Coupling Matter to Quantum Gravity via Spectral Triples

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Outline of talk

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Motivation

- Noncommutative geometry - Connes' work on the standard model of particle physics.
- Canonical quantum gravity / Ashtekar and loop variables.

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Physical Interpretation

- The construction encodes the Poisson structure of General Relativity.
- Semi-classical analysis: emergence of classical gravity coupled to interacting fermions.

Noncommutative Geometry

Noncommutative Geometry

- **A Spectral Triple** is a collection (B, H, D):
  a $*$-algebra B represented as operators in the Hilbert space H; a self-adjoint, unbounded operator D, acting in H such that:
  1. The resolvent of D, $(1 + D^2)^{-1}$, is compact.
     (spectrum of D nicely distributed)
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  - 7 "axioms", Connes 2008: reconstruction theorem.
- Thus, it is possible to reformulate Riemannian geometry in terms of algebras of functions and Dirac operators
  - Topological data stored in the algebra
  - Metric data stored in the Dirac operator

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  - $B = C^\infty(M) \otimes B_F, \quad B_F = \mathbb{C} \oplus \mathbb{H} \oplus M_3(\mathbb{C})$

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  - $H =$ fermionic content of SM
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    "almost commutative algebra"
  - $H =$ fermionic content of SM
  - $D = \not{D} \otimes 1 + \gamma_5 \otimes D_F$,
  - The classical action of the standard model coupled to gravity emerges from a certain heat kernel expansion of D.

Central point

Formulation of the classical standard model coupled to general relativity as a single gravitational theory. The standard model emerges from a modification of space-time geometry:

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Our goal

To construct a framework which combines noncommutative geometry with elements of quantum gravity/quantum field theory.

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$$\{A^i_j(x), E^k_l(y)\} = \delta^i_l \delta^k_j \delta(x-y)$$

- The Hamiltonian involves two constraints

$$H = \int N\epsilon^{ab}_c E^i_a E^j_b F^c_{ij} + N^i E^j_b F^c_{ij}$$

(Hamilton, spatial diffeomorphism)

- Shift focus from connections to holonomy and flux variables

$$h_L(A) = \mathrm{Hol}(L, A)$$

L loop on Σ

$$F_S^a(E) = \int_S \epsilon^i{}_{jk} E_i^a dx^j dx^k$$

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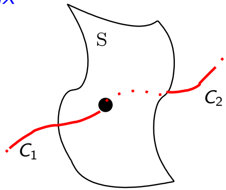

$$\{F^a_S(E), h_C(A)\} = \pm h_{C_1}(A)\sigma^a h_{C_2}(A)$$

σ^a generator of $\mathfrak{su}(2)$, $C = C_1 C_2$ are curves in Σ.

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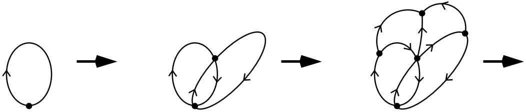

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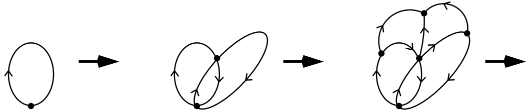

- Seen from a graph Γ the space $\mathcal{A}$ of smooth connections with gauge group G is simply the space $G^{n(\Gamma)}$

$$\mathcal{A}_\Gamma \simeq G^{n(\Gamma)}$$

where $n(\Gamma)$ is the number of edges in Γ (one copy of G for each edge in Γ).

- When one takes all graphs into account this leads to a projective system of coarse grained spaces of connections:

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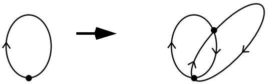
- **Example:**

$$P : G^4 \to G$$



$$(g_1, g_2, g_3, g_4) \to g_1 \cdot g_3$$

because $Hol(\nabla, \epsilon_1) \cdot Hol(\nabla, \epsilon_3) = Hol(\nabla, \epsilon_1 \cdot \epsilon_3)$

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- Open Issues:
  - $\rightarrow$ Classical limit.
  - $\rightarrow$ Coupling matter.
  - $\rightarrow$ Dynamics.
  - $\rightarrow$ ...

Our Project

- **Aim:** To construct a spectral triple that involves an algebra of holonomy loops, i.e. functions on $\mathcal{A}$:

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- **Idea:** use holonomy loops instead of Wilson loops (LQG) $\to$ Noncommutative geometry.

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  3. take a limit over graphs to obtain a spectral triple over the space of connections $\mathcal{A}$.

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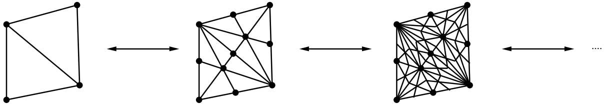

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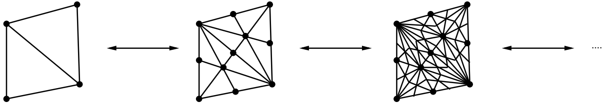

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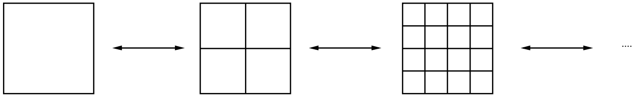

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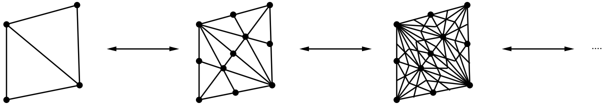

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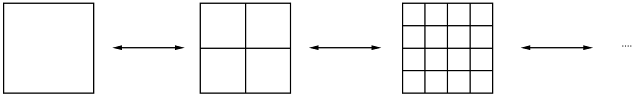

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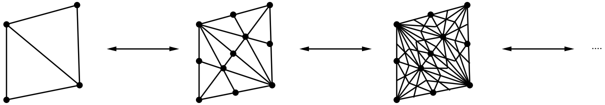

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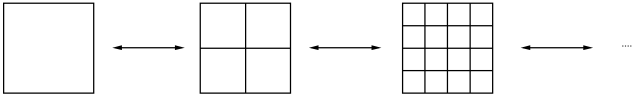

- Both systems of graphs permit a spectral triple construction.
  - But the cubic lattices turn out to be more natural (semi-classical analysis).

The construction

The construction

A single cubic lattice

- Let Γ be a finite $3D$ finite cubic lattice with oriented edges $\{\epsilon_i\}$ and vertices $\{v_i\}$.
- Assign to each edge ϵ_i a group element $g_i \in G$

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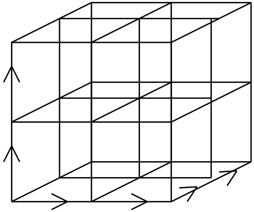

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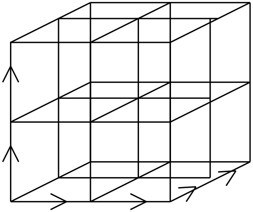

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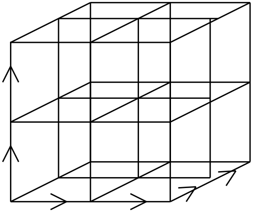

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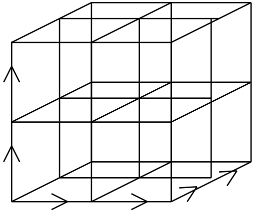

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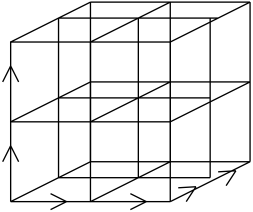

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- Think of the space $\mathcal{A}_\Gamma$ as a coarse-grained approximation of the space $\mathcal{A}$ of smooth connections.

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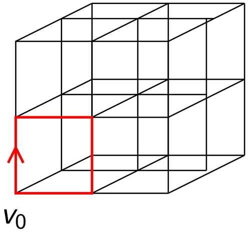

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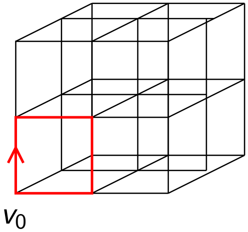

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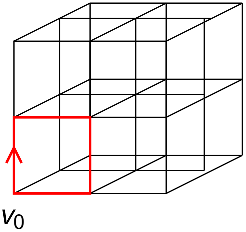
  - Involution of L by reversal of direction $L^* = L^{-1}$.



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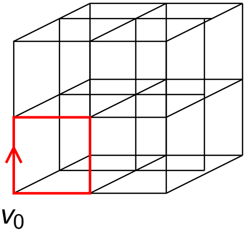
  - A loop L is a finite sequence of edges $L = \{\epsilon_{i_1}, \epsilon_{i_2}, \ldots, \epsilon_{i_n}\}$ which starts and ends in v_0.
  - Discard trivial backtracking.
  - Noncommutative product between loops by gluing them at the basepoint.
  - Involution of L by reversal of direction $L^* = L^{-1}$.
  - The algebra $\mathcal{B}_\Gamma$ is the algebra generated by loops running in Γ. A general element in $\mathcal{B}_\Gamma$ is of the form

$$a = \sum_i a_i L_i \ , \quad a_i \in \mathbb{C}$$



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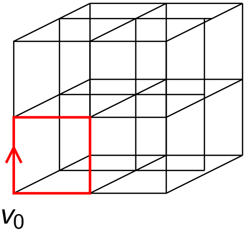

$$a = \sum_i a_i L_i \ , \quad a_i \in \mathbb{C}$$

- Define

$$\nabla(L) = \nabla(\epsilon_{i_1}) \cdot \nabla(\epsilon_{i_2}) \cdot \dots \cdot \nabla(\epsilon_{i_n})$$

Think of $\nabla(L)$ as the holonomy of a connection ∇ around a loop L.

- **Hilbert space:** There is a natural Hilbert space

$$\mathcal{H}_\Gamma = L^2(G^n, M_l(\mathbb{C}))$$

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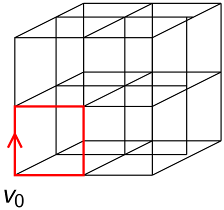
  - Example: $L = \{\epsilon_1, \epsilon_4, \epsilon_6^*, \epsilon_3^*\}$

$$f_L \sim g_1 \cdot g_4 \cdot (g_6)^{-1} \cdot (g_3)^{-1}$$



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- **All together:** we have a spectral triple $(\mathcal{B}_\Gamma, \mathcal{H}_\Gamma, D)$ at the level of the graph Γ.

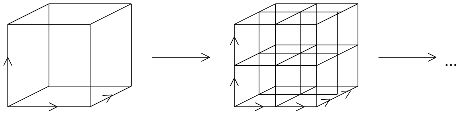
A family of lattices

A family of lattices

- Consider an infinite system of nested, 3-dimensional lattices

$$\Gamma_0 \rightarrow \Gamma_1 \rightarrow \Gamma_2 \rightarrow \dots$$

with Γ_i a subdivision of Γ_{i-1}



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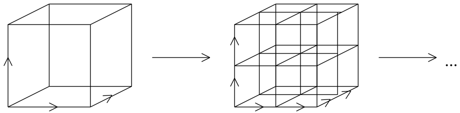

On the level of the associated manifolds $\mathcal{A}_{\Gamma_i}$ this gives rise to projections

$$\mathcal{A}_{\Gamma_0} \xleftarrow{P_{10}} \mathcal{A}_{\Gamma_1} \xleftarrow{P_{21}} \mathcal{A}_{\Gamma_2} \xleftarrow{P_{32}} \mathcal{A}_{\Gamma_3} \xleftarrow{P_{43}} \dots$$

- Consider next a corresponding system of spectral triples

$$(\mathcal{B}, \mathcal{H}, D)_{\Gamma_0} \leftrightarrow (\mathcal{B}, \mathcal{H}, D)_{\Gamma_1} \leftrightarrow (\mathcal{B}, \mathcal{H}, D)_{\Gamma_2} \leftrightarrow \dots$$

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  which are compatible with the maps between graphs.
- This requirement restricts the choice of D.
- At the level of a graph Γ, a compatible operator has the form

$$D = \sum_k a_k D_k$$

  where the sum runs over different copies of G and where

$$D_k(\xi) = \sum_a \mathbf{e}_k^a \cdot d_{e_k^a}(\xi) \qquad \xi \in L^2(G, Cl(TG))$$

  where $d_{e_k^a}$ are left-translated vectorfields on the k'th copy of G and $\mathbf{e}_k^a$ are elements in the Clifford algebra (next slide). The a_n's are free parameters related to the level of refinement (The sum over copies of G is wrt a change of variables).

- To accommodate this Dirac operator we extend the Hilbert space $\mathcal{H}_\Gamma$

$$\mathcal{H}_\Gamma = L^2(G^n, Cl(T^*G^n) \otimes M_l(\mathbb{C}))$$

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  - One copy of $SU(2)$: $Cl(T^*G) = Cliff(3)$ generated by three elements $\mathbf{e}^a$,

$$\{\mathbf{e}^a, \mathbf{e}^b\} = 2\delta^{ab} , \quad \langle \mathbf{e}^a \rangle = 0 , \quad \langle \mathbf{e}^a \mathbf{e}^b \rangle = \delta^{ab} , \quad \ldots$$

  - Several copies of $SU(2)$: graded tensor product.

The limit

- In the limit of repeated subdivisions, this gives us a candidate for a spectral triple

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- Result: For a compact Lie-group G the triple $(\mathcal{B},\mathcal{H},D)_{\overline{\mathcal{A}}}$ is a semi-finite* spectral triple:
  - D's resolvent $(1+D^2)^{-1}$ is compact (wrt. trace) and
  - the commutator $[D,b]$ is bounded

  provided the sequence $\{a_i\}$ approaches ∞.

**semi-finite: everything works up to a symmetry group with a trace (CAR algebra)* [Carey, Phillips, Sukochev].

What physical interpretation does this spectral triple construction have?

Spaces of connections

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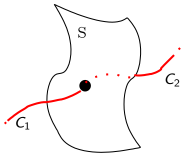
- This result holds for many different systems of ordered graphs. Fx triangulations with barycentric subdivisions.

The Poisson structure of GR

- Recall the Poisson bracket between loop and flux variables in LQG:

$$\{F_S^a(E), h_C(A)\} = \pm h_{C_1}(A)\sigma^a h_{C_2}(A)$$



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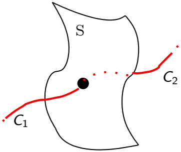

- First, for an edge ϵ_i corresponding to the $i'th$ copy of G in G^n we find

$$[d_{e_i^a}, \nabla(\epsilon_i)] = [d_{e_i^a}, g_i] = g_i\sigma^a$$

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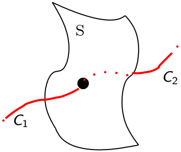

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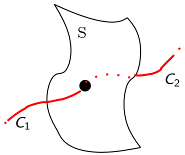

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- **In short**: the action of $d_{e_i^a}$ is to insert a Lie algebra generators at a vertex in the loop.

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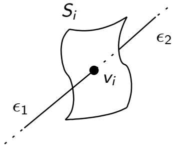
- This means that D can be interpreted as a sum of flux operators, one for each copy of G.



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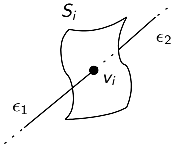

- The corresponding surfaces are 'dummy' in the sense that only the intersection points play any role in the following.

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- **Point:** the spectral triple construction captures information about the *kinematical* part of GR.

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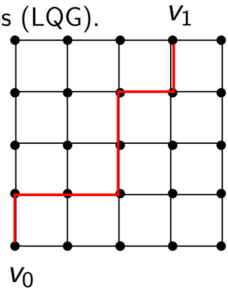

- Let L be a loop based in v_0. To shift L to a loop L' based in v_1 we need a parallel transport between v_0 and v_1

$$L' = \mathcal{U}_p(v_0, v_1) L \mathcal{U}_p^*(v_0, v_1)$$

where $p = \{\epsilon_{i_1}, \epsilon_{i_2}, \dots, \epsilon_{i_n}\}$ is a path from v_0 to v_i and $\mathcal{U}_p$ the corresponding parallel transport along p

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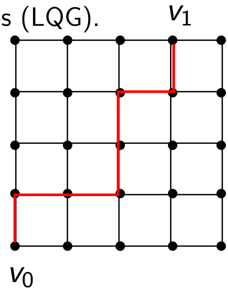

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- **Aim**: to find states which exhibit an independency on the choice of basepoint.

- Introduce the operators

$$\tilde{\mathcal{U}}_p = \tilde{\mathcal{U}}_{i_1}\tilde{\mathcal{U}}_{i_2} \cdot \ldots \cdot \tilde{\mathcal{U}}_{i_n}$$

with

$$\tilde{\mathcal{U}}_i = \frac{\mathrm{i}}{2}\left(\mathbf{e}_i^a \sigma^a + \mathrm{i}\mathbf{e}_i^1\mathbf{e}_i^2\mathbf{e}_i^3\right) g_i$$

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- These operators are unitary and mutually orthogonal

$$\langle \tilde{\mathcal{U}}_p | \tilde{\mathcal{U}}_{p'} \rangle = \begin{cases} 1 & \text{if} \quad p = p' \\ 0 & \text{if} \quad p \neq p' \end{cases}$$

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- Recall that

$$[D, \nabla(\epsilon_i)] = a_n\left(\mathbf{e}_i^a g_i \sigma^a\right)$$

  which suggest that $\tilde{\mathcal{U}}_p$ is something like an n-form.

- Consider now the object

$$\xi_k(\psi) = \frac{1}{N}\sum_i \tilde{\mathcal{U}}_{p_i}\psi(v_i)\mathcal{U}_{p_i}^{-1}$$

where $\psi(v_i)$ is an arbitrary 2x2 matrix associated to the vertex v_i (to become a spinor), and where the sum runs over vertices in $\Gamma_k \backslash \Gamma_{k-1}$.

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- These states are gauge covariant objects.

Semi-Classical States

- Pick a point (A, E) in phase-space (Ashtekar variables). Coherent states $\phi^{t,k}_{(E,A)}$ in $L^2(\mathcal{A}_{\Gamma_k})$ are given by $(t \sim l_P^2)$

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- Consider now states

$$\Psi^t_k(\psi, E, A) = \xi_k(\psi) \Phi^{t,k}_{(A,E)}$$

- This is a natural sequence of states $\{\Psi^t_k\}$ assigned to each level of subdivision of lattices .

The Dirac operator in 3 dimensions

The Dirac operator in 3 dimensions

- The expectation value of D on the states Ψ_k^t will only involve terms of the form (due to Clifford elements)

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- Important: the gravitational variables emerges from our loop/flux operators.

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- This is the principal part of the Dirac Hamiltonian in 3+1 D.

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- Note: for simplicity, we call the double limit $\lim_{k\to\infty}\lim_{t\to 0}$ for the *semi-classical limit*.

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- When we compute the expectation value of the Dirac type operator D_M on these states we obtain, in the semi-classical limit, a system of fermions coupled to the gravitational field, with an additional "interaction" (here, $n = 2$, $M = 1_2$)

$$\stackrel{\text{cl + cont.}}{\longrightarrow} \int_\Sigma dx \int_\Sigma dy \,\mathrm{Tr}\big(\mathcal{U}(y,x)\,(\nabla\psi^*_2(x))\,\psi_1(x)\mathcal{U}^{-1}(y,x)\psi^*_1(y)\psi_2(y)\big)$$

$$+ \text{'symmetric terms'}$$

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- **Question:** what interactions (local, non-local) emerge through perturbation around this flat limit?

A bosonic sector

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[work in progress]

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$$H_M = \sum_i M_i [D^2, [D^2, L_i - L_i^{-1}]]$$

where L_k, $k \in \{1,2,3\}$, are loops in a plaquet in $\Gamma_k \backslash \Gamma_{k-1}$, will descent to the Hamilton

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- GR can be recovered in a classical limit
- Key question: does the constraint algebra close (semi-classically)?

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$$D_M + H_M$$

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- Question: Why the operator $D_M + H_M$?

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- Given a spectral triple $(\mathcal{B}, \mathcal{H}, D)$ over a manifold $\mathcal{M}$ the distance formula reads

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- If they differ only on short scales, then the distance will be 'small' (difference weighted with large a's - small distance).
- The spectral triple construction is a metric structure on a configuration space of connections. This idea goes back to Feynman, Singer ...

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- A "not too complicated" operator H_M can be constructed. This leads to the pure gravity Hamiltonian in the semi-classical limit.
- The sum of the Dirac operator and H_M entails in this limit a system of interacting fermions coupled to gravity.

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- Can we take the continuum limit without the semi-classical approximation?