Shadows of trans-planckian physics on cosmology and the role of the zero-point energy density

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GM, Phys. Rev. D82, 043519 (2010)
arXiv:1005.2758 [astro-ph.CO]

GR applied to the Universe expansion: succesfull theory tested from MeV down to low redshifts ($z \approx 1$) + inflation paradigm

BBN
CMB anisotropies
Large scale inhomogeneities (Power spectrum, BAO)

Very accurate probes today (WMAP, SDSS,...) and in the near future (Planck, weak lensing,...)

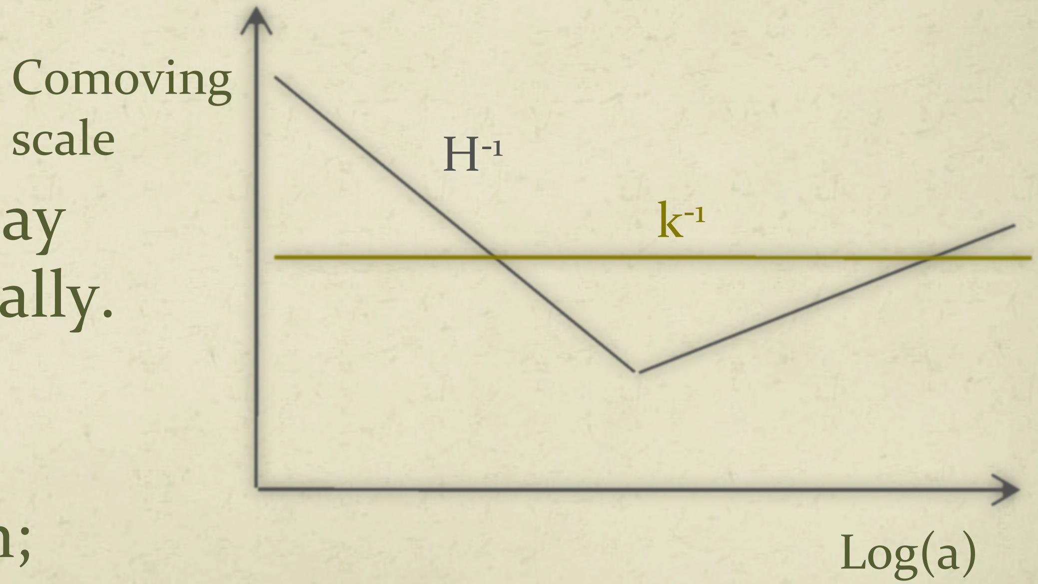
Two regimes of the theory raise questions:

- High energy: quantum gravity at Planck scale $E \approx m_{Pl}$
- Low energy: the Cosmological Constant/Dark Energy at $E \approx H_{0,}$ ($z \leq 1$)

High energy

at $E \approx m_{Pl} \approx 10^{19}$ GeV quantum gravity effects are expected to be relevant (trapped surfaces DFR): imprints on inflation?

•flatness;
•perturbation spectrum close to HZ, $n_s = 0.963 \pm 0.012$ (WMAP);
•gaussianity.



Observable large scale today maybe sub-Planckian initially.

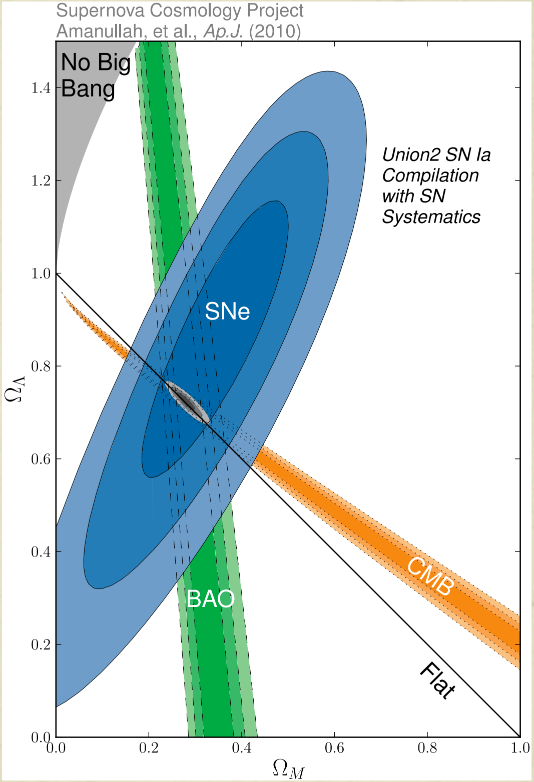
Bunch-Davis vacuum?
Mode creation mechanism;
change of dispersion relation $\omega(k)$.

Low energy

robust evidence for late acceleration of the Universe expansion (SN-Ia) at $z \leq 1$ and @ $E \approx H_0$

A new inflation?
Brans-Dicke (now popular as F(R)) ?

Cosmological constant today:
the long standing hierarchy problem



The zero – point energy density

flat spacetime: $$\sum_k \frac{\hbar\omega_k}{2}$$ regularize

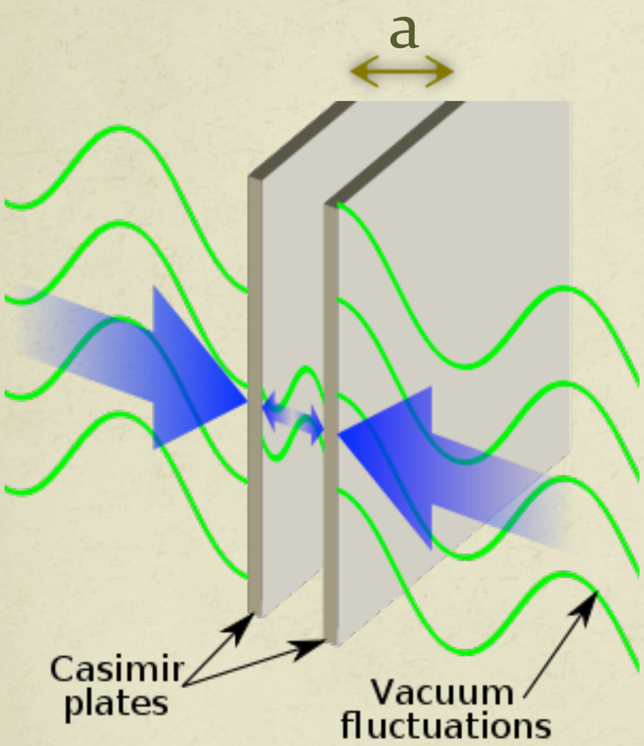
normal ordering subtraction

$\mathcal{O} \rightarrow :\mathcal{O}:$

(preferred vacuum state,
Poincare’ invariance)

curved spacetime: no preferred vacuum state (in general)
regularization and renormalization?

Effects of zero-point energy: the Casimir effect



$$\sum_k \frac{\hbar\omega_k}{2} e^{-\delta\omega_k} = \sum_k \frac{\hbar\omega_k}{2} \mu(k,\delta)$$

$E^{(\delta)}{}_M(a,\delta)$: Minkowski vacuum

$E^{(\delta)}(a,\delta)$: boundary conditions

$E^{ren}(a) = \lim_\delta \left(E^{(\delta)}(a,\delta) - E^{(\delta)}{}_M(a,\delta)\right)$

$$E^{ren}(a) = -\frac{\pi\hbar c}{24a}$$

$$F(a) = -\frac{\pi\hbar c}{24a^2}$$

Regularization schemes on curved spacetimes

General attitude: try to keep relativistic invariance

I – ***dimensional regularization*** (Akhmedov 2002)

$$\rho = \frac{1}{2}\mu^{4-d}\int \frac{d^{d-1}k}{(2\pi)^{d-1}}\sqrt{k^2+m^2} = \mu^4 \frac{\Gamma(-d/2)}{2(4\pi)^{(d-1)/2}\Gamma(-d/2)}\left(\frac{m^2}{\mu^2}\right)^{d/2} = -P$$

$$\rho \to -\frac{m^4}{64\pi^2}\left(\log\frac{\Lambda^2}{m^2}+\frac{3}{2}\right) \qquad \overline{\mathrm{MS}}$$

II – ***adiabatic renormalization*** (see e.g. Birrel & Davis 1982)

mode expansion

$$\ddot{\chi}_k + \omega^2(k)\chi_k = 0$$

WKB

$$\frac{1}{\sqrt{2W}} e^{-i\int W d\eta'}$$

$$W = \omega^2 - \frac{1}{2}\left(\frac{\ddot{W}}{W} - \frac{3}{2}\frac{\dot{W}^2}{W^2}\right), \qquad W^{(0)} = \omega$$

adiabatic parameter $\eta \to \eta/\tau, \quad \tau \to \infty$

Expand T_{ab} in inverse power of τ

$$T_{ab} = \sum_n \tau^{-n} t_{ab}$$

$$\lim_{\tau\to\infty} T_{ab} = \; :T_{ab}^{Mink}: \; = \; 0$$

On FRWL, subtraction of terms n=0 (Minkowski) & 2,4 (still divergent)

$T^{(ren)}{}_{ab} = \Sigma_{n>4}\, \tau^{-n}\, t^{n}{}_{ab}\big|_{\tau=1}$

III – ***ADM inspired renormalization*** (Maggiore 2010)

Cut-off regularization $\mu(k,\Lambda)$, e.g. $\mu(k,\Lambda) = \Theta(\Lambda\text{-}k)$ in the comoving frame

subtract terms still present in the Minkowski limit (like definition of energy (mass) as eigenvalue of Hamiltonian in ADM, or Casimir effect procedure)

- Λ seen as a "physical" scale (Planck mass) a' la Wilson;
- a slightly different adiabatic renormalization procedure; only Minkowski term subtracted
- Breaking of Lorentz invariance: broken by a preferred frame anyway (comoving frame).

$$\phi_k'' + 2\frac{a'}{a}\phi_k' + k^2\phi_k = 0$$

$$\rho = \frac{1}{2}\int \frac{d^3k}{(2\pi)^3 2k}\left(\left|\dot{\phi}_k\right|^2 + \frac{k^2}{a^2}\left|\phi_k\right|^2\right)$$

$$P = \frac{1}{2}\int \frac{d^3k}{(2\pi)^3 2k}\left(\left|\dot{\phi}_k\right|^2 - \frac{k^2}{3a^2}\left|\phi_k\right|^2\right)$$

k comoving momentum

Λ cut off in physical momentum p=k/a

Example: de Sitter phase

$$\rho = \frac{1}{4\pi^2}\int_0^{\Lambda a} dk\, k^2\left(\frac{k}{a^4}+\frac{H^2}{2a^2k}\right) = \cancel{\frac{\Lambda^4}{16\pi^2}} + \frac{H^2\Lambda^2}{16\pi^2}$$

$$P = -\rho/3$$

$$P = \frac{1}{4\pi^2}\int_0^{\Lambda a} dk\, k^2\left(\frac{k}{3a^4}-\frac{H^2}{6a^2k}\right) = \cancel{\frac{\Lambda^4}{48\pi^2}} - \frac{H^2\Lambda^2}{48\pi^2}$$

Similarly

$$\rho = \cancel{\frac{\Lambda^4}{16\pi^2}} + \frac{H^2\Lambda^2}{16\pi^2} \qquad P = \rho \qquad \text{Radiation}$$

$$\rho = \cancel{\frac{\Lambda^4}{16\pi^2}} + \frac{H^2\Lambda^2}{16\pi^2} + O\left(H^4 \log\Lambda\right) \qquad P = 2\rho/3 \qquad \text{Matter}$$

Breaking of Lorentz invariance $\qquad T_{ab} \neq \lambda\, g_{ab}$

A Paradox

Consider e.g. a de Sitter phase, H = const.

$$\rho \propto H^2 \ = \ \text{const.}$$

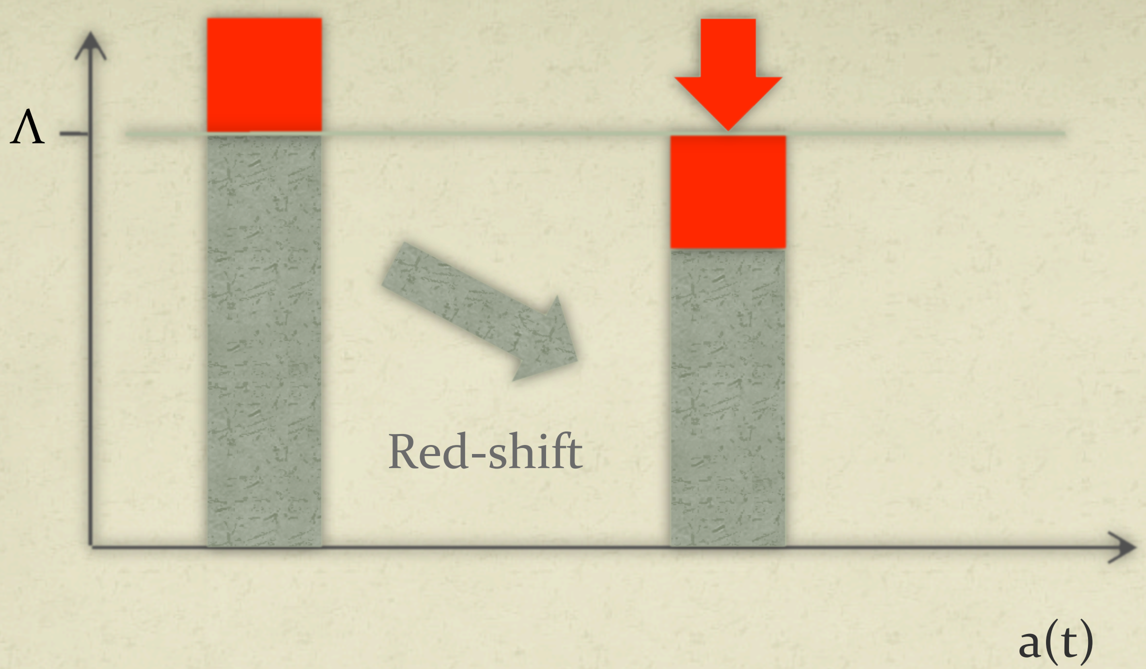
On the other hand $\nabla_a T^{ab} = 0$

$$\Rightarrow \quad \dot{\rho} + 3H(\rho + P) = \dot{\rho} + 3H\rho(1 - 1/3) = \dot{\rho} + 2H\rho = 0$$

To mantain covariant conservation of total T_{ab} and for consistency one should require that the zero-point energy/momentum is coupled with other "species"
Ad hoc ?

Maggiore 2010, Sloth 2010

But, does covariant conservation of T_{ab} hold, if we cut off our theory at Λ ?



Λ is physical momentum scale. Flow of trans-Planckian momenta at all time due to red-shift. Source term due to ***mode creation***!

G.M. 2010
Keski-Vakkuri & Sloth 2003

Simple example: the Λ^4 term

$$\frac{d}{dt}\rho = \frac{d}{dt} a^{-4} \int_0^{\Lambda a} dk\, k^3 = -4H\rho + J$$

$$J = (\Lambda a)^3 \Lambda \frac{d}{dt} a = 4H\rho$$

Source term for the (renormalized) zero-point energy/momentum tensor

$$\nabla_a T^{ab} = J^b$$

$$J^b = (J^0(t), \vec{0})$$ homogeneity and isotropy

Example: de Sitter

$$\dot{\rho} + 3H\rho(1-1/3) = \dot{\rho} + 2H\rho = J^0 = 2H\rho$$

→ ρ = const consistent with equation of state since the system is not isolated

Same conclusion for matter and radiation dominated Universes

- Same result for more sophisticated cut-off procedure with a non trivial momentum measure $\mu(k)$, e.g. $\mu(k) = \exp(-k/\Lambda)$

- Effect negligible for standard matter and radiation, characterized by a thermal distribution with temperature T, at least for $T \ll \Lambda$

$$J^0 = \frac{H}{2\pi^2}\Lambda^3 E(\Lambda)\ \exp(-E(\Lambda)/T)$$

Effective Einstein equation

Introducing the zero-point energy contribution $T_o^{(\Lambda)ab}$ due to momenta up to Λ means that effective Einstein equation should look like different

$$G^{ab} = 8\pi\, G\, T^{ab}$$

$$\nabla_a G^{ab} = 0 = \nabla_a T^{ab}$$

inconsistent for T_o^{ab} sourced by J^b.

Effective Einstein equation

$$G^{ab} + \Sigma^{(\Lambda)ab} = 8\pi\, G\, (T^{ab} + T_0^{(\Lambda)ab})$$

simplest choice $\Sigma^{(\Lambda)ab} = \sigma^{(\Lambda)}(t) g^{ab}$

$$\nabla_a \Sigma^{(\Lambda)ab} = 8\pi\, G\, \nabla_a T_0^{(\Lambda)ab} = 8\pi\, G\, J^b$$

$$\Rightarrow \quad \dot{\sigma}^{(\Lambda)} = -8\pi G J^{0}$$

defining the ***Cosmological non-Constant*** σ

Friedmann equation in the FRWL case:

$$H^{2} = \frac{8\pi G}{3}\left(\rho + \rho_{0} - 2\int \frac{da}{a}\rho_{0}\right)$$

Observational consequences?

de Sitter phase: inflation

0-order ρ_{dS}=const, $\rho_0 \approx H^2 \approx$ const

1-order

$$H^2 = \frac{8\pi G}{3}\left[\rho_{dS} + \rho_0\left(1 - 2\log\frac{a}{a_0}\right)\right]$$

or defining

$$\kappa = \frac{G\Lambda^2}{6\pi} = \frac{1}{6\pi}\frac{\Lambda^2}{m_{Pl}^2}$$

$$H^2 = \frac{8\pi G}{3}\rho_{dS}\left(1 - 2\kappa\log\frac{a}{a_0}\right)$$

logarithmically decreasing Hubble parameter

Consistent picture? For a logarithmic behaviour still $\rho_0 \approx H^2$?

Mode equation $$\phi_k'' + 2\frac{a'}{a}\phi_k' + k^2\phi_k = 0$$

$$a(\eta) = -\frac{1+\kappa}{\eta H_0}\left(\frac{\eta}{\eta_0}\right)^{-\kappa}$$

for a Bunch-Davis vacuum

$$\phi_k(\eta) = i\sqrt{\frac{\pi}{2}}\sqrt{k\eta}\left[i\, J_{3/2+\kappa}(k\eta) + Y_{3/2+\kappa}(k\eta)\right]\frac{1}{a(\eta)}$$

$\Rightarrow$ $\rho = (\Lambda\, H(\eta))^2 / 16\pi^2 + O(\kappa^2)$

Evolution of H leaves its imprint on the amplitude of scalar perturbations @ horizon crossing, k=aH:

$P(k)\, k^3 = P(k_o)\, k_o^3\, (1\text{-}2\, \kappa \log k/k_o)$

$$\mathbf{n_s = 1\text{-}2\, \kappa = 1\text{-}2\, (\Lambda/m_{Pl})^2/(6\pi)}$$

for $\Lambda = m_{Pl}$ $\quad$ $\mathbf{n_s \approx 0.9}$

or viceversa using WMAP result $n_s = 0.963 \pm 0.012$ (68 C.L.)

$$\mathbf{\Lambda/m_{Pl} = 0.5 - 0.7}$$

"running" spectral index: $dn_s/d\log(k)$ order κ^2
(WMAP observations fully consistent with a constant n_s)

Late stages: zero-point energy as dark energy?

Friedmann equation during radiation and matter domination

$$(1-\kappa)H^2 = \frac{8\pi G}{3}\rho_{R,M} - 2\kappa\int\frac{da}{a}H^2$$

$$H^2 = \frac{8\pi G}{3}\left(\frac{\rho_R}{1-3\kappa/2} + \mathscr{A}\, a^{-2\kappa/(1-\kappa)}\right)$$

$$H^2 = \frac{8\pi G}{3}\left(\frac{\rho_M}{1-5\kappa/3} + \mathscr{A}\, a^{-2\kappa/(1-\kappa)}\right)$$

the extra term corresponds to a fluid with effective equation of state

$$w \approx -1 + \frac{2\kappa}{3} = -1 + \frac{1}{9\pi}\frac{\Lambda^2}{m_{Pl}^2}$$

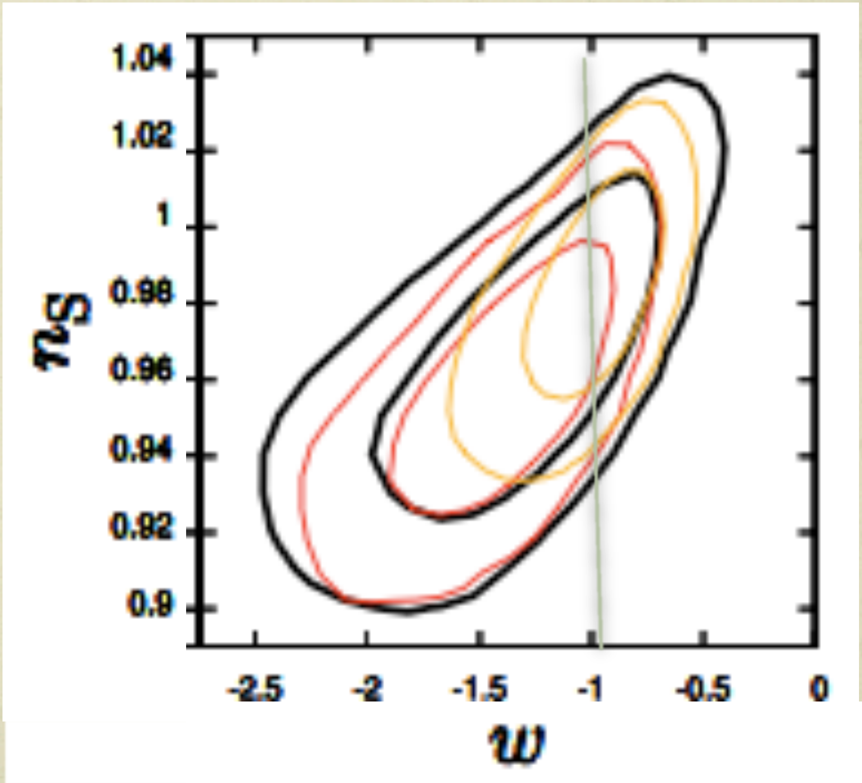
No solution (till now) of the coincidence problem:
$\mathscr{A}$ should be tuned to provide the correct ratio of $\Omega_M / \Omega_\Lambda$ today

Using $\Lambda/m_{Pl} = 0.5 - 0.7$ (from WMAP)

$$w_{DE} = -0.96$$

A new relation:

$$w_{DE} = -(2+n_s)/3$$



Hamann et al 2010

Conclusions I

Puzzles:

- high and low energy regimes of cosmological models (and General Relativity)
- Vacuum energy and hierarchy

Regularization and renormalization on curved spacetime tricky. Ultimately, answers in (hopefully) future experimental tests (as Casimir effect for Minkowski)

Conclusions II

In a specific (reasonable) scheme, there is an intriguing emerging role of the zero-point energy density

• grasping something about the ultraviolet behaviour of gravity

$$\mathbf{n_s = 1 - 2\,(\Lambda/m_{Pl})^2/(6\pi) \approx 0.964}$$

• describing a possible candidate for Dark Energy

$$\mathbf{w_{DE} = 1 - (\Lambda/m_{Pl})^2/(9\pi) \approx -\,0.96}$$

$$\mathbf{w_{DE} = -(2+n_s)/3}$$