Università degli Studi di Napoli “Federico II”
Dipartimento di Scienze Fisiche
14 maggio 2010

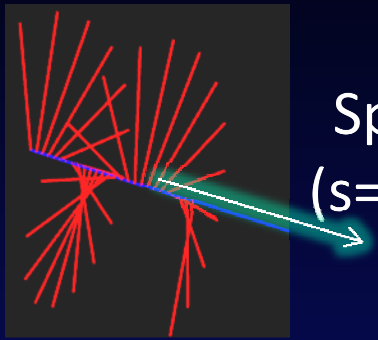
Quantum optics with photon orbital angular momentum

Enrico Santamato

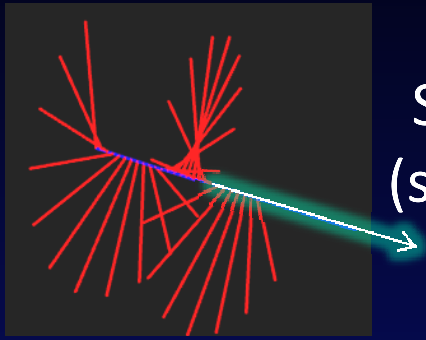
SAM and OAM

- SAM

Spin angular momentum may take two values of s=–1 and s=+1.



Spin (s=+1)

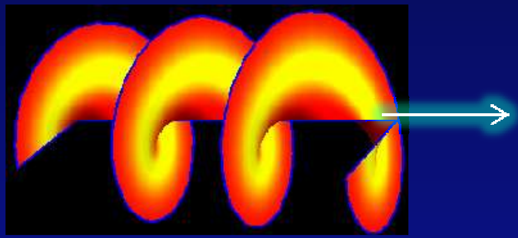


Spin (s=–1)

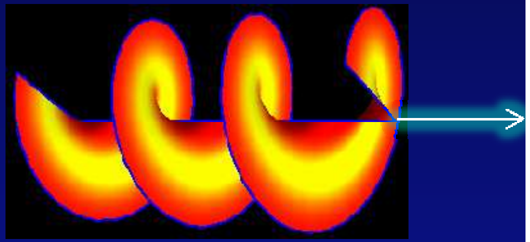
- OAM

Orbital angular momentum may take any of the infinite values *m* = 0, ±1, ±2,…

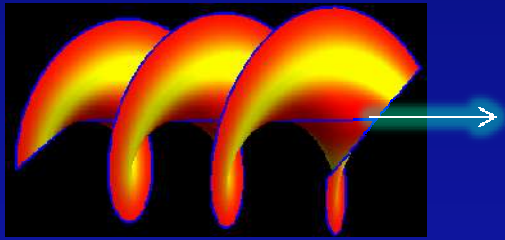
OAM, *m*=+1



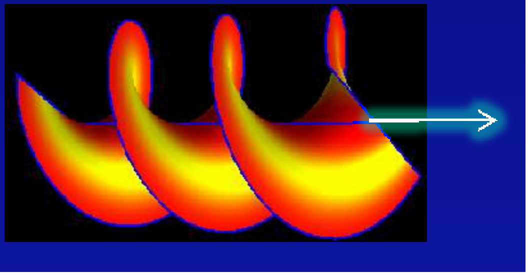
OAM, *m*=–1



OAM, *m*=+2



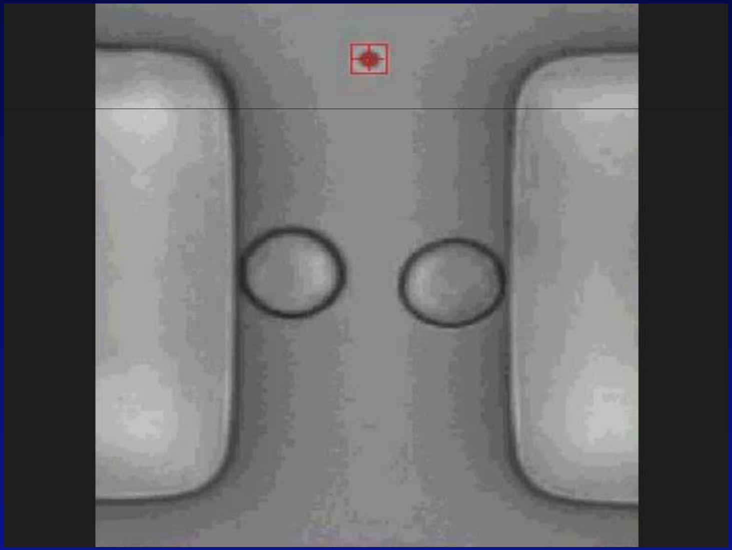
OAM, *m*=–2

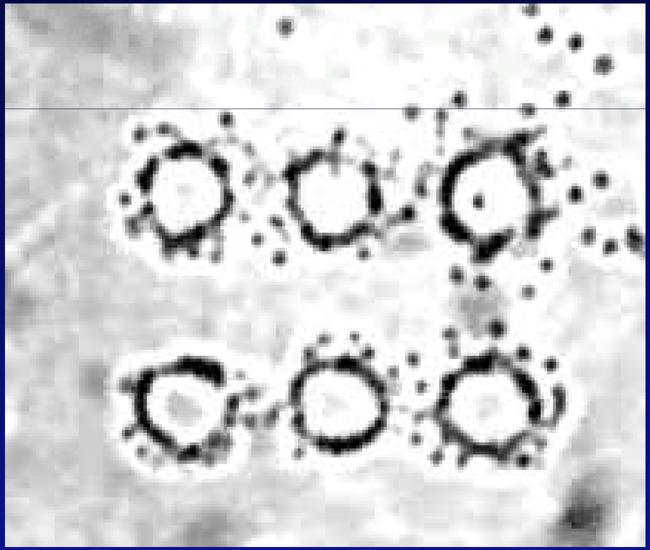


Optical micro-pumps

SAM

OAM



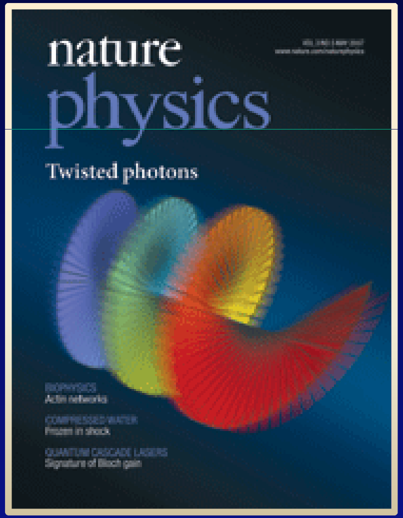


Photons can carry Spin Angular Momentum (SAM) and Orbital Angular Momentum (OAM)

nature physics | VOL 3 | MAY 2007 |

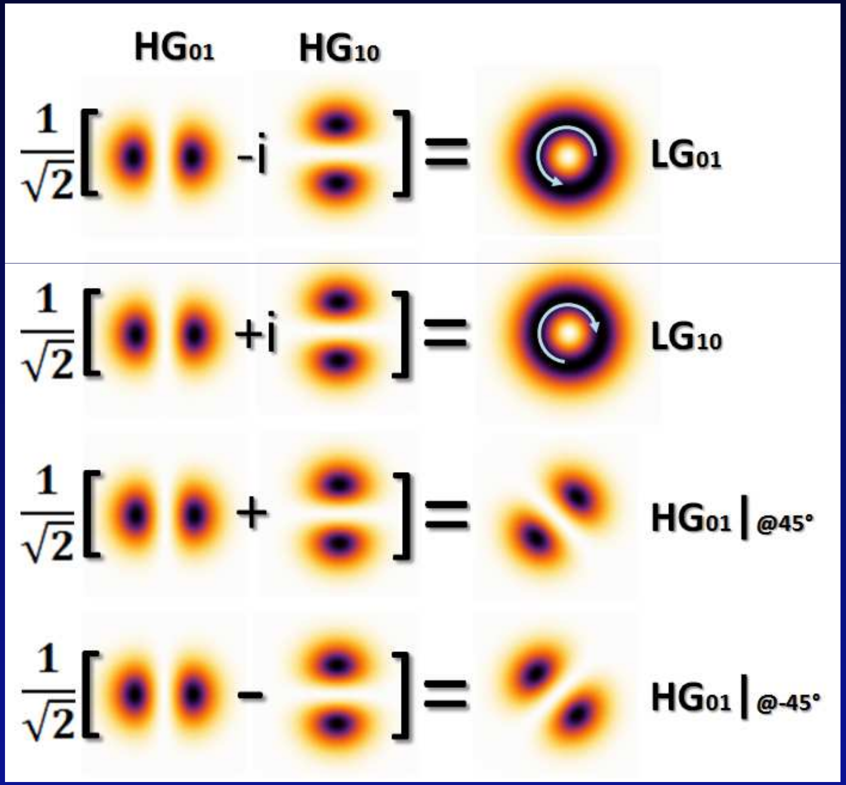
GABRIEL MOLINA-TERRIZA[1,2], JUAN P. TORRES[1,3*]
AND LLUIS TORNER[1,3]

Twisted photons



Photon OAM eigenstates

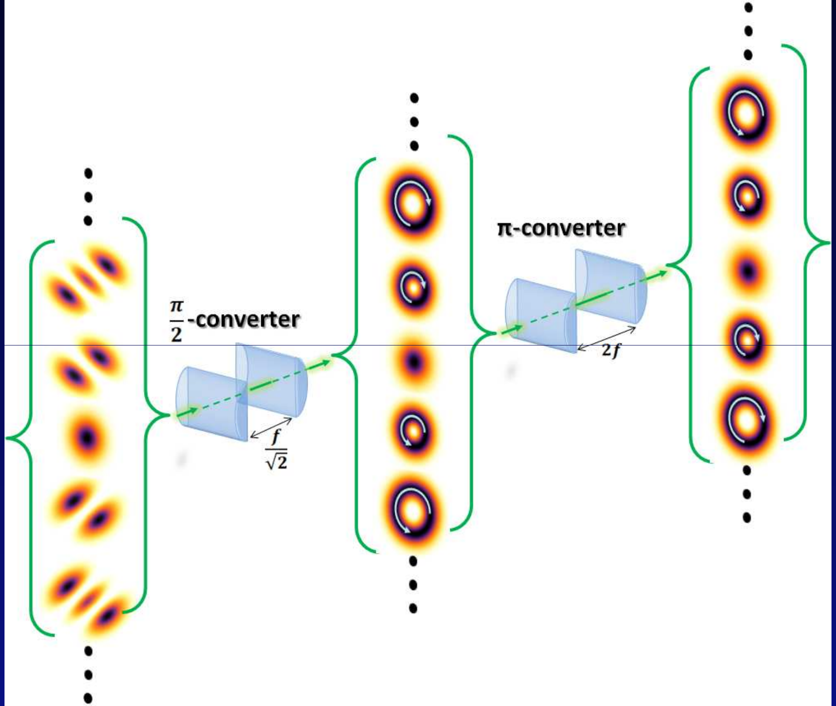
$E \propto e^{im\varphi} \quad (m = 0, \pm 1, \pm 2, \ldots)$



Complete set

1. HG modes
2. LG modes

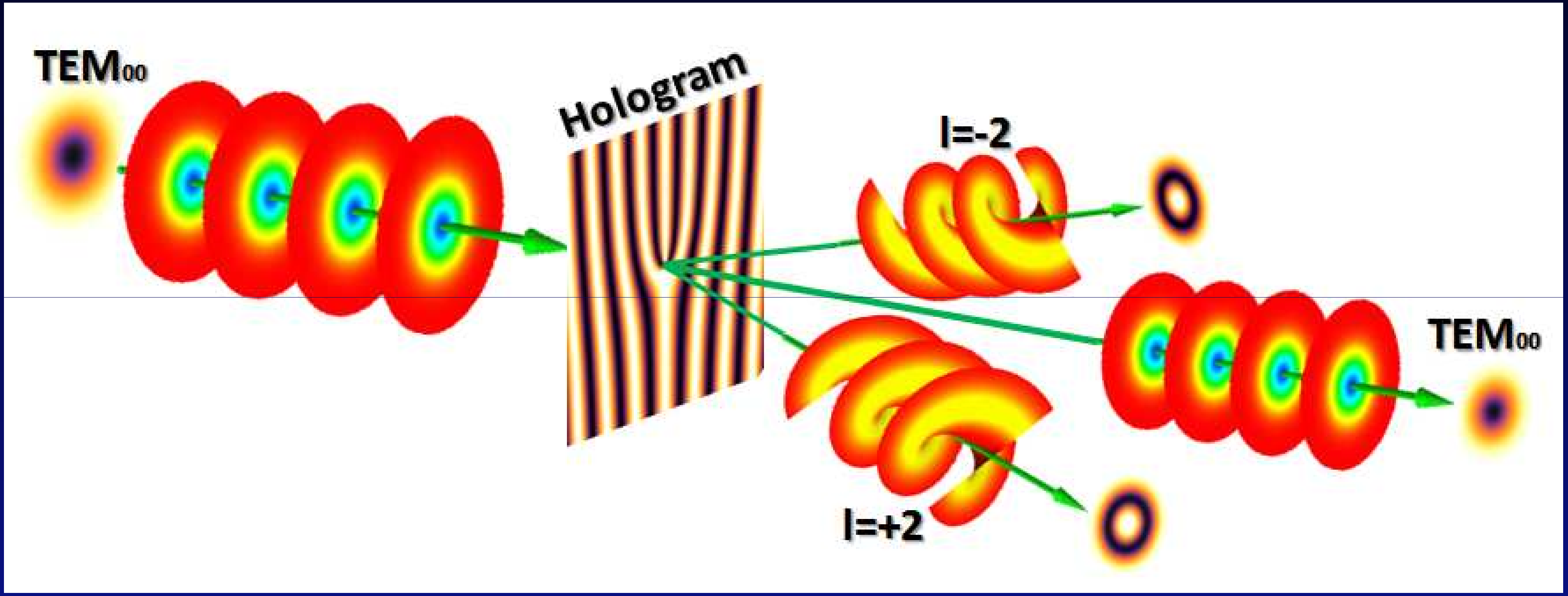
Mode converters



To obtain LG modes higher-order HG modes are required

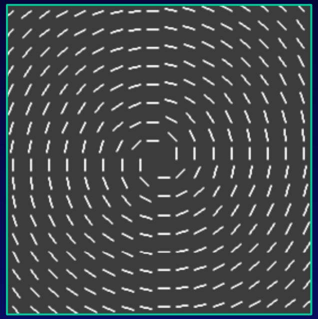
Impossible to convert TEM$_{00}$ mode into LG modes

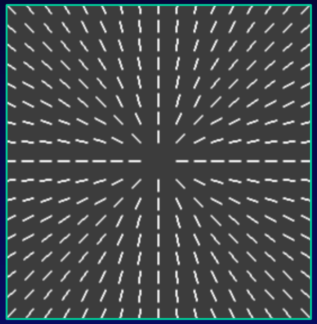
Pitch-fork hologram

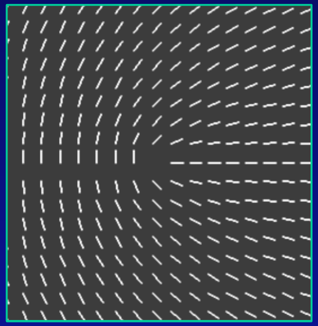


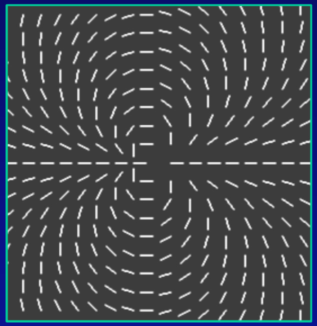
The “q-plate”

The q-Plate is a liquid crystal-based birefringent optical device which imprints a topological <u>singularity</u> into the phase of the optical field.

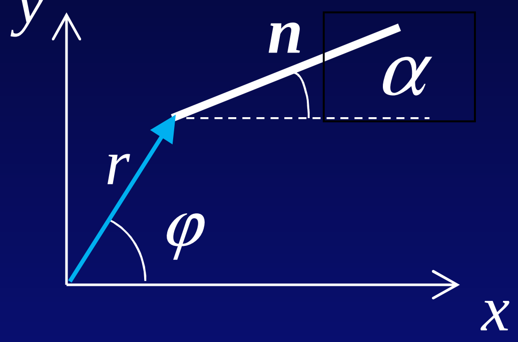

$\alpha = \varphi + \pi/2$

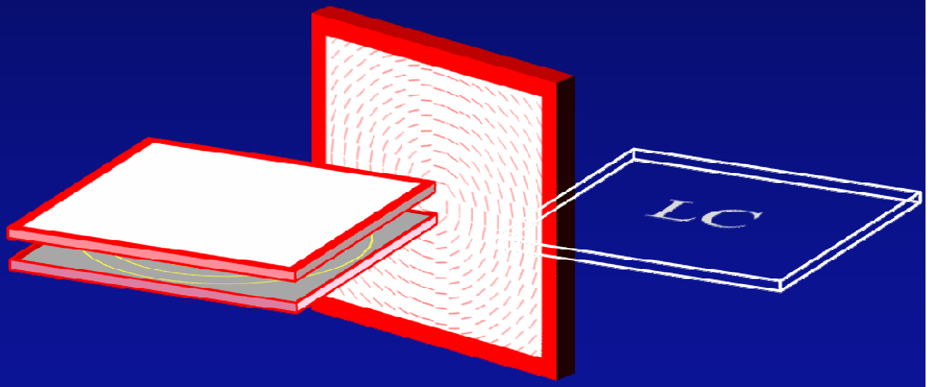

$\alpha = \varphi$


$\alpha = \varphi/2$


$\alpha = 2\varphi$

$$\boxed{\alpha = q\varphi + \alpha_0}$$





L. Marrucci et al., PRL **96**, 163905 (2006)

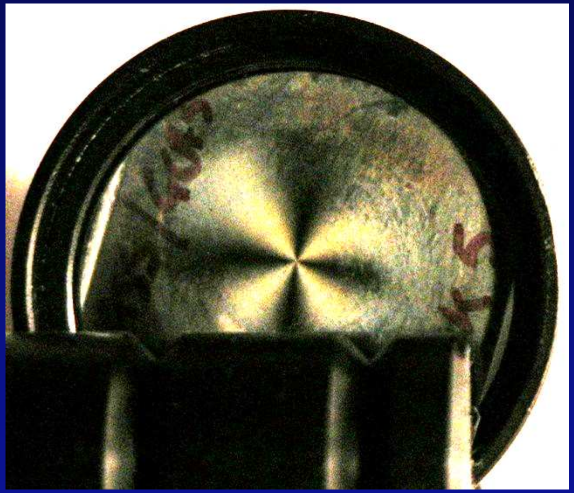
Q-plate fabrication

1. Surface rubbing
2. UV surface orienting
3. UV bulk polymer orienting

We used this

and this

We made only QP with topological charge $q = 1$



The q-plate

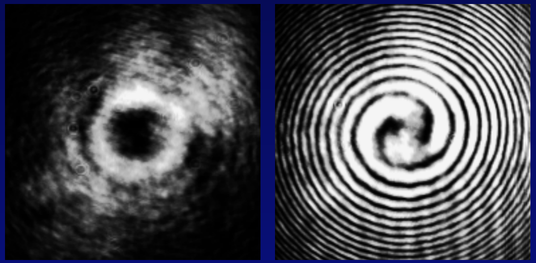
$$|L,0\rangle \xrightarrow{QP} \cos\frac{\delta}{2}|L,0\rangle - i\sin\frac{\delta}{2}|R,2\rangle$$

$$|R,0\rangle \xrightarrow{QP} \cos\frac{\delta}{2}|R,0\rangle - i\sin\frac{\delta}{2}|L,-2\rangle$$

δ = optical retardation

$\delta = \pi$ full STOC conversion

q-plate tuning



Optical vortex

Total light angular momentum is conserved

The q-plate transfers the photon SAM state into the OAM and *viceversa*

Any manipulation of SAM is directly inscribed into the OAM

A qubit can be transferred from SAM to OAM (STOC process)

L. Marrucci et al., Phys. Rev. Lett., **96**, 163905 (2006)

Quantum optics with OAM

Università
di Napoli
Federico II

B. Piccirillo, E. Karimi, L. Marrucci,
E. Santamato

SAPIENZA
UNIVERSITÀ DI ROMA

E. Nagali, F. Sciarrino, L. Sansoni,
F. De Martini

q-plate action on single photons

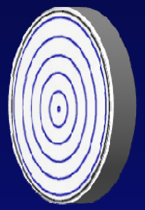
- Generation of photon spinorbit states
- Generation of photon spinorbit entangled Bell’s states
- Generation of photon spinorbit ququarts

q-plate action on single photons

Elliptically polarized
TEM$_{00}$ mode

Entangled spinorbit state

$$|\psi\rangle = (\alpha|L\rangle_\pi + \beta|R\rangle_\pi)|0\rangle_o \quad \Rightarrow$$



$$\Rightarrow \quad \alpha|R\rangle_\pi|+2\rangle_o + \beta|L\rangle_\pi|-2\rangle_o$$

$\alpha = 0$ pure spinorbit state $|L, -2\rangle$

$\beta = 0$ pure spinorbit state $|R, +2\rangle$

Notations

SAM ($s = \pm 1$)

$$|H\rangle = \tfrac{1}{\sqrt{2}}\left(|L\rangle + |R\rangle\right)$$

$$|V\rangle = \tfrac{1}{i\sqrt{2}}\left(|L\rangle - |R\rangle\right)$$

$$|A\rangle = \tfrac{1}{\sqrt{2}}\left(|H\rangle + |V\rangle\right)$$

$$|D\rangle = \tfrac{1}{\sqrt{2}}\left(|H\rangle - |V\rangle\right)$$

OAM ($m = \pm 2$)

$$|h\rangle = \tfrac{1}{\sqrt{2}}\left(|2\rangle + |-2\rangle\right)$$

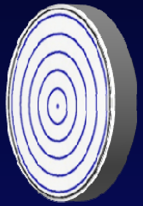
$$|v\rangle = \tfrac{1}{i\sqrt{2}}\left(|2\rangle - |-2\rangle\right)$$

$$|a\rangle = \tfrac{1}{\sqrt{2}}\left(|h\rangle + |v\rangle\right)$$

$$|d\rangle = \tfrac{1}{\sqrt{2}}\left(|h\rangle - |v\rangle\right)$$

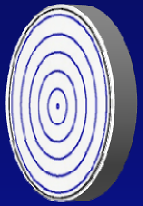
q-plate action on single photons

Linearly polarized TEM$_{00}$ input (H or V)



$$|H\rangle_\pi |0\rangle \rightarrow \quad \rightarrow \frac{1}{i\sqrt{2}}\left(|L,-2\rangle + |R,2\rangle\right) = \frac{1}{i\sqrt{2}}\left(|H,h\rangle + |V,v\rangle\right)$$

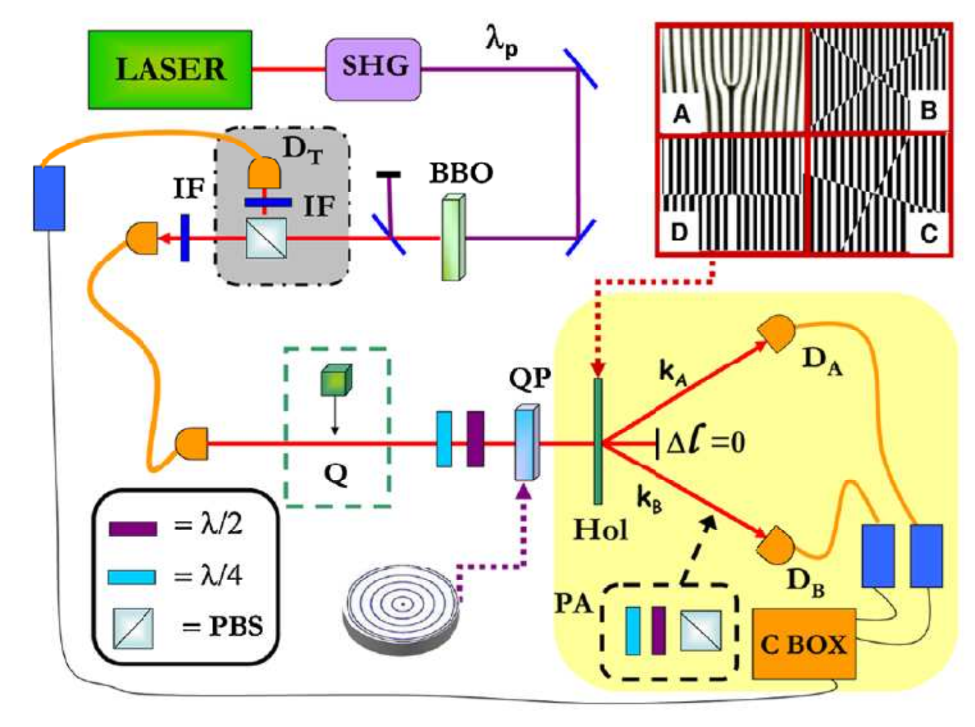
Maximally entangled
Bell’s spinorbit states



$$|V\rangle_\pi |0\rangle \rightarrow \quad \rightarrow \frac{1}{\sqrt{2}}\left(|L,-2\rangle - |R,2\rangle\right) = \frac{1}{i\sqrt{2}}\left(|H,v\rangle + |V,h\rangle\right)$$

q-plate action on single photons

Experiment with single heralded photons



E. Nagali,, et al., Optics Express, **17**, 18745 (2009)

q-plate quantum action on single photons

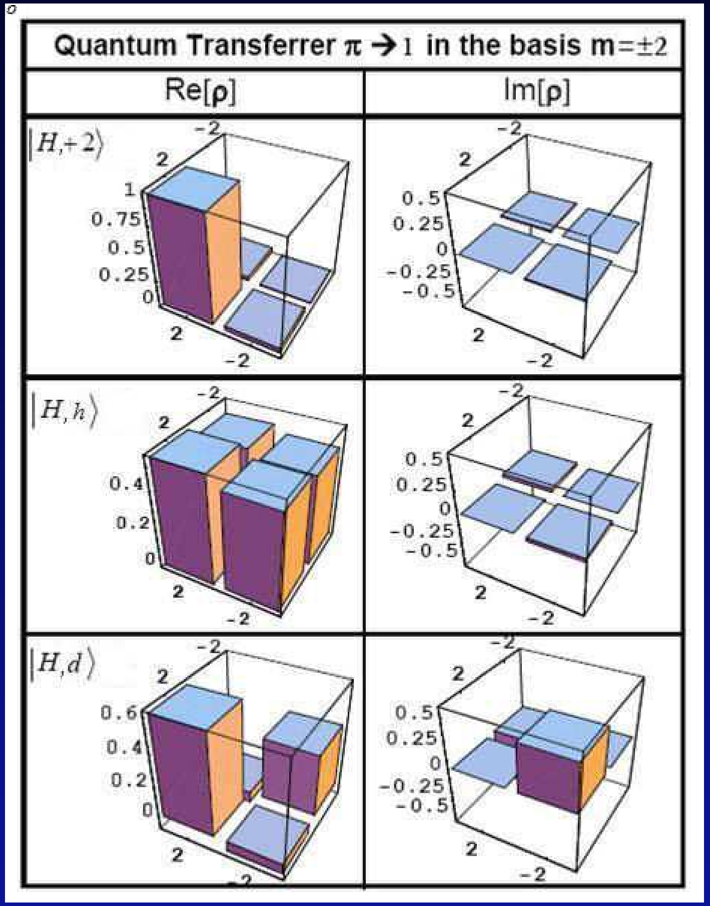
Typical quantum tomography results (SAM→OAM)

Typical experimental fidelities = 98%!

$$\rho_{\mathrm{OAM}} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

$$\rho_{\mathrm{OAM}} = \frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$

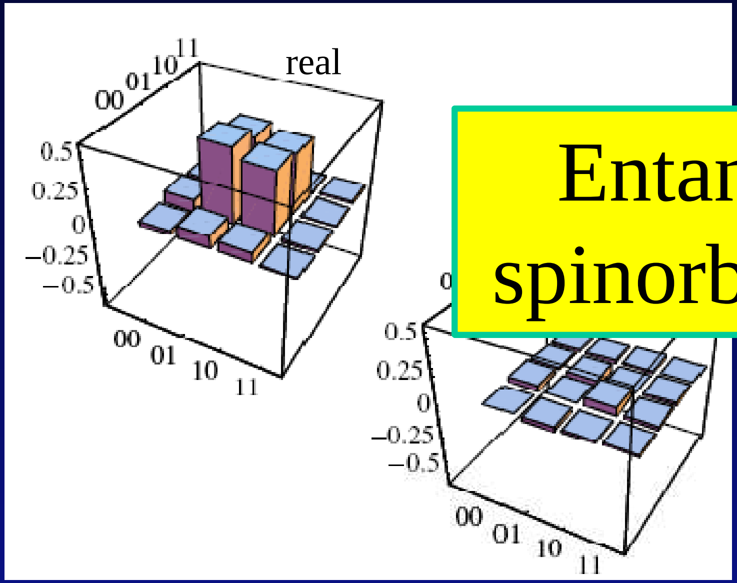
$$\rho_{\mathrm{OAM}} = \frac{1}{2}\begin{pmatrix} 1 & -i \\ i & 1 \end{pmatrix}$$

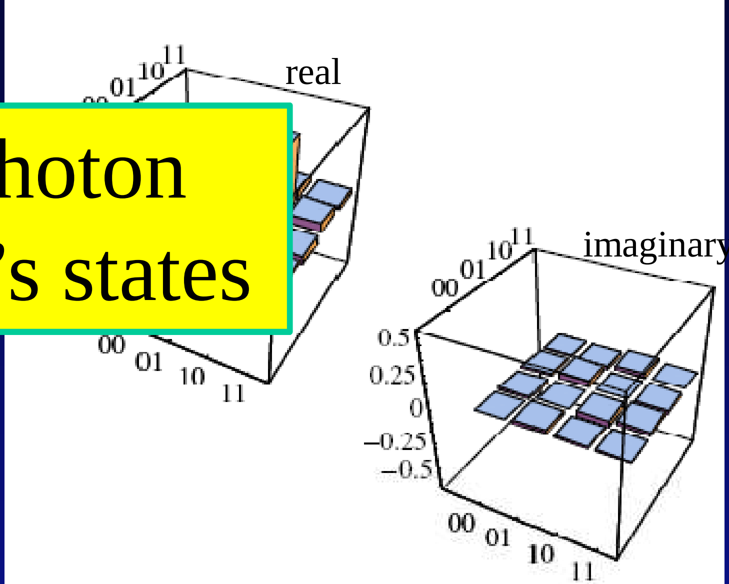


q-plate quantum action on single photons

Input *H* photons

Input *V* photons





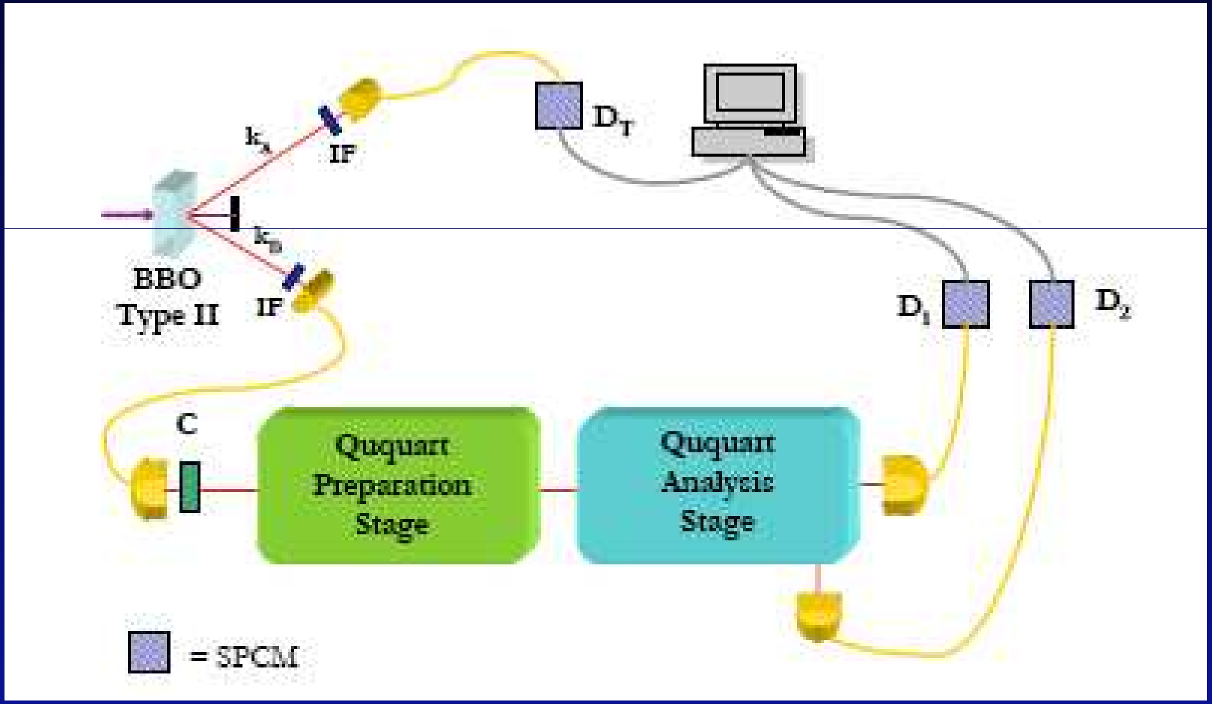
Entangled photon spinorbit Bell's states

$$|\psi\rangle = \tfrac{1}{\sqrt{2}}\left(|R,2\rangle + |L,-2\rangle\right)$$

$$|\psi\rangle = \tfrac{1}{\sqrt{2}}\left(|R,2\rangle - |L,-2\rangle\right)$$

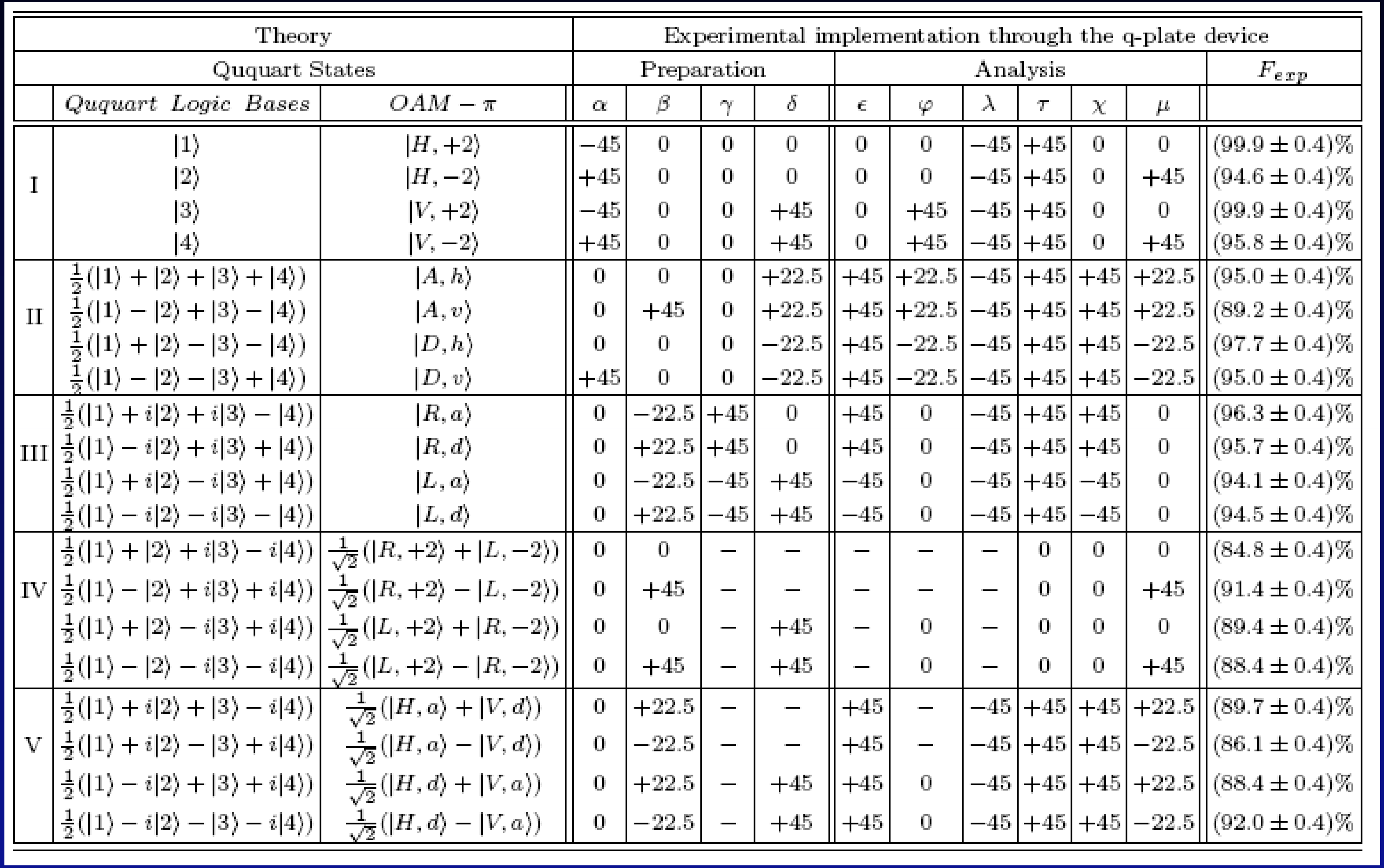
q-plate quantum action on single photons

Generation of qu-quart spinorbit states



E. Nagali et al., Phys. Rev. A, (2010), accepted

Experimental results



Theory			Experimental implementation through the q-plate device										
Ququart States			Preparation				Analysis						F_{exp}
	Ququart Logic Bases	$OAM-\pi$	α	β	γ	δ	ϵ	φ	λ	τ	χ	μ	
I	$\|1\rangle$	$\|H,+2\rangle$	−45	0	0	0	0	0	−45	+45	0	0	$(99.9\pm0.4)\%$
	$\|2\rangle$	$\|H,-2\rangle$	+45	0	0	0	0	0	−45	+45	0	+45	$(94.6\pm0.4)\%$
	$\|3\rangle$	$\|V,+2\rangle$	−45	0	0	+45	0	+45	−45	+45	0	0	$(99.9\pm0.4)\%$
	$\|4\rangle$	$\|V,-2\rangle$	+45	0	0	+45	0	+45	−45	+45	0	+45	$(95.8\pm0.4)\%$
II	$\frac{1}{2}(\|1\rangle+\|2\rangle+\|3\rangle+\|4\rangle)$	$\|A,h\rangle$	0	0	0	+22.5	+45	+22.5	−45	+45	+45	+22.5	$(95.0\pm0.4)\%$
	$\frac{1}{2}(\|1\rangle-\|2\rangle+\|3\rangle-\|4\rangle)$	$\|A,v\rangle$	0	+45	0	+22.5	+45	+22.5	−45	+45	+45	+22.5	$(89.2\pm0.4)\%$
	$\frac{1}{2}(\|1\rangle+\|2\rangle-\|3\rangle-\|4\rangle)$	$\|D,h\rangle$	0	0	0	−22.5	+45	−22.5	−45	+45	+45	−22.5	$(97.7\pm0.4)\%$
	$\frac{1}{2}(\|1\rangle-\|2\rangle-\|3\rangle+\|4\rangle)$	$\|D,v\rangle$	+45	0	0	−22.5	+45	−22.5	−45	+45	+45	−22.5	$(95.0\pm0.4)\%$
III	$\frac{1}{2}(\|1\rangle+i\|2\rangle+i\|3\rangle-\|4\rangle)$	$\|R,a\rangle$	0	−22.5	+45	0	+45	0	−45	+45	+45	0	$(96.3\pm0.4)\%$
	$\frac{1}{2}(\|1\rangle-i\|2\rangle+i\|3\rangle+\|4\rangle)$	$\|R,d\rangle$	0	+22.5	+45	0	+45	0	−45	+45	+45	0	$(95.7\pm0.4)\%$
	$\frac{1}{2}(\|1\rangle+i\|2\rangle-i\|3\rangle+\|4\rangle)$	$\|L,a\rangle$	0	−22.5	−45	+45	−45	0	−45	+45	−45	0	$(94.1\pm0.4)\%$
	$\frac{1}{2}(\|1\rangle-i\|2\rangle-i\|3\rangle-\|4\rangle)$	$\|L,d\rangle$	0	+22.5	−45	+45	−45	0	−45	+45	−45	0	$(94.5\pm0.4)\%$
IV	$\frac{1}{2}(\|1\rangle+\|2\rangle+i\|3\rangle-i\|4\rangle)$	$\frac{1}{\sqrt{2}}(\|R,+2\rangle+\|L,-2\rangle)$	0	0	−	−	−	−	−	0	0	0	$(84.8\pm0.4)\%$
	$\frac{1}{2}(\|1\rangle-\|2\rangle+i\|3\rangle+i\|4\rangle)$	$\frac{1}{\sqrt{2}}(\|R,+2\rangle-\|L,-2\rangle)$	0	+45	−	−	−	−	−	0	0	+45	$(91.4\pm0.4)\%$
	$\frac{1}{2}(\|1\rangle+\|2\rangle-i\|3\rangle+i\|4\rangle)$	$\frac{1}{\sqrt{2}}(\|L,+2\rangle+\|R,-2\rangle)$	0	0	−	+45	−	0	−	0	0	0	$(89.4\pm0.4)\%$
	$\frac{1}{2}(\|1\rangle-\|2\rangle-i\|3\rangle-i\|4\rangle)$	$\frac{1}{\sqrt{2}}(\|L,+2\rangle-\|R,-2\rangle)$	0	+45	−	+45	−	0	−	0	0	+45	$(88.4\pm0.4)\%$
V	$\frac{1}{2}(\|1\rangle+i\|2\rangle+\|3\rangle-i\|4\rangle)$	$\frac{1}{\sqrt{2}}(\|H,a\rangle+\|V,d\rangle)$	0	+22.5	−	−	+45	−	−45	+45	+45	+22.5	$(89.7\pm0.4)\%$
	$\frac{1}{2}(\|1\rangle+i\|2\rangle-\|3\rangle+i\|4\rangle)$	$\frac{1}{\sqrt{2}}(\|H,a\rangle-\|V,d\rangle)$	0	−22.5	−	−	+45	−	−45	+45	+45	−22.5	$(86.1\pm0.4)\%$
	$\frac{1}{2}(\|1\rangle-i\|2\rangle+\|3\rangle+i\|4\rangle)$	$\frac{1}{\sqrt{2}}(\|H,d\rangle+\|V,a\rangle)$	0	+22.5	−	+45	+45	0	−45	+45	+45	+22.5	$(88.4\pm0.4)\%$
	$\frac{1}{2}(\|1\rangle-i\|2\rangle-\|3\rangle-i\|4\rangle)$	$\frac{1}{\sqrt{2}}(\|H,d\rangle-\|V,a\rangle)$	0	−22.5	−	+45	+45	0	−45	+45	+45	−22.5	$(92.0\pm0.4)\%$

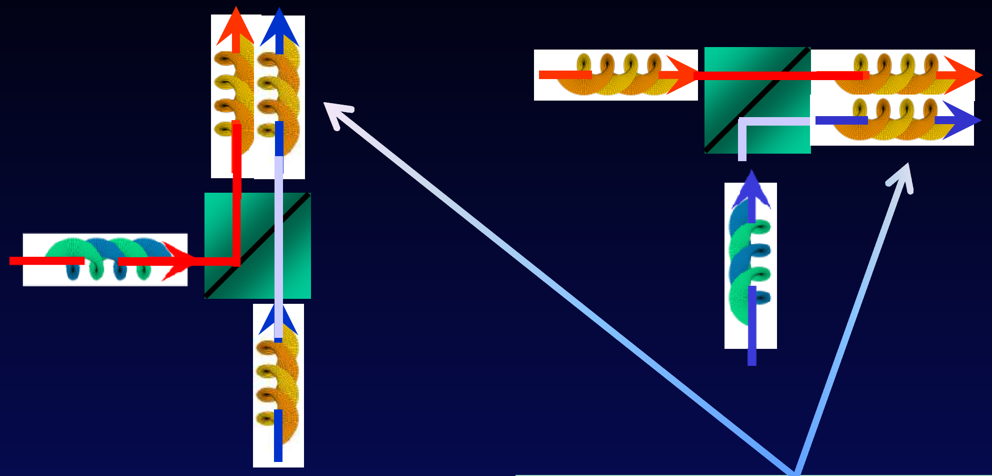
q-plate action on photon pair

- Photon-photon coalescence (Hong-Ou-Mandel effect) in the OAM space
- Optimal cloning of OAM photon states

Photon-photon coalescence in OAM

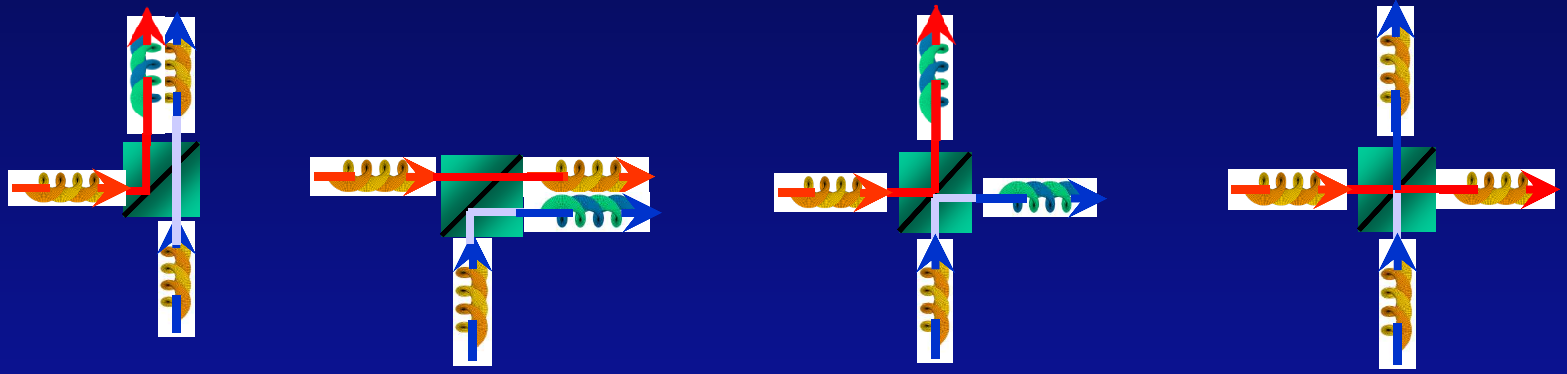
Different input OAM

Notice:
the optical reflection
inverts the OAM sign



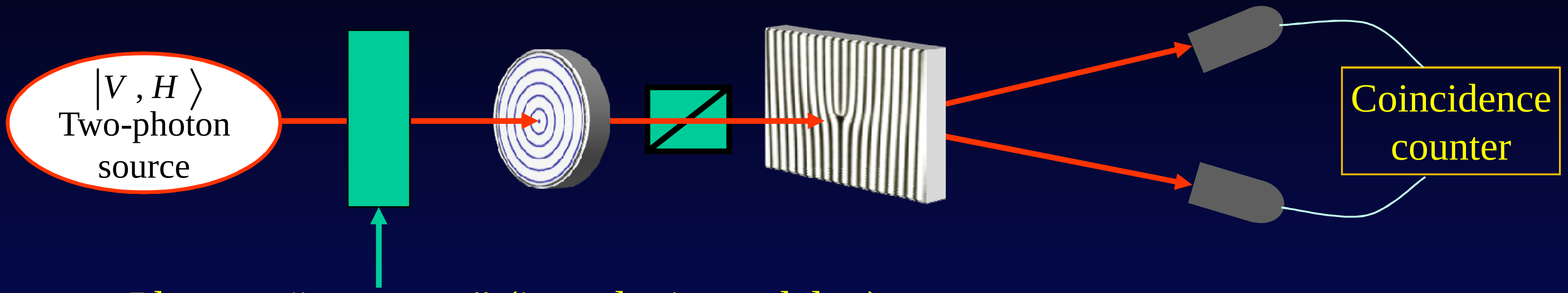
Coalescence!

Equal input OAM



Different final states, no coalescence!

Photon-photon coalescence in OAM



Photons “separator” (introducing a delay)

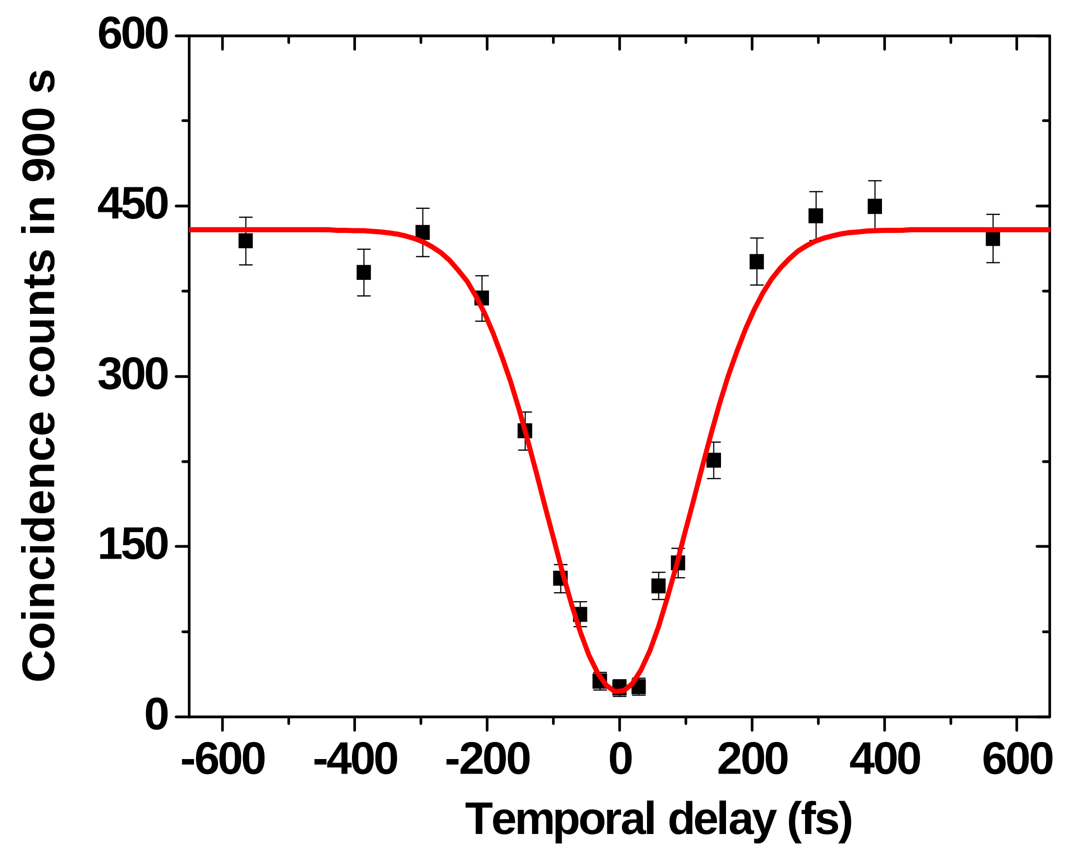
E. Nagali,, et al., Phys. Rev. Lett., **103**, 013601 (2009)

Obtained a 2-photon state with OAM quantum entanglement!

$$|\psi\rangle = \frac{1}{i\sqrt{2}}\left(|+2\rangle|+2\rangle - |-2\rangle|-2\rangle\right)$$

No coincidences when the photons are identical

Our experimental results:

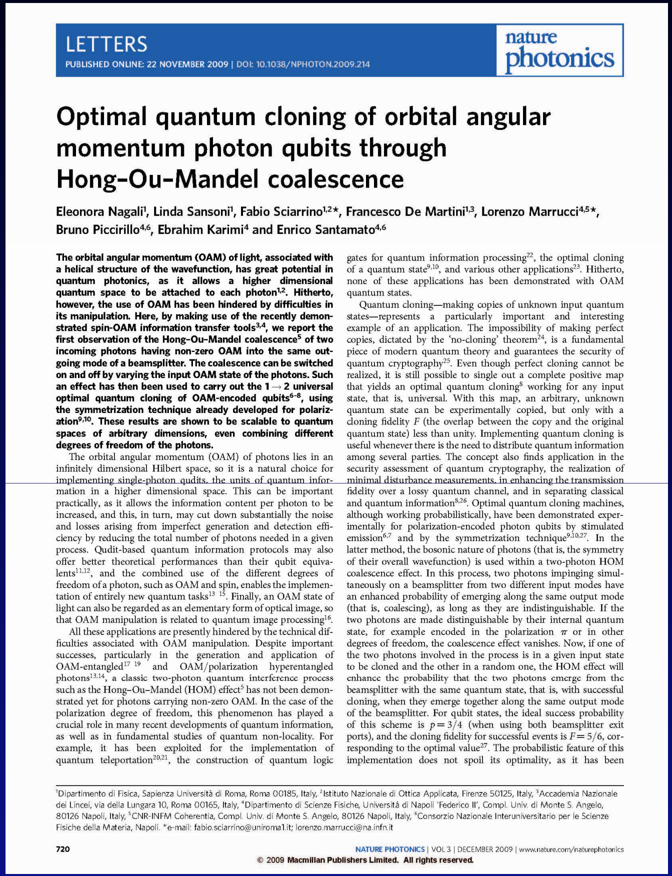


Optimal quantum cloning of OAM state

Quantum no-cloning theorem prevents a **deterministic** cloning of photon state.

Yet, not deterministic quantum cloning is possible.

The quantum cloning is optimal when the success probability is maximized.



LETTERS
PUBLISHED ONLINE: 22 NOVEMBER 2009 | DOI: 10.1038/NPHOTON.2009.214
nature photonics

Optimal quantum cloning of orbital angular momentum photon qubits through Hong–Ou–Mandel coalescence

Eleonora Nagali[1], Linda Sansoni[1], Fabio Sciarrino[1,2]*, Francesco De Martini[1,3], Lorenzo Marrucci[4,5]*, Bruno Piccirillo[4,6], Ebrahim Karimi[4] and Enrico Santamato[4,6]

The orbital angular momentum (OAM) of light, associated with a helical structure of the wavefunction, has great potential in quantum photonics, as it allows a higher dimensional quantum space to be attached to each photon[1,2]. Hitherto, however, the use of OAM has been hindered by difficulties in its manipulation. Here, by making use of the recently demonstrated spin-OAM information transfer tools[3,4], we report the first observation of the Hong–Ou–Mandel coalescence[5] of two incoming photons having non-zero OAM into the same outgoing mode of a beamsplitter. The coalescence can be switched on and off by varying the input OAM state of the photons. Such an effect has then been used to carry out the 1 → 2 universal optimal quantum cloning of OAM-encoded qubits[6–8], using the symmetrization technique already developed for polarization[9,10]. These results are shown to be scalable to quantum spaces of arbitrary dimensions, even combining different degrees of freedom of the photons.

The orbital angular momentum (OAM) of photons lies in an infinitely dimensional Hilbert space, so it is a natural choice for implementing single-photon qudits, the units of quantum information in a higher dimensional space. This can be important practically, as it allows the information content per photon to be increased, and this, in turn, may cut down substantially the noise and losses arising from imperfect generation and detection efficiency by reducing the total number of photons needed in a given process. Qudit-based quantum information protocols may also offer better theoretical performances than their qubit equivalents[11,12], and the combined use of the different degrees of freedom of a photon, such as OAM and spin, enables the implementation of entirely new quantum tasks[13–15]. Finally, an OAM state of light can also be regarded as an elementary form of optical image, so that OAM manipulation is related to quantum image processing[16].

All these applications are presently hindered by the technical difficulties associated with OAM manipulation. Despite important successes, particularly in the generation and application of OAM-entangled[17–19] and OAM/polarization hyperentangled photons[13,14], a classic two-photon quantum interference process such as the Hong–Ou–Mandel (HOM) effect[5] has not been demonstrated yet for photons carrying non-zero OAM. In the case of the polarization degree of freedom, this phenomenon has played a crucial role in many recent developments of quantum information, as well as in fundamental studies of quantum non-locality. For example, it has been exploited for the implementation of quantum teleportation[20,21], the construction of quantum logic gates for quantum information processing[22], the optimal cloning of a quantum state[9,10], and various other applications[23]. Hitherto, none of these applications has been demonstrated with OAM quantum states.

Quantum cloning—making copies of unknown input quantum states—represents a particularly important and interesting example of an application. The impossibility of making perfect copies, dictated by the 'no-cloning' theorem[24], is a fundamental piece of modern quantum theory and guarantees the security of quantum cryptography[25]. Even though perfect cloning cannot be realized, it is still possible to single out a complete positive map that yields an optimal quantum cloning[8] working for any input state, that is, universal. With this map, an arbitrary, unknown quantum state can be experimentally copied, but only with a cloning fidelity F (the overlap between the copy and the original quantum state) less than unity. Implementing quantum cloning is useful whenever there is the need to distribute quantum information among several parties. The concept also finds application in the security assessment of quantum cryptography, the realization of minimal disturbance measurements, in enhancing the transmission fidelity over a lossy quantum channel, and in separating classical and quantum information[8,26]. Optimal quantum cloning machines, although working probabilistically, have been demonstrated experimentally for polarization-encoded photon qubits by stimulated emission[6,7] and by the symmetrization technique[9,10,27]. In the latter method, the bosonic nature of photons (that is, the symmetry of their overall wavefunction) is used within a two-photon HOM coalescence effect. In this process, two photons impinging simultaneously on a beamsplitter from two different input modes have an enhanced probability of emerging along the same output mode (that is, coalescing), as long as they are indistinguishable. If the two photons are made distinguishable by their internal quantum state, for example encoded in the polarization π or in other degrees of freedom, the coalescence effect vanishes. Now, if one of the two photons involved in the process is in a given input state to be cloned and the other in a random one, the HOM effect will enhance the probability that the two photons emerge from the beamsplitter with the same quantum state, that is, with successful cloning, when they emerge together along the same output mode of the beamsplitter. For qubit states, the ideal success probability of this scheme is $p = 3/4$ (when using both beamsplitter exit ports), and the cloning fidelity for successful events is $F = 5/6$, corresponding to the optimal value[27]. The probabilistic feature of this implementation does not spoil its optimality, as it has been

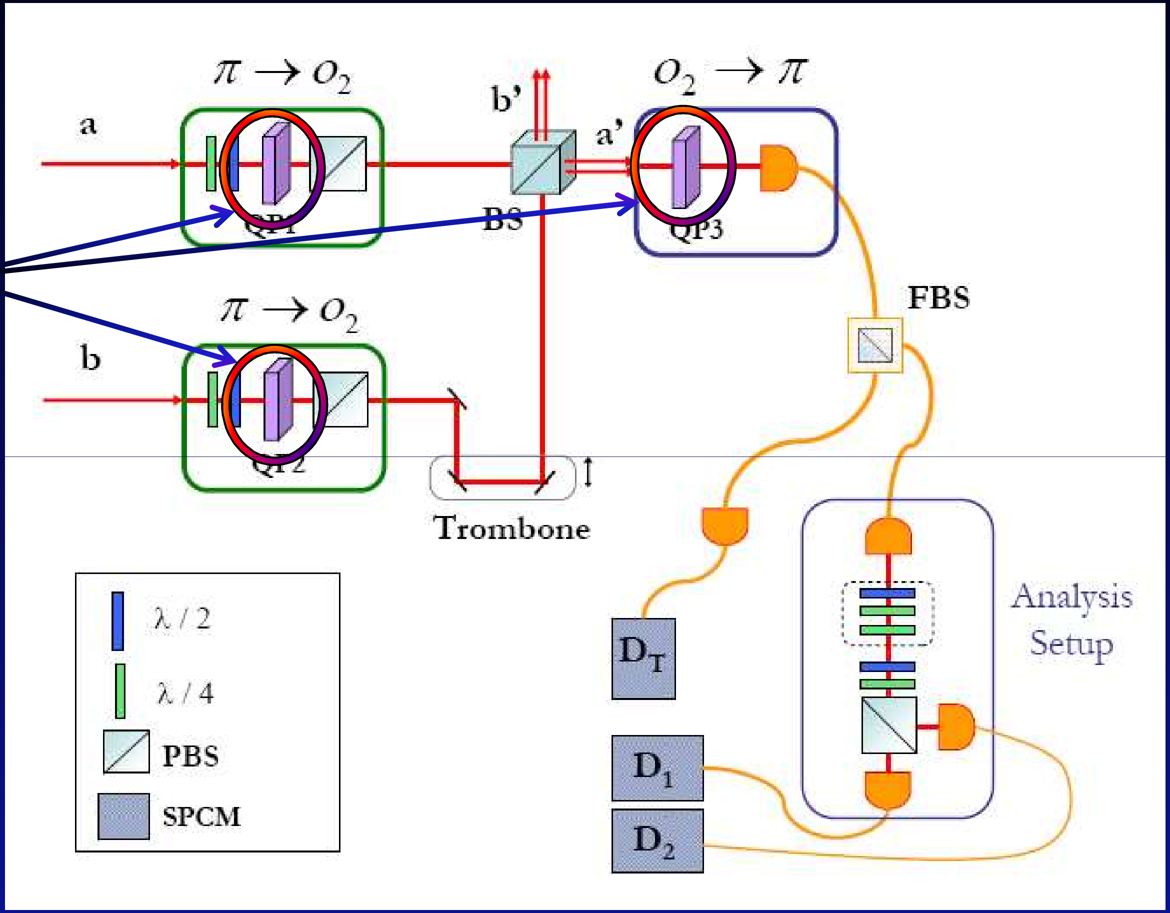
---

[1]Dipartimento di Fisica, Sapienza Università di Roma, Roma 00185, Italy, [2]Istituto Nazionale di Ottica Applicata, Firenze 50125, Italy, [3]Accademia Nazionale dei Lincei, via della Lungara 10, Roma 00165, Italy, [4]Dipartimento di Scienze Fisiche, Universitá di Napoli 'Federico II', Compl. Univ. di Monte S. Angelo, 80126 Napoli, Italy, [5]CNR-INFM Coherentia, Compl. Univ. di Monte S. Angelo, 80126 Napoli, Italy, [6]Consorzio Nazionale Interuniversitario per le Scienze Fisiche della Materia, Napoli. *e-mail: fabio.sciarrino@uniroma1.it; lorenzo.marrucci@na.infn.it

720 NATURE PHOTONICS | VOL 3 | DECEMBER 2009 | www.nature.com/naturephotonics

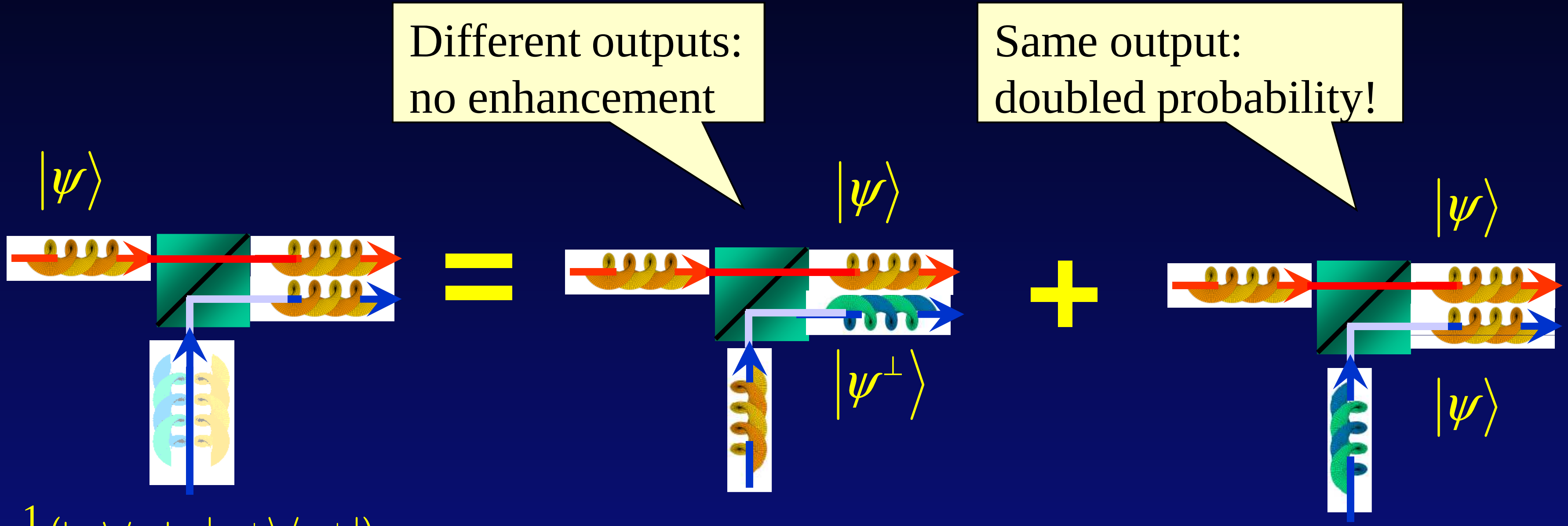
E. Nagali,, et al., Nature Photonics, **3**, 720 (2009)

Experimental setup



q-plates

Optimal cloning working principle: coalescence enhancement



Different outputs:
no enhancement

Same output:
doubled probability!

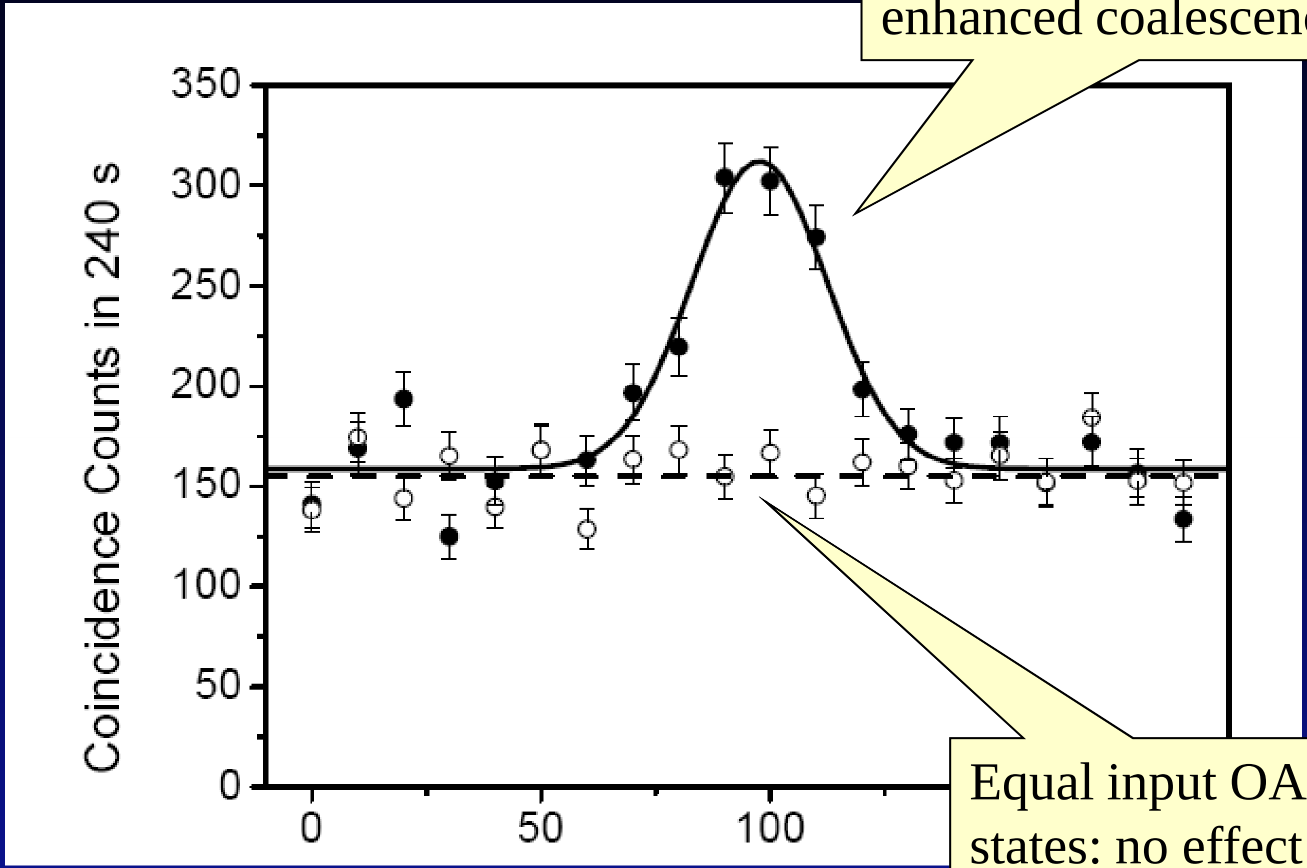
$$\frac{1}{2}\left(|\psi\rangle\langle\psi|+|\psi^{\perp}\rangle\langle\psi^{\perp}|\right)$$

P = 1/3, cloning
with 50% fidelity

P = 2/3, cloning
with 100% fidelity

Overall predicted cloning fidelity = 5/6 = optimal value!

Experimental results



Opposite input OAM states: enhanced coalescence!

Equal input OAM states: no effect

Similar results were obtained also with OAM quantum superposition states

Experimental results

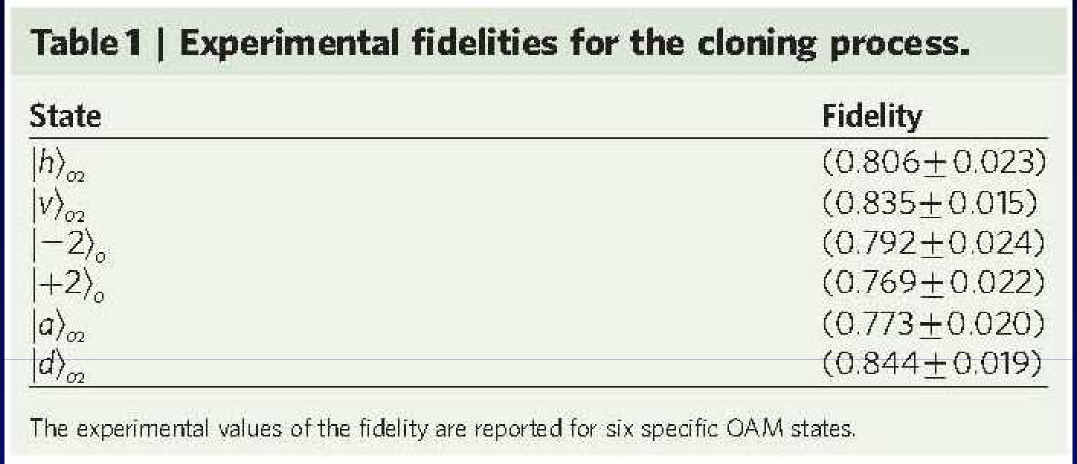
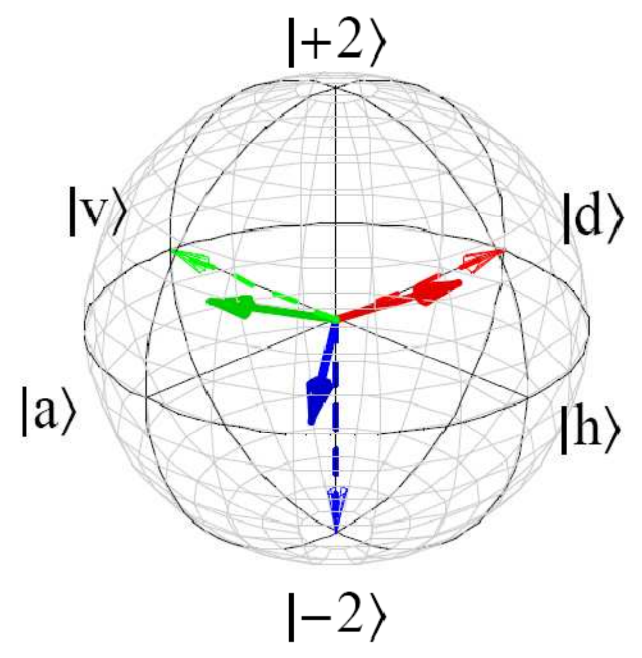


Table 1 | Experimental fidelities for the cloning process.

State	Fidelity
$\vert h\rangle_{o2}$	(0.806 ± 0.023)
$\vert v\rangle_{o2}$	(0.835 ± 0.015)
$\vert -2\rangle_{o}$	(0.792 ± 0.024)
$\vert +2\rangle_{o}$	(0.769 ± 0.022)
$\vert a\rangle_{o2}$	(0.773 ± 0.020)
$\vert d\rangle_{o2}$	(0.844 ± 0.019)

The experimental values of the fidelity are reported for six specific OAM states.



Average measured cloning fidelity: $F = 0.80 \pm 0.01$

(ideal $F = 5/6 = 0.83$)

Testing quantum contextuality and non-locality with OAM

- Non-contextuality assumes that the result of the measure of an observable *A* commuting with other observable *B* is not affected by a measurement of the latter.

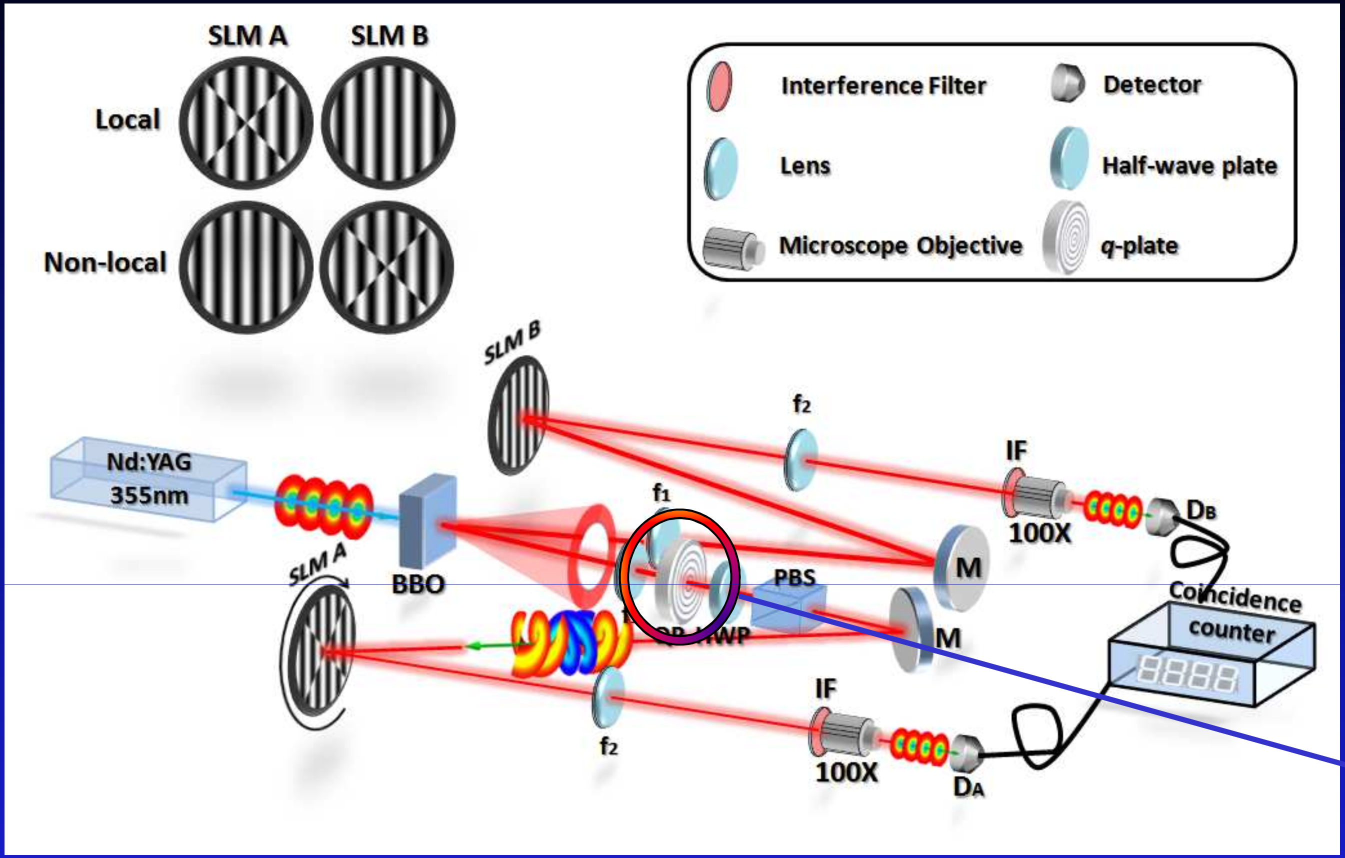
- Classical physics is non-contextual. Quantum physics is contextual.

- Quantum non-locality is a particular case of quantum contextuality

Testing quantum contextuality and non-locality with OAM

We tested quantum contextuality and non-locality in a single experiment exploiting the photon OAM.

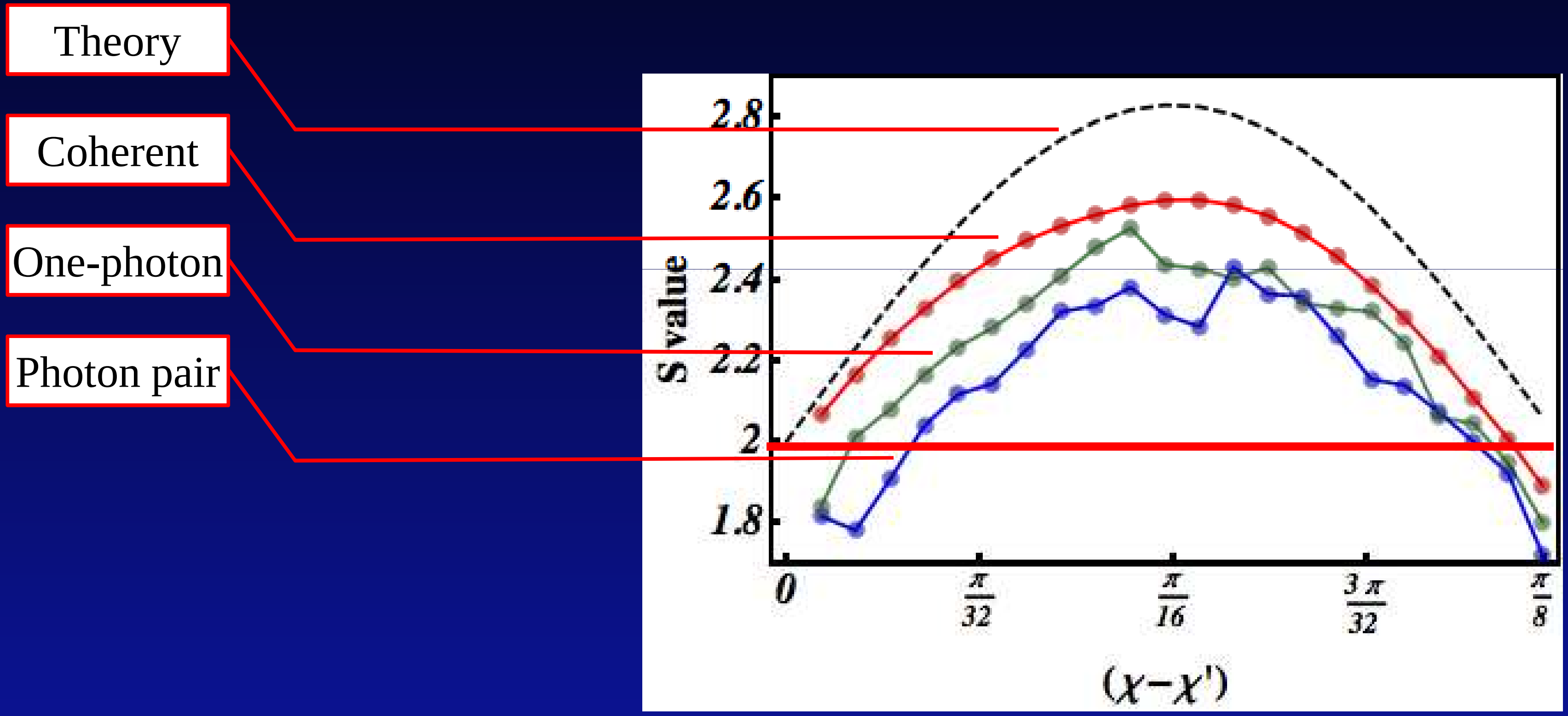
As test we used the Clasur, Horne, Shimony and Holt (CHSH) inequality.



q.plate

E. Karimi et al. submitted to Phys. Rev. Lett. (2010)

Experimental results



Theory

Coherent

One-photon

Photon pair

Work in progress on quantum optics with OAM

Single photon

- Generation of OAM eigenstates of different *m* to encode up to 8 bit in a single photon
- OAM/SAM Hardy states to test Hardy paradox
- OAM/SAM Leggett’s inequalities
- …..

Photon pair

- Generation of hyperentangled OAM/SAM states
- Optimal cloning of OAM + SAM
- Photon OAM/SAM hybridization
- ….

Hardy paradox

L. Hardy, Phys. Rev. Lett., **71**, 1665 (1993)

1. Let *A* and *B* not commuting observables of a system taking only the two eigenvalues a_i and b_i (i = 1, 2), respectively.

2. Let *A'* and *B'* not commuting observables of the same system taking only the two eigenvalues a'_i and b'_i (i = 1, 2), respectively.

3. All primed observable commute with the unprimed ones.

Hardy paradox

Let $|\Psi_H\rangle$ a state of the system so that

a) $\langle b_2, a'_2|\Psi_H\rangle = 0 \Longrightarrow B = b$ & $A' = a'$ is impossible

b) $\langle a_1, a'_1|\Psi_H\rangle = 0 \Longrightarrow$ If A $= a'$

c) $\langle b_1, b'_1|\Psi_H$

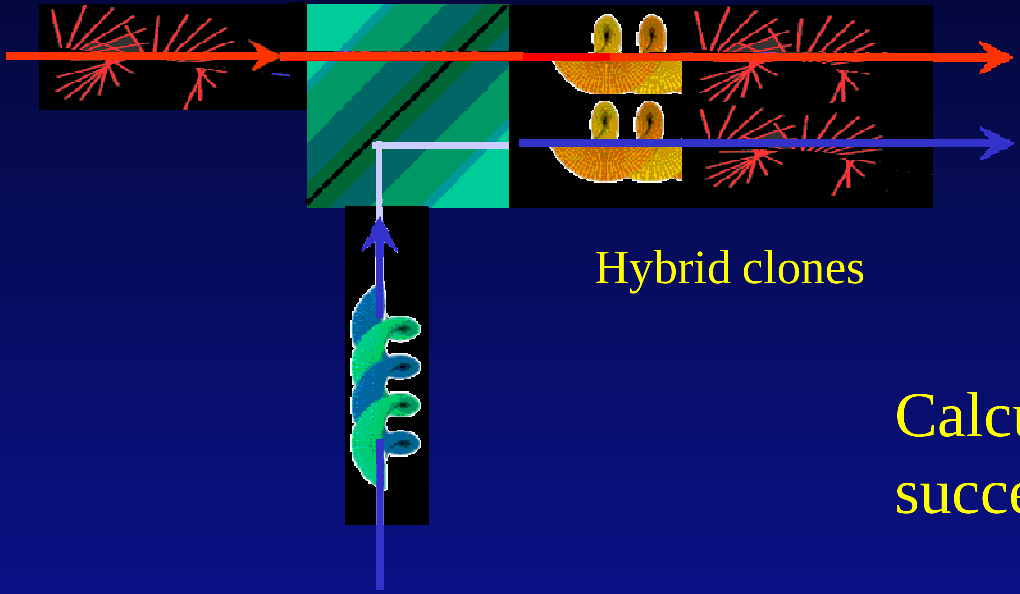
d) $\langle a_1, b'_1|$ $A = a_1$ and $B' = b'_1$

b) and c) $\Longrightarrow$ fo ha $B = b_2$ and $A' = a'_2$

But this is im ossible because of property a) !

$|\Psi_H\rangle$ exists !

Photon OAM/SAM hybridization



Maximally mixed OAM

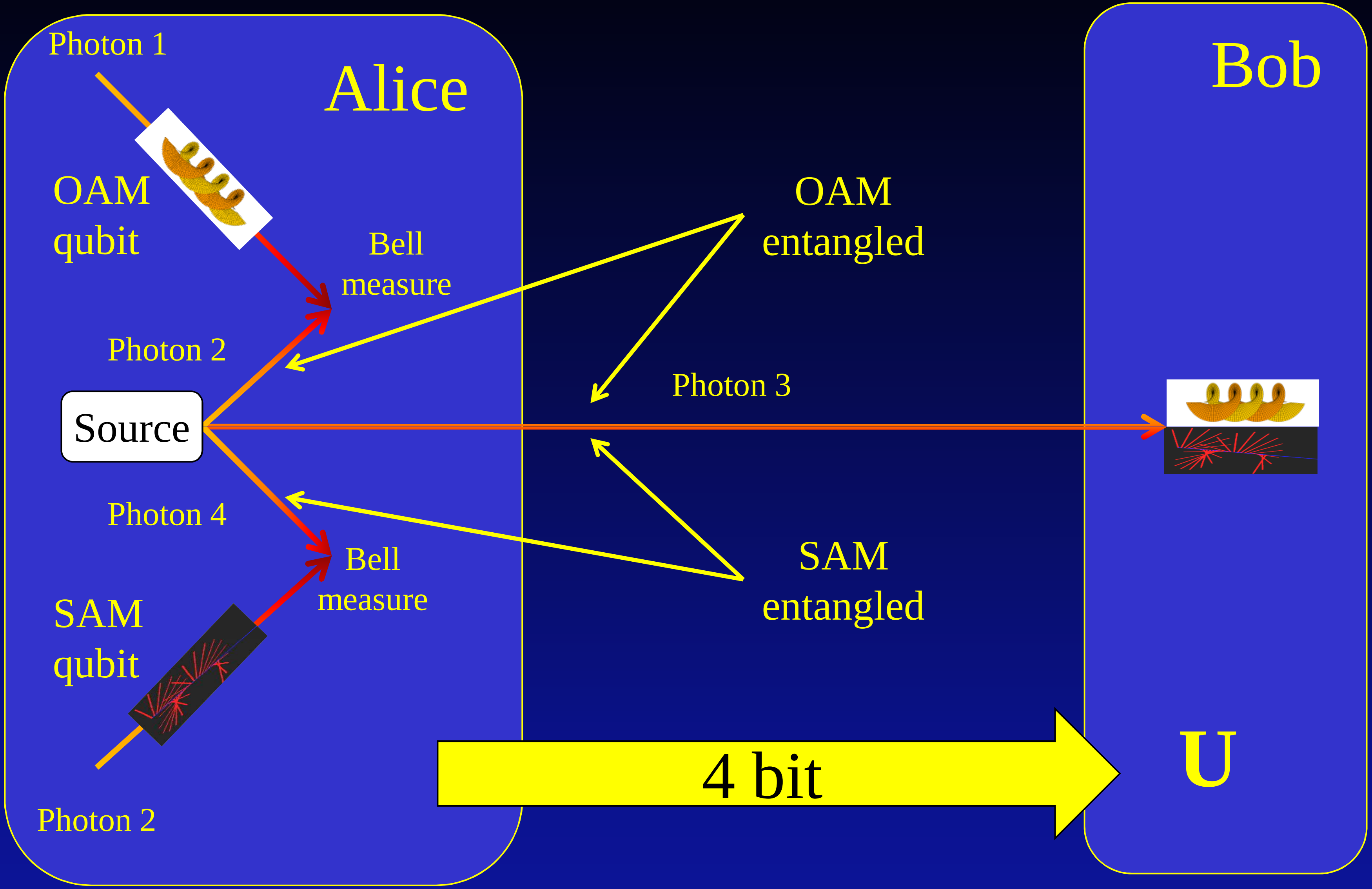
Hybrid clones

Maximally mixed SAM

Calculated cloning success probability 2/3

Calculated fidelity to the hybrid state 0.6

Two-in-one teleportation



Photon 1

Alice

OAM qubit

Bell measure

Photon 2

Source

Photon 4

Bell measure

SAM qubit

Photon 2

OAM entangled

Photon 3

SAM entangled

4 bit

Bob

U

Thank you for your attention.