PHOTON WAVE MECHANICS: A DE BROGLIE - BOHM APPROACH

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Abstract. We compare the de Broglie - Bohm theory for non-relativistic, scalar matter particles with the Majorana-Römer theory of electrodynamics, pointing out the impressive common peculiarities and the role of the spin in both theories. A novel insight into photon wave mechanics is envisaged.

1. Introduction

Modern Quantum Mechanics was born with the observation of Heisenberg [1] that in atomic (and subatomic) systems there are directly observable quantities, such as emission frequencies, intensities and so on, as well as non directly observable quantities such as, for example, the position coordinates of an electron in an atom at a given time instant. The later fruitful developments of the quantum formalism was then devoted to connect observable quantities between them without the use of a model, differently to what happened in the framework of old quantum mechanics where specific geometrical and mechanical models were investigated to deduce the values of the observable quantities from a substantially non observable underlying structure.

We now know that quantum phenomena are completely described by a complex-valued state function ψ satisfying the Schrödinger equation. The probabilistic interpretation of it was first suggested by Born [2] and, in the light of Heisenberg uncertainty principle, is a pillar of quantum mechanics itself.

All the known experiments show that the probabilistic interpretation of the wave function is indeed the correct one (see any textbook on quantum mechanics, for example [3]) and here we do not question on this. However, experimental results do not force us, as well, to consider the wave function ψ as “a mere repository of information on probabilities” [4]; it can have a more powerful role in quantum mechanics. It is certainly curious the fact that the correct theory which describes satisfactorily quantum phenomena is based on the concept of the complex-valued wave function ψ, only the squared modulus of which has a direct physical meaning. Although the situation is completely different, nevertheless the criticism reported at the beginning may be here applied as well: can a successful theory be based on a partially non observable quantity such as the wave function ψ?

The work of L. de Broglie, D. Bohm and others devoted to give an answer to this question is now well settled (for a recent excellent review see [4], [5]) and the emerging picture is the following: a coherent description of quantum phenomena exists in which the wave function ψ is an objectively realistic complex-valued quantity whose probabilistic interpretation remains but is not the only information which ψ carries. The experimental predictions of the de Broglie - Bohm theory are completely equivalent to those of usual quantum mechanics [4], so the choice of a given formulation rather than the other is, in a sense, a matter of taste. However, it has to be pointed out that the de Broglie - Bohm formulation of quantum mechanics is not free from some “peculiarities” which, even if they do not mine the coherence of the theory, make it not completely appealing. Very recent works [6], [7] (see also [8]) on this subject have, however, provided a natural explanation of these “peculiarities”. Furthermore, the probabilistic interpretation is now seen as a consequence of the theory, rather than a starting point.

Open problems in the de Broglie - Bohm theory remains, especially for the generalization to include the relativity principle and the formulation for the gauge fields. In this respect, at least for the present author point of view, the intuition of E. Majorana is fundamental. As we will show, the Majorana - Römer description of the electromagnetic field [9], [10], [11], [12], is a key starting point for a causal quantum-mechanical formulation of photon propagation which is, furthermore, completely gauge invariant. In the Majorana - Römer theory, the photon wave function is a completely observable quantity.

2. The motion of matter particles

In this section we review the main features of the causal theory of quantum phenomena (for further details see [4]) and some recent developments [7].

2.1. Introduction of the quantum potential

As we remembered in the introduction, a quantum elementary system is described by a complex wave field $\psi(\mathbf{x}, t)$ which, in the non-relativistic limit, satisfies the Schrödinger equation

$$i\hbar \frac{\partial \psi}{\partial t} \;=\; \mathrm{H}\,\psi \tag{1}$$

where H is the hamiltonian operator

$$\mathrm{H} \;=\; -\,\frac{\hbar^2\,\nabla^2}{2\mathrm{m}} \;+\; \mathrm{U} \tag{2}$$

U being the (external) potential experienced by the system of mass m (in this paper we use units in which $c \,=\, 1$). The complex equation (1) can be equivalently written as two real equations for the modulus R and the phase S of the function $\psi = Re^{iS/\hbar}$:

$$\frac{\partial S}{\partial t} \;+\; \frac{(\nabla S)^2}{2m} \;-\; \frac{\hbar^2}{2m}\frac{\nabla^2 R}{R} \;+\; U \;=\; 0 \tag{3}$$

$$\frac{\partial R^2}{\partial t} \;+\; \nabla\cdot\left(\frac{1}{m}\,R^2\,\nabla S\right) \;=\; 0 \tag{4}$$

The last equation is usually referred to as the continuity equation for the probability density $R^2 \;=\; |\psi|^2$. Instead, we note that Eq. (3) has the form of an Hamilton-Jacobi equation for the characteristic function S of a system described by an effective potential

$$V \;=\; U \;+\; Q \;=\; U \;-\; \frac{\hbar^2}{2m}\frac{\nabla^2 R}{R} \tag{5}$$

The term Q is called the "quantum potential"; it is the only non-classical term (i.e. proportional to the (squared) Planck constant) entering in the set of equations (3),(4).

It is then tempting (and natural) to explore the possibility that the Schrödinger theory can be regarded as a classical theory in which an "effective" (quantum) term is introduced; classical mechanics would then be obtained in the limit in which this quantum term become inoperative. Let us investigate to what extent this programme can be realized and what are the consequences in the interpretation of quantum phenomena.

2.2. Particle properties

Regarding Eq. (3) as a (classical) Hamilton-Jacobi equation for a particle driven by an effective potential V in (5), we have to identify the particle momentum field with

$$\mathbf{p} \;=\; \nabla S \tag{6}$$

Note that we are assuming that the vector field $\mathbf{v} = \mathbf{p}/m$ (the velocity field) defines at each point of space at each instant the tangent to a possible particle trajectory passing through that point. The trajectories are then orthogonal to the surfaces $S =$ constant and are given by the differential equation

$$m \, \frac{d\mathbf{x}}{dt} \;=\; \mathbf{p}(\mathbf{x}(t), t) \tag{7}$$

with $\mathbf{p}$ given in (6). We observe that, even if S is a multivalued function (being a phase, it keeps invariant the wave function ψ for $S \to S + 2\pi n$ with n an arbitrary integer number), $\mathbf{p}$ is a well defined single-valued quantity as well as the solution $\mathbf{x}(t)$ of (7) [1].

[1] The phase function S can have, in general, jump discontinuities, but we have to require that the wave function ψ has to be continuous. This implies that eventual jumps in S must occur in nodal points of ψ (defined by $\psi = 0$) where S is undefined. Then the momentum field $\mathbf{p}$ is irrotational except, in general, in nodal points [4].

We can further identify the total energy field of the particle with

$$\mathcal{H} = -\frac{\partial S}{\partial t} \tag{8}$$

which, from (3), results to be the sum of the kinetic energy $\mathbf{p}^2/2m$ and the total (classical plus quantum) potential energy.

Other particle properties can be defined accordingly; for example, the angular momentum field is given by

$$\mathbf{L} = \mathbf{x} \times \nabla S \tag{9}$$

We finally observe that Eq. (3) (deduced from the Schrödinger equation) and Eq. (7) (postulated to be valid in the present context) can be summarized in the "law of motion"

$$\frac{d\mathbf{p}}{dt} = -\nabla V \equiv -\nabla (U + Q) \tag{10}$$

taking the form of a Newton equation in which a "quantum force" $-\nabla Q$ is added to the classical force $-\nabla U$.

2.3. Initial conditions

In classical mechanics, the motion of a particle is univocally determined from the equations of motion once the initial conditions on its position $\mathbf{x}(t=0) = \mathbf{x}_0$ and velocity $\dot{\mathbf{x}}(t=0) = \mathbf{v}_0$ are given.

Conversely, in conventional quantum mechanics, since a physical system is described by the wave function $\psi(\mathbf{x}, t)$ obeying the Schrödinger equation (which is a first order in time differential equation), the state of the system is univocally determined (from the probabilistic point of view) when the initial wave function $\psi(\mathbf{x}, 0) = \psi_0(\mathbf{x})$ is given for all $\mathbf{x}$. In this framework, precisely known (initial) conditions on particle position and velocity are not allowed; we can only assign the probability that the particle passes for a given point or has a given velocity at a given time instant.

In the context of causal theory of quantum motion the situation changes. For the univocal determination of the motion we have to specify the initial wave function

$$\psi(\mathbf{x},0) \;=\; \psi_0(\mathbf{x}) \tag{11}$$

(or, equivalently, $R(\mathbf{x},0) = R_0(\mathbf{x})$, $S(\mathbf{x},0) = S_0(\mathbf{x})$) but this is not enough. In fact, the specification of the initial position

$$\mathbf{x}(0) \;=\; \mathbf{x}_0 \tag{12}$$

is also required in order to obtain univocally the solution of the equation (7). This is indeed possible in the present framework, differently from the case of conventional quantum mechanics, since we have here assumed that a particle does follow a definite trajectory (we return later on the probabilistic interpretation). Note that the specification of the initial velocity is not required, since it is uniquely given by

$$\dot{\mathbf{x}}(0) \;=\; \frac{1}{m} \nabla S_0(\mathbf{x}) \Big|_{\mathbf{x}=\mathbf{x}_0} \tag{13}$$

2.4. Introduction of the probability

In the previous subsections we have seen as, starting from Eq. (3), it is possible to construct (at least mathematically) a causal theory of quantum motion. This is, however, only half of the story, since we have not yet considered Eq. (4) which is, in a sense, the other half of the Schrödinger equation, and we also have not discussed at all the probabilistic interpretation of quantum phenomena which is, nevertheless, experimentally well settled. As it is well known, the two points are strictly related and will be now briefly considered (however, for a general discussion, we refer to [4]).

The introduction of the probability in the causal theory of quantum motion represents our ignorance of the precise initial state of the particle: in no way it impinges on the underlying dynamical process in which the particle is involved. It is assumed that $|\psi(\mathbf{x},t)|^2\, d^3\mathbf{x} = R^2(\mathbf{x},t) d^3\mathbf{x}$ is the probability that a particle described by the field $\psi(\mathbf{x},t)$ lies between the points $\mathbf{x}$ and $\mathbf{x} + d\mathbf{x}$ at time t. Note that we are dealing with the probability that a particle actually *is* at a precise location at time t, differently from the case of conventional quantum mechanics in which the probability of *finding* a particle in the volume $d^3\mathbf{x}$ at time t is involved. Then, here, this postulate is introduced to select, from all the possible motions implied by the Schrödinger equation and Eq. (7), those that are compatible with an initial distribution $R^2(\mathbf{x}, 0) = R_0^2(\mathbf{x})$. Since the function $R^2(\mathbf{x}, t)$ satisfies the continuity equation (4), if the probability density at $t = 0$ is given by $R^2(\mathbf{x}, 0) = R_0^2(\mathbf{x})$ then, at any time t, $R^2(\mathbf{x}, t)$ determines univocally the probability distribution.

Besides the very different framework in which operates, the causal theory of quantum motion, constructed upon the basic postulates featured in this section, leads exactly to the same experimental predictions of conventional quantum mechanics (we do not discuss this point; see [4]).

2.5. The spin as the source of the quantum behaviour

Very recently, a new picture is emerged in the framework of the causal theory of quantum motion [6], [7]. It has been shown that the quantum effects present in the Schrödinger equation are due to the presence of a peculiar spatial direction associated with the particle that, assuming the isotropy of space, can be identified with the spin of the particle itself. Then, the motion of a quantum particle results to be composed of an "external" (drift) motion, described by the velocity field

$$\mathbf{v}_B = \frac{1}{m} \nabla S \quad , \tag{14}$$

and an "internal" one (featured by the presence of spin), driven by $\mathbf{v}_S \times \mathbf{s}$, where $\mathbf{v}_S$ is given by

$$\mathbf{v}_S = \frac{\hbar}{2m} \frac{1}{R^2} \nabla R^2 \tag{15}$$

and $\mathbf{s}$ is the spin direction of the particle. In this framework, the quantum potential is completely determined from the velocity field $\mathbf{v}_S$ of the internal motion:

$$Q = -\frac{1}{2} m \mathbf{v}_S{}^2 - \frac{\hbar}{2} \nabla \cdot \mathbf{v}_S \quad . \tag{16}$$

The classical limit is recovered when the internal motion is negligible with respect to the drift one which is entirely ruled by classical mechanics.

The probabilistic interpretation, peculiar of all quantum phenomena, has a natural assessment too. In fact, assuming that initial conditions can be assigned only on the drift motion (which is, in some sense, the mean motion of the particle), this *requires* necessarily a probabilistic formulation of quantum mechanics. In this view, the internal motion is responsible both of the quantum effects (described, properly, by the Hamilton-Jacobi equation (3)) and of the quantum probabilistic interpretation of them (which is allowed by the continuity equation (4).

Also, some difficulties suffered by the de Broglie - Bohm theory, related to "unusual" properties of the quantum potential, can be simply solved in the novel picture (for further details see [7]).

3. The motion of gauge particles

In the previous section we have given an account of the de Broglie - Bohm quantum theory of motion, in which the wave function acquires a direct (and fundamental) physical meaning. The discussion has dealt with only matter particles, described by the Schrödinger equation (for non relativistic motion). Here we will consider the motion of gauge particles, treating the case of electromagnetism.

In quantum electrodynamics, photons are described by the electromagnetic potential A^μ: the equations of motion, the coupling with matter particles and so on always involve the 4-vector potential A^μ. The situation is, then, very similar to that of ordinary quantum mechanics: the fundamental quantity describing photons has not a direct physical meaning since it is not gauge invariant.

The starting problem can be cast as follows: what is the "wave function" for the gauge particles? The potential A^μ, even being the reference quantity in quantum electrodynamics cannot be regarded as a wave function, since it does not possess the basic features of such a quantity. In fact, for example, a probabilistic interpretation of it has no sense.

It has been suggested [10], [9], [11] that the quantity $\mathcal{E} - i\mathcal{B}$ can describe alternatively the propagation of photons, and here we will show that this is achieved in the same way in which the Schrödinger wave function $\psi = R\, e^{iS/\hbar}$ describes the motion of matter particles in the de Broglie - Bohm theory.
With this choice, the physical meaning of the photon wave function is acquired *ab initio*, and we will point out the impressive similarities of the present formulation of electrodynamics with the de Broglie - Bohm theory.

Although electromagnetism is Lorentz invariant, for the sake of simplicity we will use a formalism which is not manifestly invariant. The covariant Majorana - Römer formulation of electromagnetism in a general context is developed in [12].

3.1. Wave mechanics of photons

Let us indicate with $\mathcal{E}$ and $\mathcal{B}$ the electric and magnetic field, respectively, and consider the complex valued quantity [9], [10], [11], [12]

$$\psi \;=\; \frac{1}{\sqrt{2}}\,(\mathcal{E} \;-\; i\,\mathcal{B}) \tag{17}$$

In terms of ψ the Maxwell equations of vacuum electromagnetism then write

$$\nabla\cdot\psi \;=\; 0 \tag{18}$$

$$\frac{\partial\psi}{\partial t} \;=\; i\,\nabla\times\psi \tag{19}$$

The first relation can be regarded as a constraint on the field ψ. Instead, the second one can take an impressive form by introducing the following set of hermitian matrices

$$\alpha_1 \;=\; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & i \\ 0 & -i & 0 \end{pmatrix} \qquad \alpha_2 \;=\; \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix} \qquad \alpha_3 \;=\; \begin{pmatrix} 0 & i & 0 \\ -i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{20}$$

satisfying the commutation relation

$$[\alpha_i\;,\;\alpha_j] \;=\; -i\,\epsilon_{ijk}\,\alpha_k \tag{21}$$

In fact, Eq. (19) can be written as

$$i \frac{\partial \psi}{\partial t} = H \psi \tag{22}$$

or, carrying off the components,

$$i \frac{\partial \psi_i}{\partial t} = H_{ij} \psi_j = -i \left(\alpha_k\right)_{ij} \nabla^k \psi_j \tag{23}$$

It has, then, the form of a Schrödinger - Dirac equation. This leads us to explore the possibility of considering the field ψ as the wave function for photons. We have, thus, to require for ψ a probabilistic interpretation.

First of all, let us note that $\psi^* \cdot \psi = \frac{1}{2} \left(\mathcal{E}^2 + \mathcal{B}^2 \right)$ is directly proportional to the probability density function for a photon. In fact, if we have a beam of n equal photons each of them with energy ϵ (given by the Planck relation), since $\frac{1}{2}(\mathcal{E}^2 + \mathcal{B}^2)$ is the energy density of the electromagnetic field, then $\frac{1}{n\epsilon} \frac{1}{2}(\mathcal{E}^2 + \mathcal{B}^2)\, dS\, dt$ gives the probability that each photon has to be detected (or lies, according to the de Broglie - Bohm point of view) in the area dS at the time dt. (The generalization to photons of different energies (i.e. of different frequencies) is obtained with the aid of the superposition principle).

Secondly, by left-multiplying Eq, (23) by ψ_i^* and right-multiplying by ψ_i the conjugate equation of (23), summing the two relations, the result is the following continuity equation (Poynting theorem):

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \mathbf{J} = 0 \tag{24}$$

where

$$\rho = \psi^* \cdot \psi \qquad\qquad \mathbf{J} = \psi^* \alpha \psi \tag{25}$$

We then see that such a field can be effectively considered as the wave function for the photon. The quantity $\psi^* \cdot \psi$ is proportional to the probability density for this particle, while we observe that the probability current $\mathbf{J}$ has the same form of the Dirac current, with the α matrices in (20).

The wave function so defined does not suffer for the criticism of sect. 1, since it is constructed with the electric and magnetic fields which are observable quantities. It is then desirable to see if this formulation of electrodynamics (as photon wave mechanics) has some relation with the de Broglie - Bohm causal quantum theory of motion of matter particles discussed in the previous sections. In the following, we will see the important role played by the conservation equations in this connection.

3.2. A "quantum potential" for photons

The electromagnetic field (in vacuum) is described by the set of equations (23) plus the constraint relation (18). Here we will consider another set of fundamental equations derived from those which express the conservation laws for energy density, momentum density and angular momentum density, respectively.

We have already met the conservation law for energy density in equation (24), ρ and $\mathbf{J}$ being exactly the energy density and momentum density of the electromagnetic field, respectively. Their expressions in terms of $\mathcal{E}$, $\mathcal{B}$ are the following:

$$\rho \;=\; \frac{1}{2}\left(\mathcal{E}^2 \,+\, \mathcal{B}^2\right) \qquad\qquad \mathbf{J} \;=\; \mathcal{E}\times\mathcal{B} \tag{26}$$

Then, by left-multiplying Eq. (23) by $\psi^* \, \alpha$ and right-multiplying the conjugate equation of (23) by $\alpha \, \psi$, summing the resulting expressions, after little algebra we obtain the conservation law for momentum density:

$$\frac{\partial \mathbf{J}}{\partial t} \;=\; -\nabla\rho \,+\, \nabla_j \left(\psi^* \, \psi_j \,+\, \psi_j^* \, \psi\right) \tag{27}$$

The term in the RHS is more conventionally written as the divergence of the Maxwell stress tensor T_{ij}:

$$\partial_j \, T_{ij} \;=\; \partial_j \left(\mathcal{E}_i \, \mathcal{E}_j \,+\, \mathcal{B}_i \, \mathcal{B}_j \,-\, \frac{1}{2} \, \delta_{ij} \, \left(\mathcal{E}^2 \,+\, \mathcal{B}^2\right)\right) \tag{28}$$

Finally, the conservation law for angular momentum density follows from (27) by vector multiplying this equation by the position vector $\mathbf{x}$:

$$\frac{\partial \mathbf{L}}{\partial t} \;=\; -\,\mathbf{x}\times\nabla\rho \,+\, \nabla_j \left((\mathbf{x}\times\psi^*) \, \psi_j \,+\, \psi_j^* \, (\mathbf{x}\times\psi)\right) \tag{29}$$

with the angular momentum density given by

$$L_i \;=\; (\mathbf{x} \times \mathbf{J})_i \;=\; i\, x_j \left(\psi_i^* \, \psi_j \;-\; \psi_j^* \, \psi_i \right) \tag{30}$$

What have to do these conservation equations with a de Broglie - Bohm -like formulation of wave mechanics for photons?

As we have seen, the equation (24) has been crucial for the interpretation of ψ as a wave function.

Let us now consider Eq. (27); it is the local form, for the electromagnetic field, of the Newton equation, $\mathbf{J}$ being the momentum density. Since we are considering the case of free photons in vacuum, the term in the RHS would be considered the "quantum force" (density) for the electromagnetic field, analogous to the term derived from the quantum potential present in Eq. (10) for matter particles.

A corresponding interpretation of Eq. (29) can be given as well, too.

Obviously, differently from the Schrödinger case, for the electromagnetic case no Planck constant is present: matter particle wave mechanics has a quantum origin, while photon wave mechanics is described classically. Nevertheless, the description of the motion of both types of particles can exhibit analogies, some of which are here studied adopting a de Broglie - Bohm -like point of view.

However, some considerations are in order.

The equation (27) (and (29)) involves local field quantities, while equation (10) is written for particle quantities; more properly, referring to photons, Eq. (27) would be integrated over a given finite volume to obtain more similar quantities. Then, we note that the quantum force (density) -like term in Eq. (27) is made of two terms. The first one involves a scalar quantity (the energy density ρ), while in the second one the vector nature of the field ψ is explicit. Thus, differently from the case discussed in the previous sections, the quantum force (density) -like term for photons contains an explicit reference to the spin of the considered particle. In other words, while the first term in the RHS of Eq. (27) is very similar to the one describing the "internal motion" of Schrödinger particles, the second term is peculiar for vector gauge bosons and can be present only for these particles.

3.3. The (mean) motion of a photon

The analogy between the present formulation of electrodynamics and the de Broglie - Bohm theory of quantum motion is still incomplete, since we have not addressed the problem of photon motion (in particular, that of photon trajectories) in its completeness, namely the validity of a relation like that in Eq. (7) (generalized to the relativistic case) for the photon. This is still an open problem; in particular, the issue of a position operator for photons has been the subject of intensive studies [13], [14], [15]. It has been demonstrated [13] that it is possible to define position operators (and localized states) only for massive and scalar massless particles, but not for massless particles with non vanishing spin [2].This is related to the structure of the Poincaré group [15] and hence it is a general result. While here we don't question this important result (according to the de Broglie - Bohm point of view, we are dealing with the motion of photons and not with their localizability in space), we want to discuss a peculiar relation that, on one hand, seems to confirm the choice of $\psi = \mathcal{E} - i\mathcal{B}$ as the photon wave function, and, on the other hand, makes more close the relation between electromagnetism and Schrödinger wave mechanics. In the previous subsections we have already addressed the fact that the conservation equations (24), (27), (29) involve local field quantities, not particle properties. We now consider an integrated relation.

As well known, the integration over the whole space of the Eqs. (24), (27), (29) yields the conservation laws for the energy, momentum and angular momentum of the electromagnetic field respectively, i.e.

$$\frac{dE}{dt} = 0 \qquad \frac{d\mathcal{P}}{dt} = 0 \qquad \frac{d\mathcal{L}}{dt} = 0 \tag{31}$$

[2] As pointed out in [15], even for massive particles the localization is not perfect, because it is not relativistically invariant. However, the departures from strict localization are only exponentially small, and in the non relativistic limit the localization is restored.

where

$$E = \int \rho \, d^3\mathbf{x} = \int \psi^* \, \psi \, d^3\mathbf{x} \tag{32}$$

$$\mathcal{P} = \int \mathbf{J} \, d^3\mathbf{x} = \int \psi^* \, \alpha \, \psi \, d^3\mathbf{x} \tag{33}$$

$$\mathcal{L} = \int \mathbf{x}\times\mathbf{J} \, d^3\mathbf{x} = i \int x_j \left(\psi^* \, \psi_j - \psi_j^* \, \psi \right) \, d^3\mathbf{x} \tag{34}$$

However, there is a last conservation equation not derivable from (31) (see, for example, [16]). It follows from the conservation of the relativistic angular momentum tensor $M^{\alpha\beta\gamma} = \theta^{\alpha\beta} x^\gamma - \theta^{\alpha\gamma} x^\beta$ (with $\theta^{\mu\nu} = g^{\mu\alpha} F_{\alpha\beta} F^{\beta\nu} + \frac{1}{4} g^{\mu\nu} F_{\alpha\beta} F^{\alpha\beta}$ the energy-momentum tensor), that is

$$\partial_\alpha \, M^{\alpha\beta\gamma} = 0 \tag{35}$$

If for both β, γ we choose spatial indices, we re-obtain Eq. (29). Instead, let us consider the case in which $\beta = 0$ and $\gamma = i = 1, 2, 3$; we then have

$$\left(\frac{\partial \rho}{\partial t} + \nabla_j \, J_j \right) x_i = \left(\frac{\partial J_i}{\partial t} - \nabla_j \, T_{ij} \right) t \tag{36}$$

Integrating this relation, with a little algebra we arrive at the final result

$$\frac{d\mathcal{X}}{dt} = \frac{\mathcal{P}}{E} \tag{37}$$

with

$$\mathcal{X} = \frac{\int \psi^* \, \mathbf{x} \, \psi \, d^3\mathbf{x}}{\int \psi^* \, \psi \, d^3\mathbf{x}} \tag{38}$$

Allowing ψ to be the photon wave function, we recognize in (38) the expression for the mean position of the photon, while

$$\mathcal{V} = \frac{\mathcal{P}}{E} = \frac{\int \psi^* \, \alpha \, \psi \, d^3\mathbf{x}}{\int \psi^* \, \psi \, d^3\mathbf{x}} \tag{39}$$

is its mean velocity. The obtained result (37) is then the Ehrenfest theorem for the photon. Although it does not individuate the complete photon trajectory, as Eq, (7) does for the Schrödinger particle, nevertheless it gives us information on the mean motion of the photon. With the identification of ψ in (17) as the photon wave function, the relation (37) (and its interpretation) is an important result of the present formulation towards a comprehensive understanding of the quantum motion of both matter and gauge particles.

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