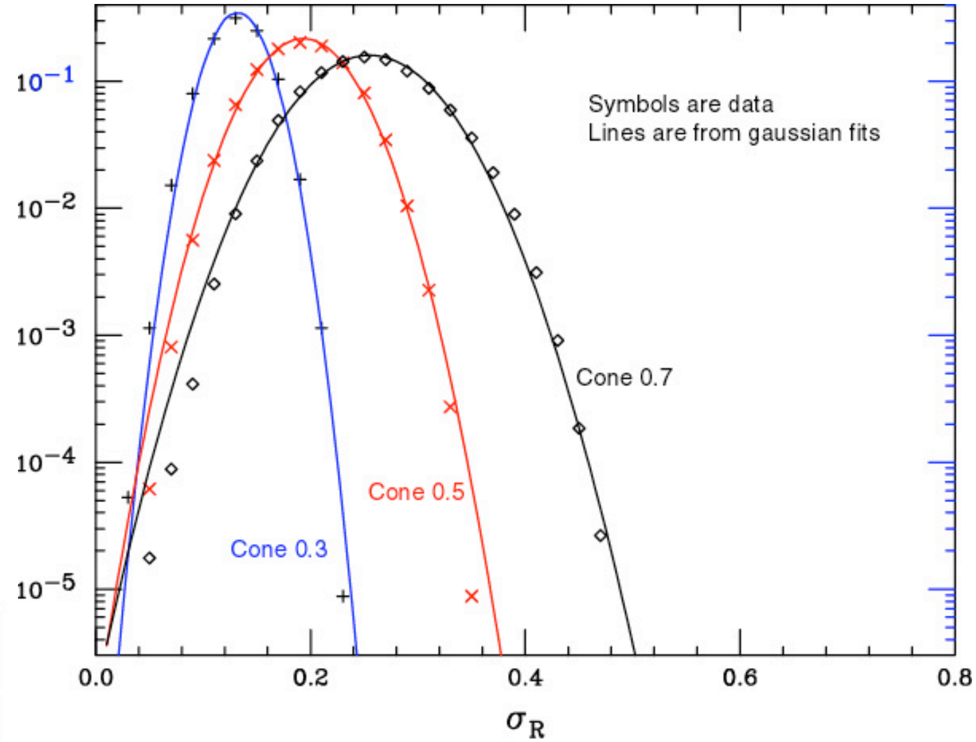
Jet Shape – E_T Weighted (cont)

- Use sample of "unmerged" jets
- Plot

$$\sigma_R = \sqrt{\frac{\sum_{particles} E'^2_{x,i} + E'^2_{y,i}}{\sum_{particles} E^2_{T,i}}}$$

  - Shape depends on cone parameter
  - Mean and widths scale linearly with cone parameter





- "Small angle" approximation pretty good
  - For Cone=0.7, distribution in σ_R has:
    - Mean ± Width =.25 ± .05
    - 99% of jets have σ_R<0.4

Jet Mass

Jet Samples

- DZero Run 1
- All pathologies eliminated (Main Ring, Hot Cells, etc.)
- $|Z_{vtx}|<60$cm
- No τ, e, or γ candidates in event
  - Checked $\eta\phi$ coords of $\tau e\gamma$ vs. jet list
  - Cut on cone size for jets
    - .025, .040, .060 for jets from cone cuttoff 0.3, 0.5, 0.7 respectively
- "UNMERGED" Sample:
  - RECO events had 2 and only 2 jets for cones .3, .5, and .7
  - Bias against merged jets but they can still be there
    - *e.g.* if merging for all cones
- "MERGED" Sample:
  - Jet algorithm reports merging

Jet Mass

- Jet is a physics object, so mass is calculated using:
  - Either one… $M_{jet}^2 = E_{jet}^2 - p_{jet}^2 = E_{T,jet}^2 - p_{T,jet}^2$
    - Note: there is no such thing as "transverse mass" for a jet
      - Transverse mass is only defined for pairs (or more) of 4-vectors…
- For large $E_{T,jet}$ we can see what happens by writing

$$M_{jet}^2 = E_{T,jet}^2 - p_{T,jet}^2 = \left(E_{T,jet} + p_{T,jet}\right)\left(E_{T,jet} - p_{T,jet}\right)$$

  - And take limit as jet narrows $\delta\eta_i \to 0$ and $\delta\phi_i \to 0$ and expand E_T and p_T

$$p_{T,jet} \to \sum E_{T,i}\left(1 - \frac{\delta\phi_i^2}{2}\right) \qquad E_{T,jet} \to \sum E_{T,i}\left(1 + \frac{\delta\eta_i^2}{2}\right)$$

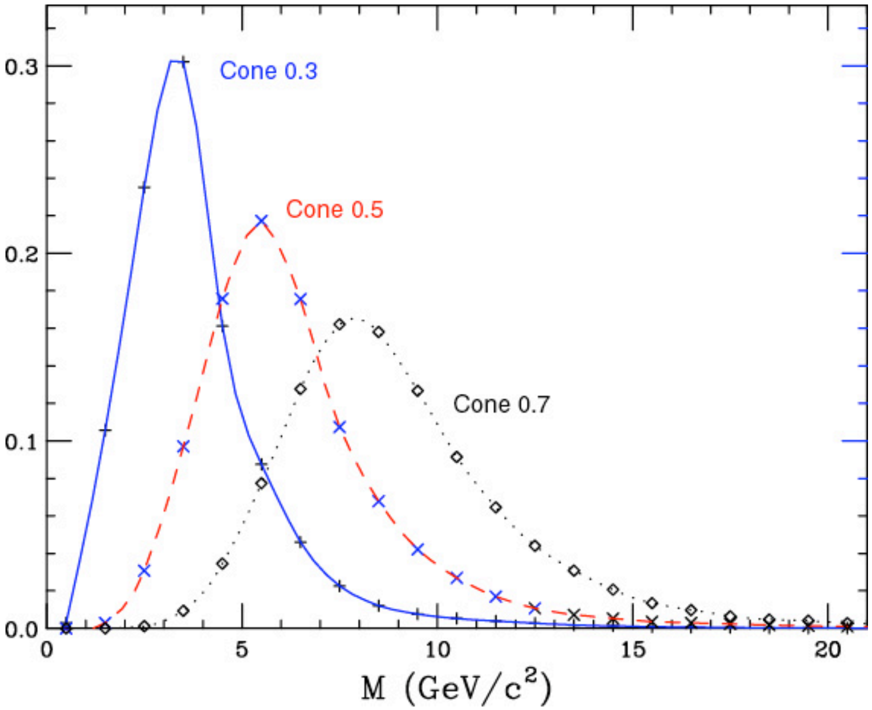
  - This gives $E_{T,jet} - p_{T,jet} = \frac{1}{2}\sum E_{T,i}\left(\delta\eta_i^2 + \delta\phi_i^2\right)$ $E_{T,jet} + p_{T,jet} = \frac{1}{2}\sum E_{T,i}\left(4 + \delta\eta_i^2 - \delta\phi_i^2\right) \approx 2\sum E_{T,i}$

Jet mass is related to jet shape!!! (in the thin jet, high energy limit)

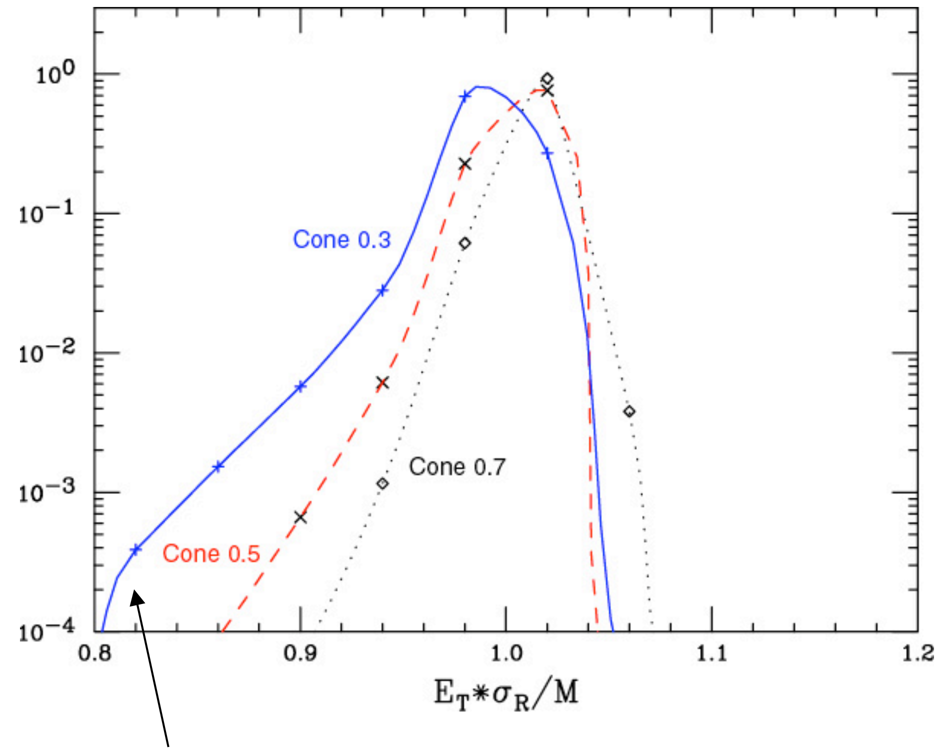
so…. $M_{jet}^2 = \sum E_{Ti} \sum E_{Ti}\left(\delta\eta_i^2 + \delta\phi_i^2\right) \to M_{jet} \cong E_{T,jet}\sigma_R$ using $E_{T,jet} \cong \sum_{particles} E_{T,i}$

Jet Mass (cont)

- Jet Mass for unmerged sample



How good is “thin jet” approximation?



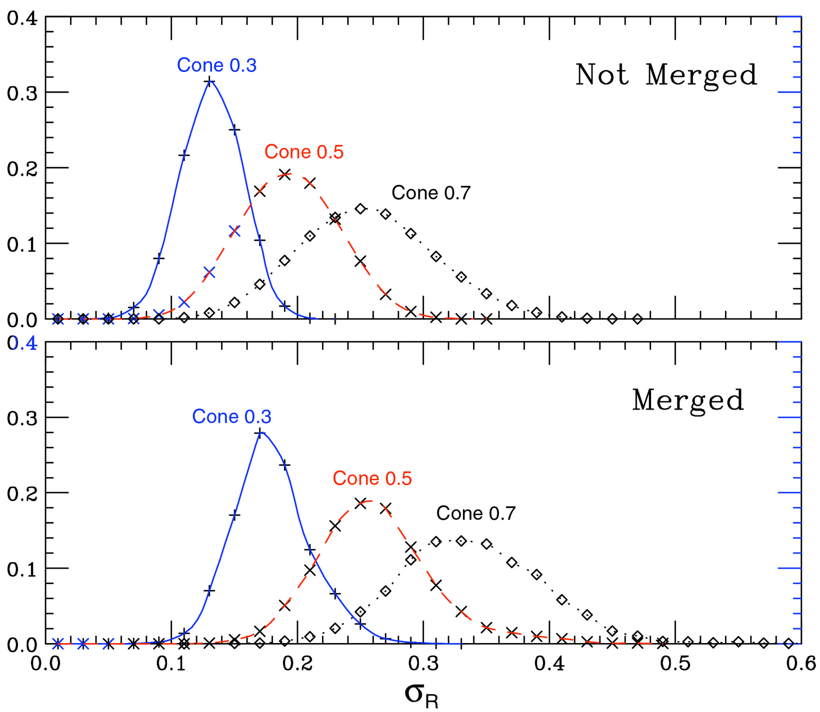
Low-side tail is due to lower E_T jets for smaller cones

(this sample has 2 and only 2 jets for all cones)

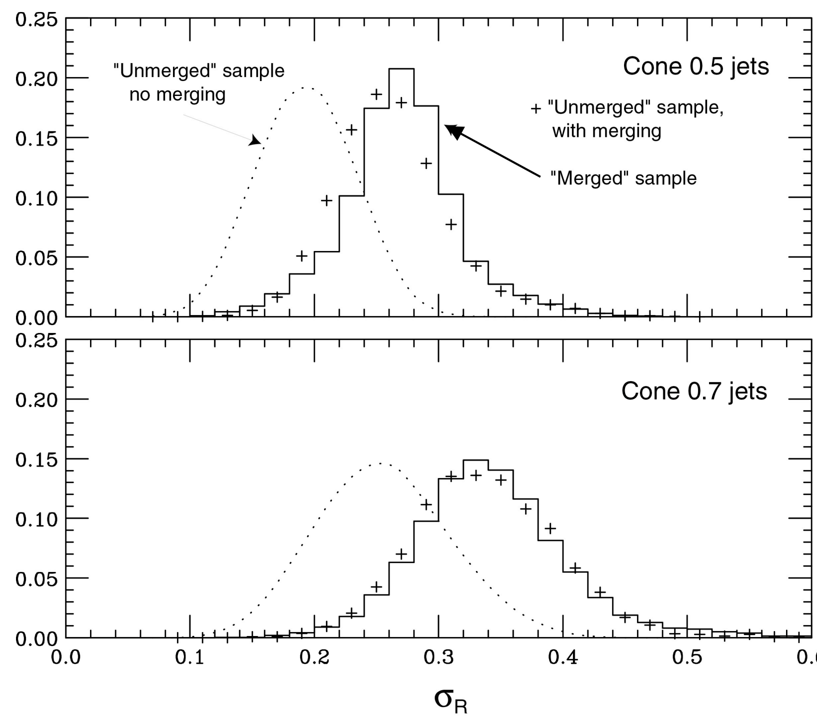
Jet Merging

- Does jet merging matter for physics?
  - For some inclusive QCD studies, it doesn't matter
  - For invariant mass calculations from *e.g.* top→Wb, it will smear out mass distribution if merging two "tree-level" jets that happen to be close
- Study σ_R…see clear correlation between σ_R and whether jet is merged or not
  - Can this be used to construct some kind of likelihood?

"Unmerged", Jet Algorithm reports merging, all cone sizes



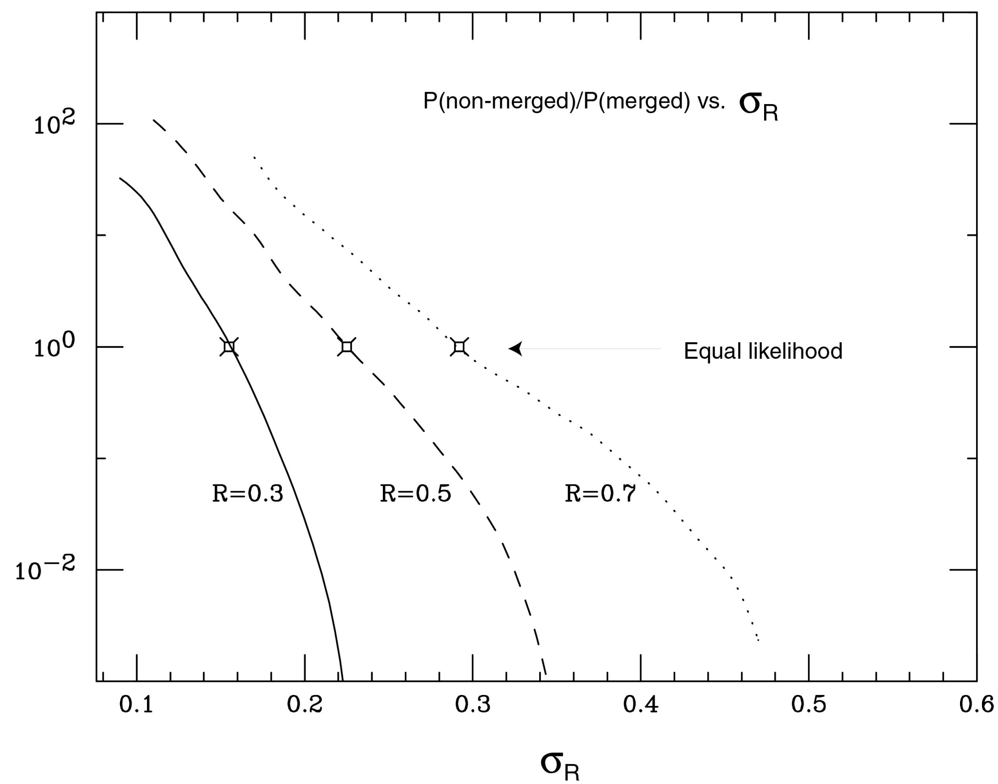
"Unmerged" v. "Merged" sample



Merging Likelihood

- Crude attempt at a likelihood
  - Can see that for this (biased) sample, can use this to pick out “unmerged” jets based on shape
  - Might be useful in Higgs search for $H \rightarrow b\bar{b}$ jet invariant mass?

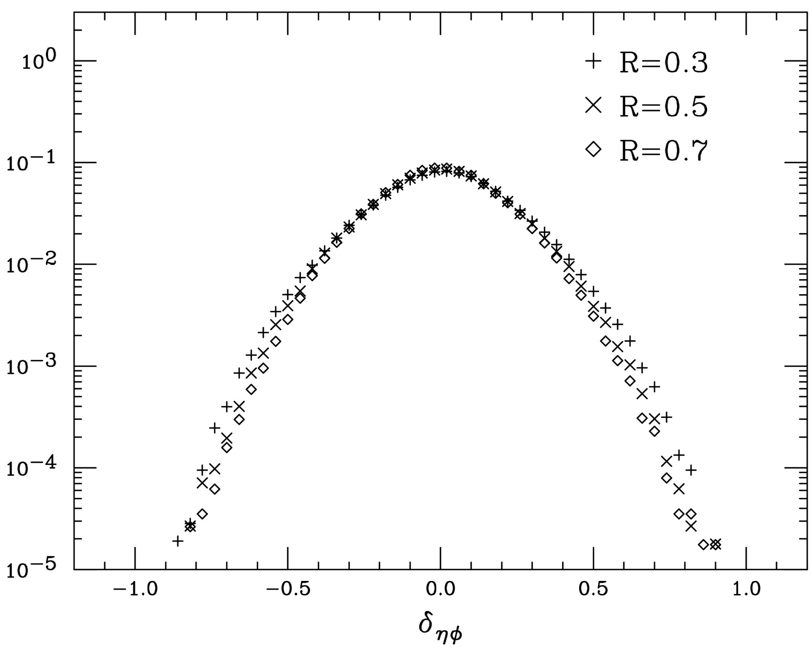
Jet cone parameter	Equal likelihood to be merged and unmerged
0.3	0.155
0.5	0.244
0.7	0.292



Merged Shape

- Width in ηϕ $\sigma_R^2 = \sigma_{\eta\eta} + \sigma_{\phi\phi}$ "assumes" circular
  - Large deviations due to merging?
  - Define $\delta_{\eta\phi} \equiv \dfrac{\sigma_{\eta\eta} - \sigma_{\phi\phi}}{\sigma_{\eta\eta} + \sigma_{\phi\phi}}$ should be independent of cone size
    - Clear broadening seen – "cigar"-shaped jets, maybe study… $\sigma_{\phi\eta} \equiv \dfrac{\sum_{particles} E_{T,i}^2 \delta\phi_i \delta\eta_i}{\sum_{particles} E_{T,i}^2}$

"Unmerged" Sample



"Merged" Sample