



The Galileo Galilei Institute for Theoretical Physics

Higgsless Models

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Outline of the talk

(Based on papers by **A. Deandrea, J. Bechi, R.C., F. Coradeschi, S. De Curtis, D. Dolce, D. Dominici, F. Feruglio, M. Grazzini, R. Gatto**)

- **Motivations for Higgsless models**
- **Example of breaking the EW symmetry without Higgs (BESS)**
- **Linear moose: effective description for extra gauge bosons**
- **Unitarity bounds and EW constraints**
- **Degenerate BESS model (DBESS)**
- **The continuum limit**
- **The continuum limit of DBESS**
- **The four site model, possibility of detection @ the LHC.**
- **Summary and conclusions**

Problems of the Higgs sector

Consider the Higgs potential $V(\varphi) = -\mu^2 |\varphi|^2 + \lambda |\varphi|^4$

$$\lambda = \frac{\mu^2}{v^2}, \quad m_H^2 = 2\lambda v^2, \quad \langle \varphi \rangle = v$$

The evolution of the coupling (neglecting gauge fields and fermions contributions) shows up a **Landau pole** at M_{Lp}

$$\lambda(M) = \frac{1}{\frac{1}{\lambda(m_H)} - \frac{3}{4\pi^2} \log \frac{M^2}{m_H^2}}$$

$$M_{Lp} = m_H e^{4\pi^2 v^2 / 3m_H^2}$$

- Or M_{Lp} pushed to infinity, but then λ goes to 0, triviality!
- Or there is a physical cutoff at a scale $M < M_{Lp}$. From this we get a bound on m_H , from $\lambda(M) > 0$



$$\lambda(m_H) < \frac{1}{\frac{3}{4\pi^2} \log \frac{M^2}{m_H^2}} \quad \longrightarrow \quad m_H^2 < \frac{4\pi^2}{3} v^2 \frac{1}{\log \frac{M}{m_H}}$$

If the cutoff is big ($M \sim 10^{16}$ GeV), $\lambda(m_H)$ is small (~ 0.2). The theory is perturbative, but the Higgs acquires a mass of order

$$\delta m_H^2 = \frac{\lambda}{8\pi^2} M^2$$

with M of the order of M_{GUT} . The naturalness problem follows and to avoid it, the quadratic divergences should cancel (SUSY).

If there is new physics at a scale M of order of TeV, then the theory has a natural cutoff at M . Then $m_H \sim M$ and $\lambda(m_H)$ is large ($\sim 3 - 4$). The theory is nonperturbative.

- 1) $\lambda \ll 1 \Rightarrow$ new particles lighter than 1 TeV (SUSY)
- 2) $\lambda \gg 1 \Rightarrow$ new particles around 1 TeV

**In the following the second option
will be considered:
new strong physics at the TeV scale**

Symmetry Breaking

- Since we are considering a strongly interacting theory: **an effective description of the SB**
- We need to break $SU(2)_L \times U(1)$ down to $U(1)_{em}$. The SB sector should be of the type

$$G \Rightarrow H,$$

$$G \supset SU(2)_L \otimes U(1), \quad H \supset U(1)_{em}$$

- In the SM the SB sector is the Higgs sector with

$$G = SU(2)_L \otimes SU(2)_R, \quad H = SU(2)_V$$

- If the SB sector is strongly interacting one can describe it at low energies making use of a general σ model of the type G/H
- For instance, in the case $SU(2)_L \times SU(2)_R / SU(2)_V$ the model can be described in terms of a field Σ in $SU(2)$ transforming as

$$\Sigma \rightarrow g_L \Sigma g_R^\dagger, \quad g_L \in SU(2)_L, \quad g_R \in SU(2)_R$$

- The strong dynamics is completely characterized by the transformation properties of the field Σ which can be summarized in the following moose diagram.



- The breaking is produced by $\langle 0 | \Sigma | 0 \rangle = 1$

$$L = \frac{v^2}{4} \text{Tr} \left(\partial_\mu \Sigma \partial^\mu \Sigma^\dagger \right), \quad \Sigma = e^{i\vec{\pi} \cdot \vec{\tau} / v}, \quad \langle \vec{\pi} \rangle = 0$$

- In this way one could describe also an explicit breaking $SU(2)_L \times SU(2)_R$ to $SU(2) \times U(1)$ through an explicit $SU(2)_R$ breaking term (the ρ -parameter is the standard one)

$$L = \frac{v^2}{4} \text{Tr} \left(\partial_\mu \Sigma \partial^\mu \Sigma^\dagger \right) + \frac{v^2}{2} (\rho - 1) \left[\text{Tr} (T_3 \Sigma^\dagger \partial^\mu \Sigma) \right]^2$$

- The model can be easily gauged to SU(2)xU(1) introducing the gauge covariant derivatives

$$D_{\mu}\Sigma = \partial_{\mu}\Sigma + igW_{\mu}\Sigma - ig'\Sigma B_{\mu}$$

- With **W** and **B** the gauge fields of SU(2) and U(1) respectively. Notice that with respect to the strong dynamics described by the σ model, **the interactions with W and B are to be considered as perturbations.**
- The σ model can be obtained as the formal limit of the SM for M_H going to infinity.

$$L = \frac{1}{4} \text{Tr} \left(D_{\mu} M D^{\mu} M^{\dagger} \right) - \frac{M_H^2}{8v^2} \left(\frac{1}{2} \text{Tr} M^{\dagger} M - v^2 \right)^2 +$$

$$+ \frac{1}{2} \text{Tr} F_{\mu\nu} (W) F^{\mu\nu} (W) + \frac{1}{2} \text{Tr} F_{\mu\nu} (B) F^{\mu\nu} (B)$$

$$M = \frac{1}{\sqrt{2}} \begin{bmatrix} \varphi_0 & -\varphi_-^* \\ \varphi_- & \varphi_0^* \end{bmatrix}$$

- Through the definition **M = S** Σ , with **S** is singlet field having a vev fixed to v in the limit of large Higgs mass.

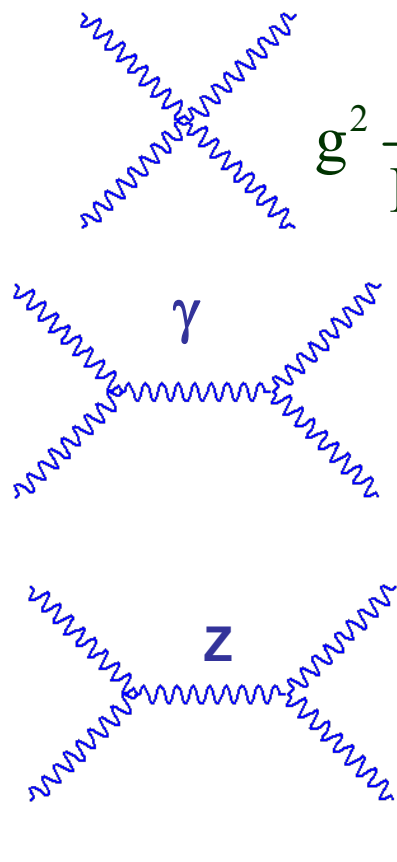
The field Σ describes the **Goldstone fields** giving mass to W and Z: **they are related to the longitudinal modes**. The interesting amplitude is $W_L W_L$ to $W_L W_L$ strictly related to the physics of the GBs.

Within the SM **no bad high-energy behaviour for light Higgs**:

- The quartic divergence is cancelled by the gauge contributions
- The quadratic part is cancelled by the Higgs boson contribution

in the high energy limit $E \gg M_W$

$$\epsilon_\mu^L \approx \frac{p_\mu}{M_W}$$



$$g^2 \frac{p^4}{M_W^4} \approx g^2 \frac{s^2}{M_W^4}$$

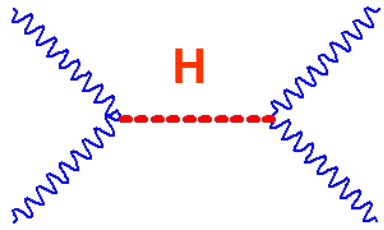
Derivative coupling

$$-e^2 \frac{s}{s} \frac{s^2}{M_W^4} = -g^2 \sin^2 \theta \frac{s^2}{M_W^4}$$

$$-g^2 \cos^2 \theta \frac{s}{s - M_Z^2} \frac{s^2}{M_W^4} \approx$$

$$\approx -g^2 \cos^2 \theta \frac{s^2}{M_W^4} - g^2 \cos^2 \theta M_Z^2 \frac{s}{M_W^4}$$

$$T_{\text{gauge}} \approx -g^2 \frac{s}{M_W^2}$$



$$\begin{aligned}
 (g^2 v)^2 \frac{1}{s - M_H^2} \frac{s^2}{M_W^4} &\approx g^2 (gv)^2 \frac{s}{M_W^4} + g^2 (gv)^2 \frac{M_H^2}{M_W^4} = \\
 &= g^2 \frac{s}{M_W^2} + g^2 \frac{M_H^2}{M_W^2}
 \end{aligned}$$

$$T = T_{\text{gauge}} + T_{\text{Higgs}} \approx g^2 \frac{M_H^2}{M_W^2}$$

$$T_{\text{gauge}} \approx -g^2 \frac{s}{M_W^2}$$

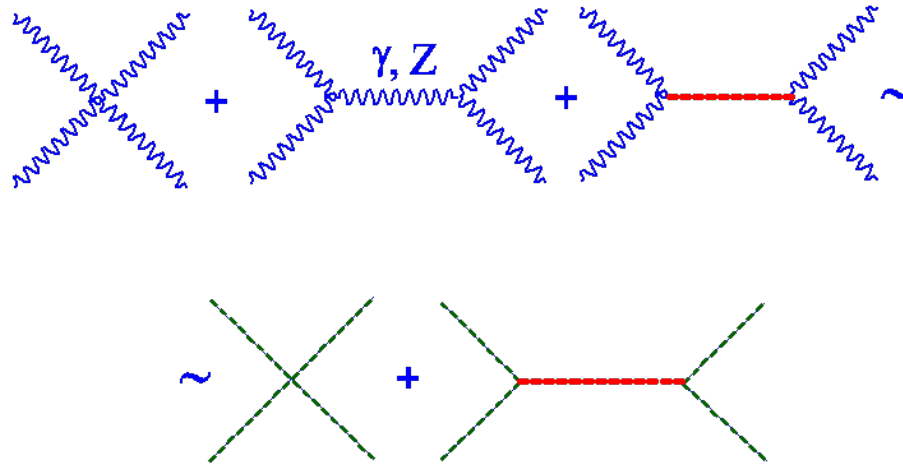
$$|a_0| = \frac{1}{16\pi} g^2 \frac{M_H^2}{M_W^2} \leq 1$$



$$M_H^2 \leq 16\pi \frac{M_W^2}{g^2} = 4\pi v^2 \approx (1 \text{ TeV})^2$$

An important result is the **Equivalence Theorem** (Cornwall, Levin & Tiktopoulos, 1974; Vayonakis, 1976):

for $E \gg M_W$ the scattering amplitudes can be evaluated by replacing the longitudinal vector bosons with the corresponding Goldstone bosons



$$T_{\text{Goldstones}} \approx \lambda + \lambda \frac{M_H^2}{s - M_H^2} \approx \lambda = \left(\frac{M_H}{v} \right)^2 = g^2 \frac{M_H^2}{M_W^2} = T_{\text{Gauge}} + T_{\text{Higgs}}$$

In the limit $M_W^2 \ll s \ll M_H^2$, the $W_L W_L$ (or Goldstone) amplitude can be represented by the non-linear σ model

$$T_{\text{Goldstones}} \approx \lambda + \lambda \frac{M_H^2}{s - M_H^2} \xrightarrow{s \ll M_H^2} -\frac{s}{v^2}$$

This coincides with the amplitude that can be extracted from the non-linear lagrangian expanding at the 4th order in the pion fields


$$L = \frac{v^2}{4} \left(\partial_\mu \Sigma \partial^\mu \Sigma^\dagger \right), \quad \Sigma = e^{i\vec{\pi} \cdot \vec{\tau}/v}$$

Since the form of the effective lagrangian depends only on the symmetries, this result is generally valid (this is the content of the **Low Energy Theorem**, LET, see **Chanowitz, Golden, Georgi, 1987**).

Due to **unitarity violation**, the validity of this description is up to

$$|a_0| = \frac{1}{16\pi} \frac{s}{v^2} \leq 1 \Rightarrow E \leq 4\sqrt{\pi}v \approx 1.7 \text{ TeV} = \Lambda_{\text{HSM}}$$

Summarizing, what have we learned so far? If we **assume that the SB sector is strongly interacting**, then:

- The low energy effective action of the strongly interacting sector is completely **fixed by the symmetries** (these can be postulated but must describe at least the breaking $SU(2) \times U(1)$ to $U(1)_{em}$)
- This allows to determine the behaviour at low energy description valid up to $E \sim 1.7$ TeV. 

Can we say something else about the strong dynamics to go beyond this low energy approximation?

- We can explore the possibility of further **bound states** as, for instance, **vector resonances**
- The vector resonances help to improve the energy behaviour of the **LET amplitudes**. This amounts to **postpone the unitarity bound**
- --- very important --- we have to satisfy the stringent experimental bounds coming from **LEP and SLC data**
- Try to learn from QCD, though we will **not adopt the point of view of TC theories**, that is **scaling QCD** up to energies of interest here

Enlarging the σ model

- We will start enlarging the non-linear σ model by introducing **vector resonances**. One of the virtues of doing this is that unitarity properties improve (as it is known from QCD). Of course one has to be consistent with the non-linear realization. This could be done by standard techniques, but a tool which is very useful is the one of **hidden gauge symmetries** (Bando, Kugo et al. 1985).

Strategy:

- Introduce a non-dynamical gauge symmetry together with a set of new scalar fields.
- The scalar fields can be eliminated by using the local symmetry and the theory is equivalent to the non linear model.
- Promoting the local symmetry to a dynamical one allows to introduce in a simple way dynamical vector resonances (the gauge fields of the new gauge interaction).
- The new vector resonances are massive due to the breaking of the local symmetry implied by the non-linear realization.

In order to realize this program, we do the following:

- Introduce a mapping $g(x)$ from the space-time to the group G :

$$g(x) \in G$$

- Construct a lagrangian invariant under

$$g(x) \rightarrow g'(x) = g_0 g(x) h(x), \quad g_0 \in G, \quad h(x) \in H, \quad H \subset G$$

$$L(g, \partial_\mu g) = L(g', \partial_\mu g')$$

- L depends only on the fields defined on the coset G/H . In fact, locally

$$g(x) = \xi(x) h(x), \quad \xi \in G/H, \quad h \in H$$

and using the invariance of L :

$$L(g, \partial_\mu g) = L(\xi, \partial_\mu \xi), \quad g(x) \rightarrow g(x) h^{-1}(x)$$

The theory formulated in G with the (non-dynamical) local symmetry H is equivalent to the non-linear model formulated over G/H

The BESS model

The simplest enlargement of the non-linear model based on $SU(2) \times SU(2)/SU(2)$ is the **BESS (Breaking Electroweak Symmetry Strongly)** model (RC, De Curtis, Dominici & Gatto, 1985, 1987, + Feruglio 1988) which introduces a local group $G_1 = SU(2)$ with two scalar fields L and R transforming as

$$\Sigma_1(x) \rightarrow g_L \Sigma_1(x) h^\dagger(x), \quad \Sigma_2(x) \rightarrow h(x) \Sigma_2(x) g_R^\dagger, \quad h \in G_1$$

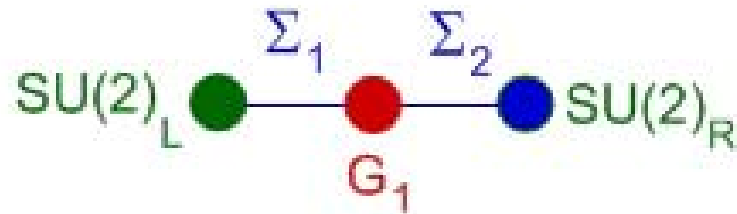
Introduce covariant derivatives, with V the gauge field associated to the local group G_1

$$D_\mu \Sigma_1 = \partial_\mu \Sigma_1 + \Sigma_1 V_\mu, \quad D_\mu \Sigma_2 = \partial_\mu \Sigma_2 - V_\mu \Sigma_2, \quad V^\dagger = -V$$

and build up the invariant lagrangian (this is not the most general one):

$$L = f_1^2 \text{Tr} \left[D_\mu \Sigma_1^\dagger D^\mu \Sigma_1 \right] + f_2^2 \text{Tr} \left[D_\mu \Sigma_2^\dagger D^\mu \Sigma_2 \right]$$

Also in this case the transformation properties of the fields can be condensed in a moose diagram



Notice that in L we have inserted only terms corresponding to the two links. In principle there is another invariant coupling of the type

$$\text{Tr} \left[(\Sigma_1^\dagger D_\mu \Sigma_1) (\Sigma_2 D^\mu \Sigma_2^\dagger) \right]$$

which we will not consider here.

Since V is not a dynamical field, it can be eliminated through its e.o.m.

$$V^\mu = - \frac{f_1^2 \Sigma_1^\dagger D^\mu \Sigma_1 + f_2^2 \Sigma_2 D^\mu \Sigma_2^\dagger}{f_1^2 + f_2^2}$$

Substituting inside L one gets back the usual non-linear σ model after the identifications

$$\Sigma = \Sigma_1 \Sigma_2 \rightarrow g_L \Sigma_1 h^\dagger h \Sigma_2 g_r^\dagger = g_L \Sigma g_r^\dagger \quad \frac{4}{v^2} = \frac{1}{f_1^2} + \frac{1}{f_2^2} = \frac{1}{f^2}$$

On the other hand, we can make V a dynamical field and by construction the lagrangian will preserve the total symmetry. It is enough to introduce the kinetic term for V in a gauge invariant way

$$L = f_1^2 \text{Tr} \left[D_\mu \Sigma_1^\dagger D^\mu \Sigma_1 \right] + f_2^2 \text{Tr} \left[D_\mu \Sigma_2^\dagger D^\mu \Sigma_2 \right] - \frac{1}{2} \text{Tr} [F_{\mu\nu}(V) F^{\mu\nu}(V)]$$

This model describes **6 scalar fields and 3 gauge bosons**. After the breaking of **$SU(2)_L \times SU(2)_R \times SU(2)_{local}$ to $SU(2)$** we get **3 Goldstone bosons** (necessary to give mass to W and Z after gauging the EW group) and **3 massive vector bosons** with mass

$$M_V^2 = \left(f_1^2 + f_2^2 \right) g_1^2, \quad g_1 = \text{gauge coupling of } V$$

Electroweak corrections

(Burgess et al.; Anichini, RC, De Curtis)

LEP I puts very stringent bounds on models of new physics. These limits, assuming universality among different generations, are coded in 3 parameters (using G_F , m_Z and α as input parameters)

$$\Delta r_W : \quad \frac{m_W^2}{m_Z^2} = \left[\frac{1}{2} + \sqrt{\frac{1}{4} - \frac{\pi\alpha(m_Z)}{\sqrt{2}G_F m_Z^2 (1 - \Delta r_W)}} \right]^2$$

And from the modifications of the Z couplings to fermions:

$$\mathcal{L}_{\text{neutral}} = -\frac{e}{s_\theta c_\theta} \left(1 + \frac{\Delta\rho}{2} \right) \bar{\Psi} \left(g_V \gamma^\mu + g_A \gamma^\mu \gamma_5 \right) \Psi Z_\mu$$

$$g_V = \frac{1}{2} (T_3)_L - \bar{s}_\theta^2 Q_{\text{em}}, \quad g_A = -\frac{1}{2} (T_3)_L$$

$$\bar{s}_\theta^2 = s_\theta^2 (1 + \Delta k), \quad c_\theta^2 = \frac{1}{2} + \sqrt{\frac{1}{4} - \frac{\pi\alpha(m_Z)}{\sqrt{2}G_F m_Z^2}}$$

It is usual to introduce another set of parameters ϵ_i , $i=1,2,3$ (Altarelli, Barbieri, 1991), or **S,T,U** (Peskin, Takeuchi, 1990), much more convenient on the theoretical side

$$\epsilon_1 = \Delta\rho, \quad \epsilon_2 = c_\theta^2 \Delta\rho + \frac{s_\theta^2}{c_{2\theta}} \Delta r_W - 2s_\theta^2 \Delta k, \quad \epsilon_3 = c_\theta^2 \Delta\rho + c_{2\theta} \Delta k$$

At the lowest order in the EW corrections the parameters ϵ_1 and ϵ_2 vanish if the SB sector has a **SU(2) custodial symmetry** (as it is the case for the BESS model). At the same order, ϵ_3 has a convenient **dispersive representation**

$$\epsilon_3 = -\frac{g^2}{4\pi} \int_0^\infty \frac{ds}{s^2} [\text{Im} \Pi_{VV}(s) - \text{Im} \Pi_{AA}(s)], \quad \Pi_{VV(AA)} = \langle J_{V(A)} J_{V(A)} \rangle_0$$

Assuming vector dominance:

$$\text{Im} \Pi_{VV(AA)}(s) = -\pi g_{V(A)}^2 \delta(s - M_V^2), \quad \langle 0 | J_{V(A)}^\mu | V(k) \rangle = g_{V(A)} \epsilon^\mu(k)$$

In the BESS model the decay coupling constants of the vector meson are:

$$g_{V(A)} = (f_1^2 \pm f_2^2) g_1$$

In the BESS model we get (assuming standard fermions not directly coupled to the vector boson V):

$$\epsilon_3 = \frac{g^2}{4} \frac{g_V^2}{M_V^4} = \left(\frac{g}{g_1} \right)^2 \frac{f_1^2 f_2^2}{(f_1^2 + f_2^2)^2} \xrightarrow{f_1=f_2} \frac{1}{4} \left(\frac{g}{g_1} \right)^2$$

for $g_1 \sim g \rightarrow 5g$, we get

$$g_1 = g \Rightarrow \epsilon_3 = 0.25, \quad g_1 = 5g \Rightarrow \epsilon_3 = 10^{-2}$$

Experimentally ϵ_3 of order 10^{-3} , we need an unnatural value of g_1 bigger than $10g-16g$ (not allowed by unitarity, see later).

A possible way out: couple directly the vector bosons V to the fermions introducing the following fields:

$$\chi_L = \sum_1^\dagger \psi_L, \quad \chi_R = \sum_2 \psi_R$$

Then, we can add two invariant terms

$$b \bar{\chi}_L i \gamma^\mu \left(\partial_\mu - V_\mu + \frac{i}{2} (B-L) Y_\mu \right) \chi_L + b' \bar{\chi}_R i \gamma^\mu \left(\partial_\mu - V_\mu + \frac{i}{2} (B-L) Y_\mu \right) \chi_R$$

The b' coupling is very much constrained by the K_L - K_S mass difference and it is generally ignored.

The b coupling gives rise to **a contribution to ϵ_3** which, **if $b > 0$** , is of opposite sign with respect to the gauge one

$$\epsilon_3 = \frac{1}{4} \left(\frac{g}{g_1} \right)^2 - \frac{b}{2}$$

$$\epsilon_3^{\text{exp}} \sim 10^{-3}$$

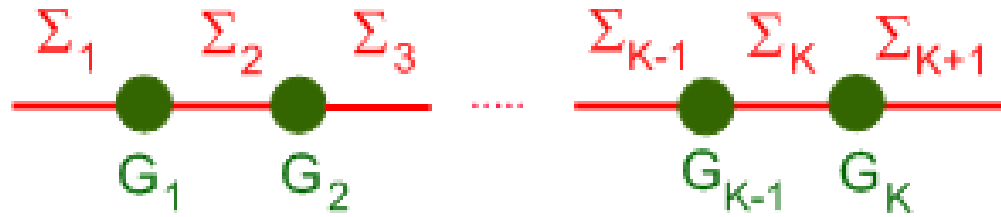
If a direct coupling of V to fermions is present, we can satisfy the bound but at the expenses of some **fine tuning**: for $g_1 \sim 5g$, **b should be fixed at the level of 2×10^{-3}** . Furthermore the new VB's would be fermiophobic.

Breaking the EW Symmetry without Higgs Fields

- Let us now generalize the moose construction: general structure given by many copies of the gauge group G intertwined by link variables Σ .
- Condition to be satisfied in order to get a Higgsless SM before gauging the EW group is the presence of **3 GB** and **all the moose gauge fields massive**.
- Simplest example: **$G_i = \text{SU}(2)$** . Each Σ_i describes three scalar fields. Therefore, in a connected moose diagram, any site (3 gauge fields) should absorb one link (3 GB's) giving rise to a massive vector field. We need:

$$\# \text{ of links} = \# \text{ of sites} + 1$$

● Example:



- The model has two global symmetries related to the beginning and to the end of the moose, that we will denote explicitly by G_L and G_R .

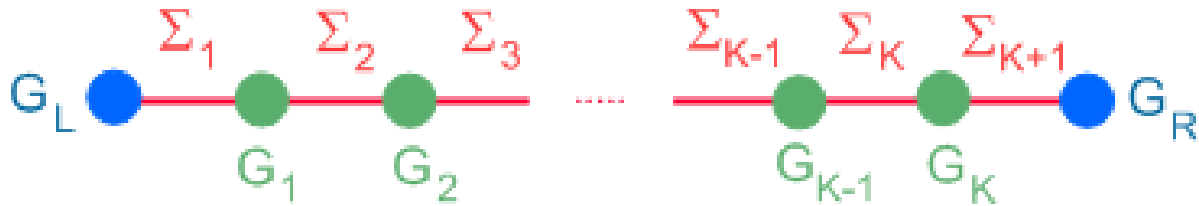


$$\begin{aligned} \Sigma_1 &\Rightarrow U_L \Sigma_1 U_L^\dagger \\ \Sigma_i &\Rightarrow U_{i-1} \Sigma_i U_i^\dagger \\ \Sigma_{K+1} &\Rightarrow U_K \Sigma_{K+1} U_R^\dagger \end{aligned}$$

- The $SU(2)_L \times SU(2)_R$ global symmetry can be gauged to the standard $SU(2) \times U(1)$ leaving us with the usual 3 massive gauge bosons, W and Z , the massless photon and $3K$ massive vectors.

- The **BESS model** can be view as a **3-site model** ($K=1$), and its generalization (RC, De Curtis, Dominici, Gatto, Feruglio, 1989) can be recast in a 4-site model ($K=2$) (see also Foadi, Frandsen, Ryttov, Sannino, 2007)

Electro-weak corrections for the linear moose



$$S_{\text{moose}} = \int d^4x \left(-\sum_{i=1}^K \frac{1}{2g_i^2} \text{Tr} \left[F_{\mu\nu}^i F^{\mu\nu i} \right] + \sum_{i=1}^{K+1} f_i^2 \text{Tr} \left[(D_\mu \Sigma_i)(D_\mu \Sigma_i)^\dagger \right] \right)$$

- If the vector fields are heavy enough one can derive a low-energy effective theory for the SM fields after gauging

$$\mathbf{SU(2)}_L \otimes \mathbf{SU(2)}_R \Rightarrow \mathbf{SU(2)} \otimes \mathbf{U(1)}$$

One has to solve finite difference equations along the link-line in terms of the SM gauge fields at the two ends (the boundary)

Low-energy limit

The low-energy limit of the theory is obtained by eliminating the A_i ($i = 1, \dots, K$) fields with the solution of the e.o.m. for $g_i \gg \tilde{g}$ corresponding to heavy masses for A_i :

$$A_i^\pm = \frac{1}{g_i}(\tilde{g}\tilde{W}^\pm z_i), \quad A_i^3 = \frac{1}{g_i}(\tilde{g}'\tilde{Y}y_i + \tilde{g}\tilde{W}^3 z_i)$$

with $z_i = \sum_{j=i+1}^{K+1} f^2/f_j^2$ and $y_i = 1 - z_i$. By substituting in $\mathcal{L}^{(2)}$:

$$\mathcal{L}_{eff}^{(2)} = -\frac{1}{4}(1+z_\gamma)\tilde{A}_{\mu\nu}\tilde{A}^{\mu\nu} - \frac{1}{2}(1+z_W)\tilde{W}_{\mu\nu}^+\tilde{W}^{-\mu\nu} - \frac{1}{4}(1+z_Z)\tilde{Z}_{\mu\nu}\tilde{Z}^{\mu\nu} + \frac{1}{2}z_{Z\gamma}\tilde{A}_{\mu\nu}\tilde{Z}^{\mu\nu}$$

$$z_\gamma = \tilde{s}_\theta^2 \sum_{i=1}^K \left(\frac{\tilde{g}}{g_i}\right)^2, \quad z_W = \sum_{i=1}^K \left(\frac{\tilde{g}}{g_i}\right)^2 (1 - y_i)^2, \quad z_Z = \frac{1}{\tilde{c}_\theta^2} \sum_{i=1}^K \left(\frac{\tilde{g}}{g_i}\right)^2 (\tilde{c}_\theta^2 - y_i)^2$$

$$z_{Z\gamma} = -\frac{\tilde{s}_\theta}{\tilde{c}_\theta} \sum_{i=1}^K \left(\frac{\tilde{g}}{g_i}\right)^2 (\tilde{c}_\theta^2 - y_i), \quad M_Z^2 = \tilde{M}_Z^2(1 - z_Z), \quad M_W^2 = \tilde{M}_W^2(1 - z_W)$$

From this, after a finite renormalization one can evaluate the EW parameters ε_i . Or else, use the dispersive representation and V-dominance:

- From vector mesons saturation one gets

$$\varepsilon_3 = \frac{g^2}{4} \sum_n \left(\frac{g_{nV}^2}{m_n^4} - \frac{g_{nA}^2}{m_n^4} \right) = g^2 g_1 g_K f_1^2 f_{K+1}^2 (M_2^{-2})_{1K} = g^2 \sum_{i=1}^K \frac{(1-y_i)y_i}{g_i^2}$$

$$y_i = \sum_{j=1}^i x_j, \quad x_i = \frac{f^2}{f_i^2}, \quad \frac{1}{f^2} = \sum_{i=1}^{K+1} \frac{1}{f_i^2} \Rightarrow \sum_{i=1}^{K+1} x_i = 1$$

- Since

$$0 \leq y_i \leq 1 \Rightarrow \varepsilon_3 \geq 0$$

(follows also from positivity of M_2)

- Example: $f_i = f_c, \quad g_i = g_c \Rightarrow \varepsilon_3 = \frac{1}{6} \frac{g^2}{g_c^2} \frac{K(K+2)}{K+1}$

- Notice that ε_3 increases with K (**more convenient small K**)

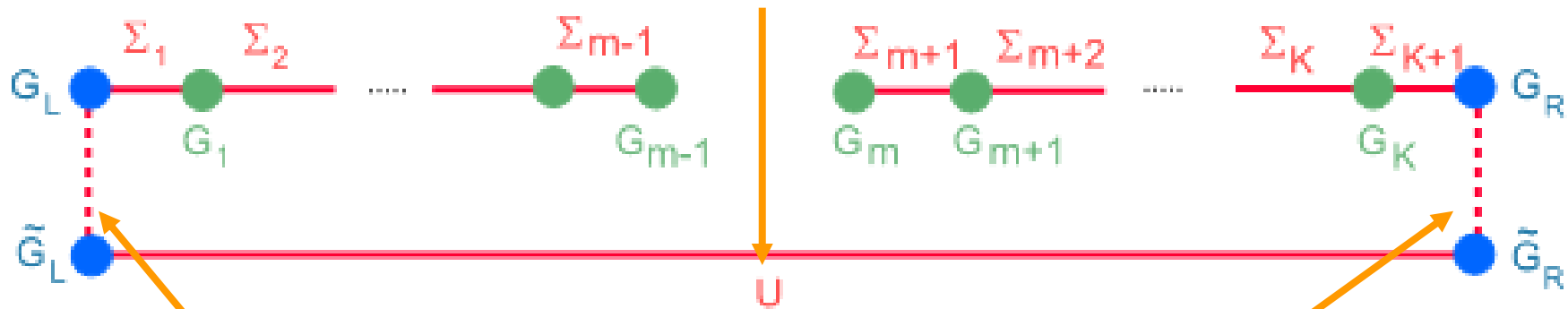
● Possible solution:

Cut a link, with $f_i = 0$, M_2 becomes block diagonal and $(M_2^{-2})_{1K} = 0$

$$\epsilon_3 = 0$$

Add a Wilson line:

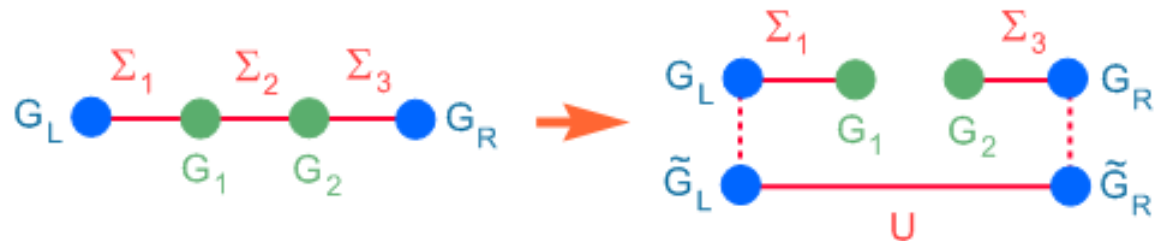
$$U = \Sigma_1 \Sigma_2 \cdots \Sigma_K \Sigma_{K+1}$$



Groups connected through weak gauging

Particular example: D-BESS model

(R.C. De Curtis, Dominici, Feruglio, Gatto, Grazzini 1995,1996)



The theory has a custodial symmetry ensuring $\epsilon_3 = 0$ (Inami, Lim, Yamada, 1992)

$$[SU(2)_L \otimes SU(2)_R]_{29}$$

Unitarity bounds for the linear moose

(Chivukula, He; Papucci, Muck, Nilse, Pilaftis, Ruckl;
Csaki, Grojean, Murayama, Pilo, Terning)

- We evaluate the scattering of longitudinal gauge bosons using the **equivalence theorem**, that is using the amplitude for the corresponding GB's.
- We choose the following parametrization for the Goldstone fields (we are evaluating $W_L W_L$ -scattering)

$$\Sigma_i = e^{i f \vec{\pi} \cdot \vec{\tau} / 2 f_i^2}, \quad \frac{1}{f^2} = \sum_{i=1}^{K+1} \frac{1}{f_i^2}$$

- The resulting 4-pion amplitude is given by

$$A_{\pi^+\pi^-\rightarrow\pi^+\pi^-} = -\frac{f^4}{4} \sum_{i=1}^{K+1} \frac{u}{f_i^6} + \frac{f^4}{4} \sum_{i,j=1}^K L_{ij} \left((u-t)(s-M_2)_{ij}^{-1} + (u-s)(t-M_2)_{ij}^{-1} \right)$$

$$L_{ij} = g_i g_j \left(\frac{1}{f_i^2} + \frac{1}{f_{i+1}^2} \right) \left(\frac{1}{f_j^2} + \frac{1}{f_{j+1}^2} \right)$$

- In the low-energy limit, $m_W \ll E \ll m_\nu$, we get the LET:

$$A_{\pi^+\pi^-\rightarrow\pi^+\pi^-} = -\frac{f^4}{4} \left(\sum_{i=1}^{K+1} \frac{1}{f_i^2} \right)^3 u = -\frac{u}{4f^2} = -\frac{u}{v^2}$$

- In the high-energy limit

$$A_{\pi^+\pi^-\rightarrow\pi^+\pi^-} = -\frac{f^4}{4} \sum_{i=1}^{K+1} \frac{1}{f_i^6} u$$

Best unitarity limit

$$f_i = f_c \rightarrow A = -\frac{u}{(K+1)^2 v^2}$$

$$\Lambda_U = (K+1)\Lambda_{\text{HSM}} \approx 1.7(K+1)\text{TeV}$$

- By taking into account all the vectors and using the equivalence theorem, the amplitudes for the Goldstones are given by

$$\left(\sum_i = e^{i\vec{\pi}_i \cdot \vec{\tau} / 2f_i} \right) \quad A_{\pi_i^+ \pi_i^- \rightarrow \pi_i^+ \pi_i^-} \rightarrow -\frac{u}{4f_i^2}$$

- The unitarity limit is determined by the smallest link coupling. By taking all equal (see also [Chivukula, He, 2002](#))

Unitarity limit

$$f_i = f_c \Rightarrow A \Rightarrow -\frac{u}{(K+1)v^2}$$

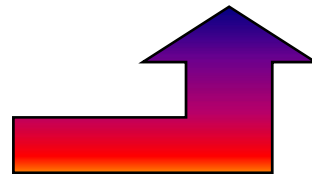
$$\Lambda_U = (K+1)^{1/2} \Lambda_{\text{HSM}} \approx 1.7(K+1)^{1/2} \text{TeV}$$

$$M_V^{\text{max}} < \Lambda_U, \text{ but roughly } M_V^{\text{max}} \approx 2\sqrt{K} \frac{g_c}{g} M_W$$



$$2\sqrt{K} \frac{g_c}{g} M_W < 1.7\sqrt{K} \text{TeV} \Rightarrow \frac{g_c}{g} < 10$$

Hardly compatible with electro-weak experimental constraints



Delocalizing fermions

(RC, De Curtis, Dolce, Dominici; Chivukula, Simmons, He, Kurachi)

- Left- and right-handed fermions, ψ_L (ψ_R) are coupled to the ends of the moose, but they can be coupled to any site by using a Wilson line

$$\chi_L^i = \Sigma_i^\dagger \Sigma_{i-1}^\dagger \cdots \Sigma_1^\dagger \psi_L, \quad \chi_L^i \rightarrow U_i \chi_L^i$$



$$b_i \bar{\chi}_L^i \gamma^\mu \left(\partial_\mu + ig_i A_\mu^i + \frac{i}{2} g' (B-L) Y_\mu \right) \chi_L^i$$

Fermion delocalization



(We avoid delocalization of the right-handed fermions. Small terms since they could contribute to right-handed currents constrained by the $K_L - K_S$ mass difference)

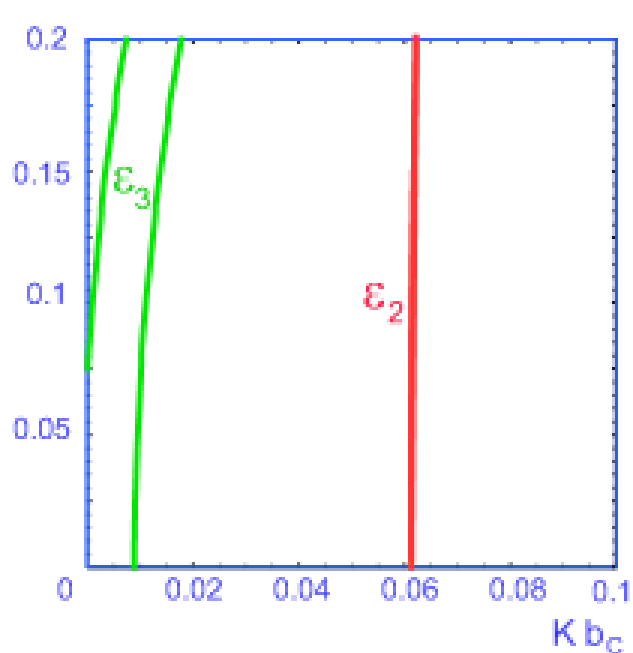
$$\varepsilon_1 \approx 0, \quad \varepsilon_2 \approx 0, \quad \varepsilon_3 \approx \sum_{i=1}^K y_i \left(\frac{g_i^2}{g_c^2} (1 - y_i) - b_i \right)$$

- Possibility of agreement with EW data with some fine tuning

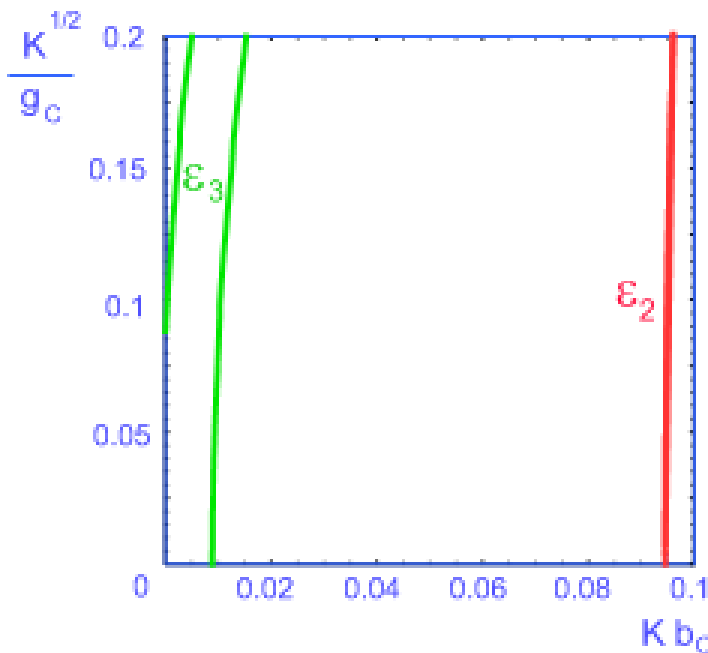
- Two cases:

$$f_i = f_c, \quad g_i = g_c, \quad b_i = b_c$$

K=1



K=10



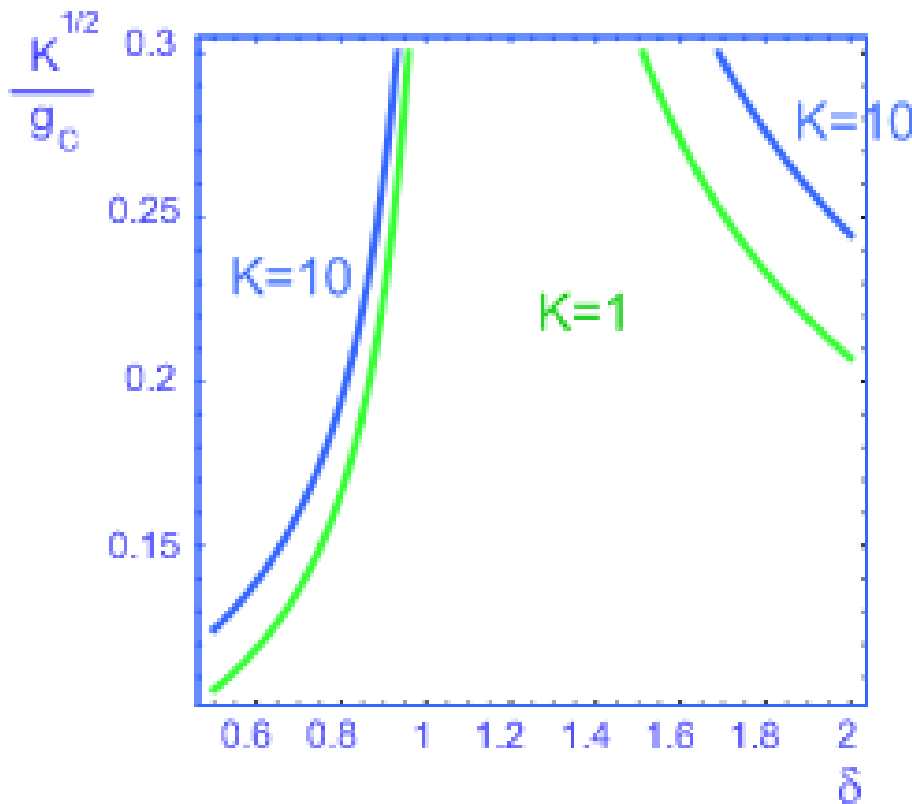
(95% CL, with rad. corr. as in the SM with 1 TeV Higgs and mass 178 GeV for the top)

Local cancellation

$$b_i = \delta \frac{g_i^2}{g_c^2} (1 - y_i), \quad \text{with} \quad g_i = g_c, f_i = f_c$$

⇓

$$b_i = \delta \frac{g^2}{g_i^2} \left(1 - \frac{i}{K+1} \right)$$



(95% CL, with rad. corr. as in the SM with 1 TeV Higgs and mass 178 GeV for the top)

The continuum limit

- Quite clearly the moose picture for large values of K can be interpreted as the discretization of a continuum theory along a fifth direction. The continuum limit is defined by

$$K \rightarrow \infty, \quad a \rightarrow 0, \quad Ka \rightarrow \pi R$$

$$\lim_{a \rightarrow 0} a g_i^2 = g_5^2(y), \quad \lim_{a \rightarrow 0} a f_i^2 = f^2(y), \quad \lim_{a \rightarrow 0} \frac{b_i}{a} = b(y)$$

- The link couplings and a variable gauge coupling can be simulated in the continuum by a non-flat 5-dim metrics. More interestingly, in the continuum limit, the geometrical structure of the moose has an interpretation in terms of a **geometrical Higgs mechanism** in a pure 5-dimensional gauge theory.

- Consider an abelian gauge theory in 4+1 dim

$$L = -\frac{1}{2g_5^2} F_{AB} F^{AB} = -\frac{1}{2g_5^2} F_{\mu\nu} F^{\mu\nu} - \frac{1}{g_5^2} F_{\mu 5} F^{\mu 5}$$

$$F_{\mu 5} = \partial_\mu A_5 - \partial_5 A_\mu$$

through the gauge transformation: $A_B \Rightarrow A_B - (\partial_5)^{-1} (\partial_B A_5)$

we get $A_5 = 0, F_{\mu 5} = -\partial_5 A_\mu, (A_\mu(x, x_5) \approx \sum_n e^{inx_5/R} A_\mu^n(x))$

With a compactified 5 dim on a circle S^1 of length $2\pi R$, the non zero eigenmodes A_μ^n acquire a mass:

$$M_n = \frac{n}{R} \quad \text{absorbing the mode } A_5^n$$

The zero mode remains massless and a GB is present

- Massless modes can be eliminated compactifying on an orbifold, that is

$$S^2 / Z, \quad Z : \mathbf{x}_5 \rightarrow -\mathbf{x}_5$$

- This allows to define fields as eigenstates of parity:

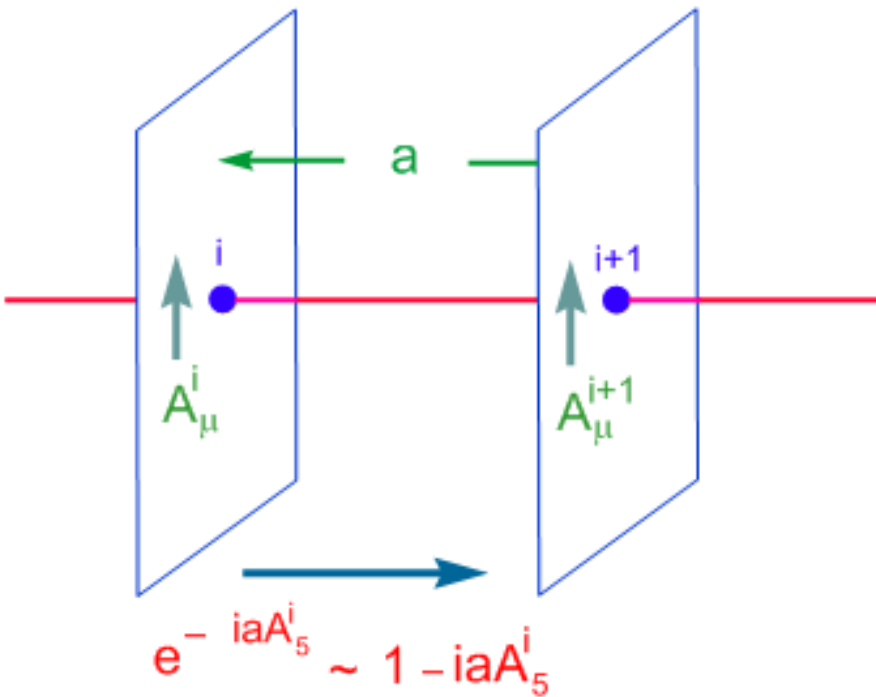
$$A_B(\mathbf{x}_\mu, \mathbf{x}_5) = \pm A_B(\mathbf{x}_\mu, -\mathbf{x}_5)$$

- Various possibilities, for instance by choosing:

A_B odd \Rightarrow no zero mode \Rightarrow only massive gauge bosons
in the spectrum

By making the theory discrete along the fifth dimension one gets back the moose structure. In this case one speaks of a deconstructed gauge theory (Hill, Pokorski, Wang, 2001).

- A gauge field is nothing but a connection: a way of relating the phases of the fields at nearby points. Once we discretize the space the connection is naturally substituted by a **link variable** realizing the parallel transport between two lattice sites



A generalized σ - model where the Higgs mechanism is realized in a standard way in terms of a Σ - field (chiral field)

$$\Sigma \approx 1 - iaA_5 \approx e^{-iaA_5}$$

$$\Sigma \Sigma^\dagger = 1$$

- More exactly:

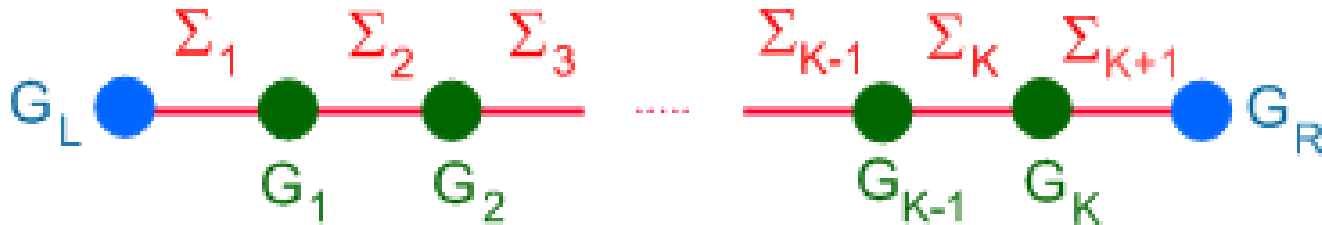
$$\Sigma_i = 1 - iaA_5^{i-1}, \quad \Sigma_i \xrightarrow{U_i \in G_i} U_{i-1} \Sigma_i U_i^\dagger$$

$$D_\mu \Sigma_i = \partial_\mu \Sigma_i - iA_\mu^{i-1} \Sigma_i + i \Sigma_i A_\mu^i = -iaF_{\mu 5}^{i-1}$$

$$F_{\mu 5}^i = \partial_\mu A_5^i - \partial_5 A_\mu^i - i[A_\mu^i, A_5^i]$$

$$S = \int d^4x \frac{a}{g_5^2} \left(-\frac{1}{2} \sum_i \text{Tr} [F_{\mu\nu}^i F^{\mu\nu i}] + \frac{1}{a^2} \text{Tr} [(D_\mu \Sigma_i)(D_\mu \Sigma_i)^\dagger] \right)$$

- Sintetically described by a moose diagram (Georgi, 1986 –Arkani-Hamed, Cohen, Georgi, 2001)



- In order to describe completely the moose structure including also the breaking, one needs also some kinetic terms on the branes plus BC's. In the case of a conformally flat metrics along the fifth direction the complete action for a SU(2)-moose would be

$$S = -\frac{1}{4} \int d^4x \int_0^{\pi R} dz e^{-A(z)} \frac{1}{g_5^2(z)} \left[(F_{\mu\nu}^a)^2 - 2(F_{\mu 5}^a)^2 \right] +$$

$$-\frac{1}{4} \int d^4x \int_0^{\pi R} dz e^{-A(z)} \left[\frac{1}{\tilde{g}^2} (F_{\mu\nu}^a)^2 \delta(z) + \frac{1}{\tilde{g}'^2} (F_{\mu\nu}^3)^2 \delta(z - \pi R) \right]$$

$$\text{BC's: } A_{\mu}^{1,2} \Big|_{z=\pi R} = 0, \quad \partial_z A_{\mu}^a \Big|_{z=0} = 0$$

- Introducing the link variable $\Sigma_i = e^{-iaA_5^i}, \quad i = 1, \dots, K+1$

$$S_{\text{moose}} = \int d^4x \left(-\sum_{i=1}^K \frac{1}{2g_i^2} \text{Tr} \left[F_{\mu\nu}^i F^{\mu\nu i} \right] + \sum_{i=1}^{K+1} f_i^2 \text{Tr} \left[(D_{\mu} \Sigma_i)(D_{\mu} \Sigma_i)^{\dagger} \right] \right)$$

$$ae^{-A_i} / g_{5i}^2 = 1 / g_i^2, \quad e^{-A_i} / (ag_{5i}^2) = f_i^2$$

$$A_{\mu}^1 = W_{\mu}^a \tau_a / 2, \quad A_{\mu}^{K+1} = Y^{\mu} \tau_3 / 2$$

FLAT CASE:

$$f_i = f_c, \quad g_i = g_c, \quad e^{-A_i} = 1, \quad g_{5i}^2 = ag_c^2$$

$$\varepsilon_3 = \frac{1}{6} \frac{g^2}{g_c^2} \frac{K(K+2)}{K} \Rightarrow \frac{1}{6} \frac{g^2}{a g_c^2} a K \Rightarrow \frac{1}{6} \frac{g^2}{g_5^2} \pi R$$

Let us go back to the unitarity limit. The 5-dim theory has a natural cutoff proportional to $1/g_5^2$, say Λ_c , which can be related to the lattice spacing:

$$a = \frac{1}{\Lambda_c} \Rightarrow K \sim \pi R \Lambda_c, \quad K a \Rightarrow \pi R$$

Then, the unitarity cutoff will be

$$\Lambda_U \sim \sqrt{K} \Lambda_{\text{HSM}} \sim \sqrt{\pi R \Lambda_c} \Lambda_{\text{HSM}}$$

If it would be possible to send a to zero (or Λ_c to infinity), the unitarity cutoff would go to infinity. In fact, in the continuum limit there is a complete cancellation of the terms increasing with the energy in the VB scattering (Csaki, Grojean, Murayama, Pilo, Terning). However the 5-dimensional theory is not renormalizable, and therefore the cutoff Λ_c makes the longitudinal vector boson scattering amplitudes not unitary (see later)

The 5-dimensional cutoff turns out to be

$$\Lambda_c = \frac{24\pi^3}{g_5^2} = \frac{12\pi^2}{Rg_4^2} \left(\frac{1}{g_5^2} = \frac{1}{2\pi Rg_4^2} \right)$$

Using $m_W < \frac{1}{R}$ It follows $\Lambda_c > \frac{12\pi^2 m_W}{g_4^2}$

$$\Lambda_c > \frac{3\pi}{2} \frac{g}{g_4^2} \sqrt{\pi} \Lambda_{\text{HSM}} \approx 10 \frac{g}{g_4^2} \Lambda_{\text{HSM}} \left(m_W = \frac{g\Lambda_{\text{HSM}}}{8\sqrt{\pi}} \right)$$

and

$$\Lambda_U \sim \sqrt{\pi R \Lambda_c} \Lambda_{\text{HSM}} = \frac{\sqrt{12\pi^3}}{g_4} \Lambda_{\text{HSM}} \approx \frac{20}{g_4} \Lambda_{\text{HSM}}$$

The cutoff is the smaller between the two. On the other hand

$$\epsilon_3 = \frac{1}{6} \left(\frac{g}{g_5} \right)^2 \pi R = \frac{1}{12} \left(\frac{g}{g_4} \right)^2$$

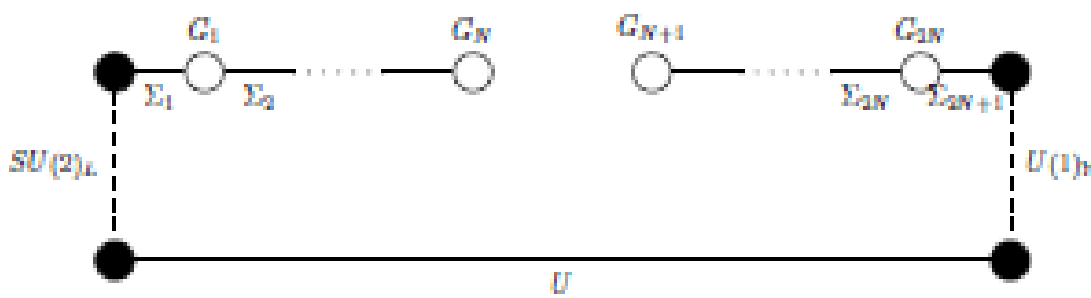
- Summarizing

$$\Lambda_c > 10 \left(\frac{g}{g_4} \right) \Lambda_{\text{HSM}}, \quad \Lambda_U > 20 \frac{1}{g_4} \Lambda_{\text{HSM}},$$

$$\varepsilon_3 = \frac{1}{12} \left(\frac{g}{g_4} \right)^2$$

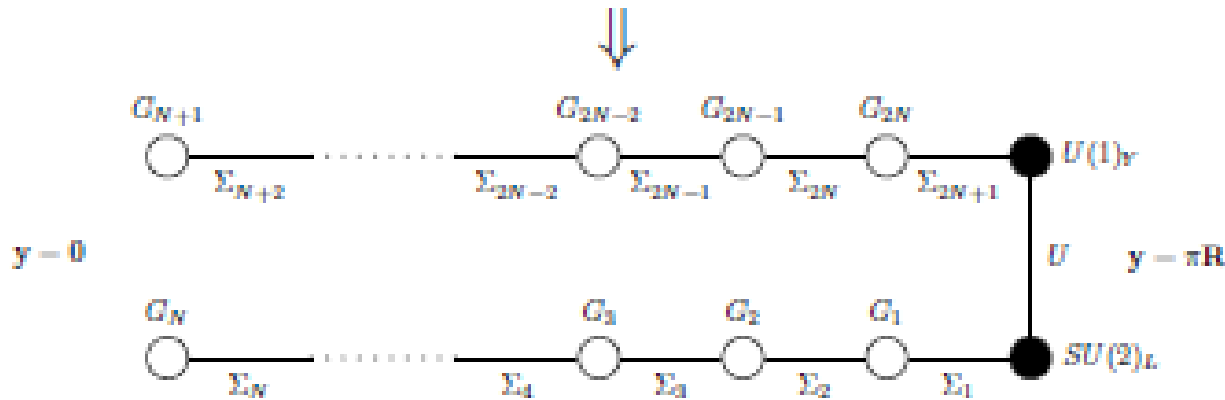
- Therefore we could increase the cutoff of the theory within a perturbative context ($g_4 \sim g$), but this would be unacceptable from the point of view of LEP bounds. On the contrary, if we want to satisfy the LEP bounds we need $g_4 \sim 10 g$, making the cutoff of the order of Λ_{HSM} .

- Introducing fermions in the bulk one gets a positive contribution (Contino, Pomarol; Panico, Serone, Wulzer; Foadi Schmidt) in analogy to the discrete case. Therefore with the help of some fine tuning one can solve both problems.



D – BESS in the continuum limit

(F. Coradeschi, S. De Curtis, D. Dominici, 2010)

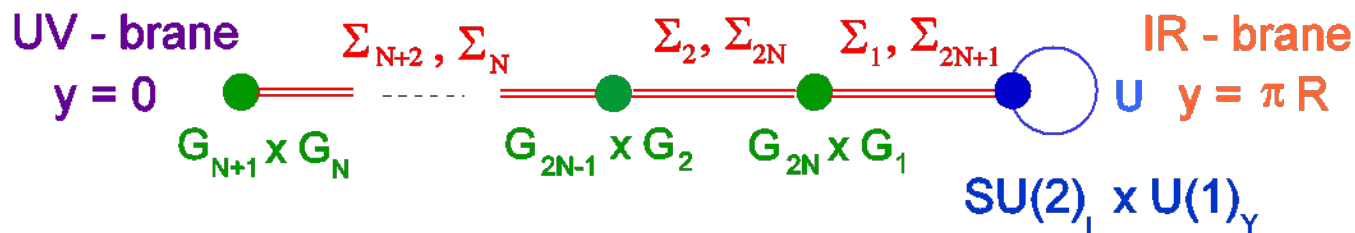


$$G_i = \text{SU}(2)_R \quad i = 1, \dots, N$$

$$G_i = \text{SU}(2)_L \quad i = N+1, \dots, 2N$$

$$\Sigma_i \sim R_5 \quad i = 1, \dots, N$$

$$\Sigma_i \sim L_5 \quad i = N+2, \dots, 2N+1$$

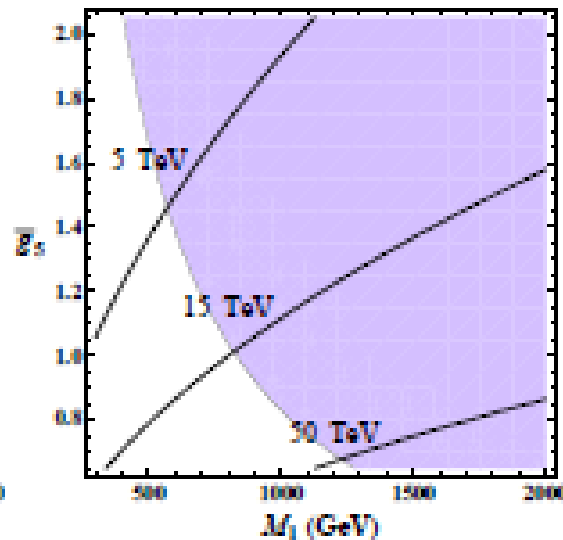
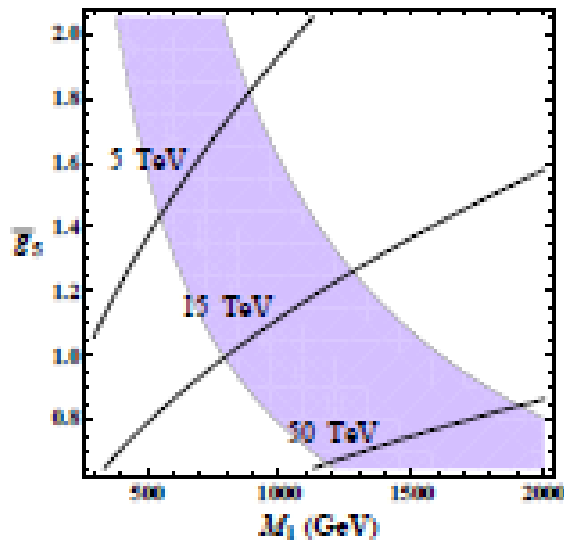
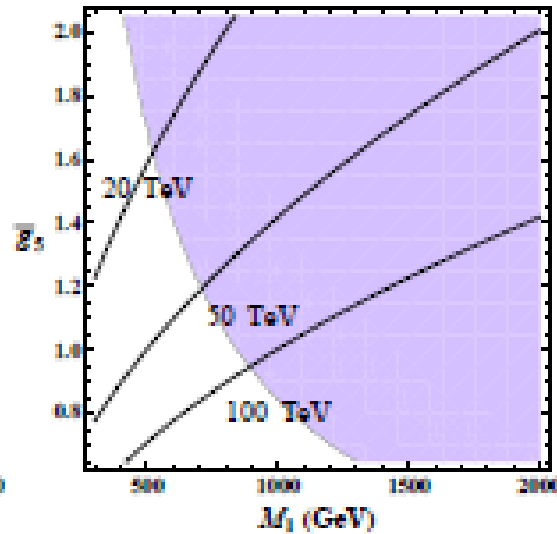
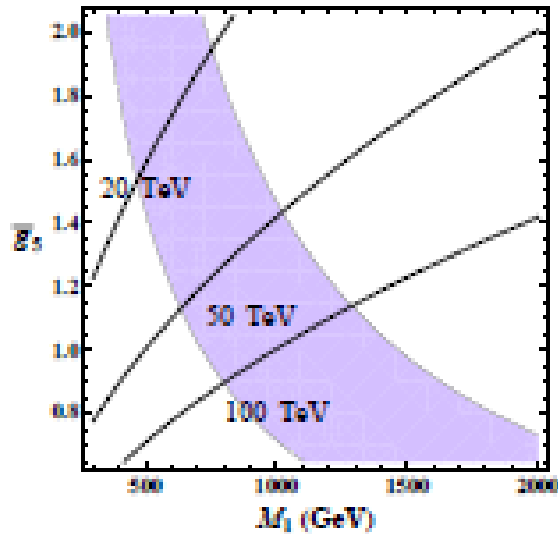


Unitarity limit about 1 TeV as in the Higgsless SM. However one can introduce a **composite Higgs field S** on the IR brane. In this case the UL gets postponed.

$$U \rightarrow SU$$

$1 \text{ TeV} > m_s > 300 \text{ GeV}$

Flat case



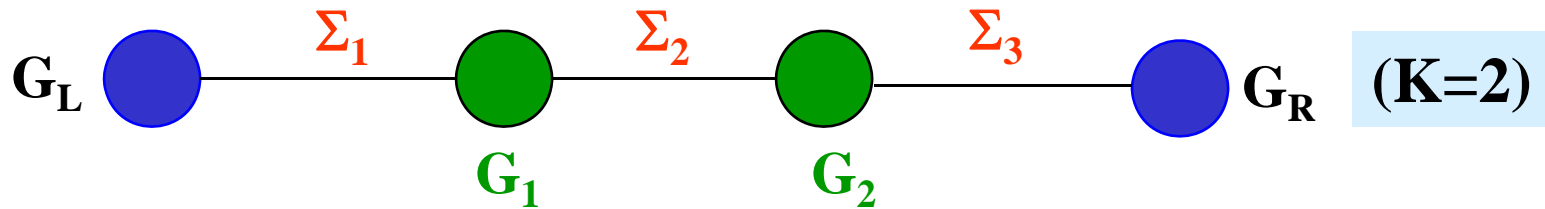
Randall Sundrum case
(In both cases
physical region below
the chosen unitarity
limit)

In practice not much difference between the continuum and discrete moose models, for phenomenological reasons it is simpler the analysis made in the last case. The simplest possibility would be BESS (3-site model), but in order to respect the EW bounds one has to make the VB's almost fermiophobic. However this is not a general feature, for instance, in the 4-site model one can satisfy the EW bounds without having fermiophobic new VB's.

The Higgsless 4-site Linear Moose model

(Accomando, De Curtis, Dominici, Fedeli, 2009)

- 2 gauge groups $G_i = \text{SU}(2)$ with global symmetry $\text{SU}(2)_L \otimes \text{SU}(2)_R$ plus LR symmetry: $g_2 = g_1, f_3 = f_1$
- 6 extra gauge bosons $W_{1,2}^{\prime}$ and $Z_{1,2}^{\prime}$ (have definite parity when $g = g' = 0$)



- 5 new parameters $\{f_1, f_2, b_1, b_2, g_1\}$ related to their masses and couplings to bosons and fermions (one is fixed to reproduce M_Z)

$$f_1, f_2 \rightarrow M_1, M_2$$

$$M_1 = f_1 g_1$$

$$M_2 = \frac{M_1}{z} > M_1$$

$$z = \frac{f_1}{\sqrt{f_1^2 + 2f_2^2}} < 1$$

charged and neutral gauge bosons almost degenerate

$$M_{1,2}^{c,n} \sim M_{1,2} + O\left(\frac{e^2}{g_1^2}\right)$$

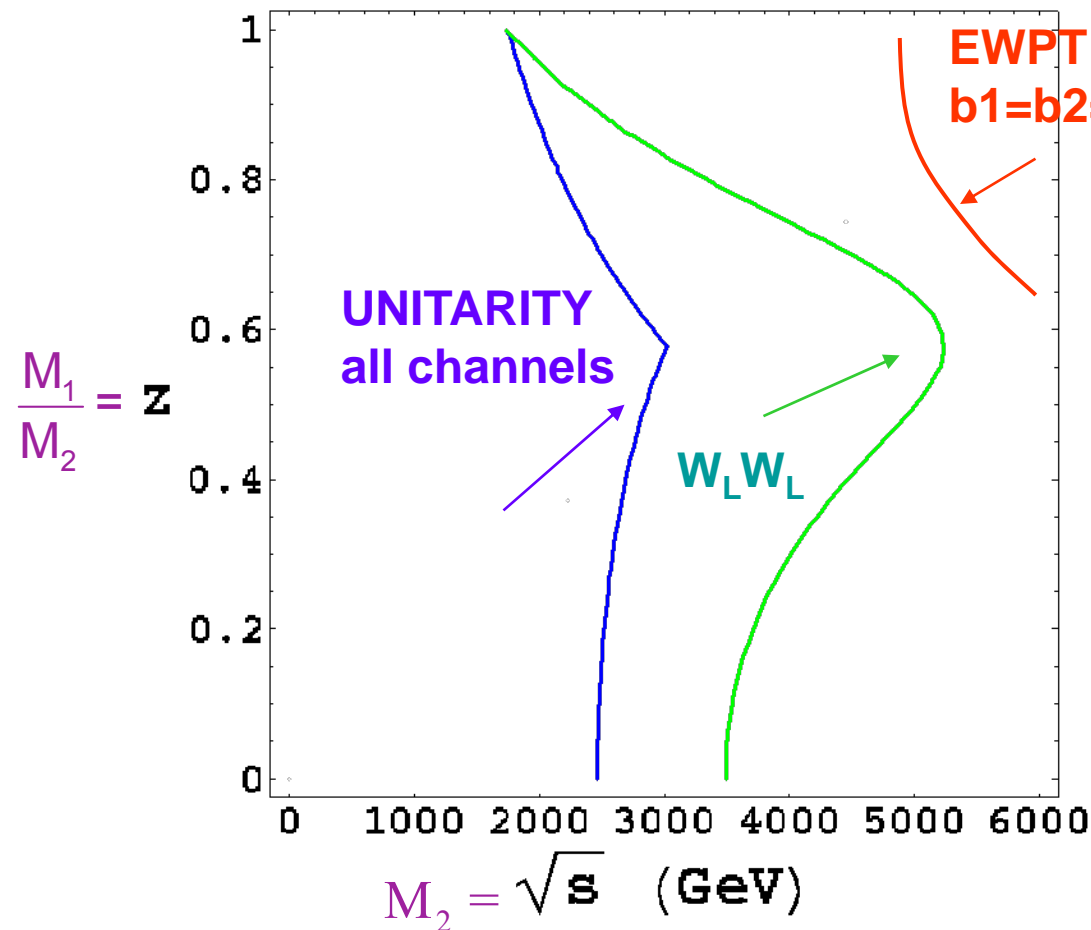
The Higgsless 4-site Linear Moose model

Unitarity and EW precision tests

$$\varepsilon_1 \approx 0 \quad \varepsilon_2 \approx 0, \quad \varepsilon_3 \approx \left(\frac{g^2}{2g_1^2} (1 - z^4) \right)$$

$$O(e^2/g_1^2), \quad b_1=b_2=0$$

Best unitarity limit
for $f_1=f_2$ or $z=1/\sqrt{3}$



Unitarity and EWPT are
not compatible !

A direct coupling of the
new gauge bosons to
ordinary matter must be
included: $b_{1,2} \neq 0$

The Higgsless 4-site Linear Moose model

EW precision tests

Calculations $O(e^2/g_1^2)$, exact in b_1, b_2

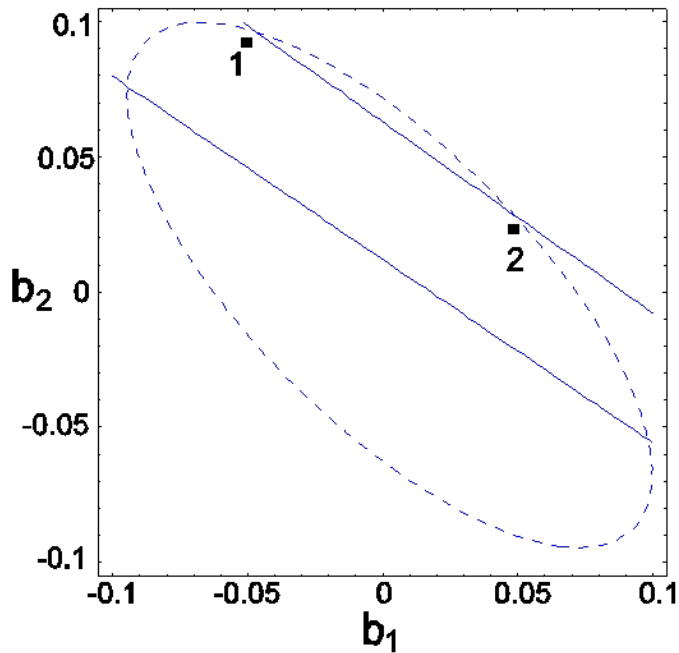
$$\varepsilon_{1,2} \approx O(b^2), \quad \varepsilon_3 \approx \left(\frac{g^2}{2g_1^2} (1 - z^4) - \frac{b}{2} \right)$$

$$b = \frac{b_1 + b_2 - (b_1 - b_2)z^2}{1 + b_1 + b_2}$$

$$M_2 = M_1/z$$

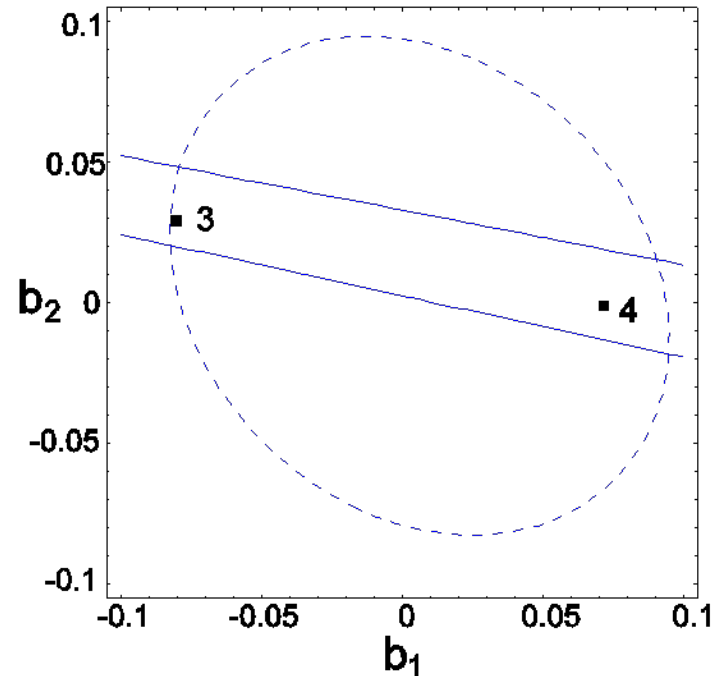
$500 < M_1 < 1000 \text{ GeV}$

$z=0.4$

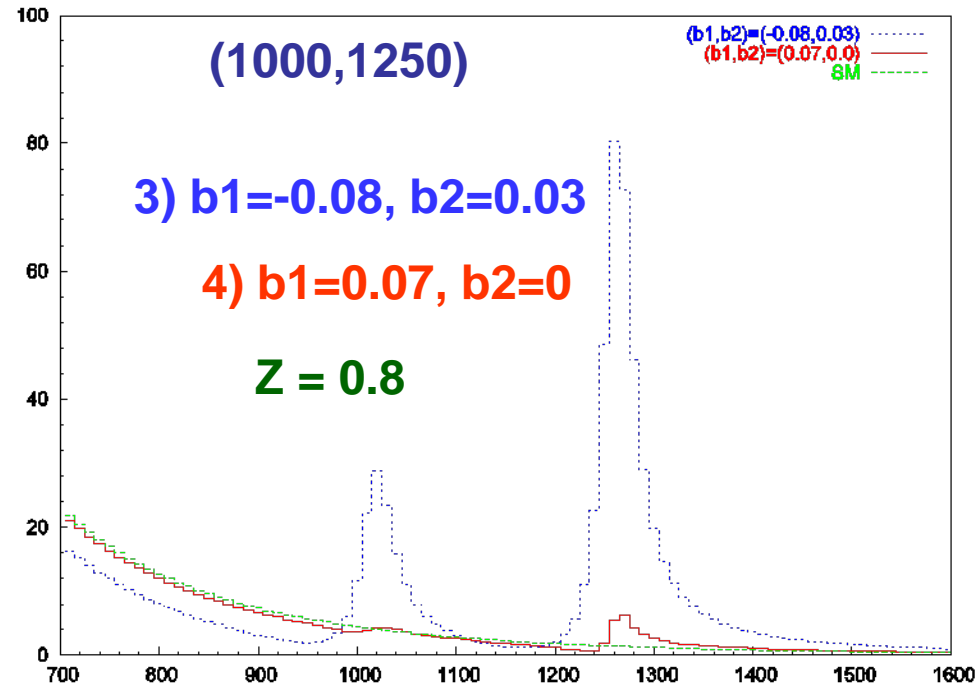
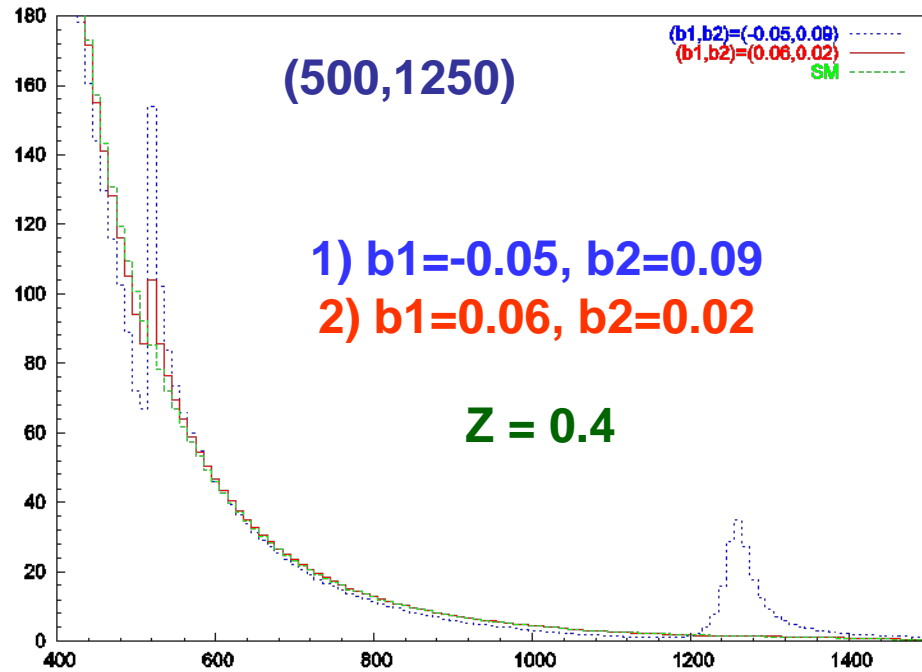


$700 < M_1 < 1600 \text{ GeV}$

$z=0.8$



The Higgsless 4-site Linear Moose model, $Z_{1,2}$ production



Total # of evts in a 10GeV-bin versus $M_{inv}(l+l-)$ for $L=10\text{fb}^{-1}$. Sum over e, μ

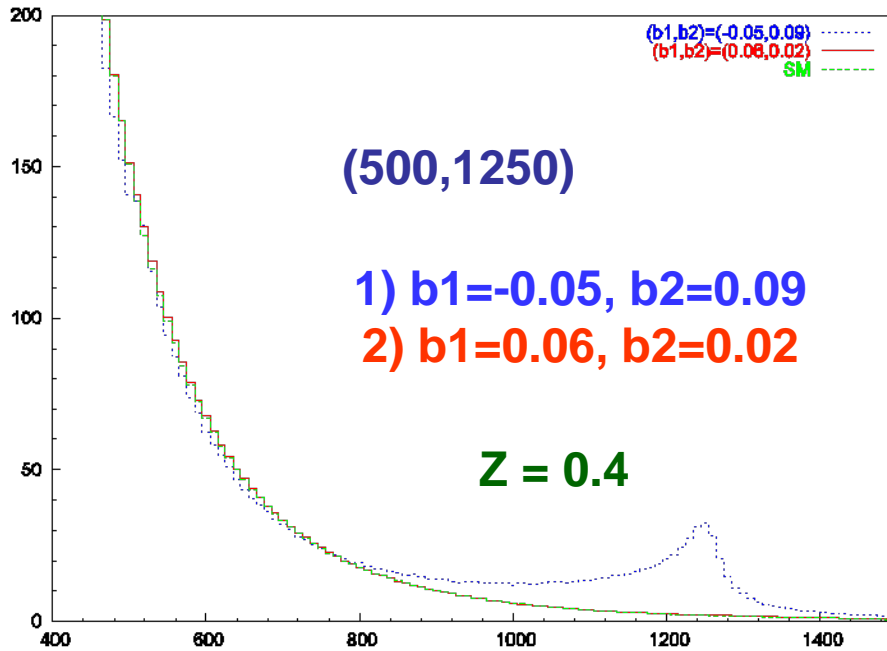
The Higgsless 4-site Linear Moose model, $Z'_{1,2}$ production

	$M_{1,2}$ (GeV)	$b_{1,2}$	$N_{\text{evt}}^{\text{sig}}(Z_1)$	$N_{\text{evt}}^{\text{tot}}(Z_1)$	$\sigma(Z_1)$	$N_{\text{evt}}^{\text{sig}}(Z_2)$	$N_{\text{evt}}^{\text{tot}}(Z_2)$	$\sigma(Z_2)$
1	500,1250	-0.05,0.09	47	154	3.8	134	143	11.2
2	500,1250	0.06,0.02	11	123	1.0	0	9	0.0
3	1732,3000	-0.07,0.04	7	10	2.2	7	8	2.5
4	1732,3000	0.03,-0.04	5	9	1.7	6	6	2.4
5	1000,1250	-0.08,0.03	108	119	9.9	291	302	16.7
6	1000,1250	0.07,0.0	3	28	0.0	15	22	3.2

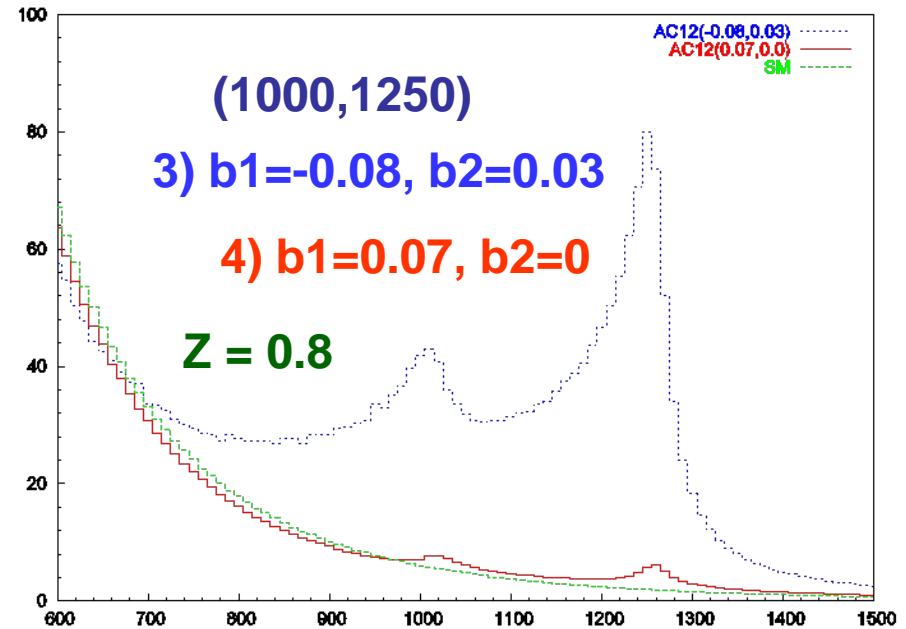
of evts for the $Z'_{1,2}$ DY production within $|M_{\text{inv}}(l+l)-M_i| < \Gamma_i$

$$\sigma = N_{\text{evt}}^{\text{sig}} / \sqrt{N_{\text{evt}}^{\text{tot}}} \text{ for an integrated luminosity } L=10 \text{ fb}^{-1}$$

The Higgsless 4-site Linear Moose model, $W_{1,2}$ production



$$M_2 = M_1/z$$



Total # of evts in a 10GeV-bin versus $M_t(l\nu)$ for $L=10\text{fb}^{-1}$. Sum over e, μ

The Higgsless 4-site Linear Moose model, $W_{1,2}$ production

	$M_{1,2}$ (GeV)	$b_{1,2}$	M_t^{cut} (GeV)	$N_{evt}^{sig}(W_1)$	$N_{evt}^{tot}(W_1)$	$\sigma(W_1)$	$N_{evt}^{sig}(W_2)$	$N_{evt}^{tot}(W_2)$	$\sigma(W_2)$
1	500,1250	-0.05,0.09	400	36	2435	0.7	776	2214	16.5
2	500,1250	0.06,0.02	400	0	2609	0	1	1807	0
3	1732,3000	-0.07,0.04	1500	10	18	2.4	24	26	4.7
4	1732,3000	0.08,-0.04	1500	9	14	2.4	22	24	4.5
5	1000,1250	-0.08,0.03	700	808	1230	23.0	1112	1189	32.3
6	1000,1250	0.07,0.0	700	12	443	0.6	17	88	1.8

of evts for the $W_{1,2}$ DY-production for $M_t(l\nu_l) > M_t^{cut}$

$$\sigma = N_{evt}^{sig} / \sqrt{N_{evt}^{tot}} \text{ for an integrated luminosity } L=10 \text{ fb}^{-1}$$

The **statistical significance** for the W 's production is ~ a **factor 2** bigger than for the Z 's but it is **less clean**.

Neutral and charged channel are complementary

Summary and Conclusions

- Extensions of the nonlinear σ model leads to moose theories.
- Simplest case: the linear moose.
- Higher dimensional gauge theories naturally suggest the possibility of Higgsless theories.
- Difficulties in EW corrections similar to TC models.
- EW corrections and unitarity bounds push in different directions.
- Possibility of easing the theory delocalizing the fermions and using some fine tuning.