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Pontecorvo neutrino oscillations [1]

Salvatore Esposito [2]

Dipartimento di Scienze Fisiche, Università di Napoli "Federico II"
and
Istituto Nazionale di Fisica Nucleare, Sezione di Napoli
Mostra d'Oltremare Pad. 20, I-80125 Napoli Italy

Abstract

The theory of neutrino oscillations for Dirac, Majorana and Dirac-Majorana neutrinos is reviewed, illustrating the contribution of B. Pontecorvo and further generalizations.

1 Introduction

The existence of neutrino oscillations was postulated for the first time in 1957 by B. Pontecorvo [1] following the idea of a mixing between the K^0 and $\overline{K}^0$ mesons. He assumed that ν and $\overline{\nu}$ emitted in the processes $p \to n\, e^+\, \nu$ and $n \to p\, e^-\, \overline{\nu}$, respectively, are not identical and that the leptonic number is not a conserved quantity; thus the reactions $p \to n\, e^+\, \overline{\nu}$ and $n \to p\, e^-\, \nu$ can take place as well, although with a lower probability. Then, if in vacuum a ν can transform itself into a $\overline{\nu}$, in general the weak interaction states ν, $\overline{\nu}$ are linear combinations of the two mass eigenstates ν_1, ν_2, and neutrinos results to be Majorana particles.

More than 10 years later, Gribov and Pontecorvo [2] introduced flavour oscillations for Majorana neutrinos, developing quantitatively the standard theory of neutrino oscillations, which was later generalized to cover the case of Dirac neutrinos [3].

The problem on the carpet is that of neutrino mass and the related one of lepton number conservation (both individual (L_e, L_μ, L_τ) and total (L)). If neutrinos are massless, the conservation of all lepton numbers (L_e, L_μ, L_τ, L) is allowed, according to the Glashow-Weinberg-Salam theory, due to the invariance of the electroweak lagrangian for un arbitrary (global) phase transformation of the matter field. Instead, if neutrino have a non-vanishing mass and are of the

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[2] e-mail: sesposito@na.infn.it

Dirac type (particle states different from the antiparticle ones), so that to the e.w. lagrangian the following mass term is added :

$$\mathcal{L}_m^D \;=\; -\sum_{l,l'} \overline{\nu}_{l'R}\, M^D_{l'l}\, \nu_{lL} \;+\; h.c. \tag{1}$$

with $l, l' \;=\; e, \mu, \tau$ and M^D, in general, a complex non diagonal 3x3 matrix, individual lepton number is no longer conserved (L_m^D is not invariant for $\nu_l \rightarrow e^{i\alpha_l}\nu_l$), while the total lepton number is still conserved (L_m^D is invariant for $\nu_l \rightarrow e^{i\alpha}\nu_l$). However, if neutrinos are massive Majorana particles (particle states coincide whith antiparticles ones), so that the mass term is

$$\mathcal{L}_m^M \;=\; -\,\frac{1}{2}\,\sum_{l,l'} \overline{\nu}^c_{l'R}\, M^M_{l'l}\, \nu_{lL} \;+\; h.c. \tag{2}$$

with M^M, in general, a symmetric 3x3 matrix, no lepton number is conserved (L_m^M is not invariant for every (global) phase transformation). Note that, while the term (2) does not require additional neutrino states besides those present in the Standard Model, the term (1) involves right-handed neutrino states (and related antiparticles) not present in tha S.M. but predicted in many GUTs (see for example [4]).

The phenomenological consequences of eq. (1) or (2) are very intriguing.

For istance, if M^D and M^M are non diagonal (in analogy to what happens with the quark mass matrix), neutrino flavour oscillations are predicted [2], [3]. But, furthermore, many other processes involving charged lepton, which violate lepton number conservation, are allowed. For example, with Dirac neutrinos, the decays $\mu \rightarrow e\gamma$, $\mu \rightarrow 3e$, $\tau \rightarrow e\pi^0$ and the conversion μ-e in presence of nuclei, such as $\mu^- + Ti \rightarrow e^- + Ti$, can be realized, while with Majorana neutrinos neutrinoless double beta decay (Z,A) $\rightarrow$ (Z+2,A) + $2e^-$ and reaction μ^- + Ti $\rightarrow e^+$ + Ca can also occur.

More in general, one can consider a lagrangian mass term whith both Dirac and Majorana terms

$$-\mathcal{L}_m^{DM} \;=\; \sum_{l,l'} \overline{\nu}_{l'R}\, M^D_{l'l}\, \nu_{l'L} \;+\; \frac{1}{2}\,\sum_{l,l'} \overline{\nu}^c_{l'R}\, M^1_{l'l}\, \nu_{lL} \;+\; \frac{1}{2}\,\sum_{l,l'} \overline{\nu}^c_{l'R}\, M^2_{l'l}\, \nu_{lR} \;+\; h.c. \tag{3}$$

involving ν_L and ν_R^c, as well as ν_R and ν_L^c predicted in many GUTs. This scenario, from a theoretical point of view, allows to give a very small mass to neutrinos in a very natural way through the so-called "see-saw" mechanism [5]. The mass eigenstates coming from (3) are in general Majorana states, and the phenomenology previous described in the discussion of eq. (2) applies in this case as well. However, as we will show later, in this case also active - sterile neutrino - antineutrino oscilaltions can take place.

In the following section, we will review the theory of flavour oscillations for Dirac and Majorana neutrinos both in the vacuum case and for propagation in a medium. In section 3 we will instead discuss the case of Dirac-Majorana neutrinos and re-derive the Pontecorvo oscillation formula. Finally, our conclusions will follow.

2 Flavour transitions

2.1 Vacuum oscillations

Let us consider Dirac neutrinos described by the mass term in (1); for the sake of simplicity, we will discuss only the two-flavour case, $l\,,\, l' \,=\, e,\, \mu$. We can then rewrite eq. (1) in the simple compact form

$$- \mathcal{L}_m \;=\; \overline{\nu}\, M\, \nu \tag{4}$$

with

$$\nu \;=\; \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} \qquad , \qquad M \;=\; \begin{pmatrix} m_{ee} & m_{e\mu} \\ m_{e\mu} & m_{\mu\mu} \end{pmatrix} \quad . \tag{5}$$

Note that we have dropped out the superscript D, since the following will hold (*mutatis mutandis*) also for Majorana neutrinos described by the term in (2). This is because we will limit ourselves to the ultrarelativistic case, and flavour oscillations do not distinguish between Dirac or Majorana mass terms (at least in the two-flavour case, neglecting CP violating effects; see ref. [6]). We have also suppressed the chirality indices R, L and grouped right-handed and left-handed spinors in the usual Dirac bispinors. In the ultrarelativistic limit, this is unambiguous, since chirality transitions (in vacuum) are suppressed (for these, see for example [7], [8]) and flavour transitions between (sterile) right-handed states or (active) left-handed ones proceed in the same way.

Now, in the flavour eigenstate basis the evolution equation is given by

$$i \frac{d}{dt} \nu \;=\; H\, \nu \tag{6}$$

where, in the ultrarelativistic limit, the hamiltonian is

$$H \;\simeq\; k \,+\, \frac{M^2}{2k} \tag{7}$$

with k being the neutrino momentum. We can now solve eq. (6) diagonalizing the hamiltonian by means of a unitary matrix U, $UU^T \,=\, 1$;

$$i \frac{d}{dt} U^T \nu \;=\; U^T H\, U \;\; U^T \nu \tag{8}$$

implies

$$i \frac{d}{dt} \nu_m \;=\; H_m\, \nu_m \quad . \tag{9}$$

Then, the mass eigenstates basis is defined by

$$\begin{aligned} \nu_m &= U^T \nu & (10) \\ H_m &= U^T H U & (11) \end{aligned}$$

where the mixing matrix U, in the simple two-flavour case, can be parametrized as

$$U = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \quad . \tag{12}$$

By requiring H_m, and then

$$U^T M U = = \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix} \quad , \tag{13}$$

to be diagonal, we can deduce the expression of the mixing angle θ in terms of the mass parameters:

$$\tan 2\theta = \frac{2\, m_{e\mu}}{m_{ee} - m_{\mu\mu}} \quad . \tag{14}$$

The time evolution of the flavour states is now given; for example we have

$$\begin{aligned} |\nu_e(t)> &= \left(\cos^2\theta\, e^{-i(k+\frac{m_1^2}{2k})t} + \sin^2\theta\, e^{-i(k+\frac{m_2^2}{2k})t} \right) |\nu_e(0)> + \\ &+ \sin\theta\cos\theta \left(-e^{-i(k+\frac{m_1^2}{2k})t} + e^{-i(k+\frac{m_2^2}{2k})t} \right) |\nu_\mu(0)> \quad . \end{aligned} \tag{15}$$

From this we see, for example, that even if we start with a pure ν_e beam, at later times we nevertheless have also a ν_μ component in the same beam. The transition probability for these flavour oscillations is given by

$$P\,(\nu_e \to \nu_\mu) = |< \nu_\mu | \nu_e(t) >|^2 = \sin^2 2\theta \sin^2 \left(\frac{\Delta m^2}{4k} t \right) \tag{16}$$

with $\Delta m^2 = m_2^2 - m_1^2$. If $x \simeq t$ is the source-detector distance, the transition probability can be written as

$$P\,(\nu_e \to \nu_\mu) = \sin^2 2\theta \sin^2 \frac{\pi x}{L_0} \tag{17}$$

where

$$L_0 = 2\pi \frac{2k}{\Delta m^2} \simeq 2.48 \left(\frac{k}{1\, MeV} \right) \left(\frac{1\, eV^2}{\Delta m^2} \right) m \tag{18}$$

is the oscillation length (in vacuum) over which the phenomenon of flavour transition $\nu_e \to \nu_\mu$ occurs. From (18) it is easy to see the range of the parameter Δm^2 that can be probed by a given experiment. For example, for $k \sim 1\, MeV$, $L_0 \sim 10^{11}\, m$ (typical Sun - Earth distance) corresponds to $\Delta m^2 \sim 10^{-11}\, eV^2$ while for $k \sim 10\, GeV$, $L_0 \sim 10\, Km$ (typical distance crossed by atmosperic neutrinos in the Earth atmosphere) corresponds to a $\Delta m^2 \sim 1\, eV^2$.

2.2 Matter oscillations

Let us now discuss what happens to flavour oscillations when neutrinos propagate in a given dense medium.
In general, the evolution equation (6) still holds but the hamiltonian will contain a term describing neutrino interactions with the particles in the medium,

$$H \; = \; k + \frac{M^2}{2k} + V \quad , \tag{19}$$

where V is an effective potential experienced by neutrinos during their travel in matter [9], which is obviously diagonal in the flavour eigenstate (weak interaction eigenstate) basis,

$$V \; = \; \left(\begin{array}{cc} V_{\nu_e} & 0 \\ 0 & V_{\nu_\mu} \end{array} \right) \quad . \tag{20}$$

Sterile states experience no effective potential (due to lack of weak interaction); for the active left-handed neutrino states we have [9]

$$V^0_{\nu_e} \; = \; \sqrt{2}\, G_F \left(N_e - \frac{1}{2} N_n \right) \tag{21}$$

$$V^0_{\nu_\mu} \; = \; - \frac{G_F}{\sqrt{2}} N_n \tag{22}$$

for normal media, while for magnetized matter [10]

$$V^B_{\nu_e} \; = \; V^0_{\nu_e} \tag{23}$$

$$V^B_{\nu_\mu} \; = \; V^0_{\nu_\mu} - \frac{e\, G_F}{\sqrt{2}} \frac{(3\pi^2 N_e)^{1/3}}{\pi^2} \frac{\mathbf{k}\cdot\mathbf{B}}{k} \tag{24}$$

for a degenerate Fermi gas, and

$$V^B_{\nu_e} \; = \; V^0_{\nu_e} - \frac{3e\, G_F}{4\sqrt{2}} \frac{N_e}{m_e^2} \frac{\mathbf{k}\cdot\mathbf{B}}{k} \tag{25}$$

$$V^B_{\nu_\mu} \; = \; V^0_{\nu_\mu} + \frac{3e\, G_F}{4\sqrt{2}} \frac{N_e}{m_e^2} \frac{\mathbf{k}\cdot\mathbf{B}}{k} \tag{26}$$

for a classical plasma. Here G_F is the Fermi coupling constant, $\mathbf{B}$ is the applied magnetic field and N_e,N_n are the electron and neutron number density of the medium, respectively. The solution of the evolution equation can be obtained in a way analogous to the one envisaged in the previous subsection, but now introducing the matter mass eigenstate basis:

$$\widetilde{\nu}_m \; = \; \widetilde{U}^T \nu \tag{27}$$

$$\widetilde{H}_m \; = \; \widetilde{U}^T H \, \widetilde{U} \tag{28}$$

where the effective mixing matrix in matter $\widetilde{U}$ has the same form as in (12) but with an effective mixing angle given by

$$\sin 2\theta_m = \frac{\frac{\Delta m^2}{2k} \sin 2\theta}{\sqrt{\left(\frac{\Delta m^2}{2k} \cos 2\theta + V_{\nu_e} - V_{\nu_\mu}\right)^2 + \left(\frac{\Delta m^2}{2k} \sin 2\theta\right)^2}} \quad . \tag{29}$$

The transition probability for flavour oscillations in a constant density medium has the same form of (17),

$$P_m (\nu_e \to \nu_\mu) = \sin^2 2\theta_m \sin^2 \frac{\pi x}{L_m} \tag{30}$$

but now the oscillation length in matter is

$$L_m = L_0 \frac{\sin 2\theta_m}{\sin 2\theta} \quad . \tag{31}$$

Note that the amplitude of the transition probability, given by $\sin^2 2\theta_m$, can reach a maximum even if the vacuum mixing angle θ is small. This is the case if the following *resonance condition* [9]

$$\frac{\Delta m^2}{2k} \cos 2\theta = V_{\nu_\mu} - V_{\nu_e} \tag{32}$$

is satisfied. This relation depend only on the difference between the two effective potentials, and it can be realized by neutrinos *or* antineutrinos but not by both, due to the fact that $V_{\overline{\nu}} - - V_\nu$ [11]. As a matter of example, for solar neutrinos with $\Delta m^2 \sim 10^{-5}\, eV^2$ the resonance condition (32) is satisfied when the density reaches the value $\rho \sim 10\, g\, cm^{-3}$.

3 Oscillations of Dirac-Majorana neutrinos

For Dirac-Majorana neutrinos described by the mass term in (3), not only flavour oscillations but also other types of transitions, as active-sterile neutrino-antineutrino oscillations, can take place. For illustrative purposes, let us consider one flavour at a time (i.e. suppose that the mass matrices in (3) are diagonal in the flavour basis) and take equal Majorana masses for the left-handed and right-handed neutrino, assuming a minimal choice for the mass parameters. In this case we have

$$-\mathcal{L}_m^{DM} = \frac{1}{2} \left(\overline{\nu_L} \, , \, \overline{\nu_L^C}\right) \begin{pmatrix} m_D & m_M \\ m_M & m_D \end{pmatrix} \begin{pmatrix} \nu_L \\ \nu_L^C \end{pmatrix} = \overline{\nu}\, M\, \nu \tag{33}$$

where we have restored the subscripts D, M and L to avoid confusion. The evolution equation for this neutrino system is again given by (6), (7) but with the mass matrix M in (33). A straigthforward calculus now leads to the following survival probability for a ν_L against $\nu_L \to \nu_L^C$ transitions:

$$P\left(\nu_L \to \nu_L\right) \;=\; 1 \;-\; \sin^2 \frac{m_D\, m_M}{4k}\, t \quad . \tag{34}$$

This expression is known as the Pontecorvo oscillation formula. It was proposed in 1957 as the first neutrino oscillation formula [1]; since neutrinos are chiral particles (differently from K^0 mesons) the only allowed antineutrino state in which a ν_L can oscillate (without suppression factors) is a sterile (under weak interactions) ν_L^C, but this was not yet realized in 1957.
Note that both m_D and m_M have to be non zero for having non vanishing $\Delta L \,=\, 2$ (active-sterile) neutrino-antineutrino transitions.

Differently to what happens for flavour oscillations, for neutrinos propagating in a medium no resonance condition exists for this type of transitions. In fact, the amplitude of oscillations is already at a maximum in vacuum, and the interaction with the medium can only alter the oscillation length. Matter oscillation can be studied analogously to the flavour transition case by means of the effecitve potential in (21)-(26). For the transition probability in matter the result (in the ultrarelativistic limit) is [7]

$$P_m\left(\nu_L \to \nu_L^C\right) \;=\; \sin^2 \frac{m_D\, m_M}{4k} \left(1 \;-\; \frac{V}{2k}\right) t \tag{35}$$

where V is the appropriate effective potential.
The corrections due to matter interaction are quite insignificant, because of the very smallness of the effective potential with respect to neutrino momentum in normal situations. Nevertheless, they can be important for relic neutrinos [12] propagating in very dense stars, as in the study developed in [13] (however, for the last case, relation (35) would not apply because of the non relativistic propagation; the right expression obtained relaxing the assumption of ultrarelativistic propagation can be found in [7]).

4 Outlook

In this talk, the theory of neutrino oscillations has been reviewed, illustrating the contribution of B. Pontecorvo and further generalizations. We have considered flavour oscillations in vacuum as well as in matter for Dirac and Majorana neutrinos which violates the individual lepton numbers but preserve the total one L. We have then focused on the $\Delta L \,=\, 2$ oscillations of Dirac-Majorana neutrinos converting an active neutrino state into a sterile antineutrino one (without flavour change). Differently to what happens for flavour transitions, for which the presence of a dense medium can induce a resonant enhancement of the oscillations, matter corrections to the $\Delta L \,=\, 2$ transition are unimportant in many relevant situations. However this is not the case, in general, for flavour changing $\Delta L \,=\, 2$ transitions (such as, for example, $\nu_{eL} \to \nu_{\mu L}^C$), which are now currently under study [8].

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