Exotic Atoms and New Physics Searches

Seminar
Università degli Studi di Napoli Federico II

Andrzej Czarnecki University of Alberta

Outline

Lamb shift: large and small Z

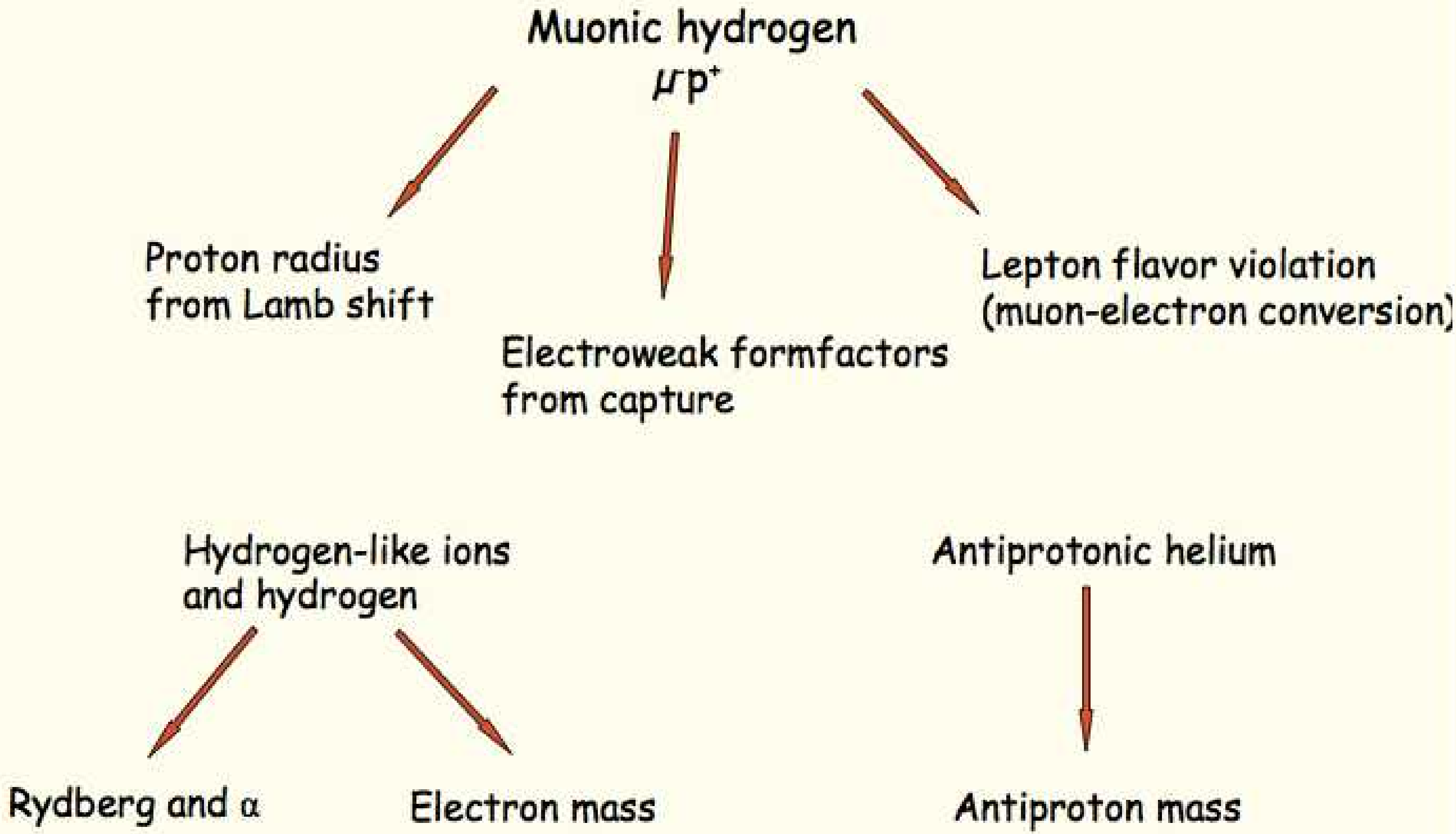
Muon decay in orbit and μe conversion

Positronium: hyperfine splitting

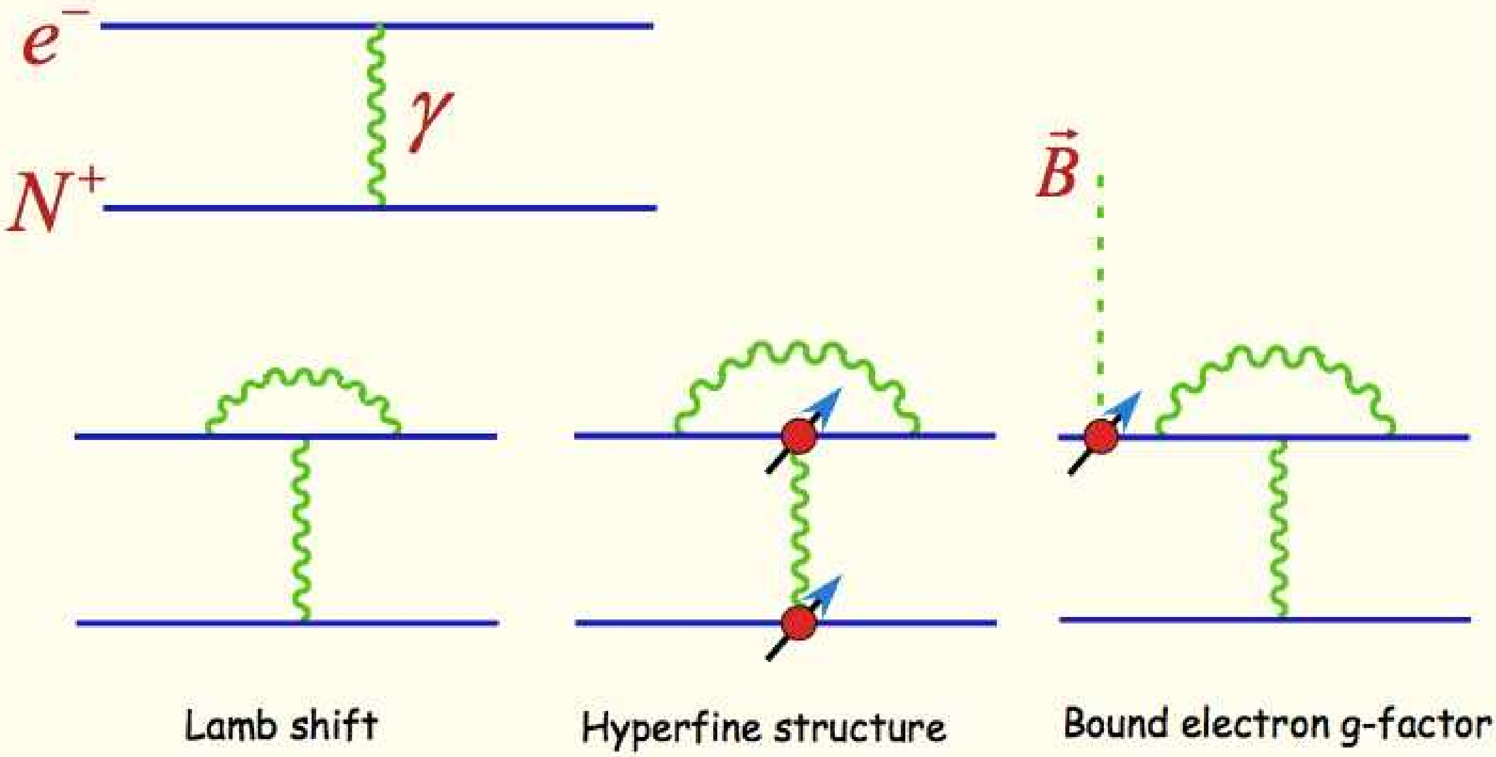
– g factor of a bound electron

Loops in few-body systems : He, Ps_2, Ps^-

Exotic atoms today



Atomic effects of interest (examples)

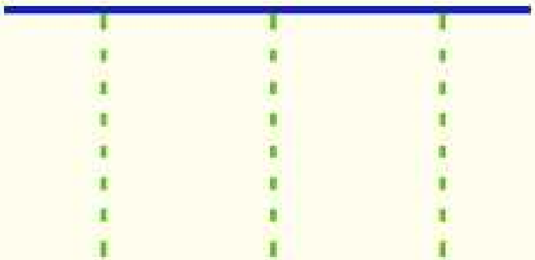


Difficulty in the theory of simple atoms: diversity of energy scales.

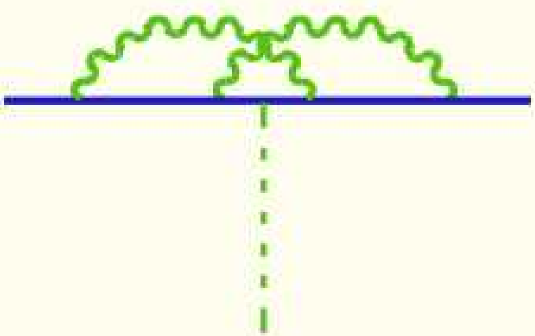
Opportunity: several expansion parameters:

$$\alpha,\ Z\alpha,\ \frac{m_e}{m_N}$$

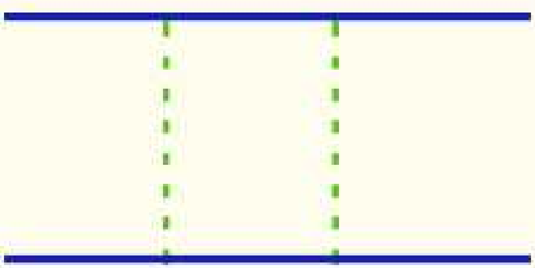
Expansion parameters



$Z\alpha$

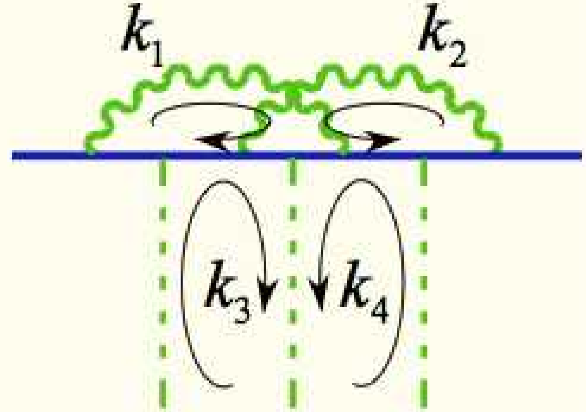


α



$\frac{m_e}{m_p}$

Example of a cutting-edge problem:



Technical tools

$$\frac{\partial}{\partial k_\mu}$$

Recurrence relations
Determination of master integrals
Expansion in small masses (for recoil effects)
Treatment of tensor integrals

Corrections of order $\alpha^2(Z\alpha)^5$ to the hyperfine splitting and the Lamb shift

MICHAEL I. EIDES AND VALERY A. SHELYUTO

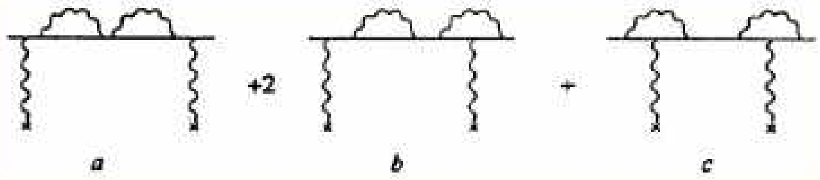


Diagram	Lamb shift $\left[\frac{\alpha^2(Z\alpha)^5}{\pi n^3}\left(\frac{m_r}{m}\right)^3 m\right]$
a	0
b	2.9551(1)
c	−2.2231(1)

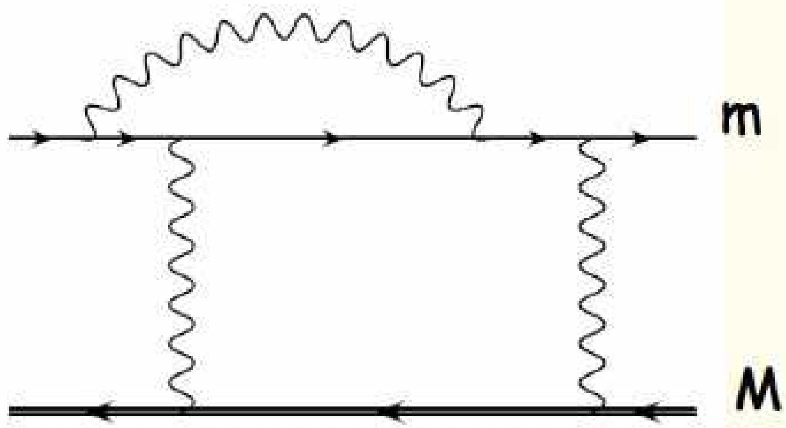
Analytical value:

$$
\begin{aligned}
&-\frac{352897}{27000}+\frac{31\pi^2}{60}-\frac{643\ln 2}{225}-\frac{248\ln^2 2}{15} \\
&\quad -\frac{26}{9\sqrt{5}}\ln\left(\frac{7-3\sqrt{5}}{2}\right)-\frac{31}{20}\ln\left(\frac{7-3\sqrt{5}}{2}\right)\ln\left(\frac{1+\sqrt{5}}{2}\right) \\
&\quad +\frac{31}{6}\ln^2\left(\sqrt{5}-2\right)-\frac{31}{15}\mathrm{Li}_2\left(2-\sqrt{5}\right)+\frac{31}{15}\mathrm{Li}_2\left(\sqrt{5}-2\right) \\
&= -2.22313.
\end{aligned}
$$

$$\frac{\partial}{\partial k_\mu}$$

Expansion in mass ratios

with Kirill Melnikov



$$J_1^{\pm} = \int \frac{[\mathrm{d}^D k_1][\mathrm{d}^D k_2]}{(k_1 Q - 1 \pm i\delta)(k_2^2 + i\delta)\left[(k_1+k_2)^2 - 1 + i\delta\right]}$$
$$= \frac{1}{(4\pi)^D}\Big[2\Gamma(1-\epsilon)\Gamma(3\epsilon-2)B(4\epsilon-3, 2\epsilon-1)$$
$$-(1\mp 1)\sqrt{\pi}\,\Gamma\left(2\epsilon-\frac{3}{2}\right)B\left(\frac{5}{2}-3\epsilon, -\frac{1}{2}+\epsilon\right)\Big]$$

$$\delta E^{\rm rad\ rec}_{\rm aver} \simeq \alpha(Z\alpha)^5 \frac{\mu^3}{m^2}\left\{\frac{139}{32} - 2\ln 2\right.$$
$$+\frac{m}{M}\left(\frac{3}{4} + \frac{6\zeta_3}{\pi^2} - \frac{14}{\pi^2} - 2\ln 2\right)$$
$$+\left(\frac{m}{M}\right)^2\left(-\frac{127}{32} + 8\ln 2\right)$$
$$+\left(\frac{m}{M}\right)^3\left(-\frac{8}{3\pi^2}\ln^2\frac{M}{m} - \frac{55}{18\pi^2}\ln\frac{M}{m} + \frac{47}{36} - \frac{3\zeta_3}{\pi^2} - \frac{85}{9\pi^2} - 2\ln 2\right)$$

-1.32402796 agrees with Pachucki

Shabaev et al.: Lamb shift
to all orders in Za

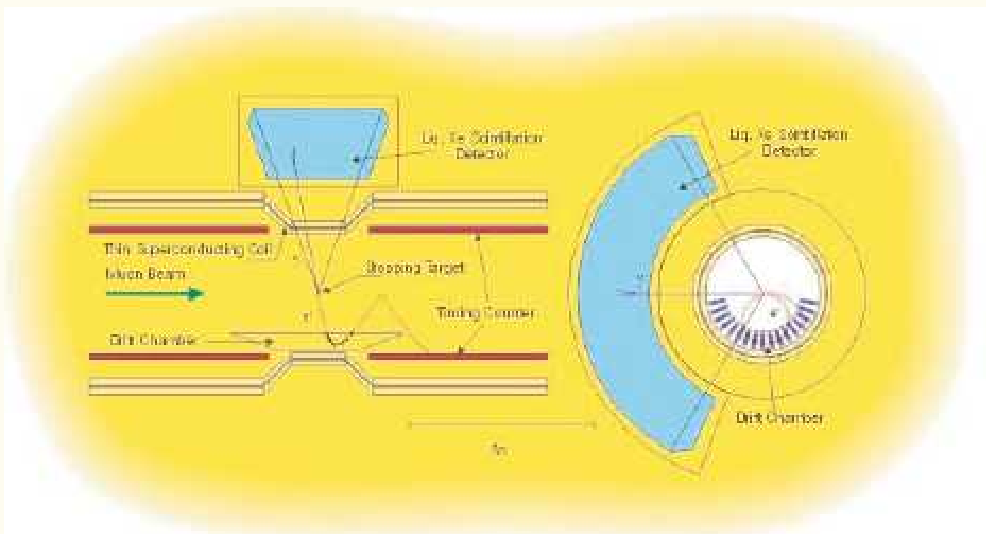
Muon decay in orbit and searches for lepton flavor violation

Muon decay to an electron and photon, μ→eγ

Until recently (MEGA @ Los Alamos): $BR(\mu \to e\gamma) < 10^{-11}$

New bound (MEG @ Paul Scherrer Institute)

$< 5.7 \cdot 10^{-13} \quad (2013)$



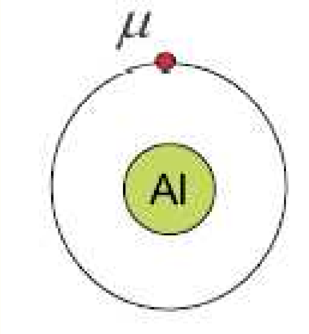
Note: unusual QED suppression ~15% (large log of the new physics scale Λ)

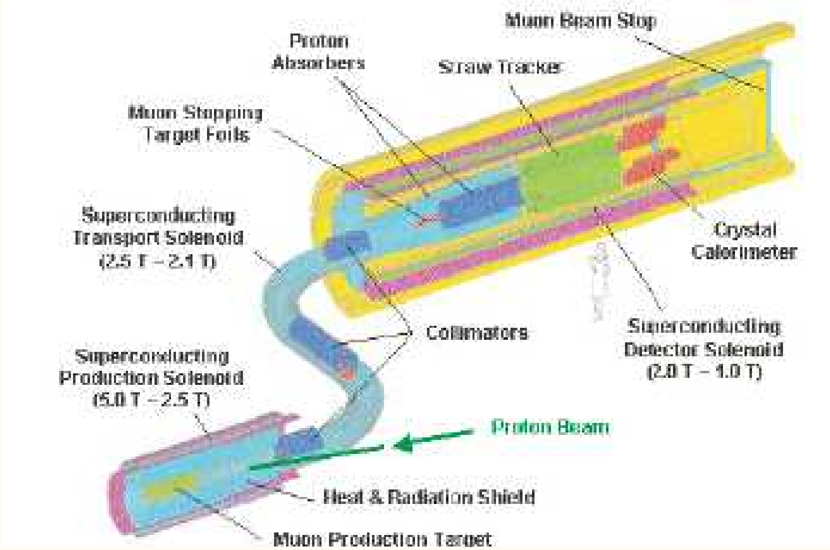
$$\Gamma(\mu \to e\gamma) \simeq \left(1 - \frac{8\alpha}{\pi} \ln \frac{\Lambda}{m_\mu}\right) \Gamma^{(0)}(\mu \to e\gamma)$$

Phys. Rev. D 65, 113004

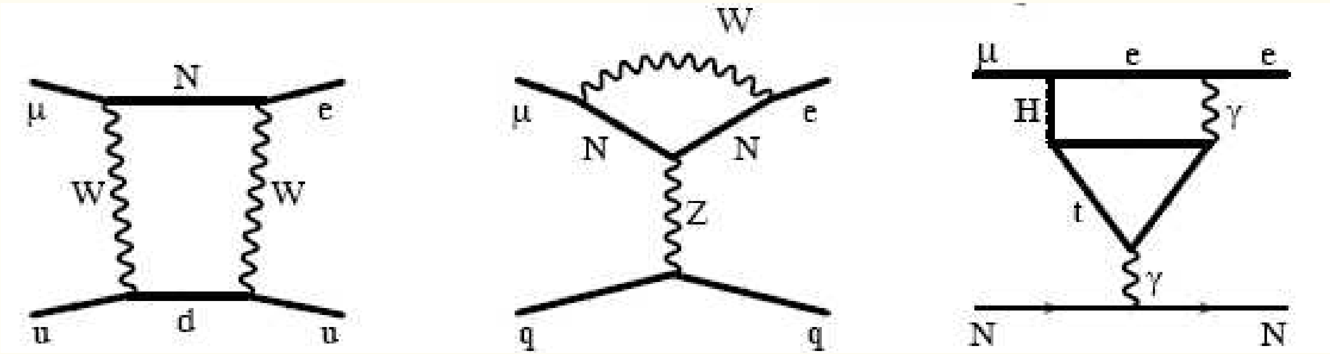
Muon-electron conversion

"The best rare process"
No accidental bkgd
(single monochromatic e^-);
10^{-17} sensitivity envisioned

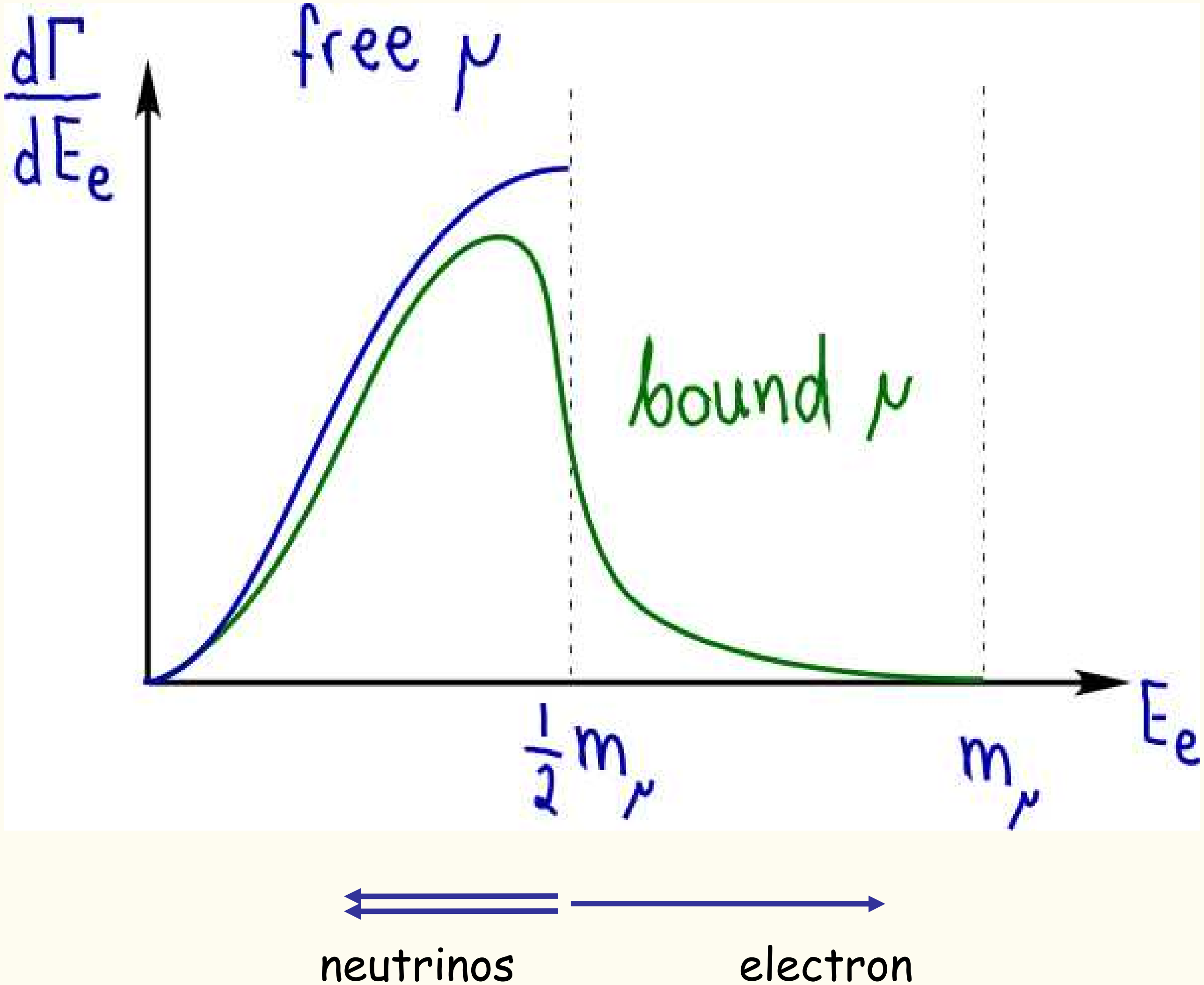




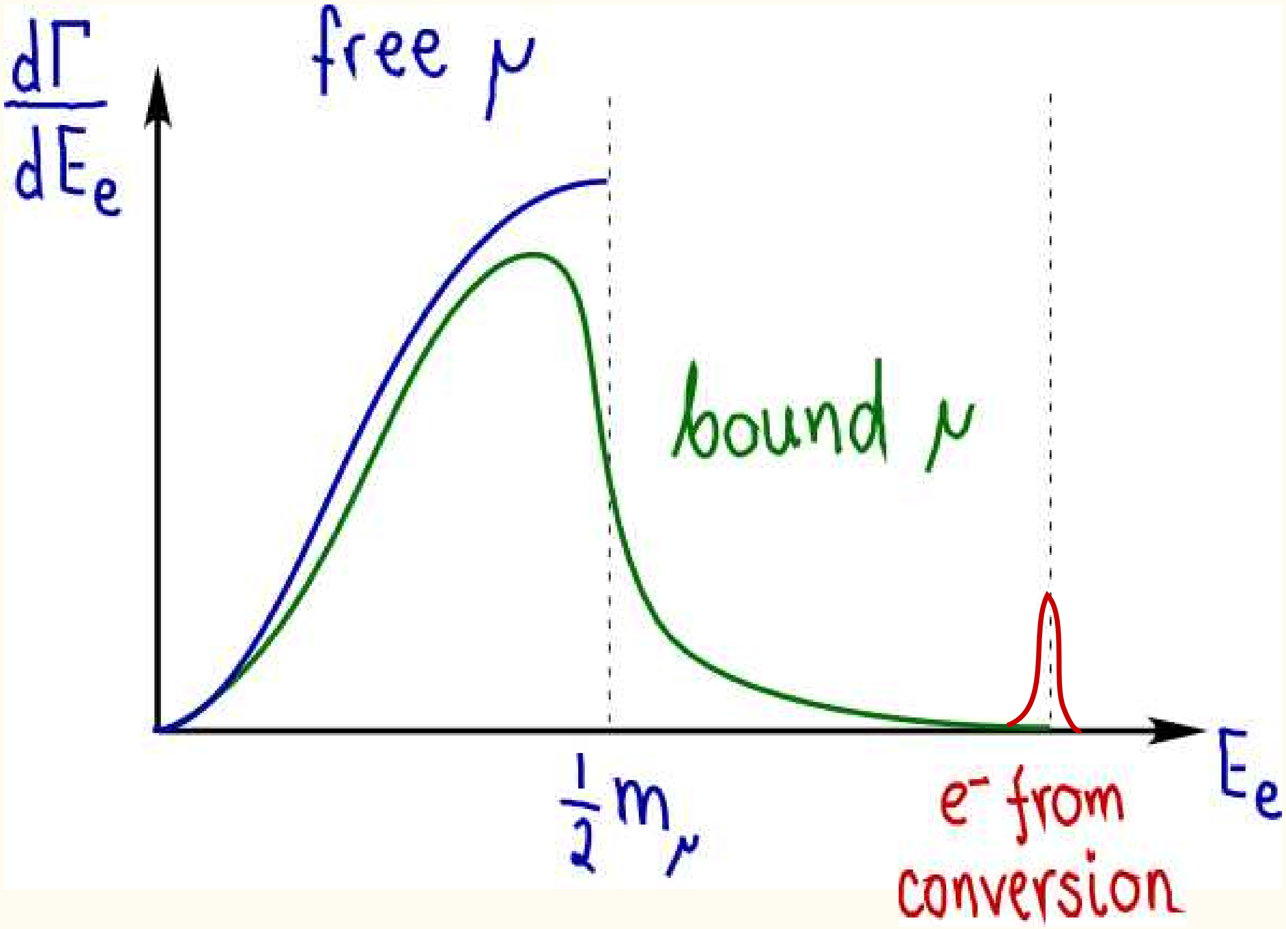
Variety of mechanisms:



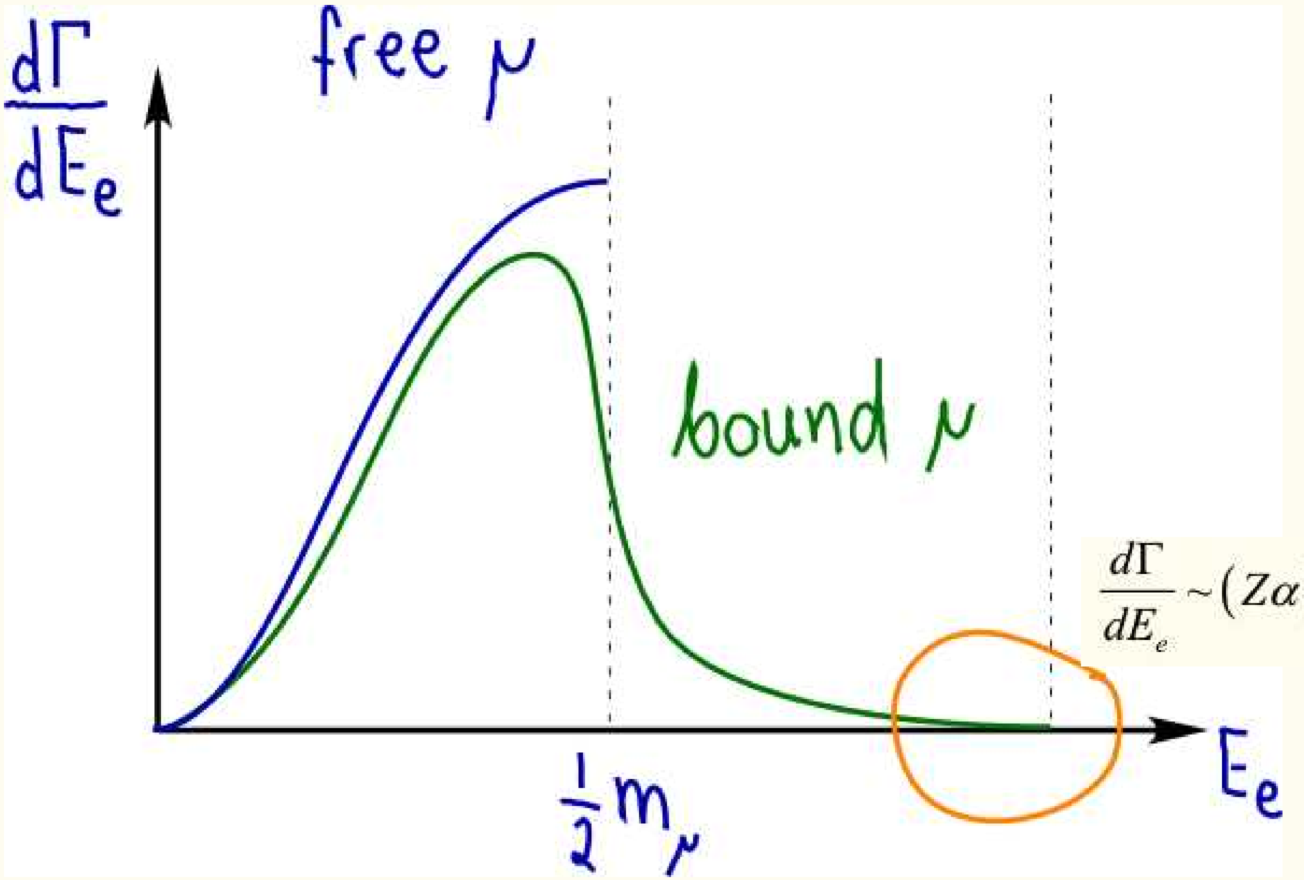
Background from the standard muon decay



Background from the standard muon decay



End point spectrum must be well understood



$$\frac{d\Gamma}{dE_e} \sim (Z\alpha)^5 (E_{\max} - E)^5$$

End point spectrum

Previous studies: Shanker & Roy, Hänggi et al., Herzog & Alder

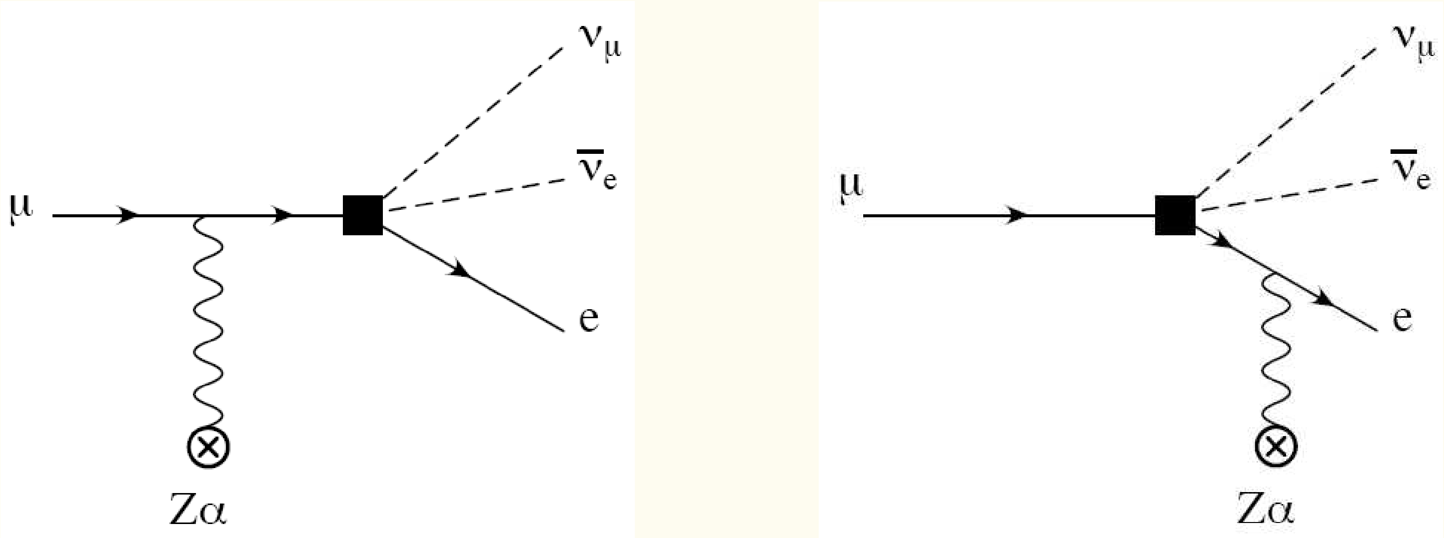
Relativistic muon wave function, nuclear size and recoil, electron final state interactions: all taken into account.

$$N(E_e)\,dE_e \simeq 0.4 \cdot 10^{-21}\left(1-\frac{E_e}{E_{\max}}\right)^5 dE_e$$

New evaluation: AC, X. Garcia i Tormo, W. J. Marciano PRD84,013006,2011

Planned energy resolution in Mu2e: ~250 keV → 0.22 background events.

How can the electron get muon's whole energy?

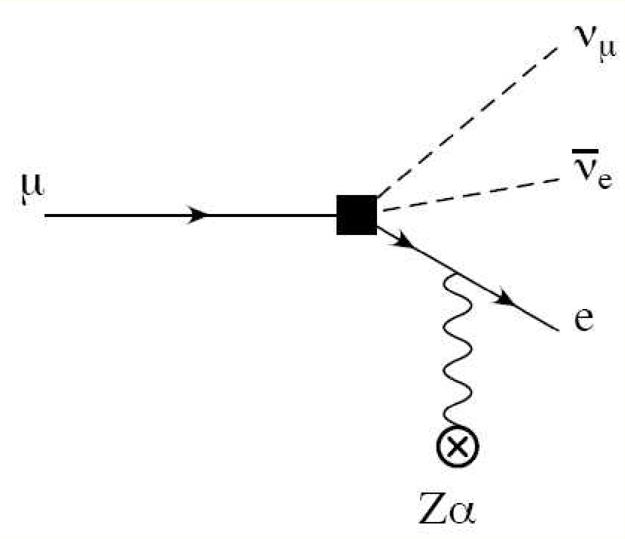


Neutrinos get no energy;
The nucleus balances electron's momentum, takes no energy.
Near the end point:

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}E_e} \sim |\psi(0)|^2 (Z\alpha)^2 \frac{\mathrm{d}^3\nu_e}{\nu_e}\frac{\mathrm{d}^3\nu_\mu}{\nu_\mu}\delta(E_{\mathrm{max}} - E_e - \nu_e - \nu_\mu)\,\mathrm{Tr}\ldots \not{\nu}_e \ldots \not{\nu}_\mu$$

$$\sim (Z\alpha)^5 (E_{\mathrm{max}} - E_e)^5$$

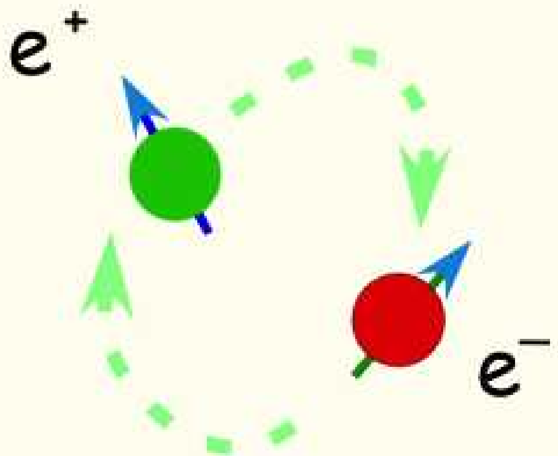
Next step: radiative corrections to the electron spectrum

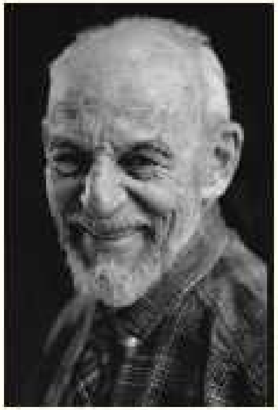


Competing effects:
- vacuum polarization in the hard photon; and
- self-energy and real radiation

Ultimate goal: smooth matching of various regions, from the bound electron at low energy, to the end-point.

Positronium





Martin Deutsch
1917 - 2002

Very similar to hydrogen, except
- no hadronic nucleus
- annihilation
- reduced mass reduced $m_e \rightarrow \frac{m_e}{2}$

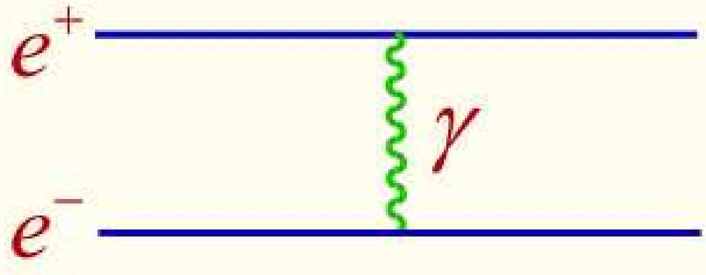
Two spin states:
singlet (para-Ps; short-lived, 0.1 ns)
triplet (ortho-Ps; long-lived, ~150 ns)

All properties can be described by QED, using one parameter: $\alpha = \frac{1}{137.036}$

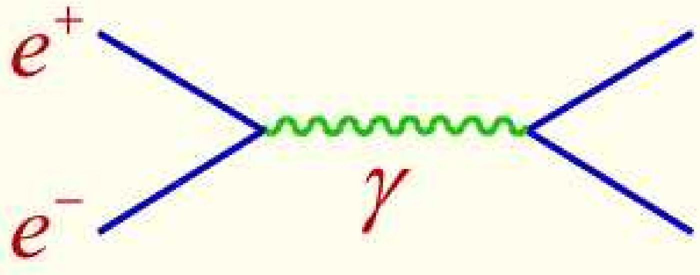
Positronium spectrum: discrepancy with QED

Tree-level QED prediction for the hyperfine splitting (HFS)

Recoil effect



Annihilation effect

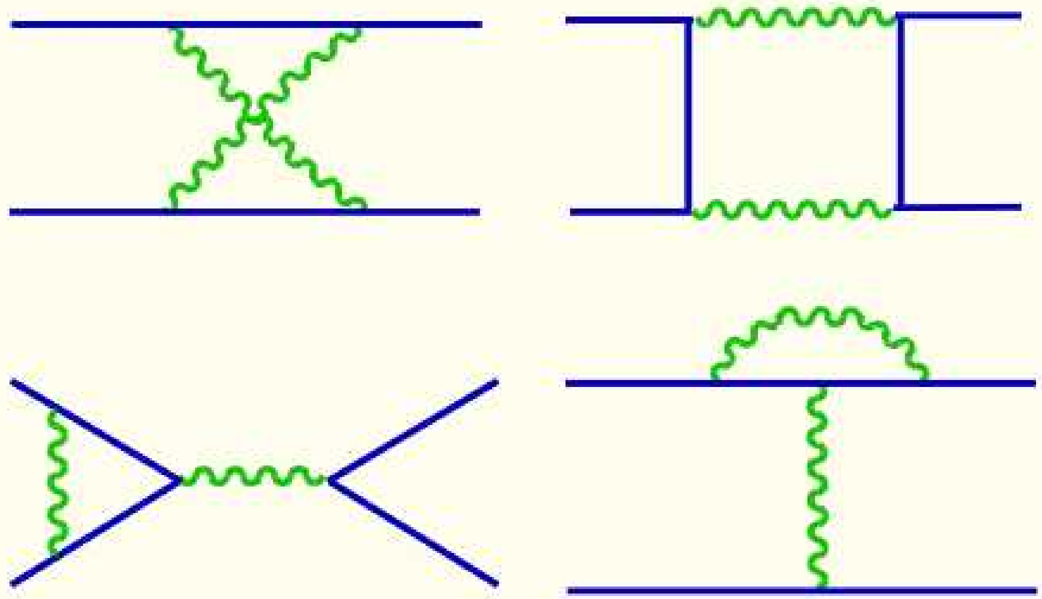


$$\gamma_\mu \otimes \gamma^\mu \to 1 \otimes 1 + \sigma \otimes \sigma$$

$$\Delta \nu_{\mathrm{HFS}} = \frac{7}{12} m_e \alpha^4 \simeq 204 \text{ GHz}$$

Quantum corrections to the HFS: one-loop

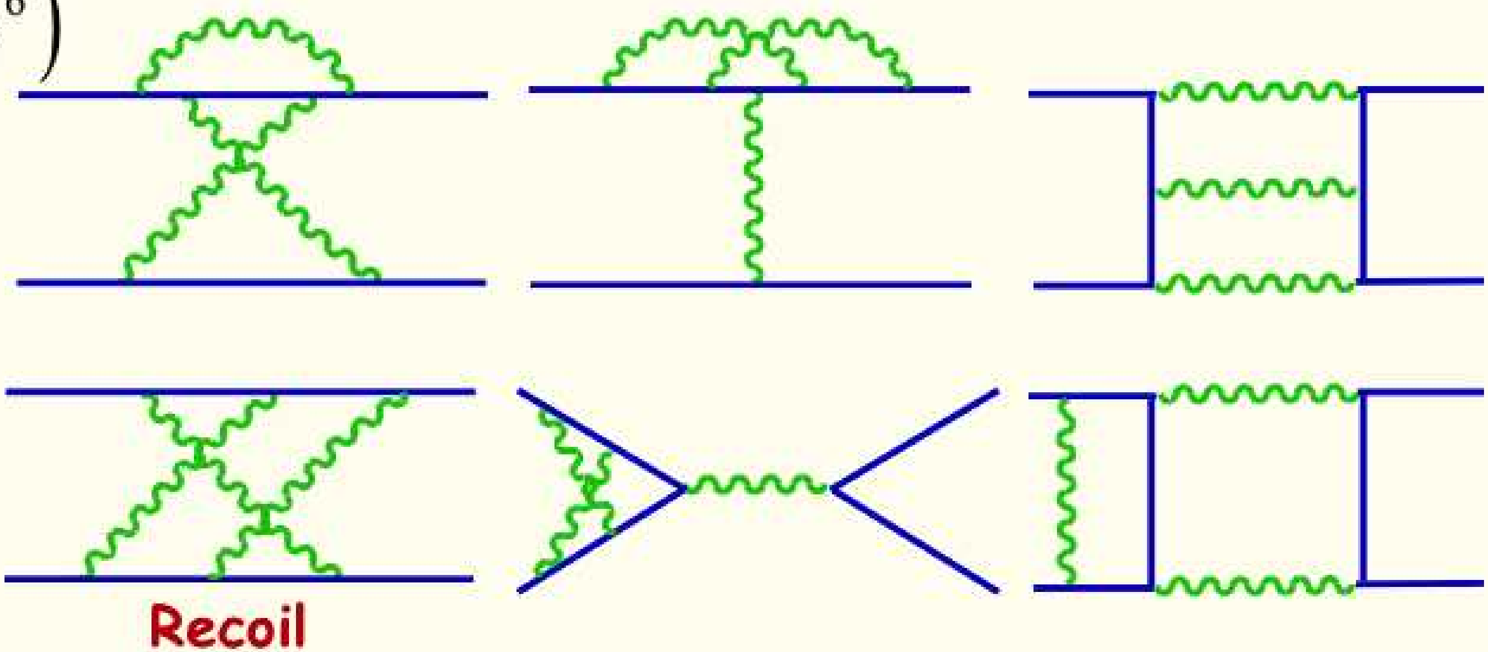
$O\left(\alpha^5\right)$



$$-\frac{m_e\alpha^5}{\pi}\left(\frac{8}{9}+\frac{\ln 2}{2}\right)\simeq -1005.5\ \text{MHz} \rightarrow -0.5\%$$

Quantum corrections to the HFS: two-loop

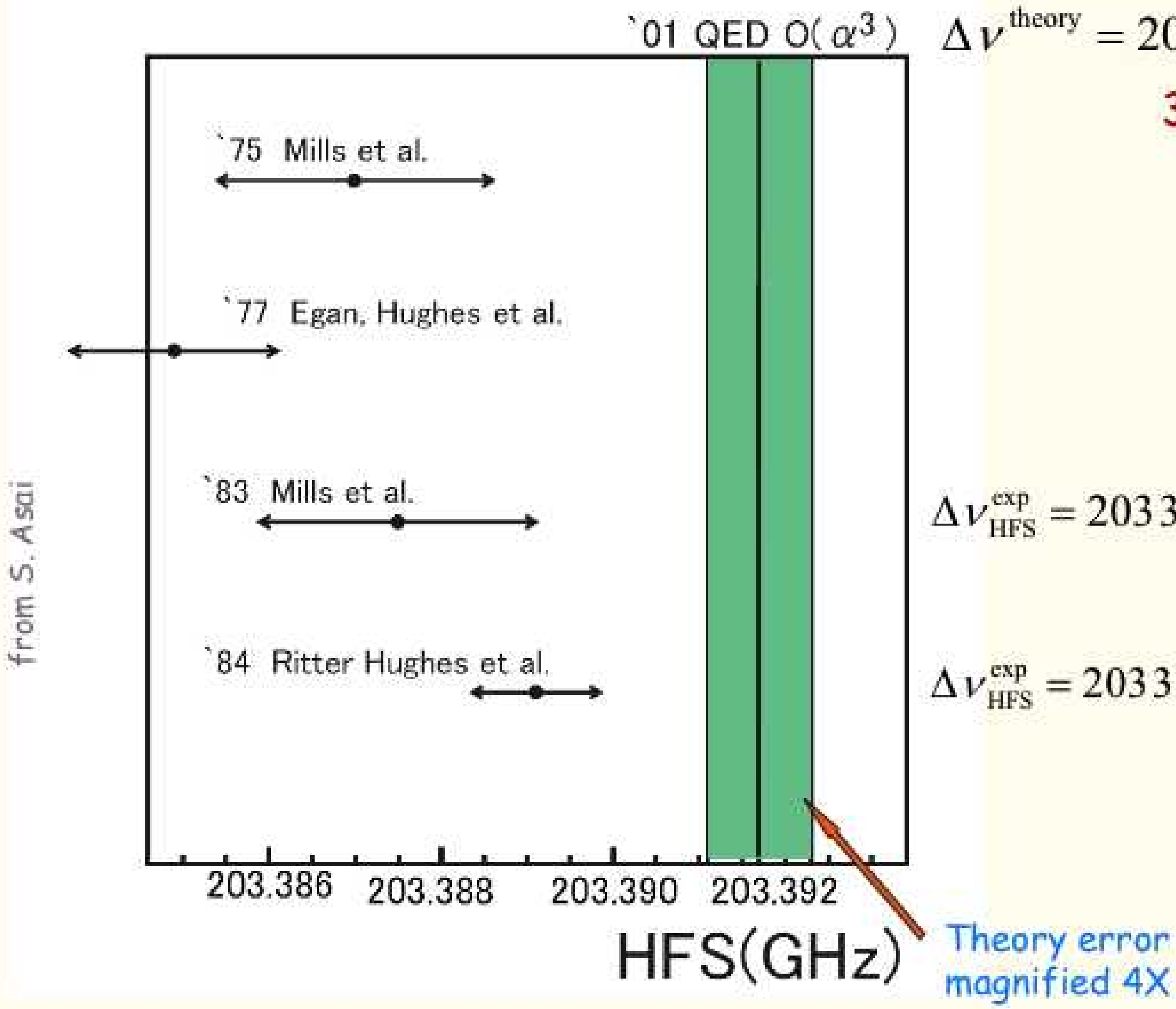
$O\left(\alpha^{6}\right)$



$$\frac{m_e\alpha^6}{\pi^2}\left[\frac{1367}{648}-\frac{5197}{3456}\pi^2+\left(\frac{1}{2}+\frac{221}{144}\pi^2\right)\ln 2-\frac{53}{32}\zeta(3)+\frac{5}{24}\pi^2\ln\frac{1}{\alpha}\right]$$

$\simeq 11.8$ MHz $\rightarrow$ 0.006% (Experimental error $\simeq 0.7$ MHz)

HFS theory vs. measurements



$$\Delta \nu^{theory} = 203391.69(16)\ \text{MHz}$$

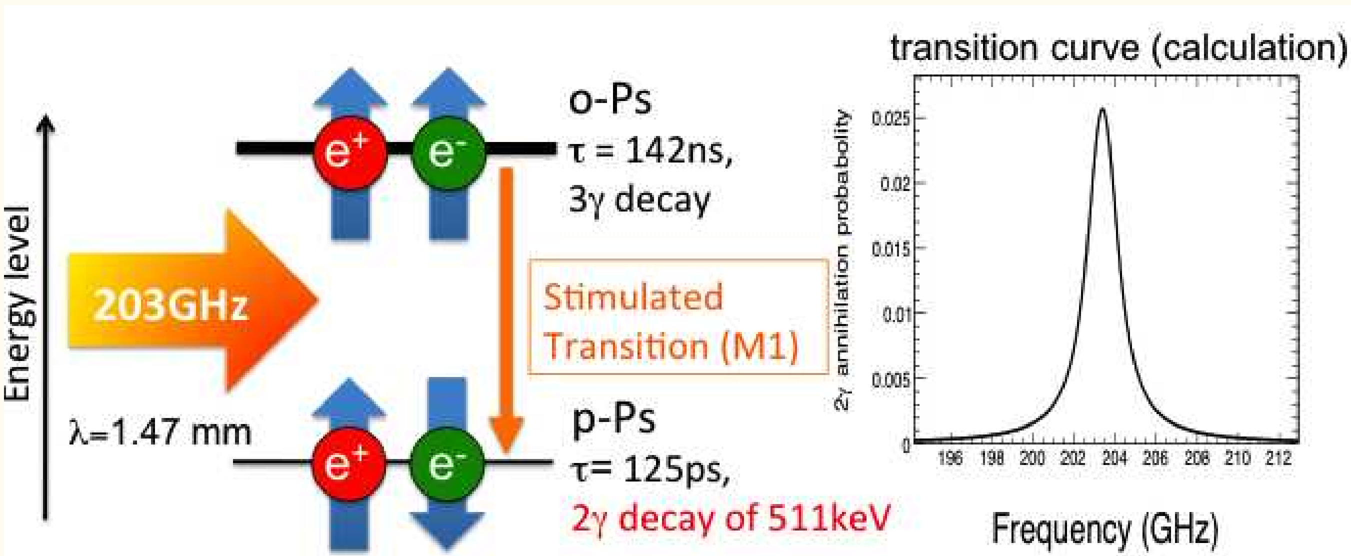
3.4σ off!

Kniehl & Penin

$$\Delta \nu^{exp}_{HFS} = 203387.5(1.6)\ \text{MHz}$$

$$\Delta \nu^{exp}_{HFS} = 203389.10(74)\ \text{MHz}$$

New experiment aims at direct transition



No one has ever observed the direct transition between these states.

Akira Miyazaki[a], T . Yamazaki[a], T. Suehara[a], T. Namba[a], S. Asai[a], T. Kobayashi[a], H . Saito[b], T. Idehara[c], I. Ogawa[c], S. Sabchevski[d]

Previous experiments: used para-ortho mixing

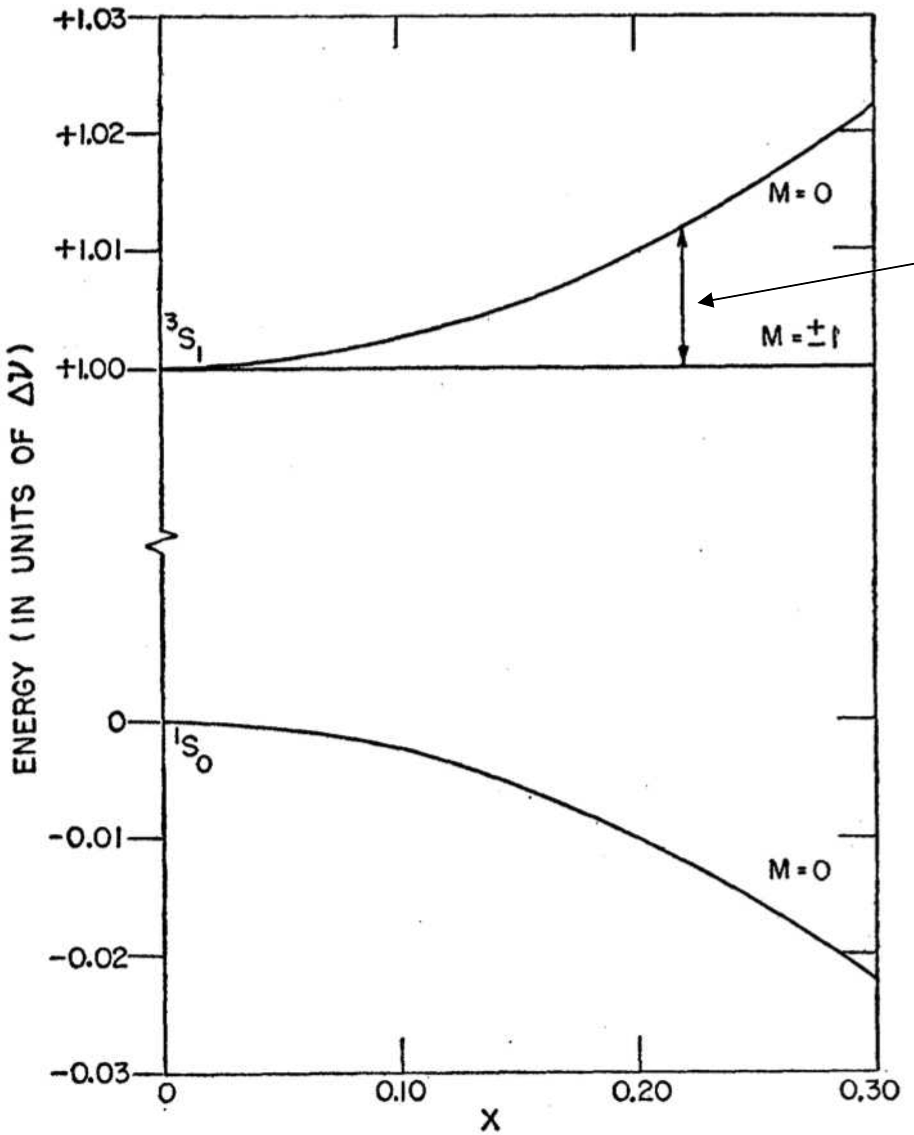
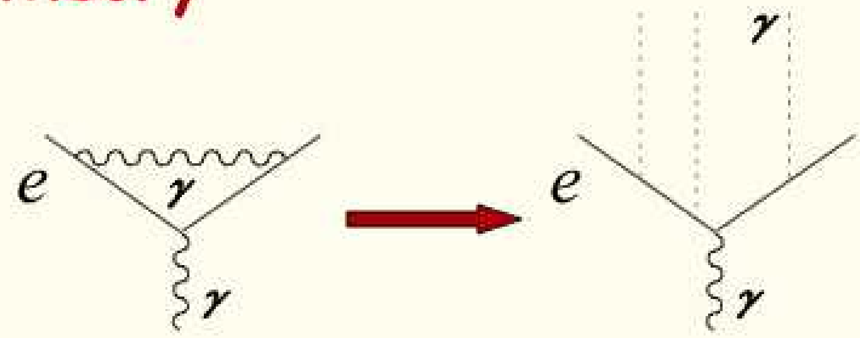


FIG. 1. Zeeman energy levels of positronium in its ground $n=1$

This splitting proportional to electron's *g*-factor, <u>in the bound state</u>

Bound-electron g-2: theory

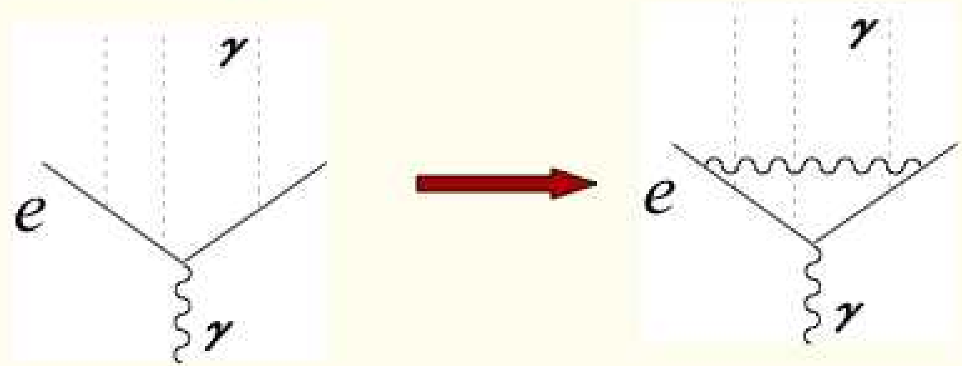


$$g = 2 - \frac{2(Z\alpha)^2}{3} - \frac{(Z\alpha)^4}{6} + O(Z\alpha)^6 = \underbrace{\frac{2}{3}\left[1 + 2\sqrt{1-(Z\alpha)^2}\right]}$$

Breit 1928 – Dirac theory

Note: Breit's calculation predates Schwinger's by 20 years

Bound-electron g-2: theory



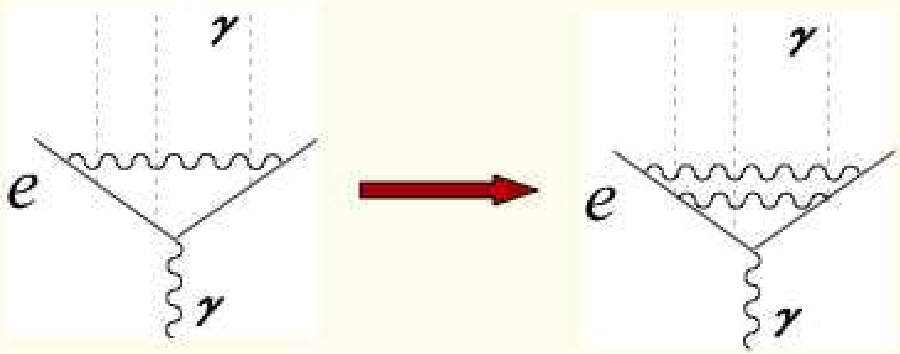
$$g = 2 - \frac{2(Z\alpha)^2}{3} - \frac{(Z\alpha)^4}{6} + O(Z\alpha)^6$$

$$\underbrace{+\frac{\alpha}{\pi}\left[1 + \frac{(Z\alpha)^2}{6} + (Z\alpha)^4\left(a_{41}\ln\frac{1}{(Z\alpha)^2} + a_{40}\right) + O(Z\alpha)^5\right]}_{\text{one-loop corrections}}$$

one-loop corrections

Pachucki, Jentschura, Yerokhin
2004

Bound-electron g-2: theory



$$g = 2 - \frac{2(Z\alpha)^2}{3} - \frac{(Z\alpha)^4}{6} + O(Z\alpha)^6$$

$$+ \frac{\alpha}{\pi}\left[1 + \frac{(Z\alpha)^2}{6} + (Z\alpha)^4\left(a_{41}\ln\frac{1}{(Z\alpha)^2} + a_{40}\right) + O(Z\alpha)^5\right]$$

$$\underbrace{+\left(\frac{\alpha}{\pi}\right)^2\left[-0.65..\left(1 + \frac{(Z\alpha)^2}{6}\right) + (Z\alpha)^4\left(b_{41}\ln\frac{1}{(Z\alpha)^2} + b_{40}\right) + ..\right]}_{\text{two-loop corrections}}$$

$$b_{41} = \frac{28}{9}$$

$$b_{40} = -16.4$$

Pachucki, AC, Jentschura, Yerokhin
2005

$$m_e\left({}^{12}C^{5+}\right) = 0.000\,548\,579\,909\,31(29)_{\text{exp}}(1)_{\text{th}}\ u$$

0.6 ppb

Theoretical error: negligible

2010: new measurement with oxygen, $^{16}O^{7+}$

Theoretical prediction:

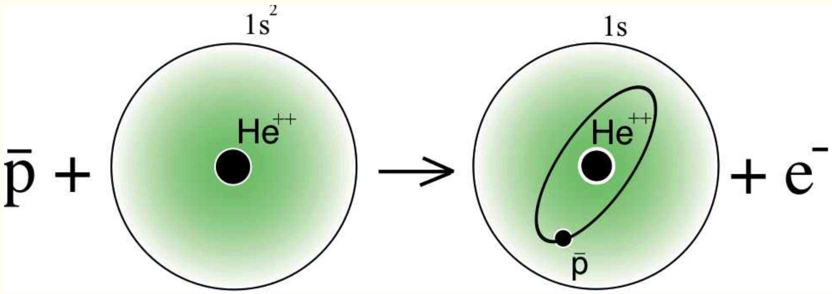
$$g^{\text{th}}(Z=8) = 2.000\,047\,020\,32(11)$$

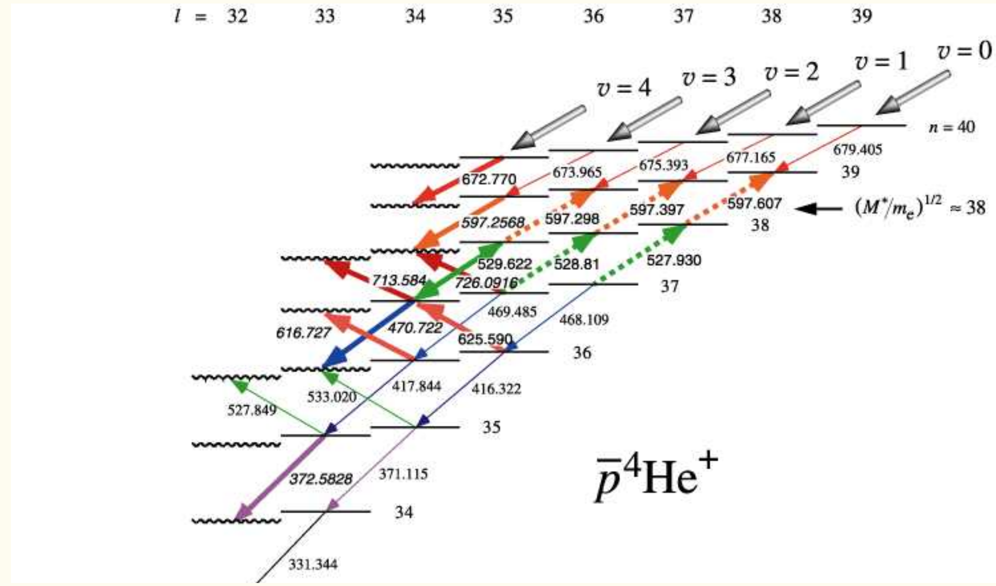
Measured value:

$$g^{\text{exp}}(Z=8) = 2.000\,047\,0201(25)$$

J. Verdú,[1] H. Häffner,[2] W. Quint,[3] T. Valenzuela,[4] and G. Werth[5] (preliminary)

Few-body systems: antiprotonic helium





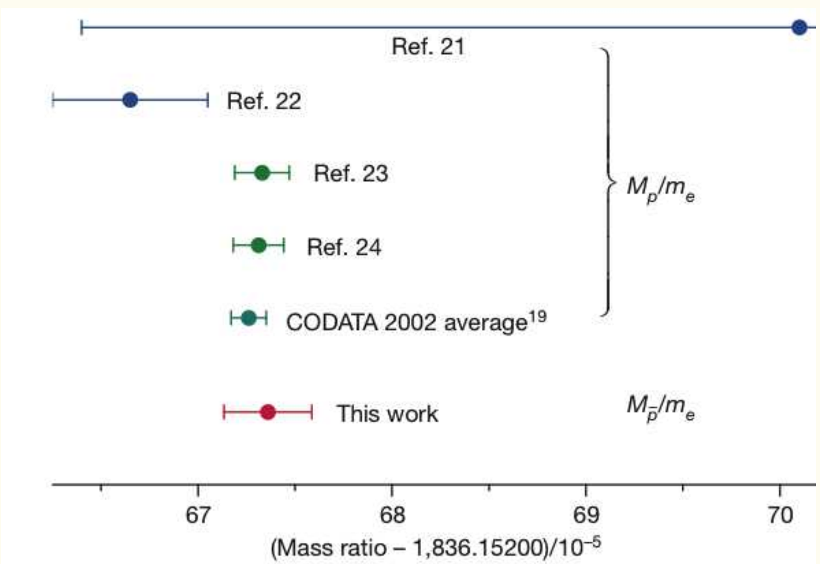


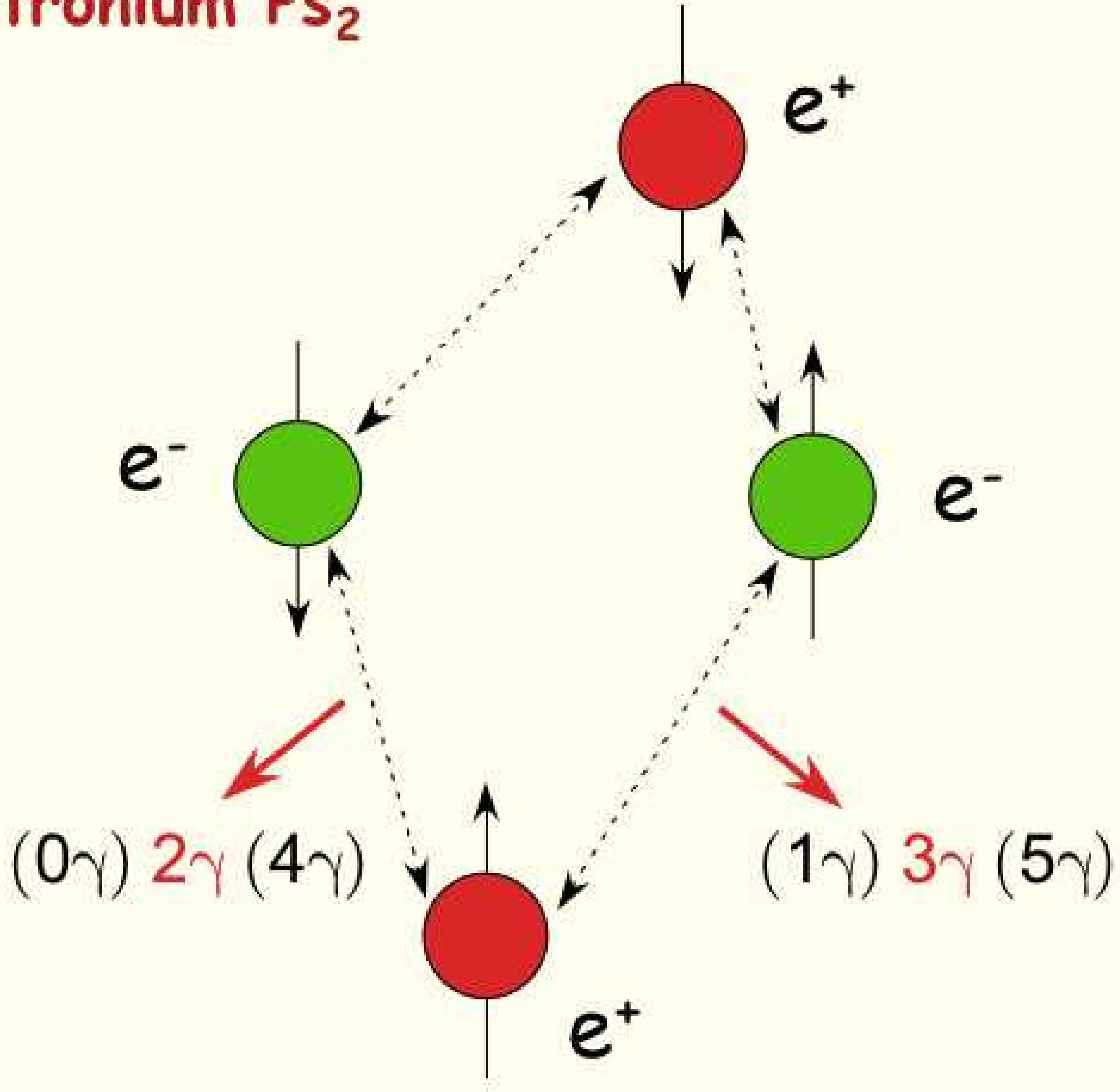
Figure 4 | Antiproton-to-electron and proton-to-electron mass ratios.

484 | NATURE | VOL 475 | 28 JULY 2011

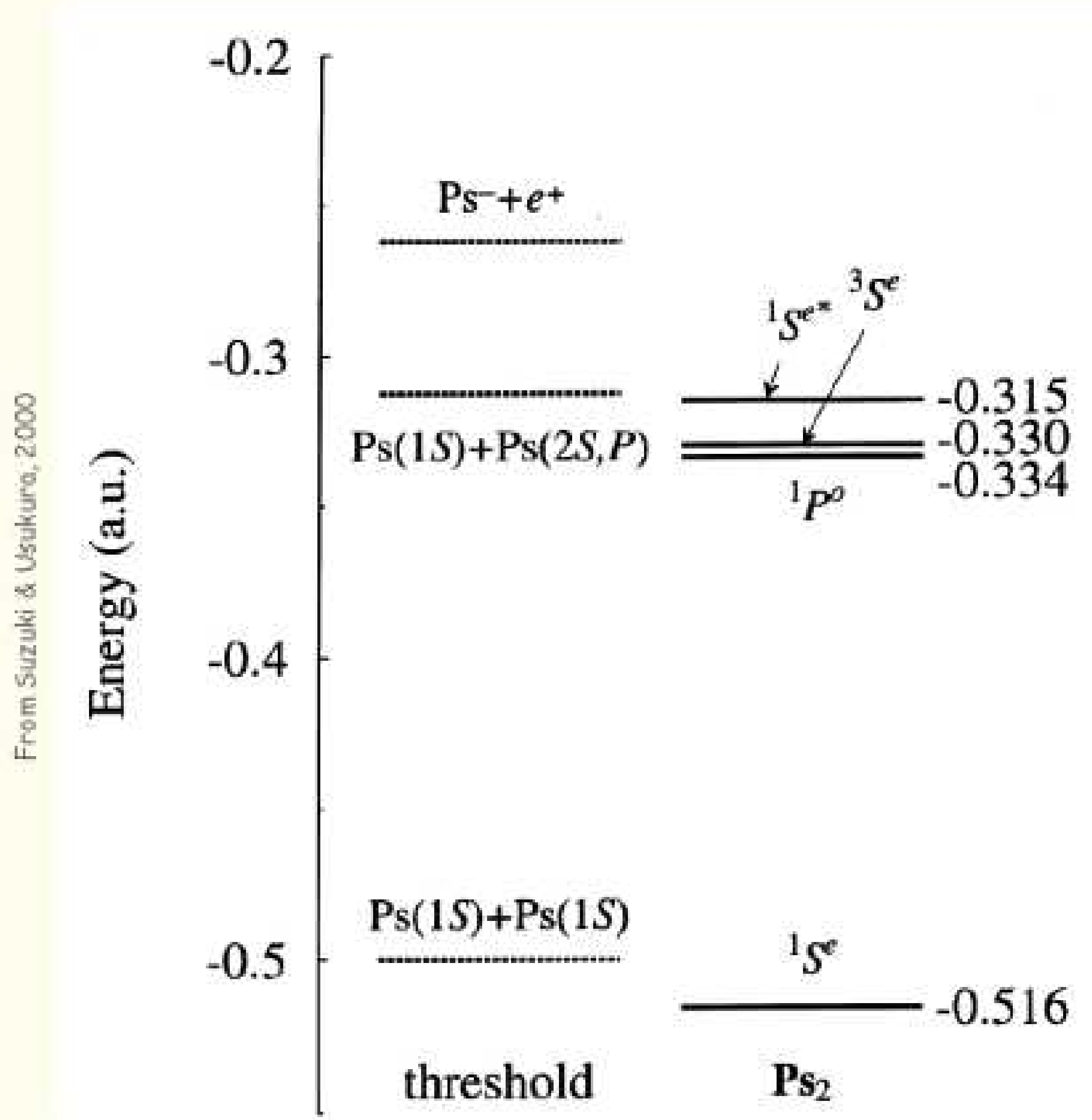
Long lived Rydberg atoms:

- no overlap with nucleus
- electron protects against other atoms

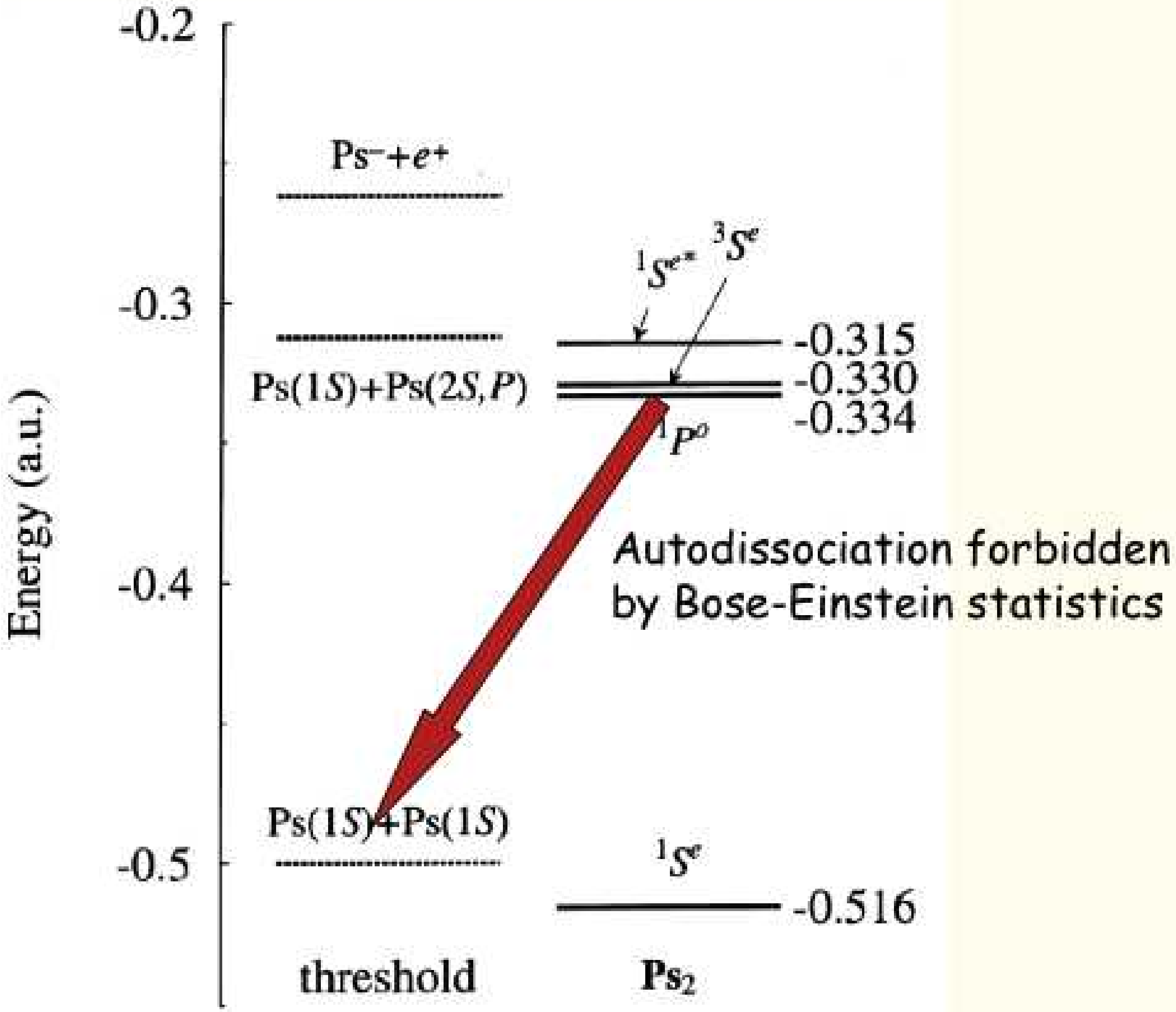
Dipositronium Ps_2



Spectrum of the molecule Ps_2

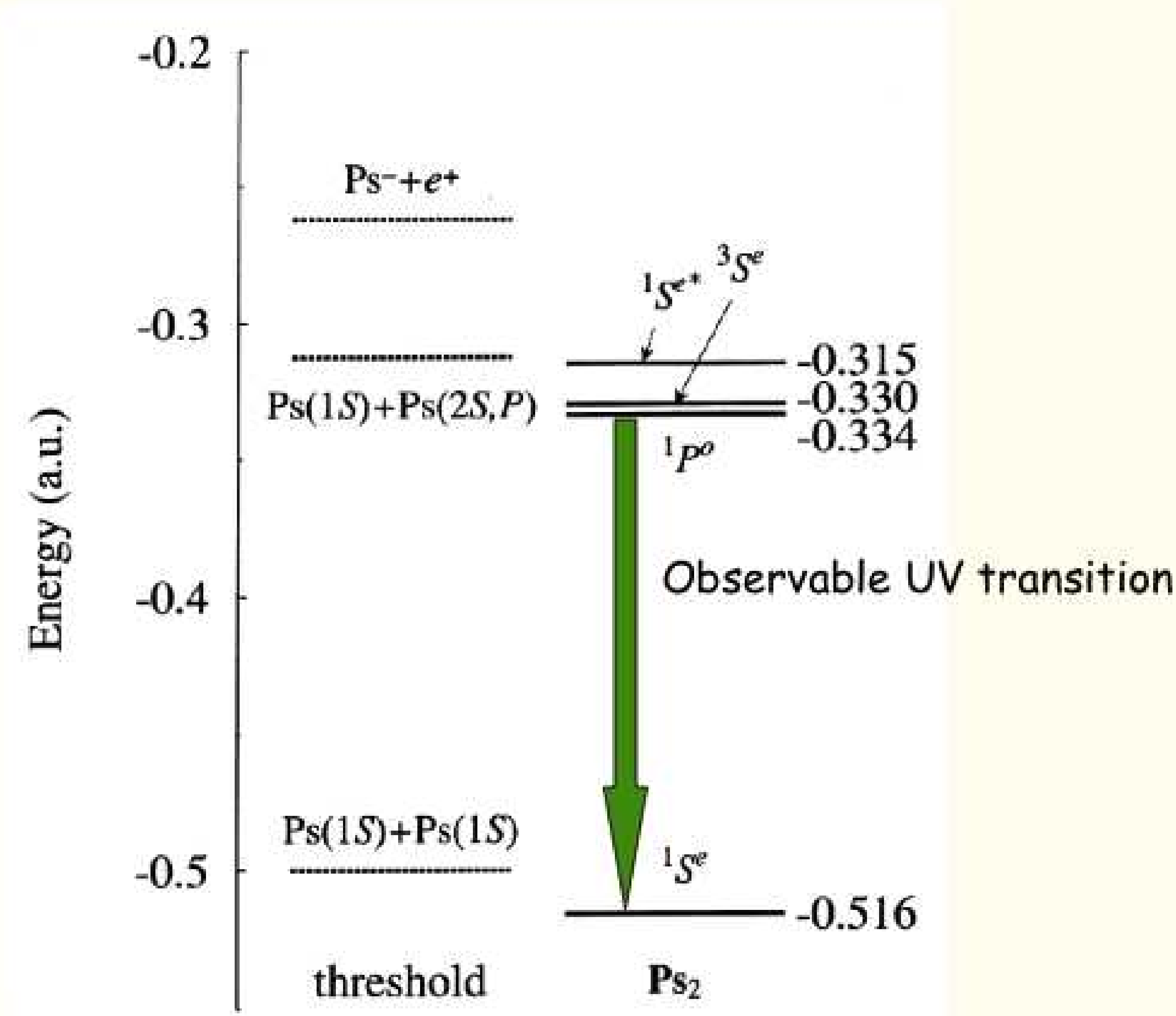


A direct signal of the molecule: transition line



From Suzuki & Usukura, 2000

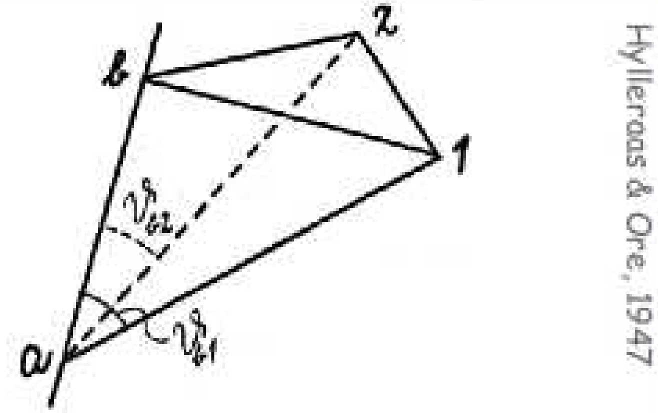
A direct signal of the molecule: transition line



From Suzuki & Usukura, 2000

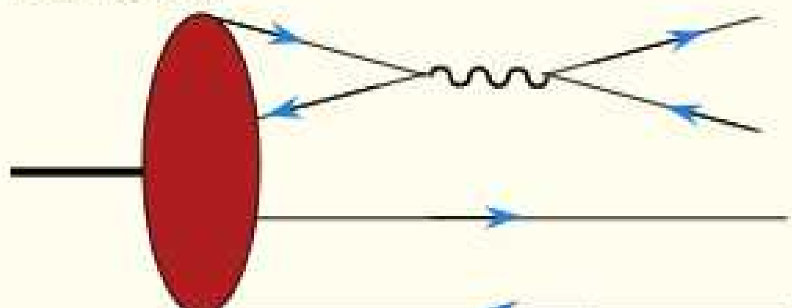
Energy levels: ground state and P-excitation

Wave function determined variationally, M. Puchalski
using Coulomb potential;



Coordinate system for the positronium molecule

Relativistic corrections: perturbations.
Annihilation dominates.



Interval P-S determined with 5 x 10^{-6} accuracy
(slightly smaller than in Ps, "dielectric effect").

Summary

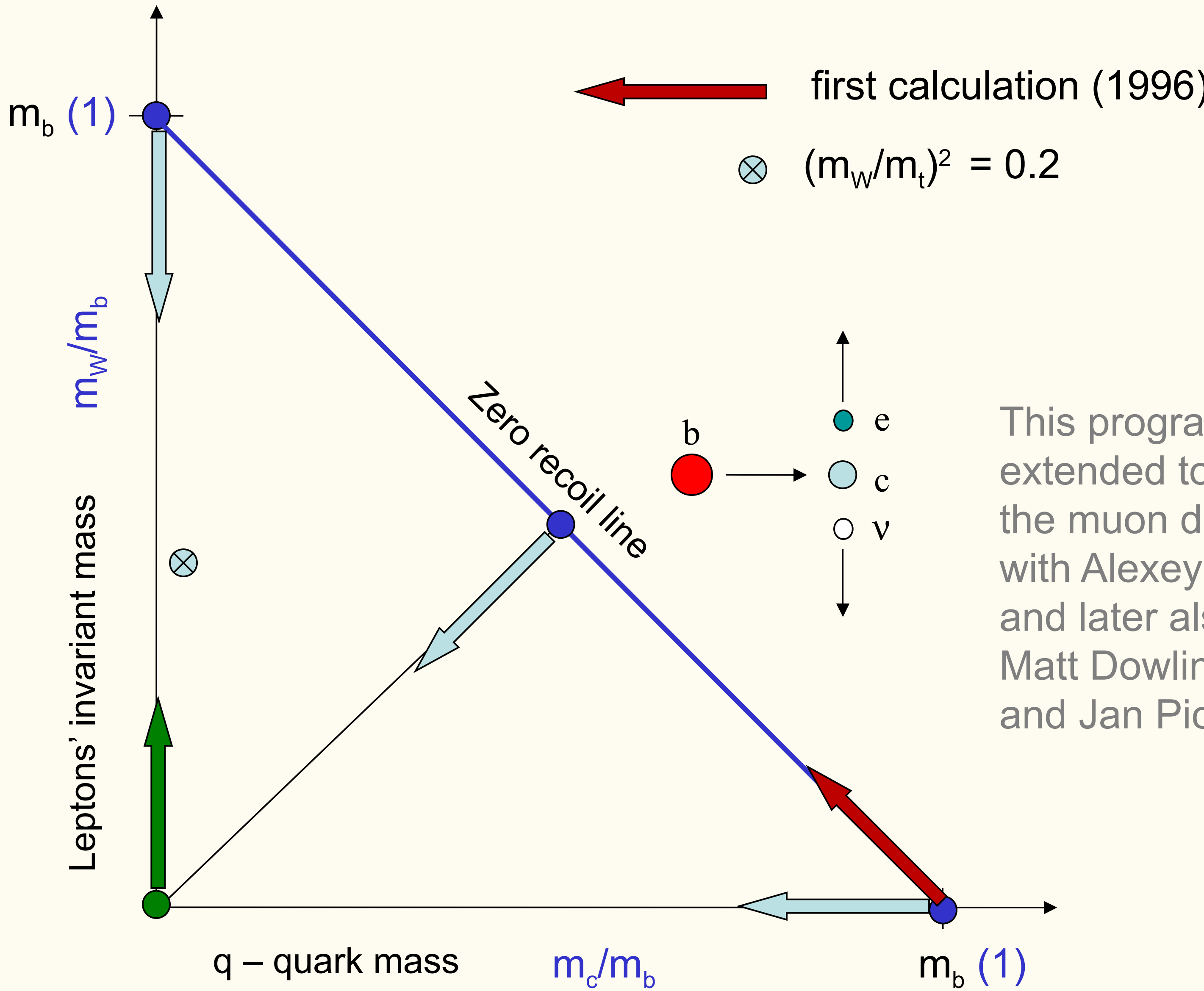
Low energy studies require (and test) precise predictions.

Here we have reviewed

* decays of bound muons;
* positronium HFS; effect of binding on the *g*-factor;
* Lamb shift;
* and few-body systems.

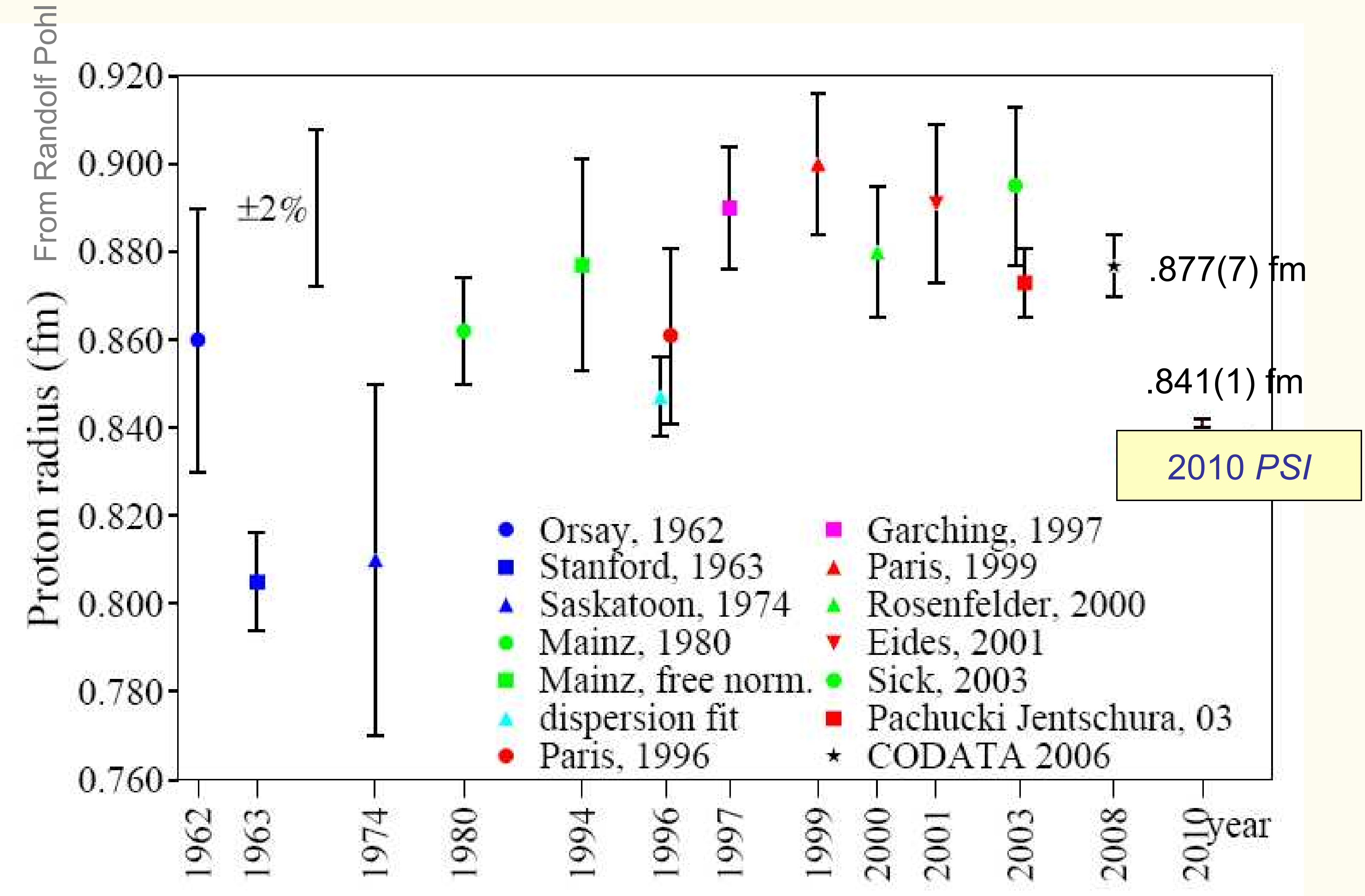
Each area needs improvements of its theory. Each has a vigorous experimental activity.

Two-loop corrections to heavy quark decays



This program was extended to the muon decay with Alexey Pak, and later also with Matt Dowling and Jan Piclum

New proton radius from muonic-H (PSI)

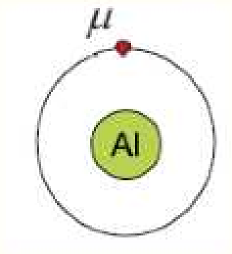


Comparison with scattering experiments

Typical luminosity in fixed-target experiments

$$\sim 10^{37\ldots38} / \left(\text{cm}^2 \cdot \text{s}\right)$$

In a single muonic atom



$$= \text{ density} \times \text{ velocity}$$

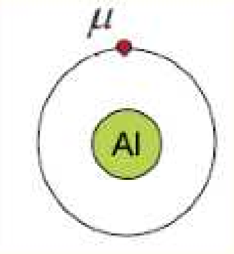
$$= \left|\psi(0)\right|^2 \cdot Z\alpha = \frac{m_\mu^3 Z^4 \alpha^4}{\pi} \sim Z^4 \cdot 4 \cdot 10^{39} / \left(\text{cm}^2 \cdot \text{s}\right)$$

Comparison with scattering experiments

Typical luminosity in fixed-target experiments

$$\sim 10^{37\ldots38} / \left(\mathrm{cm}^2 \cdot \mathrm{s}\right)$$

In a single muonic atom



$$= \text{density} \times \text{velocity}$$

$$= \left|\psi(0)\right|^2 \cdot Z\alpha = \frac{m_\mu^3 Z^4 \alpha^4}{\pi} \sim Z^4 \cdot 4 \cdot 10^{39} / \left(\mathrm{cm}^2 \cdot \mathrm{s}\right)$$

Many atoms are studied in parallel: ~10^{11} muons stopped per second, each lives about 10^{-6} seconds: 10^5 atoms present:

$$\sim 10^{49} / \left(\mathrm{cm}^2 \cdot \mathrm{s}\right)$$