

QCD transverse-momentum resummation for Higgs and Vector Boson production at the LHC

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Outline

- 1 Transverse-momentum distribution: fixed order results
- 2 Transverse-momentum resummation formalism
- 3 Resummed results
- 4 Conclusions



Motivations I

- The discovery of the Higgs boson and the study of its properties depends, in various way, by theoretical predictions for Higgs boson cross sections.
- The study of Drell-Yan lepton pair production is well motivated.

Large production rates and clean experimental signatures:

Important for detector calibration.

Possible use as luminosity monitor.



Motivations II

Transverse-momentum distributions needed for:

- Precise prediction for M_W .
- Beyond the Standard Model analysis (Z' non SUSY Higgs).
- Test of perturbative QCD predictions.
- Constrain for fits of PDFs.

To fully exploit the information contained in the experimental data from hadron colliders, precise theoretical predictions are needed
 \implies computation of higher-order QCD corrections.



State of the art: fixed order calculations for DY

- QCD corrections:
 - Total cross section known up to NNLO
[Hamberg, Van Neerven, Matsuura('91)], [Harlander, Kilgore('02)]
 - Rapidity distribution known up to NNLO
[Anastasiou, Dixon, Melnikov, Petriello('03)]
 - Fully exclusive NNLO calculation completed
[Melnikov, Petriello('06)], [Catani, Cieri, de Florian, G.F., Grazzini('09)]
 - Vector boson transverse-momentum distribution known up to NLO
[Ellis, Martinelli, Petronzio('83)], [Arnold, Reno('89)],
[Gonsalves, Pawlowski, Wai('89)]
- Electroweak correction are know at $\mathcal{O}(\alpha)$
[Dittmaier et al.('02)], [Baur et al.('02)], [Carloni Calame,
Montagna, Nicosini, Vicini('06)]

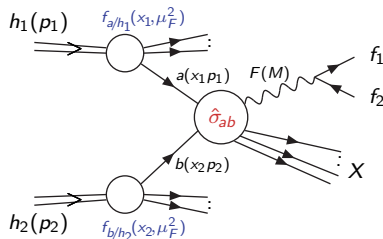


State of the art: fixed order calculations for $gg \rightarrow H$

- QCD corrections:
 - Total cross section known up to NNLO QCD corrections in the large- m_t approx. [Harlander,Kilgore('02)], [Anastasiou,Melnikov,Petriello('04)] [Ravindran,Smith,Van Neerven('03)].
 - NNLO QCD corrections beyond large- m_t approx. [Marzani,Ball,Del Duca,Forte,Vicini('08)] [Harlander,Mantler,Marzani,Ozeren('09)], [Pag,Rogal,Steinhauser('09)]. Accuracy of large- m_t approximation better than 1% for $100 \leq m_H \leq 300$ GeV.
 - Large QCD corrections: $\sim +100\%$ at NLO, $\sim +25\%$ at NNLO. The bulk due to threshold ($\hat{s} \rightarrow m_H^2$) soft/collinear emissions: NNLL resummation gives a further $\sim +10\%$ correction [Catani,de Florian,Grazzini,Nason('03)].
 - Fully exclusive NNLO QCD calculation available [Anastasiou,Melnikov,Petriello('04)], [Catani,Grazzini('07)]
 - Transverse-momentum distribution known up to NLO [de Florian,Grazzini,Kunszt,('99)], [Ravindran,Smith,Van Neerven('02)], [Glosser,Schmidt('02)]
- NLO EW corrections $\sim +5\%$ [Aglietti,Bonciani,Degrassi,Vicini('04)], [Actis,Passarino,Sturm,Uccirati('08)].



Transverse-momentum distribution



$$h_1(p_1) + h_2(p_2) \rightarrow F(M, q_T) + X \rightarrow f_1 + f_2 + X$$

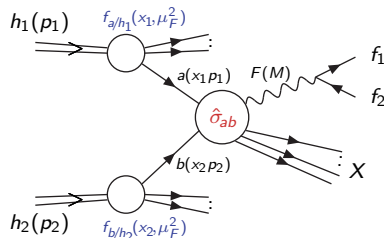
where $F = \gamma^*/Z^0, W^\pm, H$ and $f_1 f_2 = l^+ l^-, l\nu_l, \gamma\gamma$

Theoretical framework: QCD factorization formula

$$\frac{d\sigma}{dq_T^2}(q_T, M, s) = \sum_{a,b} \int_0^1 dx_1 \int_0^1 dx_2 f_{a/h_1}(x_1, \mu_F^2) f_{b/h_2}(x_2, \mu_F^2) \frac{d\hat{\sigma}_{ab}}{dq_T^2}(q_T, M, \hat{s}; \alpha_S, \mu_R^2, \mu_F^2).$$



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$$\int_{Q_T^2}^{\infty} dq_T \frac{d\hat{\sigma}_{q\bar{q}}}{dq_T^2} \sim \alpha_S \left[c_{12} \log^2(M^2/Q_T^2) + c_{11} \log(M^2/Q_T^2) + c_{10}(Q_T) \right] \\ + \alpha_S^2 \left[c_{24} \log^4(M^2/Q_T^2) + \dots + c_{21} \log(M^2/Q_T^2) + c_{20}(Q_T) \right] + \mathcal{O}(\alpha_S^3)$$

The logarithmic corrections are the residue of the cancellation of the real-virtual infrared singularities:

$$\bar{q} \quad F \quad q_T \quad = \quad \frac{\alpha_S}{2\pi} \int \frac{dk_T^2}{k_T^2} \int_{k_T/M}^1 \frac{d\omega}{\omega} C_F (2 - 2\omega + \omega^2) \left[\delta^2(\vec{q}_T - \vec{k}_T) - \delta^2(\vec{q}_T) \right] \\ q \quad g \quad \omega, k_T \quad = \quad \frac{\alpha_S}{2\pi} \int \frac{dk_T^2}{k_T^2} C_F \left(\log k_T^2/M^2 - \frac{3}{2} + \mathcal{O}(k_T/M) \right) \left[\delta^2(\vec{q}_T - \vec{k}_T) - \delta^2(\vec{q}_T) \right]$$

Fixed order calculation theoretically justified only in the region $q_T \sim M_F$

For $q_T \rightarrow 0$, $\alpha_S^n \log^m(M^2/q_T^2) \gg 1$: need for resummation of logarithmic corrections

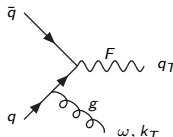


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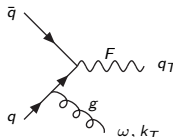


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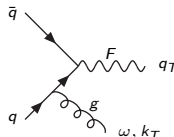


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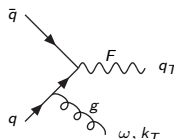


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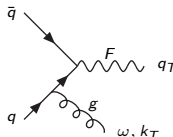


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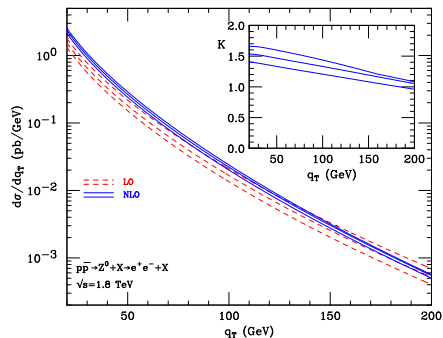
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Fixed order results: q_T spectrum of Z boson at the Tevatron

For $q_T \sim M_F$ fixed order perturbative expansion feasible: Ellis, Martinelli, Petronzio ('83); Arnold, Reno ('89); Gonsalves, Pawlowski, Wai('89)



- LO: pdf=MRST02 LO, 1-loop α_s
NLO: pdf=MRST04 NLO, 2-loop α_s
- Factorization and renormalization scale variations:
 $\mu_F = \mu_R = m_Z, \quad m_Z/2 \leq \mu_F, \mu_R \leq 2m_Z,$
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 $q_T \sim m_Z : LO \pm 25\%, NLO \pm 8\%$
 $q_T \sim 20 \text{ GeV} : LO \pm 20\%, NLO \pm 7\%$
- q_T dependent K -factor:

$$K(q_T) = \frac{d\sigma/dq_T_{NLO}(\mu_F, \mu_R)}{d\sigma/dq_T_{LO}(\mu_F = \mu_R = m_Z)}$$

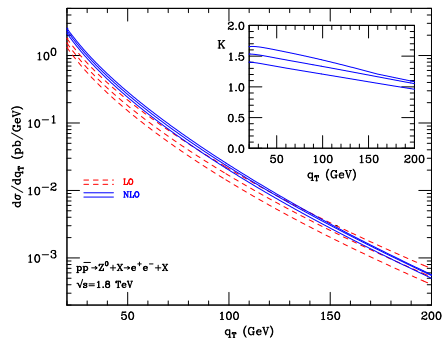
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LO and NLO scale variations bands overlap only for $q_T > 70 \text{ GeV}$



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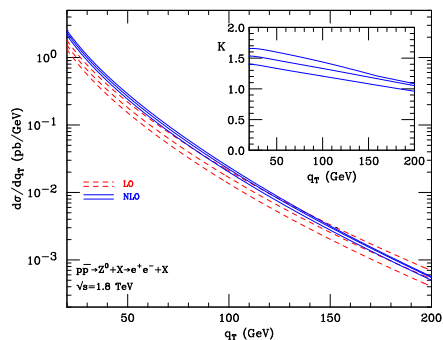
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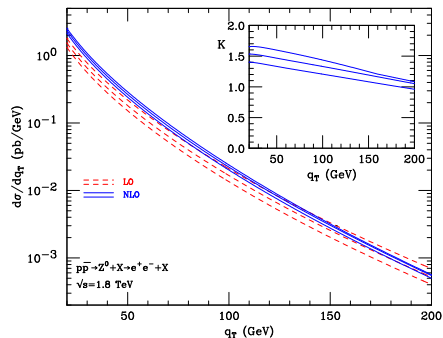
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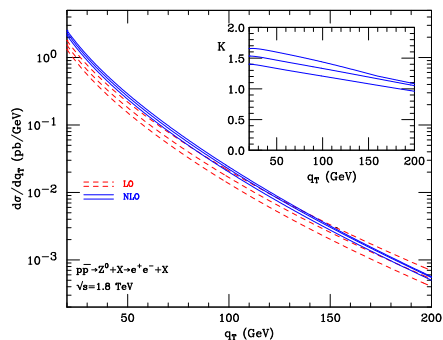
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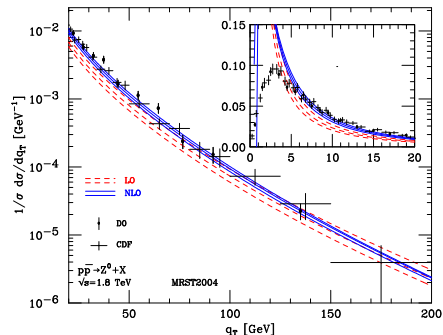
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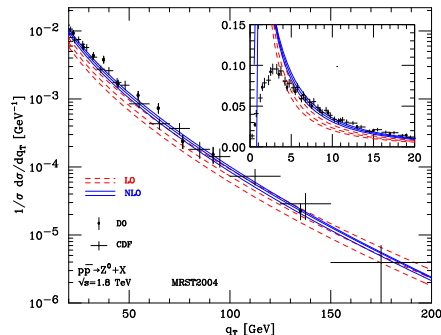
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 $\sigma_{tot} = 248 \pm 11 \text{ pb}$
 [CDF Collaboration ('00)]
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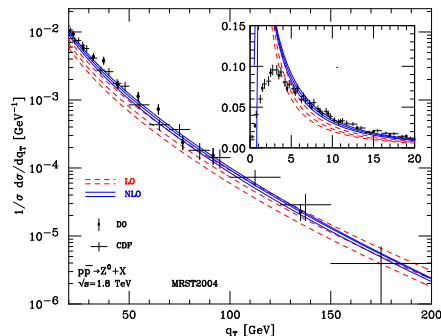


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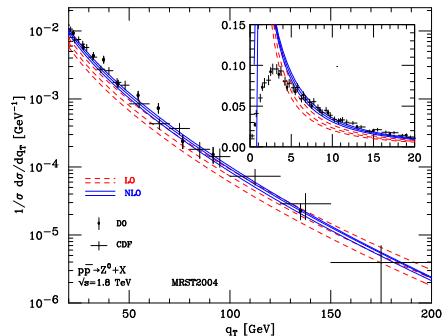
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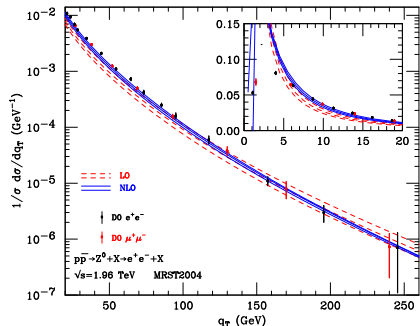
Fixed order results: q_T spectrum of Z boson at the Tevatron



- CDF data: $66 \text{ GeV} < M^2 < 116 \text{ GeV}$,
 $\sigma_{tot} = 248 \pm 11 \text{ pb}$
 [CDF Collaboration ('00)]
 D0 data: $75 \text{ GeV} < M^2 < 105 \text{ GeV}$,
 $\sigma_{tot} = 221 \pm 11 \text{ pb}$
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Fixed order results: q_T spectrum of Z and Higgs boson

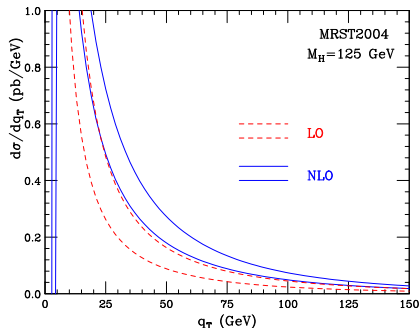


- D0 data [D0 Coll. ('08, '10)].
- Scale variations as before: $\mu_F = \mu_R = m_Z$, $1/2 \leq \{\mu_F/m_Z, \mu_R/m_Z, \mu_F/\mu_R\} \leq 2$,
- Experimental errors very small but bins are larger.
- Qualitatively same situation of Tevatron Run I data.
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Idea of Resummation

Idea of large logs (Sudakov) resummation: reorganize the perturbative expansion by all-order summation ($L = \log(M^2/q_T^2)$).

$\alpha_S L^2$	$\alpha_S L$	\dots	\dots	\dots	$\mathcal{O}(\alpha_S)$
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\dots	\dots	\dots	\dots	\dots	\dots
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- Ratio of two successive rows $\mathcal{O}(\alpha_S L^2)$: fixed order expansion valid when $\alpha_S L^2 \ll 1$.
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- Dynamics factorization: general propriety of QCD matrix element for soft emissions based on colour coherence. It is the analogous of the independent multiple soft-photon emission is QED:

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State of the art: transverse-momentum resummation

- The method to perform the resummation of the large logarithms of q_T is known
 [Parisi,Petronzio.(’79)], [Kodaira,Trentadue(’82)], [Altarelli, Ellis, Greco, Martinelli(’84)], [Collins,Soper,Sterman(’85)], [Catani,de Florian,Grazzini(’01)]
- Various phenomenological studies of the vector boson transverse momentum distribution exist
 [Balasz,Qiu,Yuan(’95)], [Balasz,Yuan(’97)], [Ellis et al.(’97)], [Kulesza et al.(’02)]
- In this study we apply the transverse-momentum distribution the resummation formalism developed by [Catani,de Florian, Grazzini(’01)] .
- Recently various results for transverse-momentum resummation in the framework of Effective Theories appeared
 [Gao,Li,Liu(’05)], [Idilbi, Ji, Yuan(’05)], [Mantry,Petriello(’10)], - [Becher,Neubert(’10)], [García, Idibli, Scimemi(’11)].



Transverse-momentum resummation in pQCD

$$\frac{d\hat{\sigma}}{dq_T^2} = \frac{d\hat{\sigma}^{(res)}}{dq_T^2} + \frac{d\hat{\sigma}^{(fin)}}{dq_T^2}; \quad \int_0^{Q_T^2} dq_T^2 \left[\frac{d\hat{\sigma}^{(res)}}{dq_T^2} \right]_{f.o.} \stackrel{Q_T \rightarrow 0}{\sim} 1 + \sum_n \sum_{m=1}^{2n} c_{nm} \alpha_S^n \log^m \frac{M^2}{Q_T^2}$$

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Transverse-momentum resummation in pQCD

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The q_T resummation formalism

Main distinctive features of the formalism [Catani,de Florian, Grazzini('01)], [Bozzi,Catani,de Florian, Grazzini('03,'06,'08)]:

- Resummation performed at partonic cross section level: PDF evaluated at $\mu_F \sim M$: no PDF extrapolation in the non perturbative region, study of μ_R and μ_F dependence as in fixed-order calculations.
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- Perturbative unitarity constrain and resummation scale Q :

$$\ln(M^2 b^2) \rightarrow \tilde{L} \equiv \ln(Q^2 b^2 + 1)$$

- avoids unjustified higher-order contributions in the small- b region.
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- We have performed the resummation up to **NNLL+NLO**. It means that our complete formula includes:
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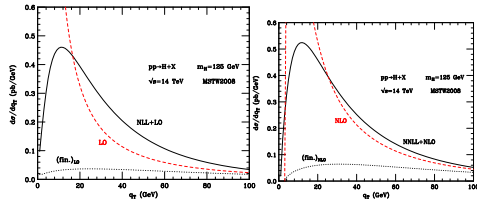
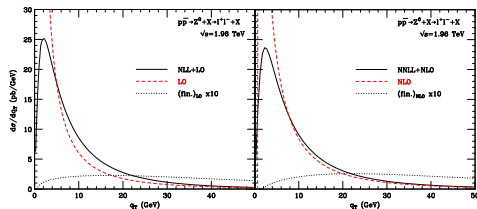


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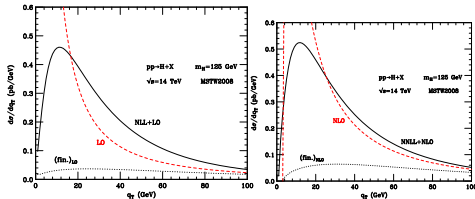
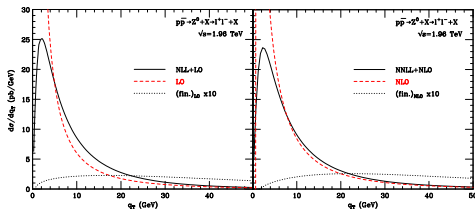
Resummed results



- Left side: NLL+LO result compared with fixed LO result. Resummation cures the fixed order divergence at $q_T \rightarrow 0$.
- Right side: NNLL+NLO result compared with fixed NLO result.
- The q_T spectrum of Z boson is slightly harder at NNLL+NLO accuracy than at NLL+LO accuracy.
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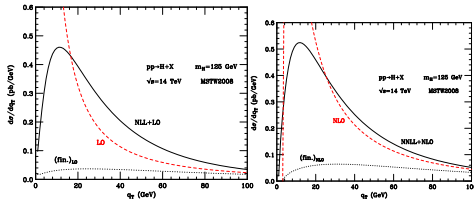
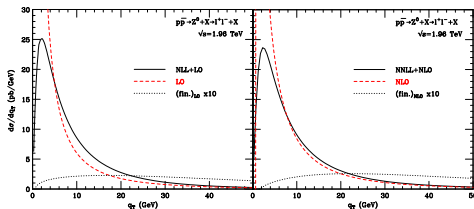
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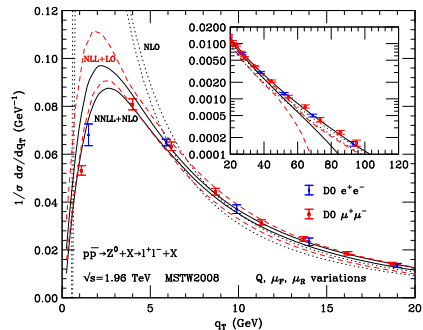
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Resummed results: q_T spectrum of Z boson at the Tevatron

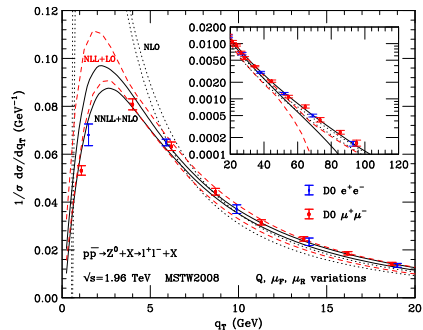


D0 data for the Z q_T spectrum compared with perturbative results.

- Uncertainty bands obtained varying μ_R , μ_F , Q independently:
 $1/2 \leq \{\mu_F/m_Z, \mu_R/m_Z, 2Q/m_Z, \mu_F/\mu_R, Q/\mu_R\} \leq 2$
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- Significant reduction of scale dependence from NLL+LO to NNLL+NLO for all q_T .
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 The perturbative uncertainty of the NNLL+NLO results is comparable with the experimental errors.



Resummed results: q_T spectrum of Z boson at the Tevatron

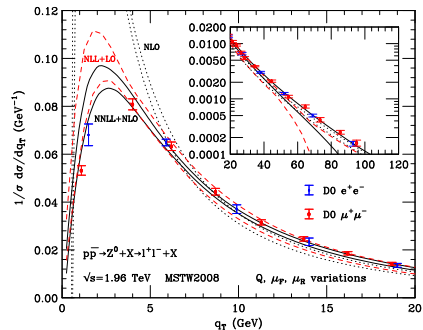


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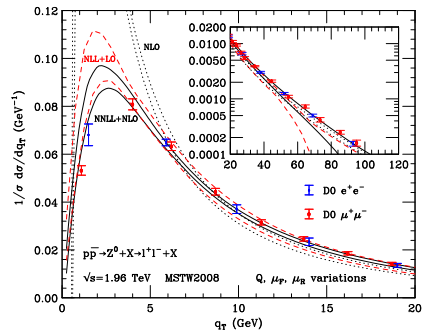


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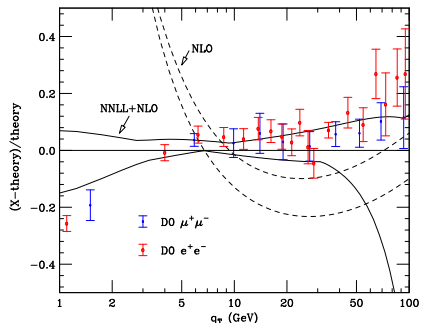


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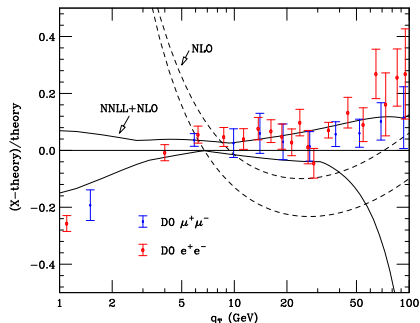


D0 data for the Z q_T spectrum: Fractional difference with respect to the reference result: NNLL+NLO, $\mu_R = \mu_F = 2Q = m_Z$.

- NNLL+NLO scale dependence is $\pm 6\%$ at the peak, $\pm 5\%$ at $q_T = 10$ GeV and $\pm 12\%$ at $q_T = 50$ GeV. For $q_T \geq 60$ GeV the resummed result loses predictivity.
- At large values of q_T , the NLO and NNLL+NLO bands overlap. At intermediate values of transverse momenta the scale variation bands do not overlap.
- The resummation improves the agreement of the NLO results with the data. In the small- q_T region, the NLO result is theoretically unreliable and the NLO band deviates from the NNLL+NLO band.



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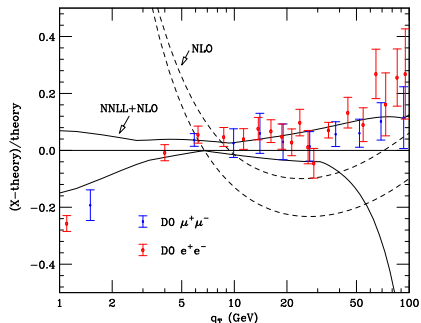


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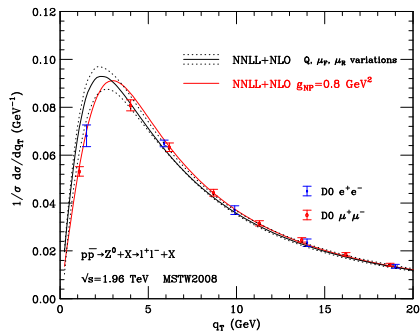


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Non perturbative effects: q_T spectrum of Z boson at the Tevatron



D0 data for the Z q_T spectrum.

- Up to now result in a complete perturbative framework.
- Non perturbative effects parametrized by a NP form factor $S_{NP} = \exp\{-g_{NP}b^2\}$:

$$\exp\{\mathcal{G}_N(\alpha_S, \tilde{L})\} \rightarrow \exp\{\mathcal{G}_N(\alpha_S, \tilde{L})\} S_{NP}$$

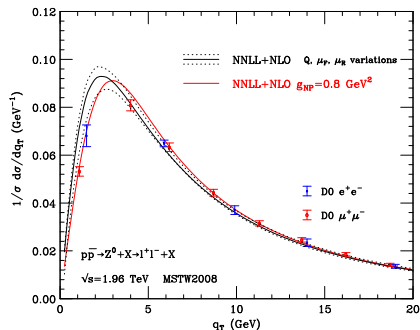
$$g_{NP} = 0.8 \text{ GeV}^2 \quad [\text{Kulesza et al. ('02)}]$$

- With NP effects the q_T spectrum is harder.

Quantitative impact of such NP effects is comparable with perturbative uncertainties.



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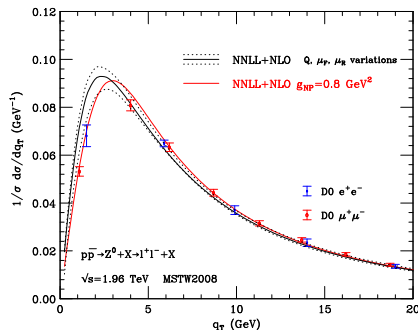
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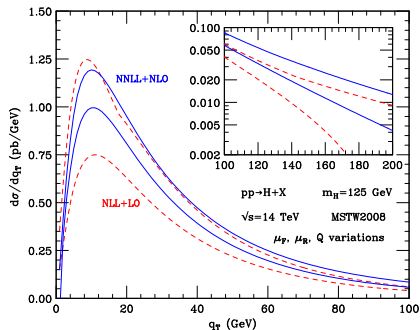
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Resummed results: q_T spectrum of H boson at the LHC $\sqrt{s} = 14$ TeV

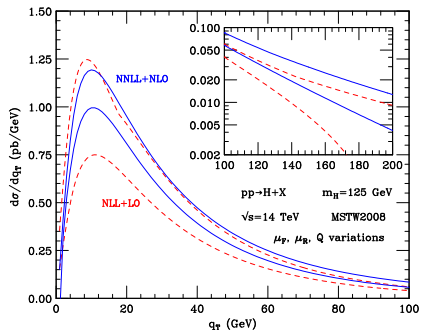


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Higgs q_T spectrum for $m_H = 125$ GeV at LHC.



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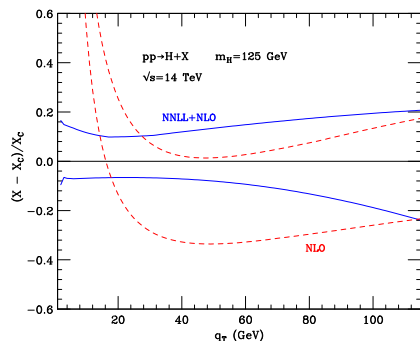


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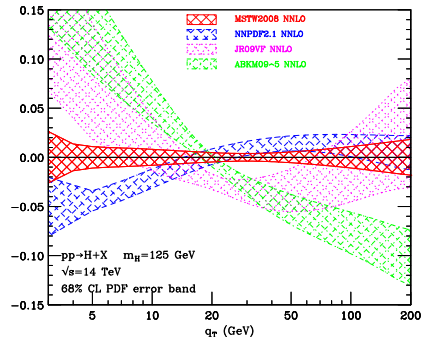


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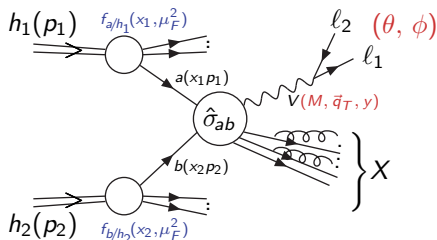


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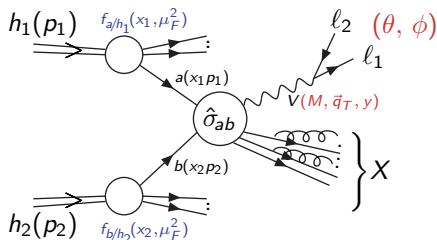
q_T -resummation with decay variables dependence



- Experiments have finite acceptance: **important to provide exclusive theoretical predictions.**
- Analytic resummation formalism inclusive over soft-gluon emission: **not possible to apply selection cuts on final state partons.**
- We have included the full dependence on the decay variables: **possible to apply cuts on vector/Higgs boson and decay products.**
- To construct the “finite” part we rely on the fully-differential NNLO result from the codes `HNNLO/DYNNLO` [Catani, Cieri, de Florian, Ferrera, Grazzini ('09)], [Catani, Grazzini ('09)].
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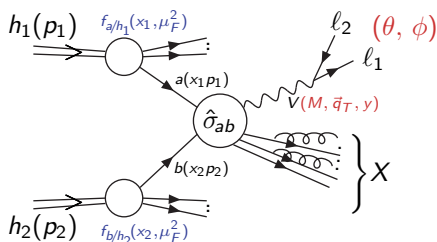
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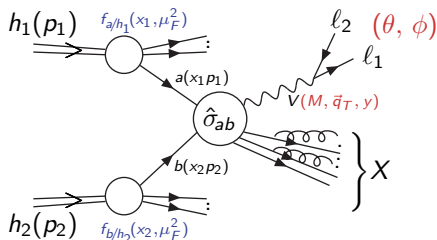
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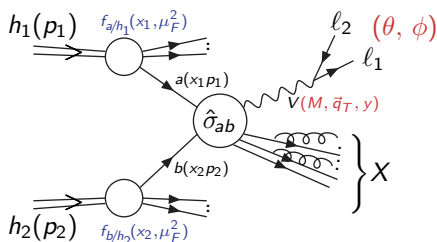
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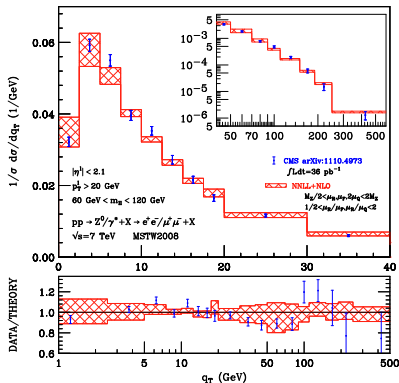
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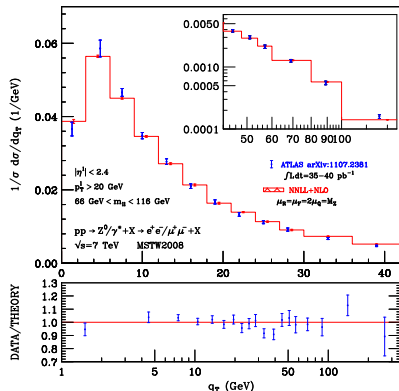


DY q_T -resummation with V boson decay



CMS data for the Z q_T spectrum compared with NNLL+NLO result.
Scale variation:

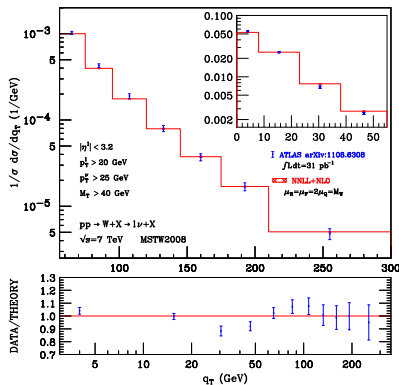
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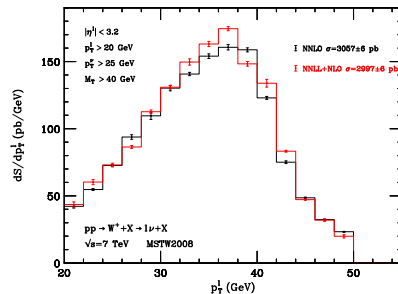
ATLAS data for the Z q_T spectrum compared with NNLL+NLO result.



DY q_T -resummation with V boson decay



ATLAS data for the W q_T spectrum compared with NNLL+NLO result.

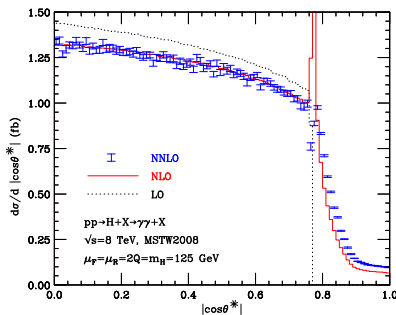


Lepton p_T spectrum from W^+ decay. NNLL+NLO result compared with the NNLO result.

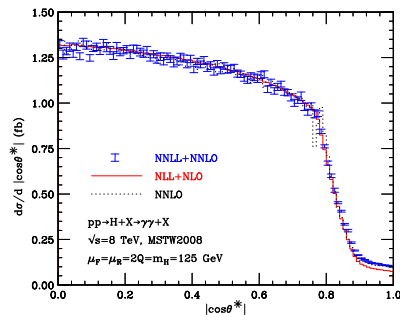
Important spectrum for the measurement of M_{W^+} at the LHC.



$gg \rightarrow H$ q_T -resummation with H boson decay



Fixed order results for
 $|\cos \theta^*| = \sqrt{1 - 4p_{T,\gamma}^2/m_H^2}$ distribution
 at the LHC.



Resummed results for
 $|\cos \theta^*| = \sqrt{1 - 4p_{T,\gamma}^2/m_H^2}$ distribution
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Conclusions

- **NNLL+NLO q_T -resummation** for Drell-Yan and Higgs production in gluon fusion.

Reduction of scale uncertainties from NLL+LO to NNLL+NLO accuracy. The NNLL+NLO results for Drell-Yan consistent with the experimental data in a wide region of q_T .

- Added full kinematical dependence on the vector/Higgs boson and on the final state leptons/photons.
- Preliminary comparison with LHC data (implementing experimental cuts): good agreement between data and NNLL+NLO results without any model for Non Perturbative effects.
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Back up slides



Fully-Exclusive Cross Sections at NNLO in hadron-collisions

- Experiments have finite acceptance, in particular VH experimental analyses performed in extreme kinematical regimes (e.g. boosted analysis with jet veto): **important to provide exclusive theoretical predictions**.
- At NLO general algorithms (e.g. Dipole formalism [Catani, Seymour('98)]) allow (relative) straightforward fully-exclusive calculations.
- At NNLO in hadronic collisions only few fully exclusive calculations exist:
 - Sector decomposition:** [Binoth, Heinrich('00)]
 $gg \rightarrow H$ [Anastasiou, Melnikov, Petriello('04)] \rightarrow FEHIP
 Drell-Yan [Melnikov, Petriello('06)] \rightarrow FEWZ
 - q_T -subtraction:**
 $gg \rightarrow H$ [Catani, Grazzini('07)] \rightarrow HNNLO
 Drell-Yan [Catani, Cieri, de Florian, G.F., Grazzini('09)] \rightarrow DYNLO
 Associated WH production [G.F., Grazzini, Tramontano('11)] \rightarrow WNNLO
 Diphoton prod. [Catani, Cieri, de Florian, G.F., Grazzini('11)] \rightarrow 2γ NNLO



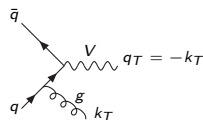
The q_T -subtraction formalism at NNLO

$$h_1(p_1) + h_2(p_2) \rightarrow V(M, q_T) + X$$

V is one or more **colourless** particles (vector bosons, leptons, photons, Higgs bosons, ...) [Catani, Grazzini('07)].

- **Key point I:** at LO the q_T of the V is exactly zero.

$$d\sigma_{(N)NLO}^V|_{q_T \neq 0} = d\sigma_{(N)LO}^{V+\text{jets}},$$



for $q_T \neq 0$ the NNLO IR divergences cancelled with the NLO subtraction method.

- The only remaining NNLO singularities are associated with the $q_T \rightarrow 0$ limit.
- **Key point II:** treat the NNLO singularities at $q_T = 0$ by an additional subtraction using the universality of logarithmically-enhanced contributions from q_T resummation formalism [Catani, de Florian, Grazzini('00)].

$$d\sigma_{N^2LO}^V \xrightarrow{q_T \rightarrow 0} d\sigma_{LO}^V \otimes \Sigma(q_T/M) dq_T^2 = d\sigma_{LO}^V \otimes \sum_{n=1}^{\infty} \sum_{k=1}^{2n} \left(\frac{\alpha_S}{\pi}\right)^n \Sigma^{(n,k)} \frac{M^2}{q_T^2} \ln^{k-1} \frac{M^2}{q_T^2} d^2 q_T$$

$$d\sigma^{CT} \xrightarrow{q_T \rightarrow 0} d\sigma_{LO}^V \otimes \Sigma(q_T/M) dq_T^2$$



The final result valid also for $q_T = 0$ is:

$$d\sigma_{(N)NLO}^V = \mathcal{H}_{(N)NLO}^V \otimes d\sigma_{LO}^V + \left[d\sigma_{(N)LO}^{V+jets} - d\sigma_{(N)LO}^{CT} \right],$$

$$\text{where } \mathcal{H}_{NNLO}^V = \left[1 + \frac{\alpha_S}{\pi} \mathcal{H}^{V(1)} + \left(\frac{\alpha_S}{\pi} \right)^2 \mathcal{H}^{V(2)} \right]$$

- The choice of the counter-term has some arbitrariness but it must behave $d\sigma^{CT} \xrightarrow{q_T \rightarrow 0} d\sigma_{LO}^V \otimes \Sigma(q_T/M) dq_T^2$ where $\Sigma(q_T/M)$ is universal.
- $d\sigma^{CT}$ regularizes the $q_T = 0$ singularity of $d\sigma^{V+jets}$: *double real* and *real-virtual* NNLO contributions, while (the finite part of) *two-loops virtual* corrections are contained in \mathcal{H}_{NNLO}^V .
- Final state partons only appear in $d\sigma^{V+jets}$ so that NNLO IR-safe cuts are included in the NLO computation: observable-independent NNLO extension of the subtraction formalism.



Associated WH production in NNLO QCD

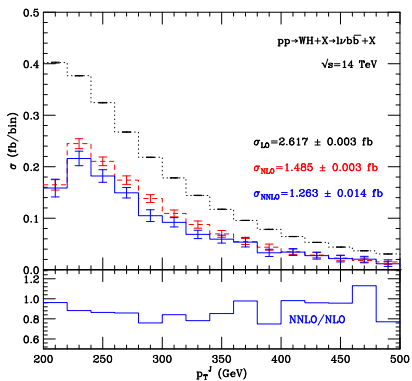
G.F., Grazzini, Tramontano arXiv:1107.1164

- A NLO calculation for $h_1 h_2 \rightarrow V + X$ requires:
 - $d\sigma_{LO}^{V+\text{jets}}$ (and $d\sigma_{LO}^V$).
 - $\mathcal{H}^{V(1)}$ [de Florian, Grazzini('01)]: contains the finite-part of the one-loop amplitude $c\bar{c} \rightarrow V$.
 - $d\sigma_{LO}^{CT}$: depends by the (universal) q_T -resummation coeff. A_1 and B_1 .
- A NNLO calculation for $h_1 h_2 \rightarrow V + X$ requires also:
 - $d\sigma_{NLO}^{V+\text{jets}}$.
 - $\mathcal{H}^{V(2)}$: contains the finite-part of the two-loops amplitude $c\bar{c} \rightarrow V$.
 - $d\sigma_{NLO}^{CT}$: depends by the (universal) q_T -resummation coeff. A_2 and B_2 .
- WH production at NNLO within q_T -subtraction:
 - $d\sigma_{NLO}^{WH+\text{jets}}$.
 - $\mathcal{H}^{DY(2)}$ [Catani, Cieri, de Florian, G.F., Grazzini('12)].

Fully-exclusive NNLO calculation, implemented in the parton-level Monte

Carlo code: [G.F., Grazzini, Tramontano('11)]



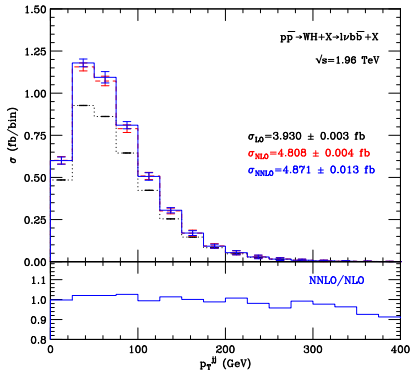


$pp \rightarrow WH(\rightarrow l\nu b\bar{b})$

p_T spectra of the fat jet at the LHC@14TeV for $m_H = 120\text{GeV}$ at LO (dots), NLO (dashes) and NNLO (solid).

- Selection strategy of [Butterworth et al. ('08)]: search a large- p_T Higgs boson through a collimated $b\bar{b}$ pair decay.
Cuts:
Leptons: $p_T^l > 30\text{GeV}$, $|\eta^l| < 2.5$,
 $p_T^{\text{miss}} > 30\text{GeV}$, $p_T^W > 200\text{GeV}$.
Jets: Cambridge/Aachen algorithm with $R=1.2$.
Fat jet (contain the $b\bar{b}$) $p_T^J > 200\text{GeV}$,
 $|\eta^J| < 2.5$
Jet veto: No other jets with $p_T > 20\text{GeV}$ and $|\eta| < 5$.
- Large negative higher-order corrections: NLO (NNLO) effects -52%/-36% (-6%/-19%), depending on the scale choice (factor two around $\mu_F = \mu_R = m_W + m_H$).
- Jet veto strongly affect the higher order corrections \Rightarrow stability of fixed order calculation challenged.





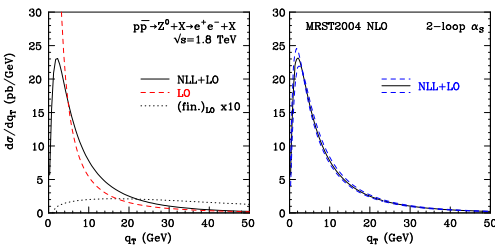
$p\bar{p} \rightarrow WH(\rightarrow l\nu b\bar{b})$

p_T spectra of the dijet system at the Tevatron for $m_H = 120\text{ GeV}$ at LO (dots), NLO (dashes) and NNLO (solid).

- Cuts:
Leptons: $p_T^l > 20\text{ GeV}$, $|\eta^l| < 2$, $p_T^{\text{miss}} > 20\text{ GeV}$.
Jets: k_T algorithm with $R=0.4$.
Exactly two jets (with $p_T > 20\text{ GeV}$ and $|\eta| < 2$) at least one of them has to be a b jet (with $|\eta| < 1$).
- Higher-order corrections: NLO (NNLO) effects from +13% to +30% (from -1% to +4%) depending on the scale choice (factor two around $\mu_F = \mu_R = m_W + m_H$). The scale dependence is at the level of about $\pm 1\%$ both at NLO and NNLO.
- The shape of the distribution is stable against perturbative corrections. Perturbative expansion under good control.



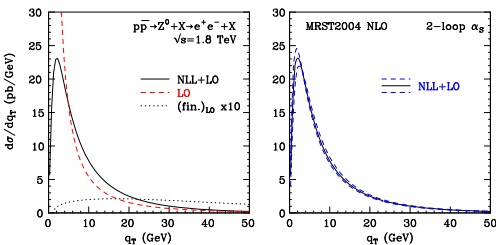
Resummed results: q_T spectrum of Drell-Yan e^+e^- pairs at $\sqrt{s} = 1.8$ TeV



- Left side: NLL+LO result compared with fixed LO result. Resummation cures the fixed order divergence at $q_T \rightarrow 0$.
- Right side: variation of factorization and renormalization scales as in customary fixed-order calculations: $\sim 5\%$ at low q_T , $\sim 9\%$ at $q_T \sim 50$ GeV.
- Finite LO component contribution is: $\lesssim 1\%$ near the peak, $\sim 8\%$ at $q_T \sim 20$ GeV, $\sim 60\%$ at $q_T \sim 50$ GeV.
- Integral of the NLL+LO curve reproduces the total NLO cross section to better than 1% (check of the code).



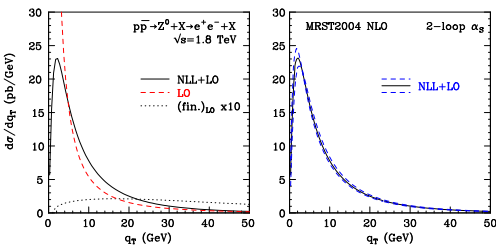
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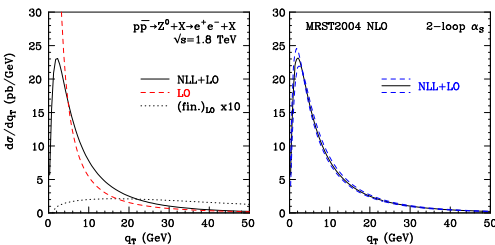
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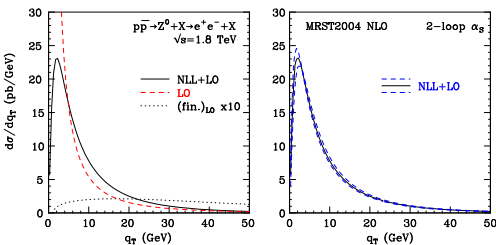
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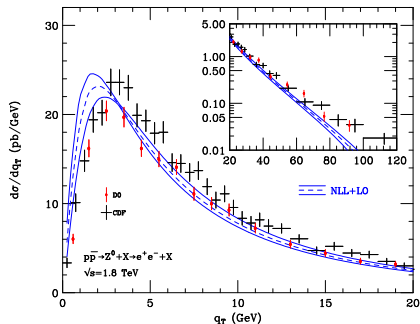
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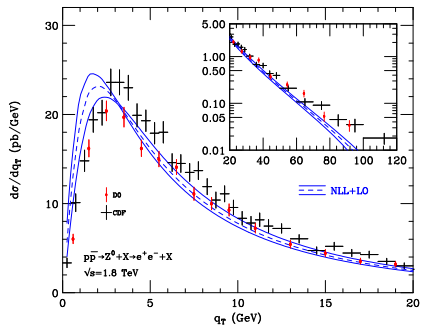
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- CDF data: $66 \text{ GeV} < M^2 < 116 \text{ GeV}$,
 $\sigma_{tot} = 248 \pm 11 \text{ pb}$
 [CDF Collaboration ('00)]
 D0 data: $75 \text{ GeV} < M^2 < 105 \text{ GeV}$,
 $\sigma_{tot} = 221 \pm 11 \text{ pb}$
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- Our calculation implements γ^*Z interference and finite-width effects. Here we use the narrow width approximation (differences within 1% level).
- NLL+LO resummed result fits reasonably well also in the $q_T \lesssim 20 \text{ GeV}$ (without a model for non-perturbative effects).
- Suppression in the large- q_T region ($q_T \lesssim 60 \text{ GeV}$) (strong dependence from the resummation scale).



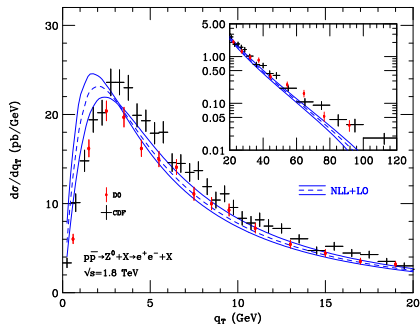
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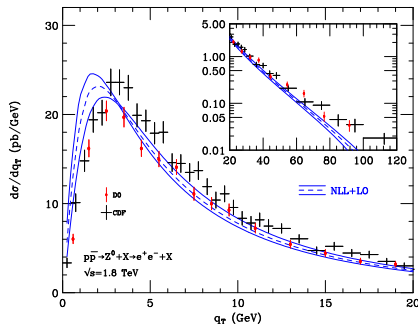
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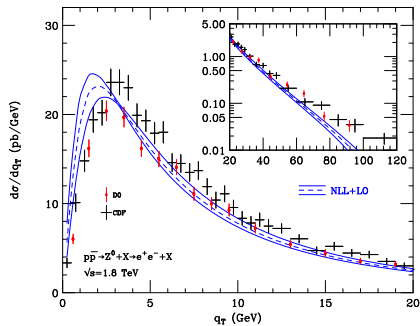
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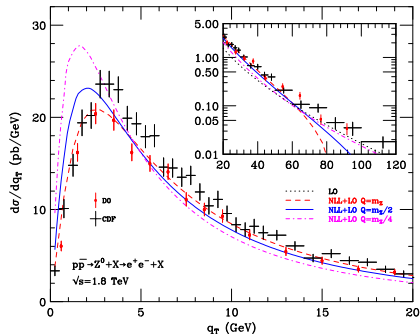
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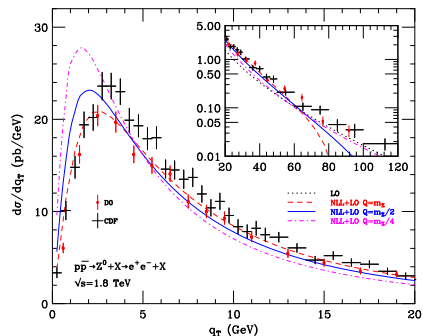
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- NLL+LO results for different values of the resummation scale Q (estimate of higher-order logarithmic contributions).
- We vary $Q = m_Z/2$, $m_Z/4 \leq Q \leq m_Z$: uncertainty $\pm 12 - 15\%$ in the region $q_T \gtrsim 20$ GeV (it dominates over the renormalization and factorization scale variations).
- We expect a sensible reduction once the complete NNLL+NLO calculation will be available.



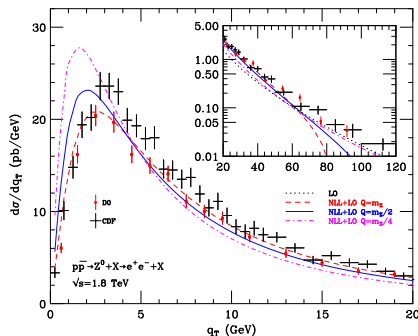
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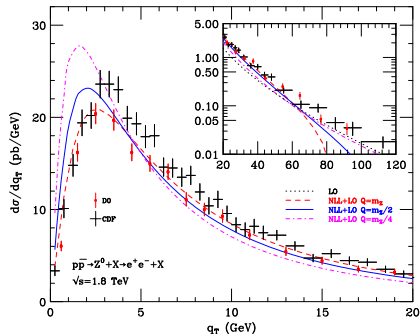
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