Selected Topics in Majorana Neutrinos

Three lectures by

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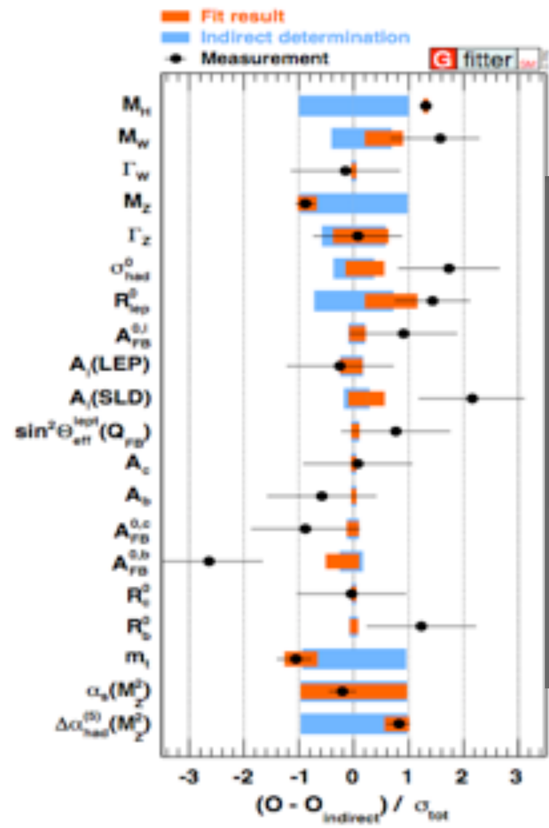
SUMMARY

1. Electroweak precision
2. See-saw neutrinos in three generations
3. A change of paradigm
4. The quark case with three families
5. The lepton case, three families and see-saw

Electroweak Fit – SM Fit Results

- Results drawn as *pull values:* → deviations to the *indirect* determinations, divided by *total error*.
- Total error: *error of direct measurement plus error from indirect determination.*
- Black: direct measurement (data)
- Orange: full fit
- Light-blue: fit excluding input from the row
- The prediction (light blue) is often more precise than the measurement!



1. Electroweak precision

Max Baak (CERN),
on behalf of the Gfitter group (*)

CERN seminar, Geneva, 23rd September, 2013

http://cern.ch/Gfitter

EPJC 72, 2205 (2012), arXiv:1209.2716

After the Higgs: Status and Prospects of The ElectroWeak fit of the SM and Beyond

- Fit result:

$S = 0.03 \pm 0.10$

$T = 0.05 \pm 0.12$

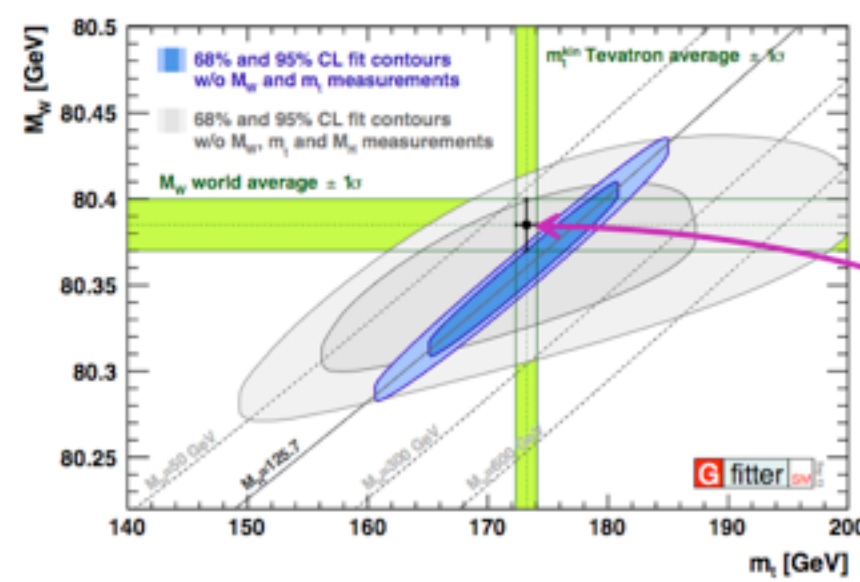
$U = 0.03 \pm 0.10$

	S	*T*	*U*
S	1	+0.89	-0.54
T		1	-0.80
U			1

- Stronger constraints from fit with U=0.
- Also available for Z→bb correction.

State of the SM: W versus top mass

- Scan of M_W vs m_t, with the direct measurements excluded from the fit.
- Results from Higgs measurement significantly reduces allowed indirect parameter space → corners the SM!

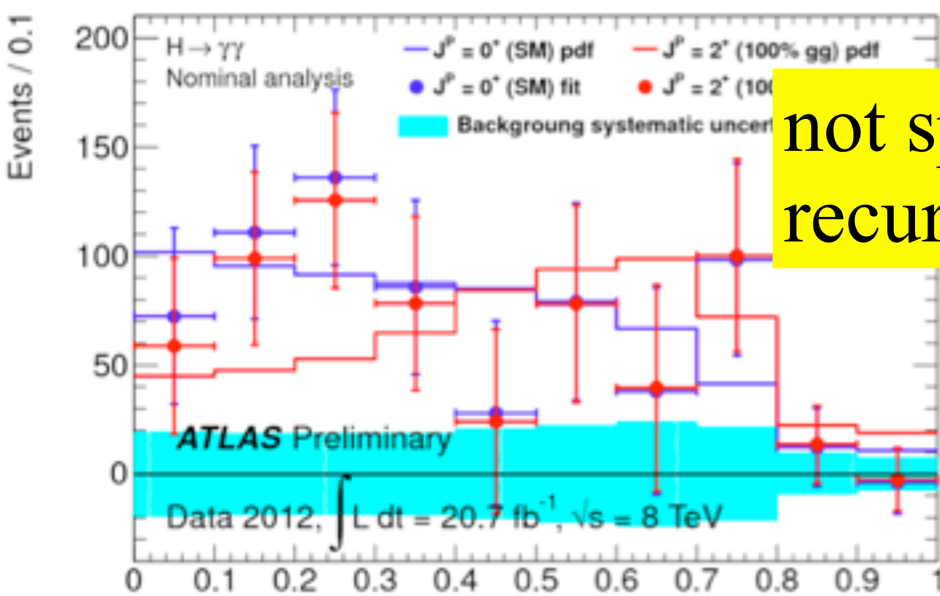


- Observed agreement demonstrates impressive consistency of the SM!

- No detectable oblique corrections: S, T, U
- Perturbative corrections from BSM physics?
- Too early to say, ...but

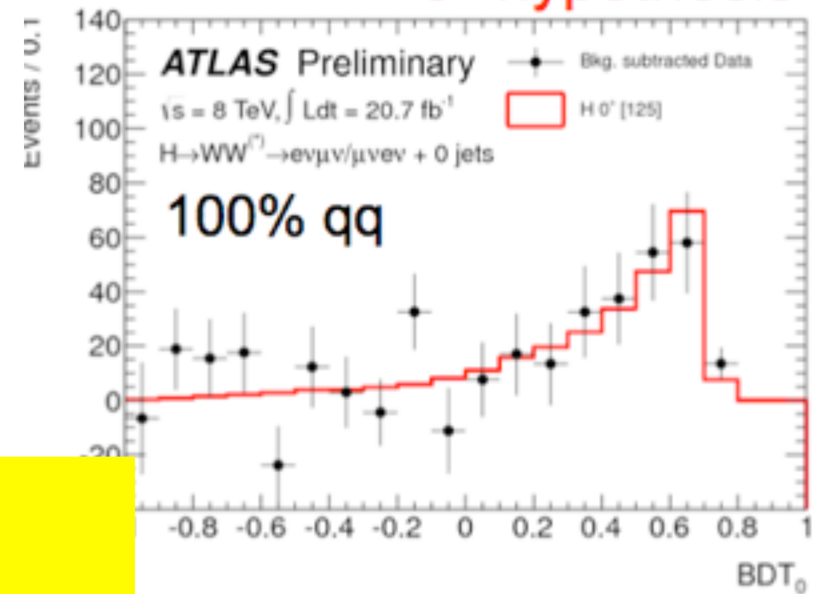
The 125 GeV particle

- Not spin 1 (decays into 2γ)
- Most likely 0^+

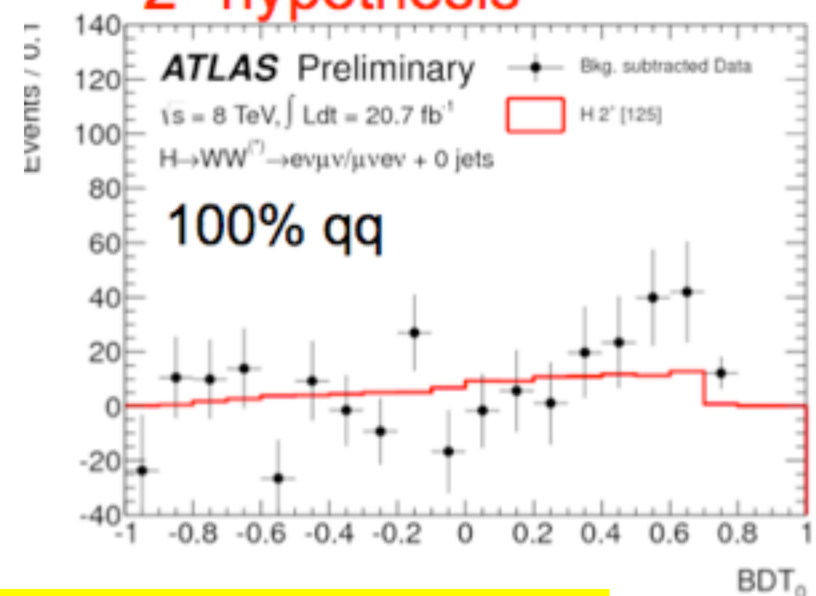


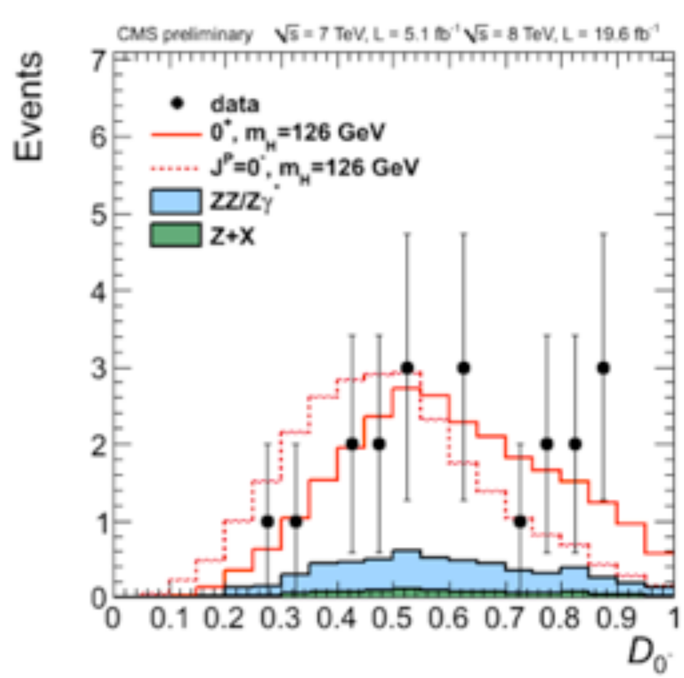
not spin 2+ (Kaluza Klein recurrence of the graviton?)

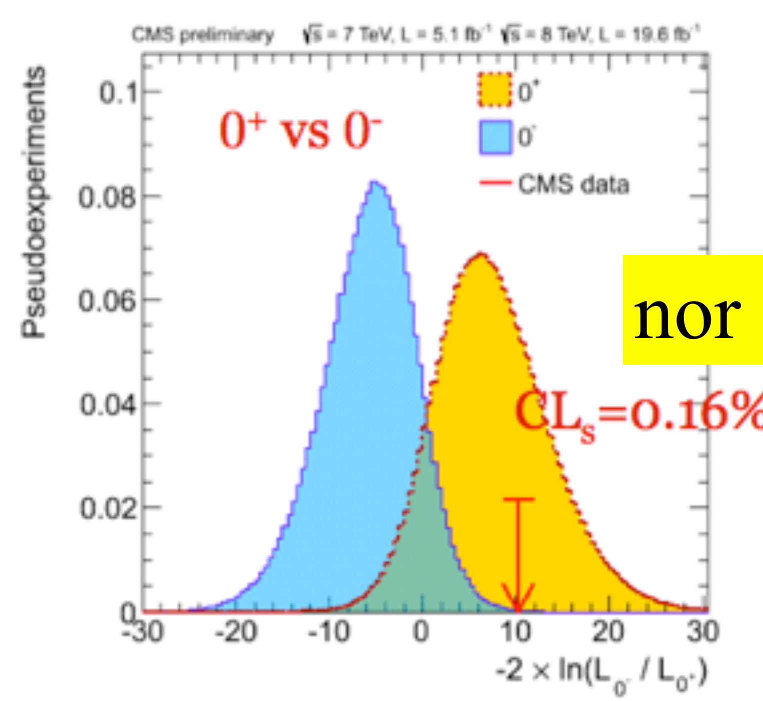
0^+ hypothesis



2^+ hypothesis

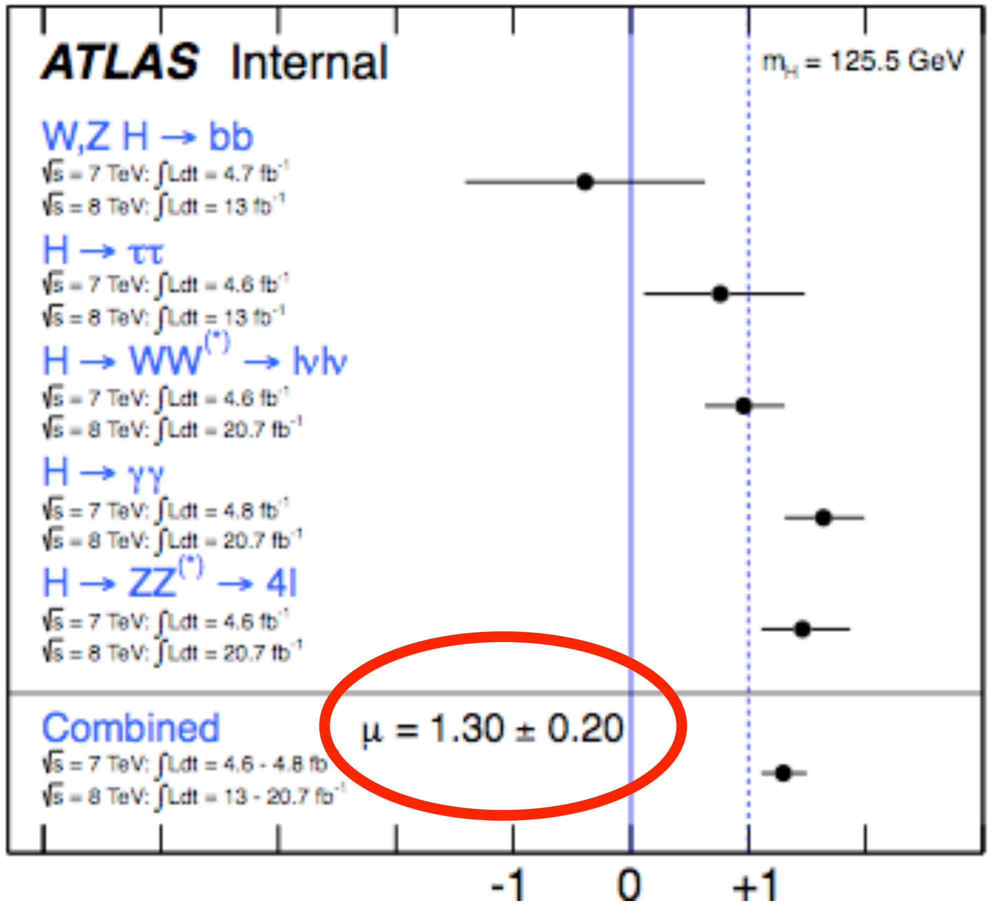




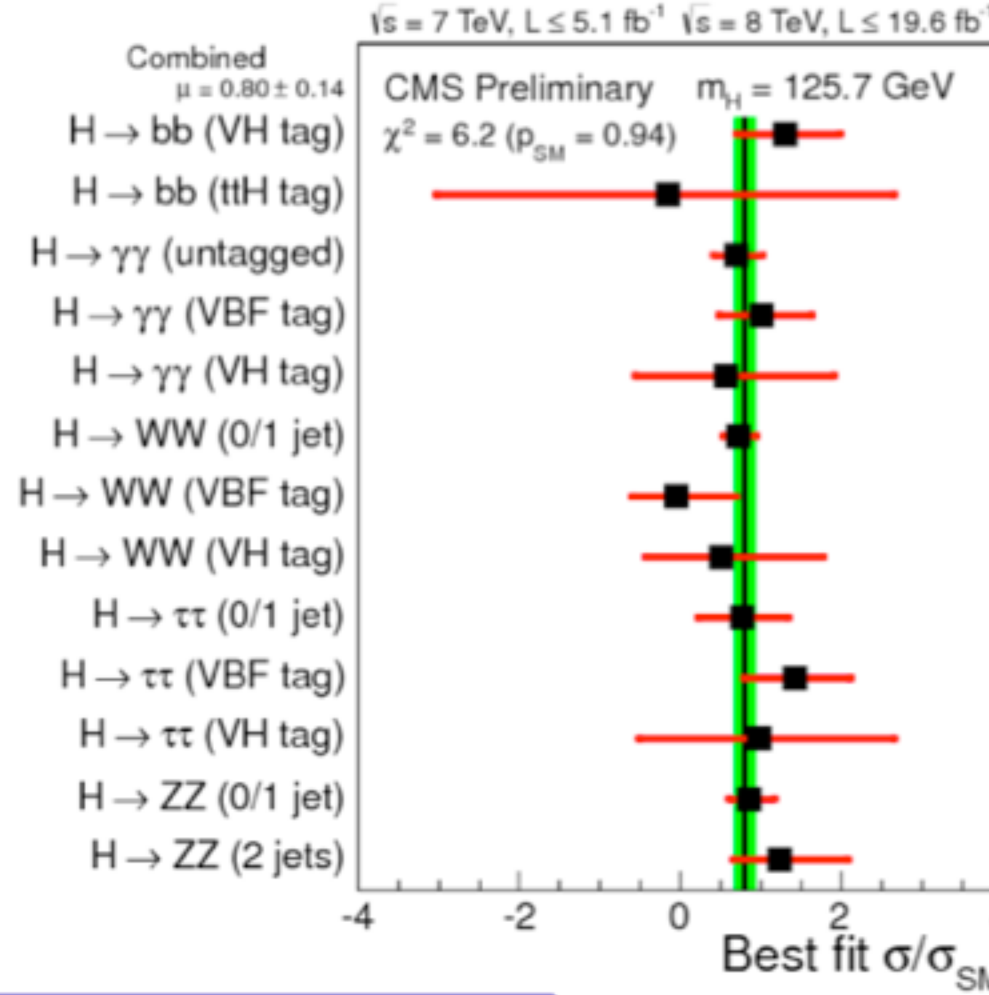


nor spin 0^- (Technipion?)

It smells like the Higgs boson



ATLAS NOTE, March 4, 2013



$\mu = 0.80 \pm 0.14$

CMS PAS HIG-13-005:

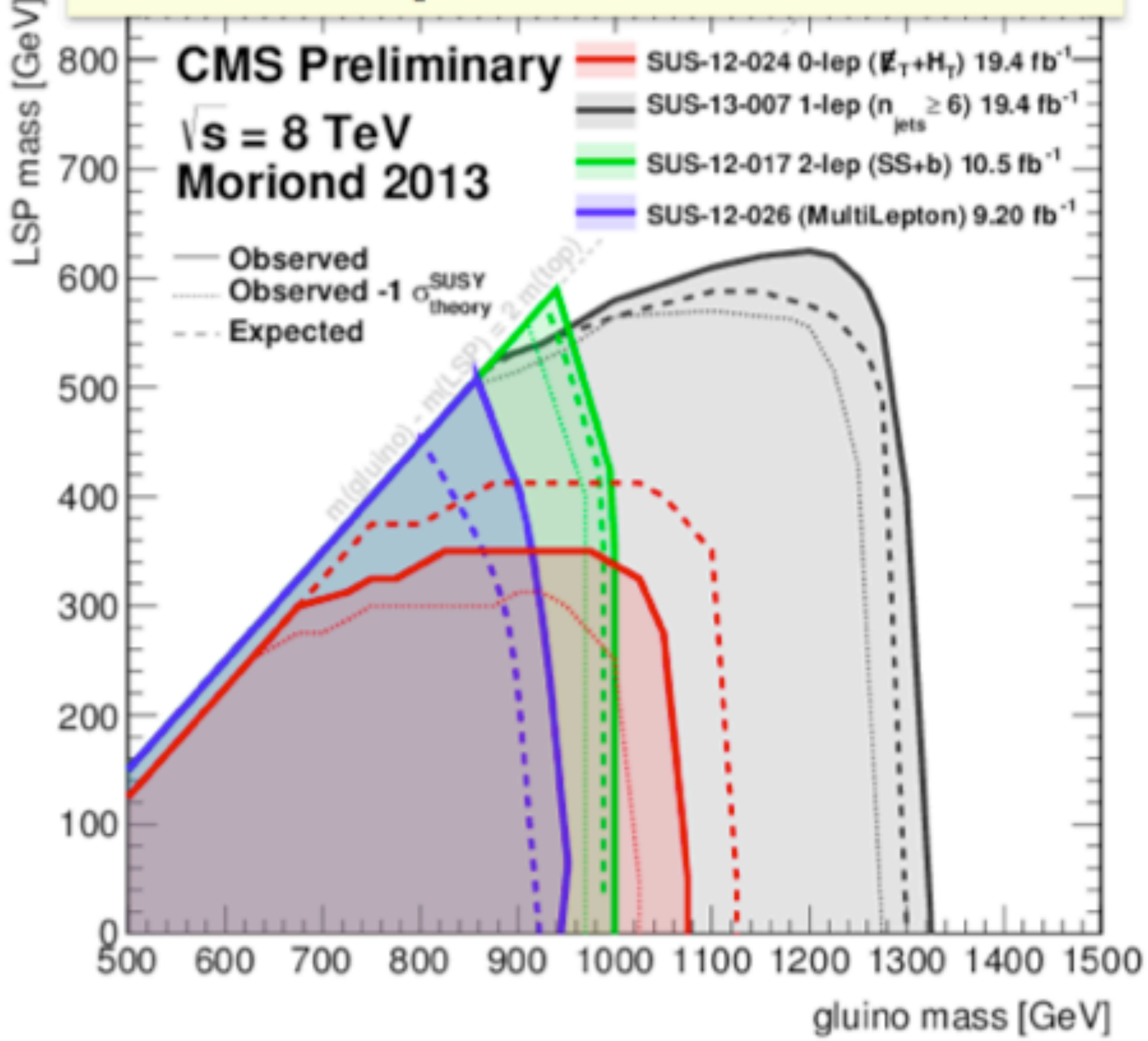
Small variations may be there, possible indications of New Physics beyond the ST: SUSY?composite? other ???
The mass of the new boson speaks for an elementary particle
SUSY prediction: M_h< 135 GeV is remarkable
SUSY at TeV may still be the best contender to solve the hierechy problem

Admittedly, No SUSY partners in sight

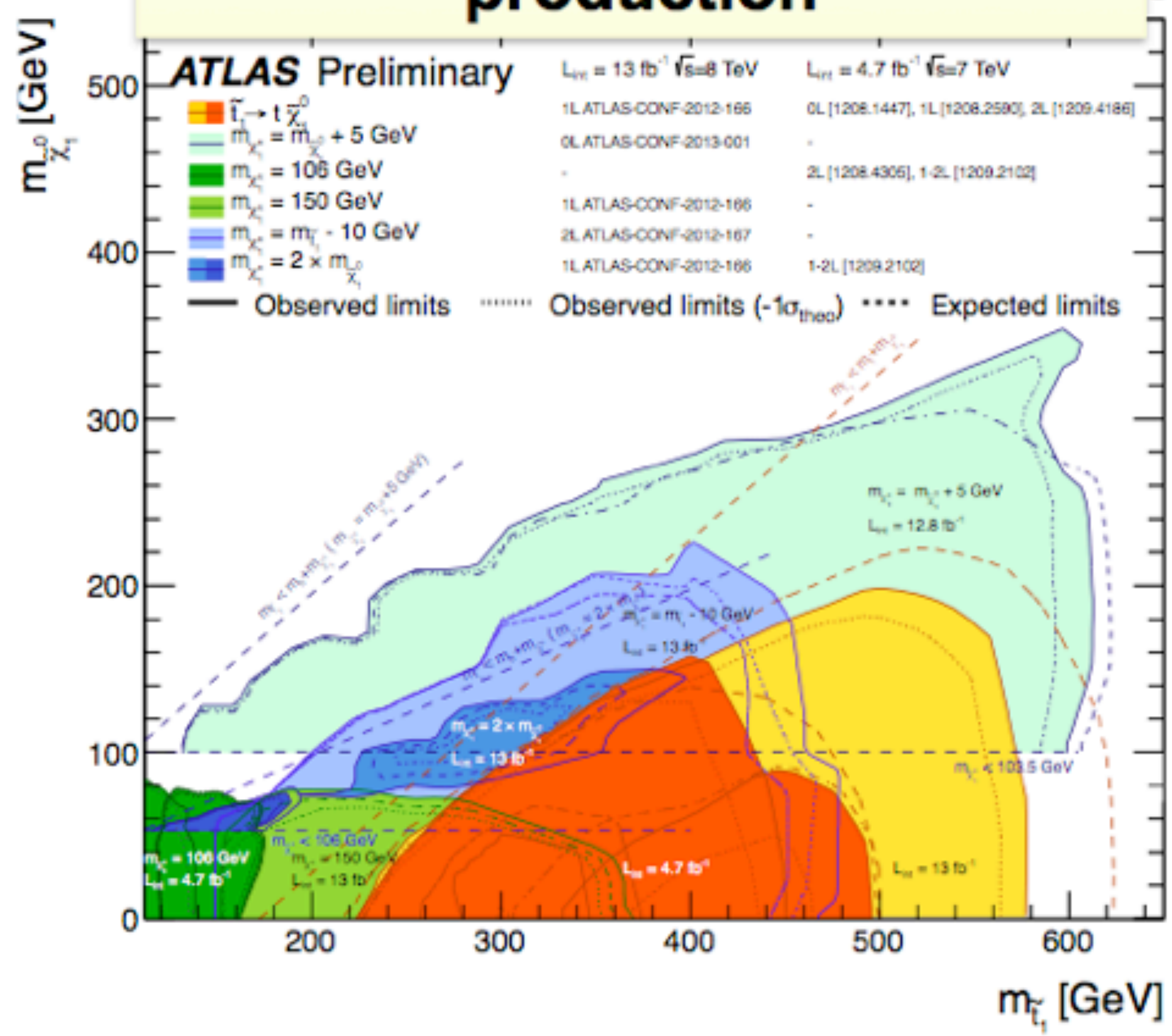
If gluino light enough:
→ gluino induced production

If third-generation squarks light enough:
→ direct production

CMS gluino-induced stop production



ATLAS direct stop production



2. Seew-saw neutrinos in 3 generations

$$\mathcal{L}_Y = \mathcal{L}_{quark} + \mathcal{L}_{ch.\ lept} + \mathcal{L}_{nu}$$
$$\mathcal{L}_{ch.\ lept} = [\bar{\ell}_L\, Y_E H E_R + \text{h.c.}]$$
$$\mathcal{L}_{nu} = \frac{M}{2} N\gamma_0 N + [\bar{\ell}_L\, Y_\nu \tilde{H} N + \text{h.c.}]$$

$$\ell_L = \begin{pmatrix} E_L \\ \nu_L \end{pmatrix}$$

- Flavor group:
  - Quark as before: G_q= SU(3)$_q$⊗SU(3)$_U$⊗SU(3)$_D$
  - Leptons: G_l= SU(3)$_l$⊗SU(3)$_E$⊗O(3)$_N$
- The effective neutrino lagrangian at low energy is obtained by integrating over N
- We write: N=ν_R+ν_R^+, so that: $\mathcal{L}_{nu} = \frac{M}{2}\nu_R\gamma^0\nu_R + \bar{\ell}\tilde{H}Y_\nu\gamma^0\nu_R +\ h.c.$
- we shift ν_R: $\nu_R \rightarrow \nu_R$+ A, and set A so as to cancel the linear term in ν_R:

$$\mathcal{L}_{nu} \rightarrow \frac{M}{2}\nu_R\gamma^0\nu_R + (M\ \ A + \bar{\ell}\tilde{H}Y_\nu)\gamma^0\nu_R + \ell_L^C\gamma^0\tilde{H}Y_\nu A$$

- note that in Majorana rep. and anticommuting fields: $A\gamma^0\nu_R = +\nu_R\gamma^0 A$
- the functional integral on ν_R is gaussian, after integration we get the effective see-saw lagrangian:

$$\mathcal{L}_{nu,\ l.e.} = \ell_L^C\gamma^0\tilde{H}Y_\nu A =$$
$$= -\ell_L^C\gamma^0\tilde{H}Y_\nu\frac{1}{M}Y_\nu^T\tilde{H}^T(\ell_L^C)^T$$

after EW symmetry breaking...

$$\mathcal{L}_{mass} = \nu_L \gamma^0 M_\nu \nu_L + h.c.;\ \ M_\nu = \frac{v^2}{M} Y_\nu Y_\nu^T$$

- By using the lepton flavor symmetry, we can reduce the leptonic Yukawa coupling to a standard, diagonal form

$$Y_E \to U_\ell Y_E U_E\ ,\quad Y_\nu \to U_\ell Y_\nu \mathcal{O}^T$$
$$Y_E = y_E\ ,\quad Y_\nu = U_L\ y_\nu\ \omega\ U_R$$

- y diagonal, real, positive; U_R unitary, ω a diagonal phase matrix of unit determinant
- the low energy neutrino mass is complex and symmetric, so is diagonalized according to:

$$M_\nu = \frac{v^2}{M} U_L (y_\nu \omega U_R U_R^T \omega y_\nu) U_L^T = U_{\rm PMNS}\ \Omega\ m_\nu\ \Omega\ U_{\rm PMNS}^T$$

- thus introducing the PMNS mixing matrix, with m_ν diagonal, real and positive, and Ω a diagonal, Majorana-phase matrix.
- We consider the Ys as the fundamental variables and YY^T as a derived quantity.

2. A change of paradigm

- The universality of Yukawa couplings postulated by the MFV principle is difficult to reconcile with the idea that the *Y*s are "just" renormalized constants.
- ***Are Yukawa couplings the VEVs of new fields, which break spontaneously the Flavor Symmetry?*** an idea pioneered by Froggat & Nielsen (arXiv:hep-ph/9905445).
- If so, Yukawa couplings can be determined by a variational principle, i.e. by the minimum of a new hidden potential
- the idea was considered in the 60's by N. Cabibbo, as a way to explain the origin of the weak angle, and was explored then by L. Michel and L. Radicati and by Cabibbo and myself;
- recent applications to neutrino masses and mixing by B. Gavela and coll.
- recent work by R. Alonso, B. Gavela, G. Isidori, L. M.

R. Alonso, B. Gavela, G. Isidori and L. Maiani, arXiv:1306.5927

Michel&Radicati, Cabibbo&Maiani (1968-69)

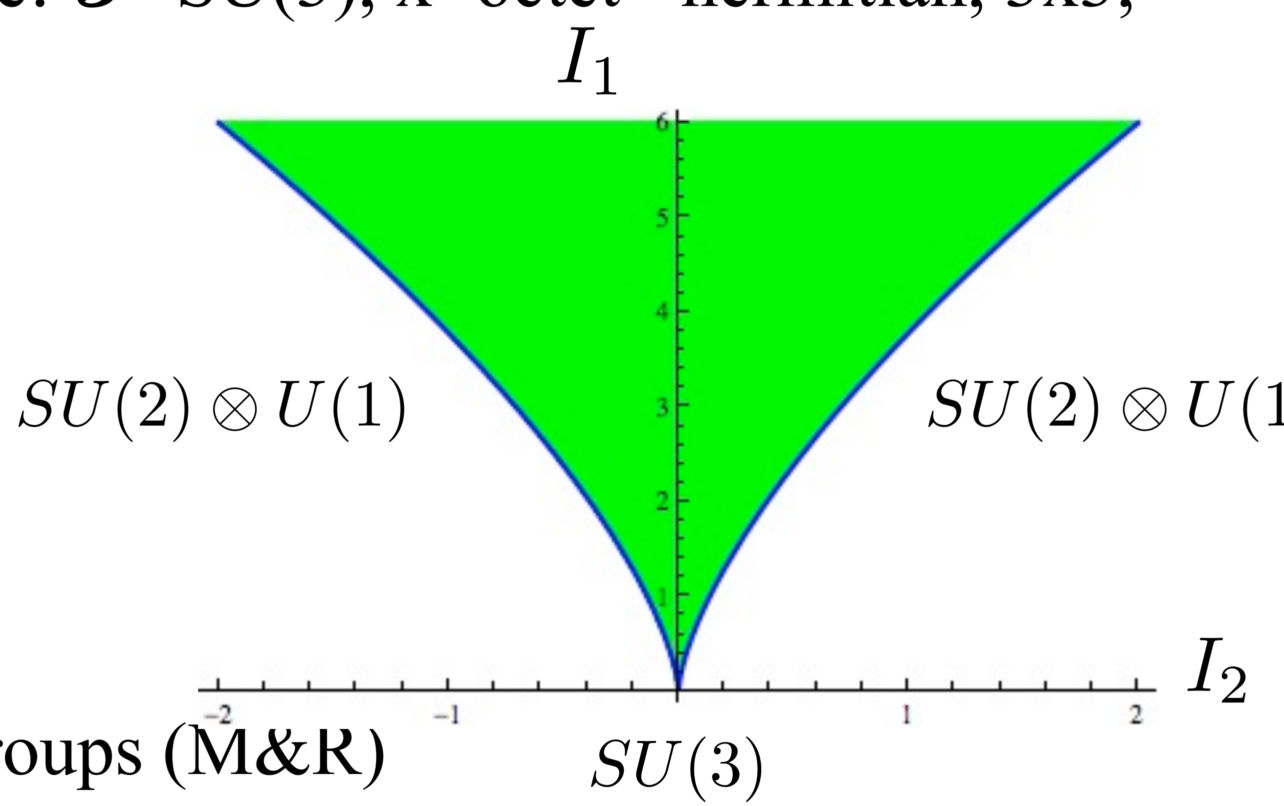
- V(x), x= components of fields transforming as a Rep. of $\mathscr{G}$, the invariance group of V;
- then V(x)=V(I_i(x)), I_i are the independent invariants one can construct out of the x (same number as the number of independent x)
- the space of the x has not boundary, but the manifold, $\mathscr{M}$, spanned by I_i (x) has boundaries;
- the situation is exemplified by the figure: $\mathscr{G}$=SU(3), x=octet= hermitian, 3x3, traceless matrix, and:

$$I_1 = \mathrm{Tr}(x^2); \quad I_2 = \mathrm{Det}(x)$$

- the manifold is the green region corresponding to:

$$I_1 \geq (54\ I_2^2)^{1/3}; \quad -\infty < I_2 < +\infty$$

- point= orbit: $x_g = g\ x\ g^{-1}$, $g \in \mathscr{G}$
- boundaries: orbits with conjugated little groups (M&R)



Classification of the "natural " extrema of the potential (C&M)

- the boundary of $\mathscr{M}$ is made of "surfaces", joined by "lines" which converge on discrete "points"...
- boundaries are identified by the rank of the Jacobian matrix: $J = \dfrac{\partial(I_1, I_2, \cdots)}{\partial(x_1, x_2, \cdots)}$
- dimension of $\mathscr{M}$=N>Rank of J= N-1 ("surface"), N-2 ("line"), N-3 (point),...
- minima of V with respect to the points of a given boundary are true minima of V(x);
- they require the vanishing of only N-1, N-2, etc. derivatives of V since J has 1 or 2, etc. vanishing eigenvectors (orthogonal to the boundary), hence are more "natural" than the generic minima, which fall inside $\mathscr{M}$
- M&R: V has always extrema on boundaries corresponding to maximal subgroups of $\mathscr{G}$
- chiral SU(3)⊗SU(3), x= quark masses, natural extrema correspond to:

$$SU(3):\ x = \begin{pmatrix} m & & \\ & m & \\ & & m \end{pmatrix};\quad SU(2)\otimes SU(2):\ x = \begin{pmatrix} 0 & & \\ & 0 & \\ & & m \end{pmatrix}$$

3. The quark case, three families

- couplings

$$\mathcal{L}_Y = \bar{Q}_L Y_U \tilde{H} U_R + \bar{Q}_L Y_D H D_R$$

$$Y_U \to U_L Y_U U_R^U;\ Y_D \to U_L Y_D U_R^D$$

$$Y_U = diag = m_U;\ Y_U = U_{CKM} \times diag = U m_D$$

- invariants:

$$\mathrm{Tr}(Y_U Y_U^\dagger), \cdots, \mathrm{Tr}(Y_D Y_D^\dagger), \cdots$$

$$\mathrm{Tr}(Y_U Y_U^\dagger Y_D Y_D^\dagger), \cdots$$

- unmixed invariants produce extrema corresponding to degenerate or hierarchical patterns (m_u=m_c=0, m_t generic);
- mixed invariants involve the CKM matrix U, e.g.:

$$\mathrm{Tr}(Y_U Y_U^\dagger Y_D Y_D^\dagger) = \sum_{ij} U_{ij} U_{ij}^\star (m_U)_i (m_D)_j = \sum_{ij} P_{ij} (m_U)_i (m_D)_j$$

- P is a bistochastic matrix (sum of elements of any row = sum of elements of any column = 1) and is extremized by ***permutation matrices*** (Birkhoff-Von Neumann theorem);
- the upshot is a relabeling of the down quark coupled to each up quark with U_{CKM} matrix =1

4. The lepton case, three families and see-saw

- couplings: $\mathcal{L}_Y = \bar{L}_L Y_E H E_R + \frac{1}{M}(\bar{L}_L Y_\nu \tilde{H}\tilde{H} Y_\nu^T L_L^c)$

  $Y_E \to U_L Y_E U_R^E;\ Y_\nu \to U_L Y_\nu \mathcal{O}$

  $\mathcal{G} = SU(3)_L \otimes SU(3)_E \otimes O(3)$

  $Y_E = diag = y_E;\ Y_\nu = U \times y_{diag} \times U_R$

- masses and phases: $M_\nu = \frac{v^2}{M} U(y_{diag} U_R U_R^T y_{diag})U = U_{PMNS}\ \Omega\ m_\nu\ \Omega\ U_{PMNS}^T$

  $\Omega = \text{diagonal Majorana phase}$

- invariants: $\mathrm{Tr}(Y_E Y_E^\dagger), \cdots, \mathrm{Tr}(Y_\nu Y_\nu^\dagger), \cdots$

  $\mathrm{Tr}(Y_E Y_E^\dagger Y_\nu Y_\nu^\dagger), \cdots$ Mixed, type 1

  $\mathrm{Tr}(Y_\nu^\dagger Y_\nu Y_\nu^T Y_\nu^\star), \cdots$ Mixed, type 2

- unmixed invariants produce extrema corresponding to degenerate or hierarchical patterns ($m_e = m_\mu = 0$, m_τ generic);
- mixed type 1 invariants contain ($U_{ij} U_{ij}^*$), like the quark case → permutation matrix;
- mixed type 2 invariants contain ($U_{Rij} U_{Rij}^*$) → a second permutation matrix.
- We may absorb the first permutation matrix in a relabeling of the neutrinos coupled to each charged lepton,
- but the second matrix ***leads to a Majorana phase and a 45^0 mixing between the two switched neutrinos***, which in addition ***are required to be degenerate*** !
- (for two families, this result was found by Alonso, Gavela, Hernandez and Merlo)
- if the third neutrino is also degenerate, we have an instable situation which may give rise to another large angle.
- ***Do the large angles observed in neutrinos indicate that neutrinos are almost degenerate***?

Group theoretical considerations

- the the solution we have found is: $Y_\nu = \begin{pmatrix} y_1 & 0 & 0 \\ 0 & \frac{y_2}{\sqrt{2}} & -i\frac{y_2}{\sqrt{2}} \\ 0 & \frac{y_3}{\sqrt{2}} & i\frac{y_3}{\sqrt{2}} \end{pmatrix}$
- it transforms, under $SU(3)_L \otimes O(3)$, according to: $Y_\nu \approx (\bar{3}, 3_V)$
- the suffix V denotes the vector representaton of $O(3)$, realized, in triplet space, by the Gell-Mann imaginary matrices $\lambda_{2,5,7}$
- one verifies that: $\frac{1}{2}(\lambda_3 + \sqrt{3}\lambda_8)Y_\nu - Y_\nu\lambda_7 = 0$
- for this solution, $SU(3)_L \otimes O(3) \rightarrow U(1)$ corresponding to simultaneous transformations by the two indicated generators;
- this U(1) is the little group of the boundary to which the solution belongs.

... more precisely

$$U = 1, \;\; y_{diag} = \mathrm{diag}(y_1, y_2, y_3)$$

$$U_R U_R^T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \; ; \; M_\nu = \begin{pmatrix} y_1^2 & 0 & 0 \\ 0 & 0 & y_2 y_3 \\ 0 & y_2 y_3 & 0 \end{pmatrix}$$

$$\Omega = \mathrm{diag}(1,1,i), \;\; U_{PMNS,0} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ 0 & -1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}$$

- with further degeneracy:

$$y_1^2 = y_2 y_3 \to m_\nu = \mathrm{diag}(m, m, -m)$$

- introduce small perturbations:

$$M_\nu = \frac{v^2 y}{M} \begin{pmatrix} 1+\delta & \epsilon+\eta & \epsilon-\eta \\ \epsilon+\eta & \delta & 1 \\ \epsilon-\eta & 1 & \delta \end{pmatrix}$$

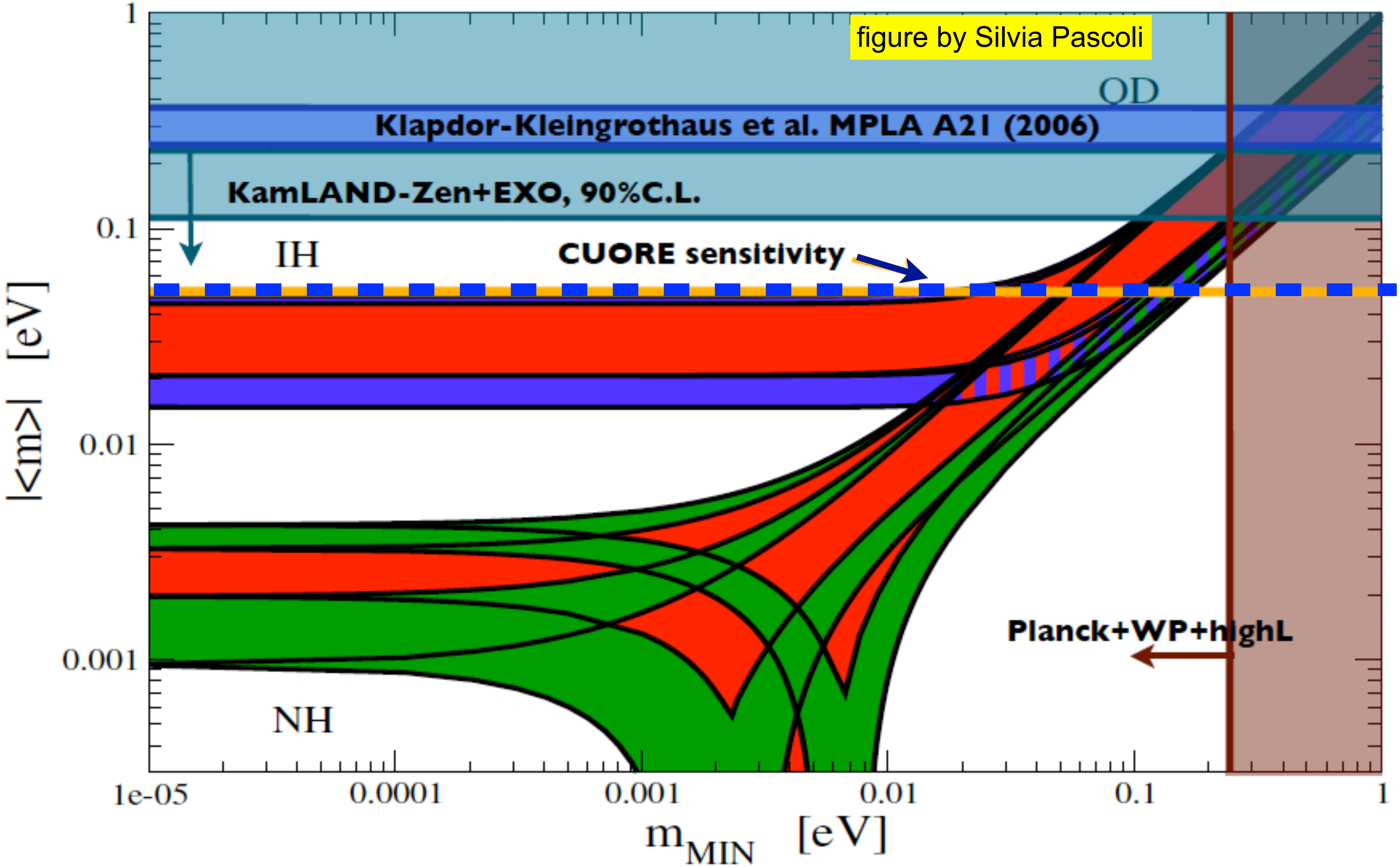
- to first order in the perturbations, we find:

$$m_\nu = m \begin{pmatrix} 1+\delta+\sqrt{2}\epsilon & 0 & 0 \\ 0 & 1+\delta-\sqrt{2}\epsilon & 0 \\ 0 & 0 & -1+\delta \end{pmatrix} \quad \mathrm{U}_{PMNS} = \begin{pmatrix} 1/\sqrt{2} & -1/\sqrt{2} & \eta/\sqrt{2} \\ 1/2(1+\eta/\sqrt{2}) & 1/2(1-\eta/\sqrt{2}) & -1/\sqrt{2} \\ 1/2(1-\eta/\sqrt{2}) & 1/2(1+\eta/\sqrt{2}) & 1/\sqrt{2} \end{pmatrix}$$

- e.g. ***normal or inverted hierarchy, with close to degenerate neutrinos, approximate bimaximal mixing (with negative θ_{12}) and a small θ_{13}.***
- ***Determining the size of the perturbations from sin θ_{13} or, equivalently, from the deviation of θ_{23} from pi/4, and assuming a similar size for the perturbations, we estimate***

$$\frac{|\Delta m^2_{\mathrm{atm}}|}{2m_0^2} \approx |\sin\theta_{13}| \approx |\theta_{12} - \frac{\pi}{4}| \approx 0.1; \;\; m \approx 0.1 \text{ eV}$$

we may be not so far from testing this prediction in 0 ν-β decay



latest from Planck....

$$\sum m_\nu = 0.22 \pm 0.09 \text{ eV}$$

Planck Collaboration: Cos

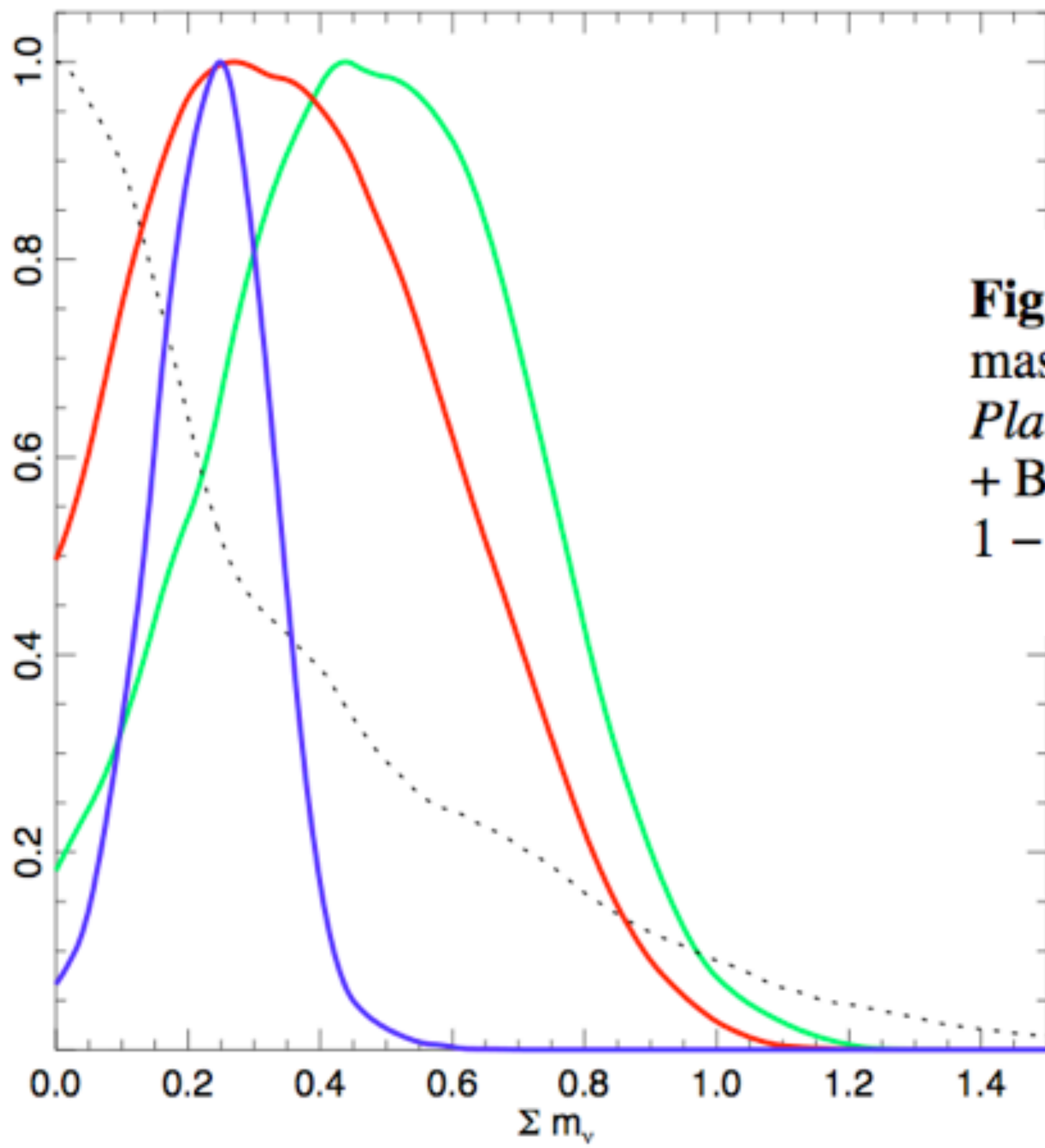


Fig. 12. Cosmological constraints when including neutrino masses $\sum m_\nu$ from: *Planck* CMB data alone (black dotted line); *Planck* CMB + SZ with $1 - b$ in [0.7, 1] (red); *Planck* CMB + SZ + BAO with $1 - b$ in [0.7, 1] (blue); and *Planck* CMB + SZ with $1 - b = 0.8$ (green).

6. Summing up

- The idea that the "Yukawa coupligs" satisfy a minimum principle with a potential symmetric under the flavor group of the Standard Theory and three generations leads to two interesting solutions:
  - *quarks: hierarchical mass pattern and unity CKM matrix;*
  - *leptons: hierarchical masses for charged leptons and degenerate Majorana neutrinos with one, potentially two, large mixing angles.*
- both solutions are close to the real situation;
- in particular, for neutrinos, small perturbations lead to a realistic pattern of mass differences and to a PNMS mixing matrix close to the bimaximal (or tribimaximal) mixing with small θ_{13}, without having to resort to artificial discrete groups;
- the prediction that large mixing angles are related to Majorana degenerate neutrinos may be amenable to experimental test in a not too distant future.