NA 14/3/13

QCD Phenomenology from Holographic Models

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Papers on

CD'A Gluon Condensates in Holographic QCD — EPJ C69 (2010) 315
CCD'A Antisymmetric tensors in " " — PRD 82 (2010) 095008
CCD'A Hadronic Light-by-Light contribution to $(g-2)_\mu$ " " — PRD 83 (2011) 093006
CCD'A Low energy weak interactions " " — PRD 85 (2012) 015003

PLAN OF THE TALK

* CHIRAL LAGRANGIANS
* HOLOGRAPHIC GAUGE/GRAVITY DUALITY
* HQCD
* SOME RESULTS
    * EFFECTS OF GLUON CONDENSATE
    * COEFFICIENTS OF WEAK CHIRAL LAGRANGIAN
    * HADRONIC LIGHT-BY-LIGHT SCATTERING CONTRIBUTION TO MUON g-2

CHIRAL SYMMETRY IN QCD

IN THE ABSENCE OF QUARK MASSES $m_q = 0$ $\quad q = \begin{pmatrix} u \\ d \\ s \end{pmatrix}$

THE QCD LAGRANGIAN $\quad \left(q_{L,R} = \frac{1 \pm \gamma_5}{2} q \right)$

$$\mathcal{L}_{QCD} = -\frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a + i \bar{q}_L \gamma^\mu D_\mu q_L + i \bar{q}_R \gamma_\mu D_\mu q_R$$

IS INVARIANT UNDER GLOBAL CHIRAL TRANSFORMATIONS

$$G \equiv SU(N_f)_L \times SU(N_f)_R \qquad \text{Parity } g_L \leftrightarrow g_R$$

$$q_L \to g_L q_L \qquad q_R \to g_R q_R \qquad g_{\substack{L\\R}} \in SU(N_f)_{\substack{L\\R}}$$

Noether theorem

$\Longrightarrow$ CONSERVED CURRENTS $\quad J^{\alpha\mu}_{L,R} = \bar{q}_{L,R} \gamma^\mu \frac{\lambda^\alpha}{2} q_{L,R} \qquad \alpha = 1 \dots 8$

$N_f = 3$

HOWEVER, CHIRAL SYMMETRY IS NOT MANIFEST AT LOW ENERGY : NO PARITY DOUBLING OF THE HADRON SPECTRUM

LOWEST MASS STATES : PSEUDOSCALAR MESONS (π's, K's)

$\in$ 8 of $SU(3)$

THEY ARE THE GOLDSTONE BOSONS OF SPONTANEOUS BREAKING OF CHIRAL SIMMETRY

$$G \equiv SU(N_f)_L \times SU(N_f)_R \longrightarrow SU(N_f)_V \equiv H$$

I.E. THE VACUUM OF THE THEORY IS NOT SIMMETRIC UNDER THE FULL G, BUT ONLY UNDER H

GOLDSTON THEOREM $\Rightarrow$ MASSLESS PARTICLES $\in G/H$

particles = # broken symmetries

LOW ENERGY QCD HADRON PHENOMENOLOGY

CHIRAL PERTURBATION THEORY (χPT)

χPT BASED ON TWO ASSUMPTIONS

- PIONS & KAONS AS GOLDSTON BOSONS OF SPONTANEOSLY BROKEN CHIRAL SYMMETRY

$$\underbrace{SU(3)_L \times SU(3)_R}_{\text{EXACT GLOBAL SYMMETRY OF } \mathcal{L}_{QCD} \text{ FOR } m_q = 0} \to SU(3)_V$$

- CHIRAL POWER COUNTING WITH EXPANSION IN

$$p^2 / \Lambda^2_{\chi SB} \qquad \Lambda_{\chi SB} \simeq 4\pi F_\pi \sim 1.2\,\text{GeV}$$

$\Rightarrow$ LOW ENERGY EFFECTIVE CHIRAL LAGRANGIAN

- NON LINEAR REALIZATION OF CHIRAL SYMMETRY

$$U(x) = e^{\frac{i\sqrt{2}}{F_\pi}\pi(x)} \; : U \longrightarrow g_L U(x) g_R^+ \quad g_{L,R} \in SU(3)_{L,R}$$

$\Rightarrow \pi(x) \to g\,\pi(x)\,g^+$ linear transf. only if $g_L = g_R = g \in SU(3)_V$

- $$\mathcal{L}^{(strong)}_{\chi PT} = \sum_{p=1}^{\infty} \mathcal{L}^{(strong)}_{2p} = \underset{O(p^2)}{\mathcal{L}^{(strong)}_2} + \underset{O(p^4)}{\mathcal{L}^{(strong)}_4} + \dots$$

$O(p^2)$ $$\mathcal{L}^{(strong)}_2 = \frac{F_\pi^2}{4} \langle \underbrace{D_\mu U D^\mu U^+}_{\text{kinetic term}} + \underbrace{\chi U^+ + U \chi^+}_{\text{mass term}} \rangle$$

$F_\pi \leftrightarrow \langle 0 | J^5_\mu | \pi \rangle$

$\chi \leftrightarrow \langle 0 | \bar{q}_L q_R | 0 \rangle$

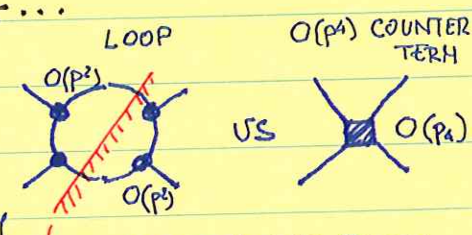
GOR $m_\pi^2 \sim \langle 0 | \bar{q}_L q_R | 0 \rangle m_q$

$O(p^4)$ $$\mathcal{L}^{(strong)}_4 = \sum_1^{12} L_i O_i = L_1 \left(\langle D_\mu U^+ D^\mu U \rangle\right)^2 + \dots$$

HOWEVER, AT THE SAME ORDER $O(p^4)$

LOOPS WITH $O(p^2)$ VERTICES

NECESSARY FOR UNITARITY



ABSORBTIVE PART

VECTOR AND AXIAL VECTOR RESONANCES IN χPT

$V(1^{--})$, $A(1^{++})$ $\quad (S(0^{++}), P(0^{+-}))$

Lowest vector meson is the ρ

$$\frac{1}{p^2 - M_\rho^2} \simeq -\frac{1}{M_\rho^2}\left(1 + \frac{p^2}{M_\rho^2} + \dots\right)$$

for $p^2 < M_\rho^2$ p^2 expansion

$\Rightarrow$ π's coupling $\propto \frac{1}{M_\rho^2}$

VECTOR MESON OCTET

$$\begin{pmatrix} \frac{1}{\sqrt{2}}\rho^0 + \frac{1}{6}\omega & \rho^+ & K^{*+} \\ \rho^- & -\frac{1}{2}\rho^0 + \frac{1}{\sqrt{6}}\omega & K^{*0} \\ K^{*-} & \overline{K^{*0}} & -\frac{2}{\sqrt{6}}\omega \end{pmatrix}$$

CCWZ :

HOW TO INCLUDE ADDITIONAL MATTER FIELDS IN THE CHIRAL LAGRANGIAN

VECTOR MESON DOMINANCE IN THE STRONG SECTOR

	EXP	V	A	V+A+ SCALARS
L_1	0.4 ± 0.3	0.6	0	0.6
L_2	1.4 ± 0.3	1.2	0	1.2
L_3	-3.5 ± 1.1	-3.6	0	-3.0
L_4	-0.3 ± 0.5	0	0	0
L_5	1.4 ± 0.5	0	0	1.4
L_6	-0.2 ± 0.3	0	0	0
L_7	-0.4 ± 0.2	0	0	-0.3
L_8	0.9 ± 0.3	0	0	0.9
L_9	6.9 ± 0.7	6.9	0	6.9
L_{10}	-5.5 ± 0.7	-10	4	-6.0

$$L_1^V = \frac{L_2^V}{2} = -\frac{L_3^V}{6} = \frac{G_V^2}{8M_V^2}$$

$$L_9^V = \frac{F_V G_V}{2M_V^2}$$

$$L_{10}^{V+A} = -\frac{F_V^2}{4M_V^2} + \frac{F_A^2}{4M_A^2}$$

IF WE USE VMD AND MATCHING WITH QCD LARGE-Q^2

$F_V \doteq 2G_V = \sqrt{2} f_\pi$

$F_A = F_\pi$

$M_A = 2M_V$

HOLOGRAPHIC QCD

THE GOOD

* A LAGRANGIAN FORMULATION (IN 5D)
* LEADING SHORT DISTANCE AGREEMENT WITH QCD IN MODELS WITH (ASYMPTOTIC) AdS METRIC
* WITH AN IR CUT-OFF, THERE ARE ∞ (KK) RESONANCES SATURATING THE CHANNELS (AS IN LARGE-N_c QCD)
* SMALL NUMBER OF FREE PARAMETERS
* MANY EXPLICIT CALCULATIONS

THE LESS GOOD

* χSB REALIZED IN DIFFERENT WAYS (BUT WE HAVE OUR FAVOURITE ONE!)
* DIFFERENT MASS SPECTRUM OF RESONANCES (REGGE VS NON-REGGE)
* ONE TOP-DOWN MODEL (SAKAI·SUGIMOTO) NOT EVEN ASYMPTOTICALLY AdS

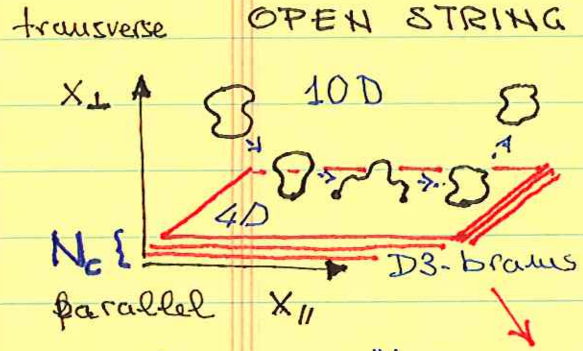
THE ORIGIN OF HOLOGRAPHIC MODELS IN STRING THEORY

POLCHINSKI '95 : D-BRANES

CLOSED STRINGS $\longrightarrow$ (SUPER) GRAVITY in 10D

contain LOW ENERGY

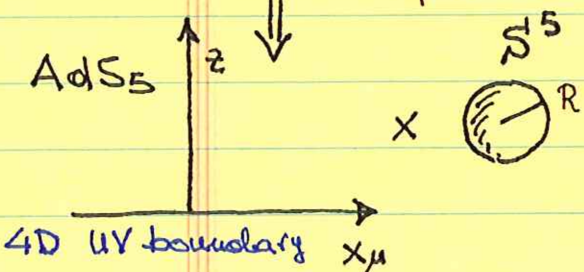
OPEN STRING SECTORS $\longrightarrow$ GAUGE THEORY in $< 10D$



ON THE D3-BRANE THE 10D GRAVITATIONAL FIELDS COUPLE TO THE 4D GAUGE FIELDS

$$S^{(4)}_{DBI} \sim \int d^4x_\parallel \; \underbrace{\phi(x_\parallel, x_\perp)\Big|_{x_\perp = 0}}_{\text{10 D SUGRA FIELD}} \; \underbrace{\psi(x_\parallel)}_{\text{4D GAUGE FIELD}}$$

LARGE N_c $\Downarrow$ curve the metric of the transverse space



$$ds_5 = \frac{1}{z^2}\left(dx_\mu^2 - dz^2\right)$$

MALDACENA CONJECTURE '97

> IN THE LARGE-N_c (AND 'T HOOFT LIMIT) THE DEGREES OF FREEDOM OF 10D GRAVITY NEAR (THE HORIZON OF) THE BRANE DECOUPLE (FROM THOSE FAR AWAY) AND DESCRIBE THE SAME (QUANTUM) THEORY OF THE 4D GAUGE THEORY ON THE N_c-D3-BRANES

$$\boxed{\mathcal{N} = 4 \text{ SYM is the same as IIB on } AdS_5}$$

PARAMETERS

AdS_5/S^5 radius $R = (4\pi g_s N_c)^{1/4} \ell_s$

LARGE$(N_c g_s) \Rightarrow$ LARGE R

$\Rightarrow$ SMALL CURVATURE $\sim 1/R$

$\Rightarrow$ CLASSICAL GRAVITY IS A GOOD DESCRIPTION

T' Hooft parameter $\lambda = g_{YM}^2 N_c$

FROM $S^{(4)}_{DBI}$ $\quad g^2_{YM} \sim g_s$

$\longrightarrow$ LARGE λ STRONG COUPLING LIMIT OF GAUGE THEORY

ONE OF THE TWO THEORIES IS AT STRONG COUPLING

Explaining the master formula of gauge/gravity duality

$$\boxed{\begin{aligned} e^{iW[s(x)]} &\equiv \left\langle e^{i\left(S_{QCD}+\int O(x)\,s(x)\,d^4x\right)} \right\rangle_{QCD} \\ &= e^{i S_{AdS_5}(\phi(x,z))^{\text{on-shell}}}\Big|_{\phi(x,0)=s(x)} \end{aligned}}$$

Evaluate correlation functions of colorless operators $O(x)$ of the 4D gauge theory from the dual 5D gravitational theory in AdS_5

Dictionary

4D operators	5D fields
$O(x)$	$\longleftrightarrow\ \phi(x,z)$
Δ	$\longleftrightarrow\ m_5$

$$\phi(x,0) = s(x)$$

5D/4D matching is done on the UV boundary

I) $s(x)$ = source coupled to the 4D operator $O(x)$
= UV boundary values of the 5D field $\phi(x,z)$

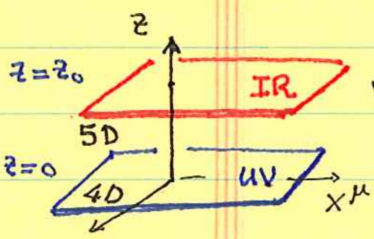
II) $W[s(x)]$ = 4D effective action
= 5D action evaluated on-shell, i.e. on fields $\phi(x,z)$ which solve the 5D EOM with the UV boundary condition above

(Explicit example in the next slide)

Comments

1) gauge/gravity dual has not been proved
2) it has passed a lot of checks when applied to highly "non-physical" theories, e.g. 5D sugra / string on $AdS_5 \times S^5$ vs $N=4$ SYM
3) it tipically relates theories in which one of them is at strong coupling
4) but the other (the gravitational in most applications) is in the weak coupling regime
5) So, we have "SOLUTIONS" (5D gravitational models) in search of their "PROBLEMS" (4D (gauge) theories at strong coupling)

EXAMPLE 5D ABELIAN GAUGE FIELD ON A CURVED SLICE



$V_M(x,z)$ $M=\mu, z$ $ds^2 = g_{MN}\, dx^M dx^N = w(z)^2\left(dx_\mu^2 - dz^2\right)$

$\underline{AdS_5}\begin{cases} w(z) = 1/z \\ z_0 \to \infty \end{cases}$

$$S^{(5)} = -\frac{1}{4g_5^2}\int d^5x\,\sqrt{g}\,g^{MN}g^{PQ}F_{MP}F_{NQ} \qquad F_{MN} = \partial_M V_N - \partial_N V_M$$

$$= -\frac{1}{4g_5^2}\int d^4x \int dz\; w(z)\left(F_{\mu\nu}^2 + (\partial_z V_\mu)^2\right)$$

gauge choice $V_z = 0$ $\partial_\mu V^\mu = 0$

INTEGRATE BY PARTS IN Z
$$= -\frac{1}{2g_5^2}\left\{\int d^4x\, w(z) V_\mu \partial_z V_\mu\Big|_0^{z_0} + \int d^5x\, w(z) V_\mu\left(\Box_x V_\mu - \frac{1}{w}\partial_z\left(w\,\partial_z V_\mu\right)\right)\right\}$$

4D boundary term

5D EOM

$$\underline{S^{(5)}\left(V_\mu(x,z)\right)^{\text{on-shell}} = W^{(4)}\left(V_\mu(x,0)\right)} \equiv -\frac{1}{2g_5^2}\int d^4x\, w V_\mu \partial_z V_\mu\Big|_{z=0}$$

Note that we have to impose IR at $z = z_0$: $V_\mu \partial_z V_\mu\Big|_{z_0} = 0$ $\begin{cases} V_\mu\big|_{z_0} = 0 & D \\ \partial_z V_\mu\big|_{z_0} = 0 & N \end{cases}$

4D Fourier transform $V_\mu(x,z) \to \tilde{V}_\mu(q,z) = \hat{v}_\mu(q)\, v(z,q)$ $q^\mu \hat{v}_\mu(q) = 0$

$v(z,q)$ is the bulk-to-boundary propagator
- it solves the 5D EOM
- satisfies UV b.c. $v(0,q) = 1$

$$W^{(4)}\left(\hat{v}_\mu(q)\right) = \int d^4q \int d^4q'\,\delta(q+q')\,\hat{v}_\mu(q)\,\hat{v}_\mu(q')\Big[-w(z)\,\partial_z v(z,q)\Big]_{z=0}$$

$$\equiv \int d^4q \int d^4q'\,\delta(q+q')\,\hat{v}_\mu(q)\,\hat{v}_\nu(q') \langle J_\mu(q) J_\nu(q')\rangle$$

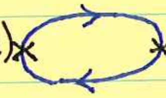
$$\langle J_\mu(q) J_\nu(-q)\rangle = \frac{\delta^2 W^{(4)}}{\delta\hat{v}_\mu(q)\,\delta\hat{v}_\nu(-q')} = \left(\eta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)\Pi_V(q^2)$$

VECTOR CURRENT
2-POINT FUNCTION

$$\boxed{\Pi_V(q^2) \sim \left(-w(z)\,\partial_z v(z,q)\right)\Big|_{z=0}}$$

For AdS_5 $\quad w(z) = 1/z \quad z_0 \to \infty$ (NO IR boundary)

$$v(z,q) = qz\, K_1(qz)$$ MODIFIED BESSEL FUNCTION

$\Pi_V(q^2) \sim \log q^2 \quad \sim \quad J_\mu(q)$  $J_\nu(-q)$ $\qquad g_5 \sim \frac{1}{N_c}$

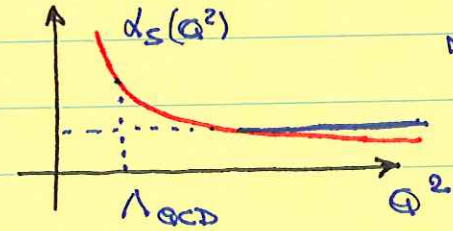
FREE PARTON LOOP

$\sim$ UV LEADING RESULTS OF QCD FOR $\Pi_V(q^2)$

AT LARGE (EUCLIDEAN) MOMENTUM

QCD "ALMOST CONFORMAL"

IN THE UV REGIME



Note that • the $\log q^2$ UV behaviour of $\Pi_V(q^2)$ holds for ~~ALMOST~~ ASYMPTOTICALLY AdS_5 metric

$$w(z) \longrightarrow 1/z \quad \text{at small } z \to 0$$

• for an AdS slice with $z_0 < \infty$ and $w(z) = 1/z$

$$\Pi_V(q^2) = -\frac{4}{3}\frac{N_c}{(4\pi)^2}\left[\log\frac{q^2}{\mu^2} - \frac{\pi\, Y_0(qz_0)}{\underbrace{J_0(qz_0)}}\right] \sim \log\frac{q^2}{\mu^2} + \sum_{n=1}^{\infty}\frac{f_n^2}{q^2 - m_n^2}$$

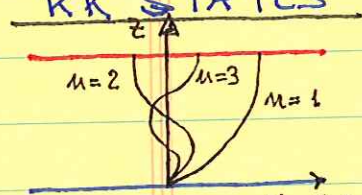
LEADING UV TERM

∞ # POLES LOCATED AT THE ZEROES OF THE BESSEL FUNCTION $J_0(\gamma_n) = 0$

$\Downarrow$

∞ SERIES OF VECTOR MESON RESONANCES with MASSES $\boxed{m_n = \frac{\gamma_n}{z_0}}$ $\qquad z_0 = \frac{\gamma_0}{m_\rho}$

KK STATES



WAVE FUNCTIONS $\psi_n(z)$

GENERALIZED VECTOR MESON DOMINANCE

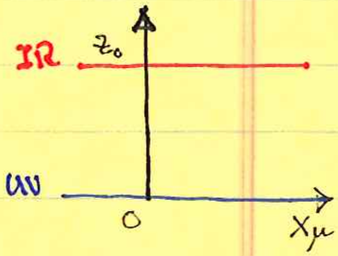
CONSISTENT WITH LARGE N_c QCD

J  $J \simeq \sum_n^{\infty}$ 

∞ stable resonances

HARD WALL MODEL WITH χSB INDUCED BY IR BOUNDARY COND'S.

HIRN, SANZ



$$S^{(5)}_{YM} = -\frac{1}{4g_5^2}\int d^4x \int_0^{z_0} dz\,\sqrt{g}\, g^{MN} g^{PQ} \langle L_{MP} L_{NQ} + R_{MP} R_{NQ} \rangle$$

$$R_{MN} = \partial_M R_N - \partial_N R_M - i\,[R_M, R_N] \qquad R_M = R_M^a \frac{t^a}{2}$$

"parity" $L_M(x,z)$, $R_M(x,z)$ gauge fields of $SU(N_f)_L$, $SU(N_f)_R$

χSB boundary conditions on the IR boundary $z = z_0$

I) $R_\mu(x, z_0) - L_\mu(x, z_0) = 0$ II) $R_{z\mu}(x, z_0) + L_{z\mu}(x, z_0) = 0$

b.c I) is preserved only by 5D gauge trans's : $g_L(x, z_0) = g_R(x, z_0) \equiv h(x)$ i.e. $\in SU(N_f)_V$ on the IR boundary

b.c II) does not impose any additional condition on $h(x)$

χSB b.c. I) and II) can be rewritten in terms of vector and axial-vectors gauge fields

$$V_M = L_M + R_M \qquad A_M = L_M - R_M$$

as

$$A_M(x, z_0) = 0 \qquad V_{z\mu}(x, z_0) = 0$$

$\Rightarrow$ * DIFFERENT b.c. $\Rightarrow$ DIFFERENT KK EXPANSION and DIFFERENT POLES AND RESIDUES OF $\Pi_V(q^2)$ AND $\Pi_A(q^2)$

* NON TRIVIAL ZERO MODE FOR THE AXIAL VECTORS $\Rightarrow$ PION $\Rightarrow$ $\Pi_A(q^2) = \dfrac{F_\pi^2}{q^2}$ + massive resonances

BETTER TO WORK IN THE "AXIAL" GAUGE

$$V_z = 0 \qquad A_z = 0$$

HOWEVER this can be done, but we have to respect the b.c. I) $\Rightarrow$ gauge transf has to vanish at $z = z_0$

Solution

Wilson lines

$$\xi^{(5)}_{\substack{L\\R}}(x,z) = P\exp\left\{-i\int_z^{z_0} \underset{R_z}{L_z}(x,z')\,dz'\right\}$$

* $\xi^{(5)}_L(x,z_0) = 0$ ok

* $\xi^{(5)}_L(x,0) = P\exp\left\{-i\int_0^{z_0} L_z(x,z')\,dz'\right\} \equiv \xi_L(x)$

is a new 4D field.

HIDDEN SYMMETRY CONSTRUCTION OF THE CHIRAL FIELD

$$U(x) = \xi_L(x)\,\xi_R^\dagger(x)$$

MOREOVER

the gauge transf's $\xi^{(5)}_{L,R}(x,z)$ are non vanishing on the UV boundary

$\Rightarrow$ they modify ("chirally rotate") the original UV b.c.

$$L_\mu(x,0) = l_\mu(x) \xrightarrow{\xi^{(5)}_L} L_\mu(x,0) = \xi_L\,(l_\mu + i\partial_\mu)\,\xi_L^\dagger$$

$$R_\mu(x,0) = r_\mu(x) \xrightarrow[\xi^{(5)}_R]{} R_\mu(x,0) = \xi_R\,(r_\mu + i\partial_\mu)\,\xi_R^\dagger$$

One can make the gauge choice $\xi_R(x) = \xi_L^\dagger(x) \equiv u(x)$

$$\Rightarrow \quad U(x) = u(x)^2$$

$$L_\mu(x,0) = u\,(l_\mu + i\partial_\mu)\,u^\dagger \equiv i\Gamma_\mu - \frac{1}{2}u_\mu \qquad u_\mu = i u^\dagger D_\mu U u$$

$$R_\mu(x,0) = u^\dagger\,(r_\mu + i\partial_\mu)\,u = i\Gamma_\mu + \frac{1}{2}u_\mu \qquad = -i u D_\mu U^\dagger u$$

LOW ENERGY CHIRAL LAGRANGIAN FROM $S^{(5)}_{YM}$

$$S^{(5)} \sim \int \Big\{ \underbrace{(\partial_z L_\mu)^2 + (\partial_z R_\mu)^2}_{O(p^2)} + \underbrace{L_{\mu\nu}^2 + R_{\mu\nu}^2}_{O(p^4)} \Big\}$$

$\sim S_{\chi PT}$

$$L_\mu(x,z) \simeq L_\mu(x,0) + \frac{1-\alpha(z)}{2} u_\mu + (\text{resonances})$$

$$R_\mu(x,z) \simeq R_\mu(x,0) + \frac{1+\alpha(z)}{2} u_\mu + (\text{resonances})$$

"pion wave funct." $\alpha(z) = 1 - \frac{z^2}{z_0}$ for AdS metric

$\Rightarrow$ parameters of the chiral Lagrangian

$O(p^2)$: $f_\pi^2 = \frac{2}{g_5 z_0}$

$O(p^4)$:

$$L_1 = \frac{1}{32 g_5^2} \int_0^{z_0} \frac{dz}{z} \left(1 - \alpha(z)^2\right)^2, \quad L_2 = 2L_1, \quad L_3 = -6L_1$$

$$L_{10} = -\frac{1}{4g_5^2} \int_0^{z_0} \frac{dz}{z} \left(1 - \alpha(z)^2\right), \quad L_9 = -L_{10} \qquad H_1 = -\frac{1}{8g_5^2} \int_0^{z_0} \frac{dz}{z} \left(1 + \alpha(z)^2\right)$$

∞# RESONANCES with MASSES m_n^2 and decay constants f_n and coupling to pions g_n contribute to the LOW ENERGY CONSTANTS (GENERALIZED VMD)

SUM RULES (from completeness of 5D wave functions $\psi_n(z)$)

$$f_\pi^2 = \sum_1^\infty f_{V_n} g_{V_n} m_{V_n}^2 \qquad L_1 = \frac{1}{8} \sum_1^\infty g_{V_n}^2 \quad \text{etc}$$

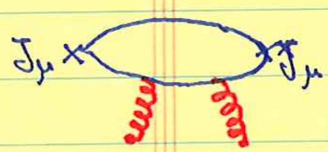
with LEADING CONTRIBUTIONS FROM THE FEW LIGHTEST ONES

EFFECT OF GLUON CONDENSATE

HIRN, SANZ, RIOS — C. D'AMBROSIO

CONDENSATES

QUESTION : ADDING THE EFFECT OF GLUON CONDENSATE DOES IMPROVE AGREEMENT WITH PHENOMENOLOGY



$$\langle \alpha_s G_{\mu\nu} G^{\mu\nu} \rangle \xrightarrow[\text{DICTIONARY}]{\text{HOLOGRAPHIE}} \text{5D METRIC DEFORMATION}$$

$$w(z) = \frac{1}{z}\left(1 + y\left(\frac{z}{z_0}\right)^4\right)$$

EVALUATE PERTURBATIVELY CORRECTION $O(y)$

$$y = \frac{3\pi}{16 N_c} z_0^4 \langle \alpha_s G_{\mu\nu} G^{\mu\nu} \rangle$$

1) to ALL LOW ENERGY CONSTANT OF THE CHIRAL LAGRANGIAN
   to RESONANCE MASSES DECAY WIDHTS , COUPLINGS

2) to THE LEADING BEHAVIOOR OF THE 2-POINT FUNCTIONS

$$\Pi_V(Q^2) \sim \log\left(\frac{Q^2}{\mu^2}\right) - \frac{\langle \pi \alpha G_{\mu\nu} G^{\mu\nu} \rangle / N_c}{Q^4}$$

3) CHECK THE CONSISTENCY OF THE "DEFORMED" SUM RULES

4) GLOBAL FIT OF THE "DEFORMED" VALUES OF

$f_\pi, L_1, L_2, L_3, L_9, L_{10}, m_\rho, f_\rho, g_\rho$

Estimate of the Gluon condensate $\left(8.7 ^{+9.4}_{-4.2}\right) \times 10^{-2}\,\mathrm{GeV}^4$

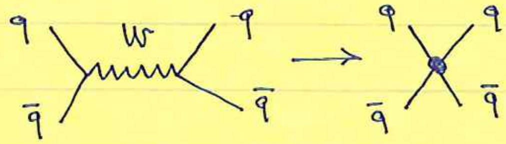
COMPATIBLE WITH "WORLD AVERAGE"

$(6.8 \pm 1.13) \times 10^{-2}\,\mathrm{GeV}^4$

$\Delta S = 1$ NON LEPTONIC WEAK INTERACTIONS

AT LOW ENERGIES $E \ll M_W$

WITH EFFECTIVE HAMILTONIAN



$$\mathcal{H}_{eff}^{\Delta S=1} = \frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \sum_i C_i(\mu)\, Q_i + \text{h.c.}$$

Wilson coefficients depending on the renormalization scale

4-quark operators

current-current

e.g. $(\bar{s}_L \gamma^\mu d_L)(\bar{u}_L \gamma_\mu u_L)$

* $Q_- = Q_2 - Q_1$

transforms as

$(8_L, 1_R)$ under the chiral group $SU(3)_L \times SU(3)_R$

and induces $|\Delta I| = 1/2$ transitions.

* $Q_1 + \frac{2}{3} Q_2 - \frac{1}{3} Q_3$ transforms as $(27_L, 1_R)$

and induces both $|\Delta I| = 1/2$ and $|\Delta I| = 3/2$

WEAK CHIRAL LAGRANGIAN

$$\mathcal{L}_2^{\Delta S=1} = -\frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \left\{ g_8 \langle \lambda L_\mu L^\mu \rangle + g_{27} \left(L_{\mu 23} L^\mu_{11} + \frac{2}{3} L_{\mu 21} L^\mu_{13} \right) + \text{h.c.} \right\}$$

$$\lambda = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \qquad L_\mu = i F_\pi^2 U^\dagger D_\mu U$$

$$\mathcal{L}_4^{\Delta S=1} = \sum_i N_i W_i$$ many more terms than in the strong sector

VECTOR MESON DOMINANCE NOT AS SUCCESSFUL AS IN THE STRONG SECTOR

IN PARTICULAR FOR $K \to 3\pi$

HARD WALL WEAK INTERACTIONS

L.C., O.CATA, G.D'AMBROSIO

THE RELEVANT OPERATOR IN $\mathcal{H}_{eff}^{|\Delta S=1|}$ IS DOUBLE TRACE IN COLOR INDECES

IN AdS/CFT:

* THERE IS A WELL DEFINED PRESCRIPTION TO DEAL WITH DEFORMATION OF THE 4D THEORY DUE TO DOUBLE TRACE OP'S

* GAZIT & YEE PROPOSED TO USE IT TO EVALUATE SEMILEPTONIC DECAYS IN HOLOGRAPHIC MODELS

* WE FULLY EXPLOIT IT IN THE HIRN-SANZ MODEL TO GET EXPRESSIONS FOR THE WEAK CHIRAL LAGRANGIAN

THE RESULT IS A WEAK DEFORMATION OF THE CHIRALLY ROTATED FIELDS

$$L_\mu(x,z) = L_\mu^{(0)} + \frac{1-\alpha(z)}{2} u_\mu + k\,\frac{1+\alpha(z)}{2}\, l_\mu^{W}$$

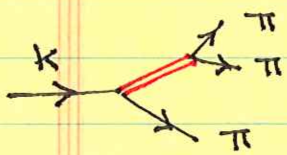
$$l_\mu^{W} = -\xi_8 \frac{f_\pi^2}{2}\left(\{\Delta, u_\mu\} - \delta_3 \langle \Delta u_\mu \rangle\right) \qquad \Delta = u \lambda u^{+}$$

* THE $O(p^4)$ constants N_i are obtained as integrals of expressions containing the pion wave funct' $\alpha(z)$

  $\Rightarrow$ THE N_i ARE RELATED TO THE L_i

* WE GET AN HOLOGRAPHIC EXPLANATION OF THE EURISTIC WEAK DEFORMATION MODEL

* AN IMPORTANT PHENOMENOLOGICAL RESULT IN $K \to 3\pi$ DECAYS



4D VMD with ρ meson $\propto L_3^V + \frac{3}{4} L_9^V = 0$

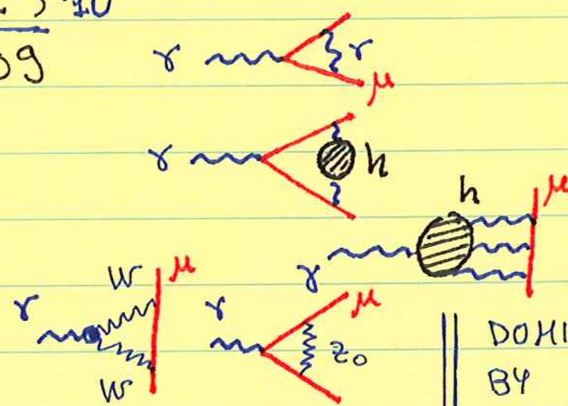
Holographic QCD $\quad L_3 + \frac{3}{4} L_9 \neq 0$

GIVES BETTER AGREEMENT WITH FENOMENOLOGY

HADRON LIGHT-BY-LIGHT SCATTERING CONTRIBUTION TO MUON g-2

CONTRIBUTIONS (STANDARD MODEL) '10

QED (leptons)		116584718.09
HVP (lo)		6914
HVP (ho)	$\times 10^{-11}$	−98
HLbyL		105
EW		152
Total SM		116591793 ± 51



DOMINATED BY PION EXCHANGE

timely study because

EXPERIMENT E989 at FERMILAB APPROVED!

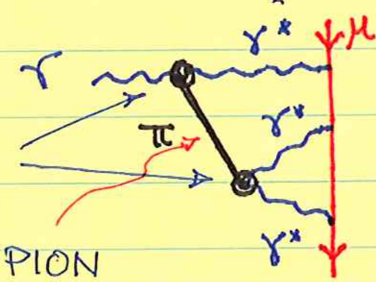
$\sigma = \pm 0.14$ ppm

4 times more precise than E821

PION FORM FACTORS

$F_{\pi^0\gamma^*\gamma^*}(Q_1, Q_2)$

OFF-SHELL PION



QUESTIONS WE ADDRESS

* WHAT PARAMETERS ENTERING PFF MOST CONTRIBUTING TO HLBL UNCERTAINTIES?
* CAN WE USE HQCD TO EVALUATE PFF?
* IS VECTOR MESON DOMINANCE A GOOD APPROX. FOR PFF?
* HOW TO TAKE CARE OF THE OFF-SHELLNESS OF THE PION?

OUR STRATEGY   L.C., O. CATA, G. D'AMBROSIO

* USE LOW ENERGY DATA AS A FILTERING OF THE HQCD MODELS
* FROM THE SELECTED ONES EXTRACT PREDICTIONS FOR PFF AL LOW-Q^2
* USE AN INTERPOLATOR TO MATCH low-Q^2 PFF with high Q^2 from PQCD
* USE THE INTERPOLATOR IN THE EVALUATION OF HLbL
* CHECK NUMERICAL STABILITY OF THE RESULT

ANOMALOUS
PION FORM FACTOR IN HQCD

ANOMALOUS INTERACTIONS FROM 5D CHERN-SIMONS TERM

$$S^{(5)}_{CS} = \frac{N_c}{24\pi^2} \int_5 \left(\omega_5(L) - \omega_5(R) \right)$$

$$\omega_5(A) \equiv \langle AF^2 - \frac{1}{3} A^3 F + \frac{1}{10} A^5 \rangle$$

* under a gauge transf. $\delta \omega_5(A) = d\alpha_4(A)$

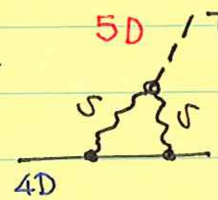
$\underbrace{\text{gauge}}_{\downarrow}$ AXIAL GAUGE (USING WILSON LINES)

$$S_{CS} = \frac{N_c}{24\pi^2} \left\{ \int_5 \omega_5(L) - \frac{1}{10} \langle (d\xi_L \xi_L^\dagger)^5 \rangle - \int_{\partial 5} \alpha_4(du^\dagger u, L) \right\} - (L \leftrightarrow R)$$

WITTEN 5D term of anomalous XL — gauged anomalous XL

EXTRACT AVV terms $\downarrow$

$$\longrightarrow K(Q_1, Q_2) = - \int_0^{z_0} \mathcal{V}(Q_1, z)\, \mathcal{V}(Q_2, z)\, \partial_z \alpha(z)\, dz$$



$\mathcal{V}(z, Q)$ is the bulk-to-boundary propagator

$$\boxed{F_{\gamma^*\gamma^*\pi^0}(Q_1^2, Q_2^2) = - \frac{N_c}{12\pi^2 f_\pi} K(Q_1^2, Q_2^2)}$$

* Low-q^2 expansion — Fixed by experiments

$$F_{\gamma^*\gamma^*\pi^0}(Q_1^2, Q_2^2) \simeq - \frac{N_c}{12\pi^2 f_\pi^2} \left[1 + \hat{\alpha}(Q_1^2 + Q_2^2) + \hat{\beta}\, Q_1^2 Q_2^2 + \hat{\gamma}\,(Q_1^4 + Q_2^4) + \dots \right]$$

* AGREEMENT WITH EXP TO SELECT AMONG DIFFERENT HQCD MODELS

* INTERPOLATOR INSTEAD OF ACTUAL HQCD PFF

$$(\text{DIP}) \quad K(Q_1^2, Q_2^2) = 1 + \lambda \left(\frac{Q_1^2}{Q_1^2 - m_V^2} + \frac{Q_2^2}{Q_2^2 - m_V^2} \right) + \mu \frac{Q_1^2 Q_2^2}{(Q_1^2 - m_V^2)(Q_2^2 - m_V^2)}$$

$\Longrightarrow$ SATISFY ANOTHER SHORT DISTANCE CONSTRAINT

$$F_{\pi^0\gamma^*\gamma^*}(Q^2, Q^2, 0) \xrightarrow[Q \to \infty]{} - \frac{f_\pi}{3} \chi_0 \leftarrow \text{ONLY THEORETICAL ESTIMATES}$$

WHAT IS THE IMPACT ON $(g-2)_\mu$?

FINAL RESULT FOR $(g-2)_\mu^{HLBL}$ AND COMPARISON WITH OTHER MODELS FOR $F_{\pi^0\gamma^*\gamma^*}$ NYFFELER '11

	$a_\mu(\pi^0) \times 10^{-11}$	
AdS/QCD/DIP	65.4 (2.5)	
mod ENJL	59 (9)	Extended Jona-Lasinio = Chiral quark model
VMD/HLS	57 (4)	VMD
LMD+V	58 (10)	VMD
	63 (10)	VMD
MV	77 (7)	Melnikov, Vainshtein NEW SHORT DISTANCE CONSTRAINT
AdS/QCD	69	Another HQCD estimate
DSE	58 (7)	Bethe Salpeter approach
JN	72 (12)	

* A THEORETICAL EFFORT SHOULD BE DONE TO MAKE THE MODELS TO TALK EACH OTHER

OUR UNCERTAINTY ESTIMATE

* WE WORKED IN A "THEORY SPACE" CONSIDERING DIFFERENT HQCD MODELS AND DISCRIMINATING AMONG THEM ON THE BASIS OF LOW-Q^2 PREDICTIONS FOR PFF

* WE INCLUDED THE EFFECTS OF χ_0 (POORLY KNOWN) (MAIN DEPARTURE FROM HQCD MODEL $\rightarrow$ DIP INTERPOLATOR)
$\Longrightarrow$ THAT ADDS 10-15% TO UNCERTAINTY

* THE MV RESULT (SENSIBLY HIGHER THAN OTHERS) SHOULD BE CHALLENGED IN A MORE COMPLETE HQCD CALCULATION (BUT THEN PROBABLY MISSING THE HANDLE ON χ_0)