

# A peculiar lecture by Ettore Majorana

## S Esposito

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### Abstract

We give, for the first time, the English translation of a manuscript by Ettore Majorana, which probably corresponds to the text for a seminar lecture delivered at the University of Naples in 1938, where he lectured on theoretical physics. Some passages reveal a physical interpretation of quantum mechanics which anticipates for several years the Feynman approach in terms of path integrals, independent of the underlying mathematical formulation.

From January to March 1938, Ettore Majorana delivered his only lectures on quantum mechanics at the University of Naples, where he obtained a position as a full-time professor of theoretical physics, a few months before he mysteriously disappeared. Although the scientific personality of such a physicist is widely recognized, here we only mention that Majorana obtained that position in the 1937 Italian national competition for three full professorships for 'great and well-deserved fame', independent of the competition itself, upon recommendation of the judging committee chaired by Enrico Fermi [1].

The interest in the course on theoretical physics, delivered by Ettore Majorana at the University of Naples in 1938, has been recently revived by the discovery of the Moreno paper [2], which is a faithful transcription of the lecture notes prepared by Majorana himself made by the student Eugenio Moreno<sup>1</sup>. Such a paper, in fact, includes some previously unknown lecture notes, whose original manuscripts seem to be missing. The handwritten notes by Majorana were reproduced some years ago [3] along with some papers initially interpreted as the notes prepared for a forthcoming lecture that, however, Majorana never delivered due to his mysterious disappearance.

Here, however, we do not focus on the university lecture notes (delivered to students) but, rather just on the last mentioned spare papers that cannot be considered as notes for academic lectures, even for an advanced course such as that by Majorana. This is testified mainly by an accurate analysis of the scientific content present in those notes, referring to

<sup>1</sup> This document is owned by the Moreno family in Naples and, at the moment, is kept by the present author.

advanced topics not at all related to those treated by Majorana in his last lectures, where he only introduced the foundations of wave mechanics and its probabilistic interpretation. In contrast, some accurate knowledge of particular arguments (such as the theory of molecular bonding) is required, and these cannot be regarded as direct applications of the novel formalism just introduced. On the other hand, the style adopted by Majorana in writing these notes is completely different from that used for the lectures addressed to the students, where the mathematical expression of physical concepts plays a relevant role. Rather, its conversational form is closer to that generally employed in the inaugural lecture, which was mainly addressed to the academic staff of the University of Naples [2]. In particular, even though not the whole set of topics (i.e. the applications) reported at the beginning of the considered manuscript was effectively discussed, the closing sentence is quite imperative and typical of a general conference: “Quantum Mechanics opens the road to the logic unification of all the sciences having the inorganic world as a common object of study”.

Although no firm documentary credits exist at present, for the sake of definiteness we regard the considered notes as prepared (by Majorana for his own personal use) for a general seminar or conference held at the University of Naples, likely at the end of January 1938, soon after the arrival of Majorana in Naples. This seminar was probably solicited by the Director of the Institute of Physics, Antonio Carrelli, who worked in that period on some experimental topics related to molecular physics, since the main theme of that conference appears exactly to be the theoretical interpretation of the molecular bonding in the framework of quantum mechanics.

The present scientific interest in that dissertation, however, is not just related to the theory of molecular bonding, but rather it is more centred on the interpretation given by Majorana about some topics of the novel, for that time, quantum theory (namely, the concept of quantum state), although direct applications of this theory to a particular case (that is, precisely, the molecular bonding) are considered. An accurate reading of the manuscript, in fact, not only discloses a peculiar cleverness of the author in treating a pivotal argument of the novel mechanics, but, keeping in mind that it was written in 1938, also reveals a net advance of at least ten years in the use made of that topic. The last point, however, is quite common for Majorana, and as only one example we refer here to the case of the Thomas–Fermi atomic model [4] (see, however, also [5]).

This point was already noted some years ago by N Cabibbo (in [3]), who saw in the Majorana manuscript a vague and approximate anticipation of the idea underlying the Feynman interpretation of quantum mechanics in terms of path integrals [6]. A more analytic study, conducted on the critical edition of that paper [7] which was not available at that time, reveals instead some intriguing surprises which we will here focus on.

In the present work we report, for the first time, an English translation (from the Italian) of the mentioned notes. These are contained into a manuscript (kept at the Domus Galilaeana in Pisa) which has been reproduced in an anastatic reprint from the original handwritten paper in [3], and whose importance has been discovered only very recently [8]. Although a historical interest is, without doubt, behind the presentation of an unpublished paper by Majorana, in the present case our aim is also to present the powerful didactic method used by the author on the given subject. Even after a very long time, in fact, his peculiar presentation of given quantum-mechanical problems could be very useful to people *now* teaching those topics (or some others related to them) and, based upon personal experience, the Majorana paper could also be of some direct benefit to (and easily understood by) students.

Since the text considered is written in a very simple and clear form (a feature which is very common in the Majorana works [5]), in the following we will only stress on few interesting points.

The starting point in Majorana's paper is to search for a meaningful and clear formulation of the concept of quantum state. This is achieved by considering some sets of 'solutions that differ for the initial conditions' which, in the Feynman language of 1948 [6], correspond precisely to the different integration paths. In fact, the different initial conditions are, in any case, always referred to the same initial time, while the determined quantum state corresponds to a fixed end time.

Moreover, the crucial point in the Feynman formulation of quantum mechanics, namely that of considering not only the paths corresponding to classical trajectories, but *all* the possible paths joining the initial point with the end one, is introduced in the Majorana manuscript after a discussion on an interesting example of the harmonic oscillator. Quite explicitly, in fact, the author points out that the wavefunction "corresponds in quantum mechanics to any possible state of the electron". Such a reference, which only superficially could be interpreted, in the common acceptance, that all the information on the physical systems is contained in the wavefunction, should instead be considered in the meaning given by Feynman, according to the comprehensive discussion made by Majorana on the concept of state.

Finally, we also stress the key role played by the symmetry properties of the physical system in the Majorana analysis; a feature which is, again, quite common in papers of this author.

Summing up, it is without doubt that no trace can be found of the formalism underlying the Feynman path-integral approach to quantum mechanics in the Majorana manuscript (contrary to what happens for a paper by Dirac in 1933 [9], probably known to Majorana). Nevertheless, it is very interesting that the main physical items, about the novel way of interpreting the theory of quanta, were realized well in advance by Majorana. And this is particularly impressive if we take into account that, in the known historical path, the interpretation of the formalism has only followed the mathematical development of the formalism itself.

Furthermore, in the Majorana paper, several interesting applications to atomic and molecular systems are present as well, where known results are deduced or reinterpreted according to the novel point of view. The search for such applications, however, will be left to the reader, who will benefit from the reading of the complete text by Majorana reported in the following<sup>2</sup>.

## The text by Majorana

### *On the meaning of quantum state*

The internal energy of a closed system (atom, molecule, etc) can take, according to quantum mechanics, discrete values belonging to a set  $E_0, E_1, E_2, \dots$  composed of the so-called energy 'eigenvalues'. To each given value of the energy we can associate a 'quantum state', which is a state where the system may remain indefinitely without external perturbations. As an example of these perturbations, we can in general consider the coupling of the system with the radiation field, by means of which the system may loose energy in the form of electromagnetic radiation, jumping from an energy level  $E_k$  to a lower one  $E_i < E_k$ . Only when the internal energy takes the minimum value  $E_0$ , it cannot be further decreased by means of radiation; in this case the system is said to be in its 'ground state' from which it cannot be removed without sufficiently strong external influences, such as the scattering with fast particles or with light quanta of large frequency.

<sup>2</sup> The original manuscript by Majorana, as it can be seen from [3], reports (at the beginning) a sort of table of contents which, however, is only partially followed by the author. For the sake of ease, we have preferred to divide the whole text in some sections, according to the reported table of contents.

What is the corresponding concept of quantum state in classical mechanics? An answer is primarily required to such a question, in order to have a correct representation of the results obtained in our field by quantum mechanics, without entering, however, in the complex computational methods adopted by this.

In classical mechanics the motion of a system composed of  $N$  mass points is entirely determined when the coordinates  $q_1 \cdots q_{3N}$  of all the points are known as a function of time:

$$q_i = q_i(t). \quad (1)$$

Equation (1) gives the dynamical equations where all the internal and external forces acting on the system are present, and they can always be chosen in such a way that at a given instant all the coordinates  $q_i(0)$  and their time derivatives  $\dot{q}_i(0)$  take arbitrarily fixed values. Thus the general solution of the equations of motion must depend on  $2 \cdot 3N$  arbitrary constants.

For a system with atomic dimensions, the classical representation no longer holds and two successive modifications have been proposed. The first one, due to Bohr and Sommerfeld and which has provided very useful results, has been completely abandoned afterwards with the emergence of the novel quantum mechanics, which has been the only one to give an extremely general formalism, fully confirmed by the experiences on the study of the elementary processes. According to the old theory of Bohr–Sommerfeld, classical mechanics still holds in describing the atom, so that the motion of an electron, for example, around the hydrogen nucleus is still described by a solution (1) of the equations of classical mechanics; however, if we consider periodic motions, such as the revolution of an electron around the nucleus, not all the solutions of the classical equations are realized in nature, but only a discrete infinity of those satisfying the so-called Sommerfeld conditions, that is, certain cabalistic-like integral relations. For example, in every periodic motion in one dimension the integral of the double of the kinetic energy over period  $\tau$ ,

$$\int_0^\tau 2T(t) dt = nh,$$

must be an integer multiple of the Planck constant ( $h = 6.55 \times 10^{-27}$ ). The combination of classical mechanics with a principle which is unrelated with it, such as that of the quantized orbits, appears so hybrid that the complete failure of that theory occurred in the last decade should not be surprising, irrespective of several favourable experimental tests which were supposed to be conclusive.

The novel quantum mechanics, primarily due to Heisenberg, is substantially more closed to the classical conceptions than the old one. According to the Heisenberg theory, a quantum state corresponds not to a strangely privileged solution of the classical equations but rather to a set of solutions which differ for the initial conditions and even for the energy, i.e. what is meant as precisely defined energy for the quantum state corresponds to a sort of average over the infinite classical orbits belonging to that state. Thus the quantum states come to be the minimal statistical sets of classical motions, *slightly different* from each other, accessible to the observations. These minimal statistical sets cannot be further partitioned due to the uncertainty principle, introduced by Heisenberg himself, which forbids the precise simultaneous measurement of the position and the velocity of a particle, which is the determination of its orbit.

A harmonic oscillator with frequency  $\nu$  can oscillate classically with arbitrary amplitude and phase, with its energy given by

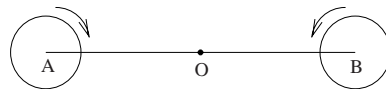
$$E = 2\pi^2 m \nu^2 A_0^2,$$

where  $m$  is its mass and  $A_0$  the maximum elongation. According to quantum mechanics the possible values for  $E$  are, as is well known,  $E_0 = \frac{1}{2}h\nu$ ,  $E = \frac{3}{2}h\nu$ ,  $\dots$   $E_n = (n + \frac{1}{2})h\nu \dots$ ;

in this case, we can say that the ground state with energy  $E_0 = \frac{1}{2}h\nu$  corresponds roughly to all the classical oscillations with energy between 0 and  $h\nu$ , the first excited state with energy  $E_1 = \frac{3}{2}h\nu$  corresponds to the classical solutions with energy between  $h\nu$  and  $2 \cdot h\nu$ , and so on. Obviously, the correspondence between quantum states and sets of classical solutions is only approximate, since the equations describing the quantum dynamics are in general independent of the corresponding classical equations, but denote a real modification of the mechanical laws, as well as a constraint on the feasibility of a given observation; however, it is better founded than the representation of the quantum states in terms of quantized orbits, and can be usefully employed in qualitative studies.

#### *Symmetry properties of a system in classical and quantum mechanics*

Systems showing some symmetry property deserve a particular study. For these systems, due to symmetry considerations alone, from one particular solution of the classical equations of motion  $q_i = q_i(t)$  we can deduce, in general, some other different ones  $q'_i = q'_i(t)$ . For example, if the system contains two or more electrons or, in general, two or more identical particles, from one given solution we can obtain another solution, which in general will be different from the previous one, just by changing the coordinates of two particles. Analogously, if we consider an electron moving in the field of two identical nuclei or atoms (denoted as  $A$  and  $B$  in the figure), starting from an allowed orbit  $q_i = q_i(t)$  described around  $A$  with a given law of motion, we can deduce another orbit  $q'_i = q'_i(t)$  described by the electron around the nucleus or atom  $B$  by a reflection with respect to the centre  $O$  of the line  $AB$ .



The exchange operations between two identical particles, reflection with respect to one point or the others corresponding to any symmetry property, keep their meaning in quantum mechanics. Thus it is possible to deduce from a state  $S$  another one  $S'$ , corresponding to the same known value of the energy, if in the mentioned two examples we exchange two identical particles between them and reflect the system with respect to point  $O$ . However, differently from what happens in classical mechanics for the *single solutions* of the dynamical equations, in general, it is no longer true that  $S'$  will be distinct from  $S$ . We can realize this easily by representing  $S'$  with a set of classical solutions, as seen above; it then suffices that  $S$  includes, for any given solution, even the other one obtained from that solution by applying a symmetry property of the motions of the systems, in order that  $S'$  results to be identical to  $S$ .

In several cases, if the system satisfies sufficiently complex symmetry properties, it is instead possible to obtain, by symmetry on a given quantum state, other different states but with the same energy. In this case the system is said to be *degenerate*, i.e. it has many states with the same energy, exactly due to its symmetry properties. The study of degenerate systems and of the conditions under which degeneration can take place will bring us too far and, in any case, it is difficult to make such a study in terms of only classical analogies. Then we will leave it completely aside and limit our attention to problems without degeneration. This condition is always satisfied if the symmetry of the mechanical system allows only a so simple transformation that its square, that is, the transformation applied twice, reduces to the identity transformation. For example, by a double reflection of a system of mass points with respect to a plane, a line or a point, we necessarily recover the same initial arrangement; analogously, the system remains unaltered by changing twice two identical particles. In all these cases

we have only simple quantum states, i.e. associated with every possible value of the energy is only one quantum state. It follows that all the quantum states of a system containing two identical particles are symmetric with respect to the two particles, remaining unaltered under their exchange. Thus the states of an electron orbiting around two identical nuclei  $A$  and  $B$  are symmetric with respect to the middle point  $O$  of  $AB$ , or remain unaltered by reflection in  $O$ , and analogously for other similar cases. Under given assumptions, which are verified in the very simple problems that we will consider, we can say that every quantum state possesses all the symmetry properties of the constraints of the system.

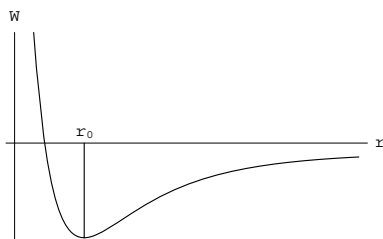
*Resonance forces between states that cannot be symmetrized for small perturbations and spectroscopic consequences. Theory of homeopolar valence according to the method of bounding electrons. Properties of the symmetrized states that are not obtained from non-symmetrized ones with a weak perturbation*

Let us consider an electron moving in the field of two hydrogen nuclei or protons. The system composed of the two protons and the electron has a net resulting charge of  $+e$  and constitutes the simplest possible molecule, that is, the positively ionized hydrogen molecule. In such a system the protons are able to move as well as the electron, but due to the large mass difference between them (mass ratio 1840:1) the mean velocity of the protons is much lower than that of the electron, and the motion of this can be studied with great accuracy by assuming that the protons are at rest at a given mutual distance. This distance is determined, by stability reasons, in such a way that the total energy of the molecule, that at a first approximation, is given by the sum of the mutual potential energy of the two protons and the energy of the electron moving in the field of the first ones, and is different for different electron quantum states, and is at a minimum.

The mutual potential energy of the protons is given by  $\frac{e^2}{r}$  if  $r$  is their distance, while the binding energy of the electron in its ground state is a negative function  $E(r)$  of  $r$  that does not have a simple analytic expression, but it can be obtained from quantum mechanics with an arbitrary large accuracy. The equilibrium distance  $r_0$  is then determined by the condition that the total energy is at a minimum:

$$W(r_0) = \frac{e^2}{r_0} + E(r_0).$$

The curve  $W(r)$  has a behaviour like that shown in the figure, if we assume that zero energy corresponds to the molecule which is dissociated into a neutral hydrogen atom and an ionized atom at an infinite distance. The equilibrium distance



has been theoretically evaluated by Burrau [10] finding  $r_0 = 1.05 \times 10^{-8}$  cm and, for the corresponding energy,  $W(r_0) = -2.75$  eV. Both these results have been fully confirmed by observations on the spectrum emitted by the neutral or ionized molecule, which indirectly depends on them.

What is the origin of the force  $F = +\frac{dW}{dr}$  that tends to get close to the two hydrogen nuclei when they are at a distance larger than  $r_0$ ? The answer given by quantum mechanics is surprising since it seems to show that, in addition to certain polarization forces which can be foreseen by classical mechanics, a predominant role is played by a completely novel kind of forces, the so-called *resonance forces*.

Let us suppose the distance  $r$  to be large with respect to the radius of the neutral hydrogen atom ( $\sim 0.5 \times 10^{-8}$  cm). Then the electron undergoes the action of either of the two protons, and around each of them can classically describe closed orbits. The system composed of an electron and a nucleus around which it orbits forms a neutral hydrogen atom, so that our molecule turns out to be essentially composed of one neutral atom and one proton at a certain distance from the first. The neutral hydrogen atom in its ground state has a charge distribution with spherical symmetry, classically meaning that all the orientations of the electronic orbit are equally possible, and the negative charge density exponentially decreases with the distance in such a way that the atomic radius can practically be considered as finite; it follows that no electric field is generated outside a neutral hydrogen atom, and thus no action can be exerted on a proton at a distance  $r$  which is large compared to the atomic dimensions. However, the neutral atom can be polarized under the action of the external proton and acquire an electric moment along the proton–neutral atom direction, and from the interaction of this electric moment with the non-uniform field generated by the proton comes out an attractive force which tends to combine the atom and the ion in a molecular system.

The *polarization forces*, which can be easily predicted with classical arguments, can give origin by themselves to molecular compounds that, however, are characterized by a pronounced fleetingness. More stable compounds can only be obtained if other forces are considered in addition to the polarization ones. In the polar molecules, composed of two ions of different sign charge, such forces are essentially given by the electrostatic attraction between the ions; for example, the HCl molecule is kept together essentially by the mutual attraction between the  $H^+$  positive ion and the  $Cl^-$  negative one. However, in a molecule composed of two neutral atoms, or by a neutral atom and an ionized one, as in the case of the molecular ion  $H_2^+$ , the *chemical affinity* is essentially driven by the phenomenon of resonance, according to the meaning assumed by this word in the novel mechanics, which has no parallel in classical mechanics.

When we study, from the quantum mechanics point of view, the motion of the electron in the field of the two protons, assumed to be fixed at a very large mutual distance  $r$ , at the first approximation we can determine the energy levels by assuming that the electron should move around the proton in  $A$  (or  $B$ ) and neglecting the influence of the other proton in  $B$  (or  $A$ ), which exerts a weak perturbative action due to its distance. For the lowest energy eigenvalue  $E_0$  we thus obtain a state  $S$  corresponding to the formation of a neutral atom in its ground state consisting of the electron and the nucleus  $A$ , and a state  $S'$  corresponding to a neutral atom composed of the electron and the nucleus  $B$ . Now if we take into account the perturbation that is exerted, in both cases, on the neutral atom by the positive ion, we again find, as long as the perturbation is small, not two eigenvalues equal to  $E_0$  but two eigenvalues  $E_1$  and  $E_2$  which are slightly different from  $E_0$  and both close to this value; however, the quantum states corresponding to them, denoted as  $T_1$  and  $T_2$ , are not separately close to  $S$  and  $S'$ , since, due to the fact that the potential field where the electron moves is symmetric with respect to the middle point of  $AB$ , the same symmetry must be shown, for what stated above, by the effective states  $T_1$  and  $T_2$  of the electron, while it is not separately shown by  $S$  and  $S'$ .

According to the model representation of the quantum states introduced above,  $S$  consists of a set of electronic orbits around  $A$ , and analogously  $S'$  consists of a set of orbits around  $B$ , while the true quantum states of the system  $T_1$  and  $T_2$  each correspond, at the first

approximation for very large  $r$ , for one half to the orbits in  $S$  and for the other half to those in  $S'$ . The computations prove that for sufficiently large nuclear distances the mean value of the perturbed eigenvalue  $E_1$  and  $E_2$  coincides closely to the single unperturbed value  $E_0$ , while their difference is not negligible and has a conclusive importance in the present as well as in infinite other analogous cases of the study of the chemical reactions. We can thus suppose that  $E_1 < E_0$  but  $E_2 > E_0$ , and then  $T_1$  will be the ground state of the electron, while  $T_2$  will correspond to the excited state with a slightly higher energy.

The electron in the  $T_1$  state, as well as in the  $T_2$  state, spends half of its time around the nucleus  $A$  and the other half around the nucleus  $B$ . We can also estimate the mean frequency of the periodic transit of the electron from  $A$  to  $B$  and vice versa, or of the neutral or ionized state exchange between the two atoms, thus finding

$$\nu = \frac{E_2 - E_1}{h}$$

where  $h$  is the Planck constant. For large values of  $r$ ,  $E_2 - E_1$  decreases according to an exponential-like curve and thus the exchange frequency rapidly tends to zero, meaning that the electron which was initially placed around  $A$  remains here for an increasingly larger time, as expected from a classical point of view.

If the electron is in state  $T_1$ , that is, in its ground state, its energy ( $E_1$ ) is lower than that it would have without the mentioned exchange effect between nuclei  $A$  and  $B$ . This occurrence originates a novel kind of attractive forces among the nuclei, in addition to the polarization forces considered above, and is exactly the dominant cause of the molecular bonding.

The resonance forces, as stated, have no analogy in classical mechanics. However, as long as the analogy leading to the correspondence between a quantum state and a statistical set of classical motions can hold, the two states  $T_1$  and  $T_2$ , where the resonance forces have opposite sign too, each are composed identically of half of both the original unperturbed states  $S$  and  $S'$ . This, however, is true only at a certain approximation, that is, exactly at the approximation where we can neglect the resonance forces. For an exact computation taking into account the resonance forces, we must necessarily use quantum mechanics, and thus find a *qualitative* difference in the structure of the two quantum states that manifest itself mainly in the intermediate region between  $A$  and  $B$  through which a periodic transit of the electron between one atom and the other takes place, according to a mechanism that cannot be described by classical mechanics. Such a qualitative difference is purely formal in nature and we can deal with it only by introducing the wavefunction  $\psi(x, y, z)$  that, as known, corresponds in quantum mechanics to any possible state of the electron. The modulus of the square of  $\psi$ , which can also be a complex quantity, gives the probability that the electron lies in the volume unit around a generic point  $x, y, z$ . The wavefunction  $\psi$  must then satisfy a linear differential equation and thus we can always multiply  $\psi$  in any point by a fixed real or complex number of modulus 1; this constraint is required by the normalization condition

$$\int |\psi|^2 dx dy dz = 1$$

which is necessary for the mentioned physical interpretation of  $|\psi|^2$ . The multiplication of  $\psi$  by a constant of modulus 1 leaves unaltered the spatial distribution of the electronic charge, and has in general no physical meaning. Now we will formally define the reflection of a quantum state with respect to the middle point  $O$  between the two nuclei  $A$  and  $B$  directly on the wavefunction  $\psi$ , by setting

$$\psi(x, y, z) = \psi'(-x, -y, -z)$$

in a coordinate frame with the origin in  $O$ . If  $\psi$  should represent a symmetric quantum state, and thus invariant by reflection in  $O$ , the reflected wavefunction  $\psi'$  must have the same physical



meaning of  $\psi$  and thus differ from  $\psi$ , for what stated, for a real or complex constant factor of modulus 1. Moreover, such a constant factor has to be  $\pm 1$ , since its square must give the unity, due to the fact that by a further reflection of  $\psi'$  with respect to the point  $O$  we again obtain the initial wavefunction  $\psi$ .

For all the states of the system we then must have

$$\psi(x, y, z) = \pm\psi(-x, -y, -z)$$

where the + sign holds for a part of them, and the – one for the others. The formal difference between the  $T_1$  and  $T_2$  states considered above consists precisely in the fact that, in the previous equation, the upper sign holds for  $T_1$  while the lower one for  $T_2$ . The symmetry with respect to one point and, in general, any symmetry property, determines a formal splitting of the state of the system into two or more sectors; an important property of this splitting is that no transition between different sectors can be induced by external perturbations respecting the symmetries shown by the constraints of the system. Thus in systems containing two electrons, we have two kinds of non-combinable states which are determined by the fact that the wavefunction, which now depends on the coordinates of both the electrons, remains unaltered or changes its sign by exchanging the two identical particles. In the special case of the helium atom, this gives rise to the well-known spectroscopic appearance of two distinct elements: parahelium and orthohelium.

The theory of the chemical affinity between the neutral hydrogen atom and the ionized one, which we have considered until now, can be extended to the study of the neutral hydrogen molecule and, more generally, of all the molecules resulting from two equal neutral atoms. Instead of only one electron moving around two fixed protons, for the neutral hydrogen molecule we should consider *two* electrons moving in the same field, neglecting at the first approximation their mutual repulsion. The stability of the molecule can then be understood by assuming that each of the two electrons lies in the  $T_1$  state, corresponding to attractive resonance forces. According to F Hund we can state that the hydrogen molecule is kept together by two ‘bounding’ electrons. However, the interaction between the two electrons is so large as to leave only a qualitative explanation for the schematic theory by Hund, but in principle we could predict exactly all the properties of the hydrogen molecule, by solving with a sufficient precision the equations introduced by quantum mechanics. In this way, with appropriate mathematical methods, we can effectively determine the chemical affinity between two neutral hydrogen atoms with only theoretical considerations, and the theoretical value agrees with the experimental one, given the precision of the computation imposed by practical reasons.

For molecules different from the hydrogen one, the theory of the chemical affinity is considerably more complex, due both to the larger number of electrons to be considered and to the Pauli principle, forbidding the simultaneous presence of more than two electrons in the same state; however, the different theories of the chemical affinity proposed in the past few years, each of which has an applicability range more or less large, practically consist in the search for approximated computation methods for a mathematical problem that is exactly determined in itself, and not in the enunciation of novel physical principles.

Then it is possible to bring the theory of the valence saturations back to more general principles of physics. Quantum mechanics opens the road to the logic unification of all the sciences having the inorganic world as a common object of study.

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