Isgur-Wise functions and the Lorentz group

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Introduction

Well-known that the transitions $H_b \to H_c \ell \nu$ like

Meson transitions $\quad \overline{B}_d \to D \ell \nu \qquad \overline{B}_d \to D^* \ell \nu$
Baryon transition $\quad \Lambda_b \to \Lambda_c \ell \nu$

are related to the exclusive determination of $|V_{cb}|$

Many form factors but Heavy Quark Symmetry $SU(2N_f)$
$\to$ form factors given by a single function $\xi(w)$ (IW function)

Tension between inclusive and exclusive determinations of $|V_{cb}|$

But my purpose is only to expose new interesting theoretical results on the properties of the Heavy Quark Effective Theory of QCD not discovered during the development of this theory in the 1990's

Heavy Quark Symmetry

Elastic meson transitions $\quad \overline{B}_d \to D\ell\nu \quad \overline{B}_d \to D^*\ell\nu$

Light cloud $\frac{1}{2}^-$ combines with heavy quark spin $s_Q = \frac{1}{2}$

$\to \quad J^P = 0^-(D)$ and $1^-(D^*)$ ground states

By spin-flavor Heavy Quark Symmetry $SU(2N_f)$ (N_f heavy flavors)

six form factors (f_0, f_+ for $\overline{B} \to D$), (V, A_0, A_1, A_2 for $\overline{B} \to D^*$)

reduce to a single Isgur-Wise function $\xi(w) \quad \left(w = \frac{m_B^2+m_D^2-q^2}{2m_B m_D}\right)$

for the light cloud ($L = 0, s_q = \frac{1}{2}$) transition $\frac{1}{2}^- \to \frac{1}{2}^-$

Excited meson transitions $\quad \overline{B}_d \to D^{**}\ell\nu \quad (L = 1,\ D^{**}$ of $P = +)$

$L = 1, s_q = \frac{1}{2}$: light cloud transitions $\frac{1}{2}^- \to \frac{1}{2}^+$ and $\frac{1}{2}^- \to \frac{3}{2}^+$

two IW functions $\tau_{1/2}(w), \tau_{3/2}(w) \qquad D^{**} : 0^+_{1/2}, 1^+_{1/2}, 1^+_{3/2}, 2^+_{3/2}$

Exclusive determination of $|V_{cb}|$

$$\frac{d\Gamma(\overline{B}\to D^*\ell\nu)}{dw} = \frac{G_F^2}{48\pi^3}(m_B - m_{D^*})^2 m_{D^*}^3 K(w,r)|V_{cb}|^2|\mathcal{F}^*(1)|^2|\xi(w)|^2$$

$$r = \frac{m_{D^*}}{m_B}, w = \frac{m_B^2+m_D^2-q^2}{2m_B m_D}, w = 1 \to q^2_{max} = (m_B - m_{D^*})^2$$

$$\mathcal{F}^*(1) = \eta_A\left(1+\delta_{1/m^2}+...\right) = 0.924 \pm 0.012 \pm 0.019 \qquad \text{(lattice QCD)}$$

$$\xi(1) = 1, \qquad \xi'(1) = -\rho^2$$

$$|V_{cb}| = (38.7 \pm 0.7 \pm 0.9) \times 10^{-3} \qquad \text{(HFAG 2007)}$$

Great dispersion of data in the $(|V_{cb}|, \rho^2)$ plane

Inclusive determination $\qquad |V_{cb}| = (41.7 \pm 0.4 \pm 0.6) \times 10^{-3}$

[Buchmüller and Flächer (2005-2007), from Bigi et al., Bauer et al.]

$m_b = 4.59\ GeV$, $m_c = 1.14\ GeV$, $\mu_G^2 = 0.35\ GeV^2$, $\mu_\pi^2 = 0.40\ GeV^2$

Different hadronic uncertainties in inclusive vs. exclusive methods

Bjorken and Uraltsev Sum Rules

Bjorken SR $\rho^2 = \frac{1}{4} + \sum_n \left[|\tau_{1/2}^{(n)}(1)|^2 + 2|\tau_{3/2}^{(n)}(1)|^2 \right] \quad \rightarrow \quad \rho^2 \geqslant \frac{1}{4}$

Uraltsev SR $\sum_n \left[|\tau_{3/2}^{(n)}(1)|^2 - |\tau_{1/2}^{(n)}(1)|^2 \right] = \frac{1}{4}$

Bjorken (1990-1991) + Uraltsev (2001) $\rightarrow \quad \rho^2 \geqslant \frac{3}{4}$

Bound obtained in Bakamjian-Thomas quark models
(Le Yaouanc et al. 1996)

- covariant for $m_Q \rightarrow \infty$
- explicit Isgur-Wise scaling
- satisfying Bjorken and Uraltsev SR

Operator Product Expansion

$T = i\int d^4x e^{-iq.x} < \overline{B}|T[J(x)J^+(0)]|\overline{B} >$ $\qquad J = \overline{c}\Gamma b$

$T \sim \sum_X \frac{|<X|J(0)|\overline{B}>|^2}{m_B - q^0 - E_X}\delta(\mathbf{p}_X + \mathbf{q}) - \sum_{X'} \frac{|<X' B\overline{B}|J^+(0)|\overline{B}>|^2}{m_B + q^0 - (E_{X'} + 2m_B)}\delta(\mathbf{p}_{X'} - \mathbf{q})$

Direct channel virtuality $\qquad \mathcal{V} = m_B - q^0 - E_X$

Choose q^0 such that $\qquad \Lambda_{QCD} \ll \mathcal{V} \ll m_B$

Crossed channel denominator $\qquad \mathcal{V} + 2m_D \gg \mathcal{V}$

Leading contribution to the OPE

$T = i\int d^4x e^{-iq.x} < \overline{B}|\overline{b}(x)\Gamma^+ S_c^{free}(x,0)\Gamma b(0)|\overline{B} > + \ O(1/m_c^2)$

Varying independently $\mathcal{V}, m_b, m_c$ and equating residues

$\sum_{X_c} | < X_c|J(0)|\overline{B} > |^2 = < \overline{B}|\overline{b}\ \overline{\Gamma}\frac{\not{v}_c' + 1}{2v_c'^0}\Gamma b(0)|\overline{B} >$

$\frac{\not{v}_c' + 1}{2v_c'^0}$: positive energy residue of c quark propagator

Isgur-Wise functions and Sum Rules in HQET

(Bjorken; Isgur and Wise; Uraltsev; Le Yaouanc et al.)

Case of mesons : consider the non-forward amplitude

$$\overline{B}(v_i) \to D^{(n)}(v') \to \overline{B}(v_f) \quad (w_i = v_i \cdot v', w_f = v_f \cdot v', w_{if} = v_i \cdot v_f)$$

SR obtained from the OPE

$$L_{Hadrons}(w_i, w_f, w_{if}) = R_{OPE}(w_i, w_f, w_{if})$$

$L_{Hadrons}$: sum over $D^{(n)}$ states $\qquad$ R_{OPE} : OPE counterpart

$$\sum_{D=P,V} \sum_n Tr[\overline{B}_f(v_f)\Gamma_f D^{(n)}(v')]\, Tr[\overline{D}^{(n)}(v')\Gamma_i B_i(v_i)]\xi^{(n)}(w_i)\xi^{(n)}(w_f)$$

$$+ \ Other\ excited\ states = -2\xi(w_{if})\ Tr[\overline{B}_f(v_f)\Gamma_f P'_+ \Gamma_i B_i(v_i)]$$

$P'_+ = \frac{1+\not{v}'}{2}$: positive energy projector on the intermediate c

Light cloud angular momentum j and bound state spin J

$\overline{B}$: pseudoscalar ground state $(j^P, J^P) = \left(\frac{1}{2}^-, 0^-\right)$
$D^{(n)}$: tower $(j^P, J^P), J = j \pm \frac{1}{2}, j = L \pm \frac{1}{2}, P = (-1)^{L+1}$

Heavy quark currents : $\overline{h}_{v'}\Gamma_i h_{v_i}$ $\overline{h}_{v_f}\Gamma_f h_{v'}$

Domain of the variables (w_i, w_f, w_{if}) :

$w_i \geq 1$ $w_f \geq 1$

$$w_i w_f - \sqrt{(w_i^2-1)(w_f^2-1)} \leq w_{if} \leq w_i w_f + \sqrt{(w_i^2-1)(w_f^2-1)}$$

For $w_i = w_f = w$, the domain becomes :

$w \geq 1$ $1 \leq w_{if} \leq 2w^2 - 1$

$$\Gamma_i = \not{v}_i \qquad \Gamma_f = \not{v}_f \qquad \rightarrow \qquad \textit{Vector SR}$$

$$(w+1)^2 \sum_{L\geq 0} \frac{L+1}{2L+1}\ S_L(w, w_{if}) \sum_n \left[\tau_{L+1/2}^{(L)(n)}(w)\right]^2$$
$$+ \sum_{L\geq 1} S_L(w, w_{if}) \sum_n \left[\tau_{L-1/2}^{(L)(n)}(w)\right]^2 = (1 + 2w + w_{if})\, \xi(w_{if})$$

$$\Gamma_i = \not{v}_i\gamma_5 \qquad \Gamma_f = \not{v}_f\gamma_5 \qquad \rightarrow \qquad \textit{Axial SR}$$

$$\sum_{L\geq 0} S_{L+1}(w, w_{if}) \sum_n \left[\tau_{L+1/2}^{(L)(n)}(w)\right]^2$$
$$+ (w-1)^2 \sum_{L\geq 1} \frac{L}{2L-1} S_{L-1}(w, w_{if}) \sum_n \left[\tau_{L-1/2}^{(L)(n)}(w)\right]^2$$
$$= -(1 - 2w + w_{if})\, \xi(w_{if})$$

IW functions $\tau_{L\pm 1/2}^{(L)(n)}(w)$: $\frac{1}{2}^- \rightarrow \left(L \pm \frac{1}{2}\right)^P, P = (-1)^{L+1}$

$S_L(w, w_{if})$ is a Legendre polynomial :

$$S_L(w, w_{if}) = \sum_{0 \leq k \leq L/2} C_{L,k} \left(w^2 - 1\right)^{2k} \left(w^2 - w_{if}\right)^{L-2k}$$

$$C_{L,k} = (-1)^k \frac{(L!)^2}{(2L)!} \; \frac{(2L-2k)!}{k!(L-k)!(L-2k)!}$$

Differentiating the Sum Rules $\left[\frac{d^{p+q}(L_{Hadrons} - R_{OPE})}{dw_{if}^p dw^q}\right]_{w_{if}=w=1} = 0$

(going to the frontier of the domain $w \to 1$, $w_{if} \to 1$)

one finds constraints on the derivatives $\xi^{(n)}(1)$, in particular

$$\rho^2 = -\xi'(1) \geq \frac{3}{4} \qquad \xi''(1) \geq \frac{1}{5}\left[4\rho^2 + 3(\rho^2)^2\right]$$

Consideration of the non-forward amplitude (Uraltsev)
$\overline{B}(v_i) \to D^{(n)}(v') \to \overline{B}(v_f)$
allows to improve Bjorken's bound $\rho^2 \geq \frac{1}{4}$

Details of the calculations of the sum rules and bounds

- The excited states of arbitrary spin (Falk 1992)
- Calculation of the polynomial $S_L(w_i, w_f, w_{if})$ (Le Yaouanc et al. 2002)
- Simple derivation of Bjorken and Uraltsev SR (Le Yaouanc et al. 2002)
- Generalizations for higher derivatives (Le Yaouanc et al. 2002)
- Proof of improved bound on the curvature (Le Yaouanc et al. 2003)
- The Isgur-Wise function in the BPS limit (Jugeau et al. 2006)
- Radiative corrections (Dorsten 2003)
- Phenomenology (Dorsten 2003)

4×4 matrices for states of arbitrary spin

L : orbital angular momentum of light clound of half-integer spin j

$k = j - \frac{1}{2}$

- $j = L + \frac{1}{2}, J = j + \frac{1}{2}$ $\qquad \mathcal{M}^{\mu_1, \cdots \mu_k}(v) = P_+ \epsilon^{\mu_1, \cdots \mu_{k+1}} \gamma_{\mu_{k+1}}$

- $j = L + \frac{1}{2}, J = j - \frac{1}{2}$ $\qquad \mathcal{M}^{\mu_1, \cdots \mu_k}(v) = -\sqrt{\frac{2k+1}{k+1}} P_+ \gamma_5 \epsilon^{\nu_1, \cdots \nu_k}$

$\times \left[g^{\mu_1}_{\nu_1} ... g^{\mu_k}_{\nu_k} - \frac{1}{k+1} \left[\gamma_{\nu_1} (\gamma^{\mu_1} - v^{\mu_1}) g^{\mu_2}_{\nu_2} ... g^{\mu_k}_{\nu_k} + g^{\mu_1}_{\nu_1} ... g^{\mu_{k-1}}_{\nu_{k-1}} \gamma_{\nu_k} (\gamma^{\mu_k} - v^{\mu_k}) \right] \right]$

- $j = L - \frac{1}{2}, J = j + \frac{1}{2}$ $\qquad \mathcal{M}^{\mu_1, \cdots \mu_k}(v) = P_+ \epsilon^{\mu_1, \cdots \mu_{k+1}} \gamma_5 \gamma_{\mu_{k+1}}$

- $j = L - \frac{1}{2}, J = j - \frac{1}{2}$ $\qquad \mathcal{M}^{\mu_1, \cdots \mu_k}(v) = \sqrt{\frac{2k+1}{k+1}} P_+ \epsilon^{\nu_1, \cdots \nu_k}$

$\times \left[g^{\mu_1}_{\nu_1} ... g^{\mu_k}_{\nu_k} - \frac{1}{k+1} \left[\gamma_{\nu_1} (\gamma^{\mu_1} - v^{\mu_1}) g^{\mu_2}_{\nu_2} ... g^{\mu_k}_{\nu_k} + g^{\mu_1}_{\nu_1} ... g^{\mu_{k-1}}_{\nu_{k-1}} \gamma_{\nu_k} (\gamma^{\mu_k} - v^{\mu_k}) \right] \right]$

Calculation of the polynomial $S_L(w_i, w_f, w_{if})$

$$S_L(w_i, w_f, w_{if}) = v_{f\nu_1} \ldots v_{f\nu_L} T^{v_{f\nu_1} \cdots v_{f\nu_L}, v_{i\mu_1} \cdots v_{i\mu_L}} v_{i\mu_1} \ldots v_{i\mu_L}$$

Projector on polarization tensor of integer spin L

$T^{v_{f\nu_1} \cdots v_{f\nu_L}, v_{i\mu_1} \cdots v_{i\mu_L}} = \sum_\lambda \epsilon'^{(\lambda)*\nu_1 \ldots \nu_L} \epsilon'^{(\lambda)\mu_1 \ldots \mu_L}$ (depends on v')

Polarization tensor $\epsilon'^{(\lambda)\mu_1 \ldots \mu_L}$ is symmetric, traceless and transverse

$g_{\mu_i \mu_j} \epsilon'^{(\lambda)\mu_1 \ldots \mu_L} = v'_{\mu_i} \epsilon'^{(\lambda)\mu_1 \ldots \mu_L} = 0$ Examples of projector :

$$L = 1 \qquad T^{\mu\nu} = -g^{\mu\nu} + v'^\mu v'^\nu$$

$$\begin{aligned} L = 2 \qquad T^{\mu\nu,\rho\sigma} = \tfrac{1}{6}[&-2g^{\mu\nu} g^{\rho\sigma} + 3\left(g^{\mu\rho} g^{\nu\sigma} + g^{\mu\sigma} g^{\nu\rho}\right) \\ &+2\left(g^{\mu\nu} v'^\rho v'^\sigma + g^{\rho\sigma} v'^\mu v'^\nu\right) + 4 v'^\mu v'^\nu v'^\rho v'^\sigma \\ &-3\left(g^{\mu\rho} v'^\nu v'^\sigma + g^{\nu\sigma} v'^\mu v'^\rho + g^{\nu\rho} v'^\mu v'^\sigma + g^{\mu\sigma} v'^\nu v'^\rho\right)] \end{aligned}$$

$$S_L(w_i, w_f, w_{if}) = \sum_{0 \leq k \leq L/2} C_{L,k} (w_i^2 - 1)^k (w_f^2 - 1)^k (w_i w_f - w_{if})^{L-2k}$$

$$C_{L,k} = (-1)^k \frac{(L!)^2}{(2L)!} \; \frac{(2L-2k)!}{k!(L-k)!(L-2k)!}$$

Sketch of the demonstration

Reduce to a three-dimensional problem at rest

$$v' = (1, \mathbf{0}), v_i = (\sqrt{1+\mathbf{v}_i^2}, \mathbf{v}_i), v_f = (\sqrt{1+\mathbf{v}_f^2}, \mathbf{v}_f) \ \rightarrow \ T^{j_1,\dots j_L, i_1 \dots i_L}$$

Couple L angular momenta $\vec{1}$ into total $\vec{L}$

$$S_L(\mathbf{v}_i^2, \mathbf{v}_f^2, \mathbf{v}_i.\mathbf{v}_f) = \sum_{j_1\dots j_L} \sum_{k_1\dots k_L} v_f^{k_1} \dots v_f^{k_L} T^{k_1,\dots k_L, j_1 \dots j_L} v_i^{j_1} \dots v_i^{j_L}$$

$$= \frac{2^L (L!)^2}{(2L+1)!} 4\pi \sum_{M=-L}^{M=L} \mathcal{Y}_L^M(\mathbf{v}_f)^* \mathcal{Y}_L^M(\mathbf{v}_i) = \frac{2^L (L!)^2}{(2L)!} |\mathbf{v}_i|^L |\mathbf{v}_f|^L P_L(\hat{\mathbf{v}}_i.\hat{\mathbf{v}}_f)$$

$$S_L(\mathbf{v}_i^2, \mathbf{v}_f^2, \mathbf{v}_i.\mathbf{v}_f) = \sum_{0 \leqslant k \leqslant \frac{L}{2}} \frac{(L!)^2}{(2L)!} (-1)^k \frac{(2L-2k)!}{k!(L-k)!(L-2k)!} (\mathbf{v}_i^2)^k (\mathbf{v}_f^2)^k (\mathbf{v}_i.\mathbf{v}_f)^{L-2k}$$

Covariant $\quad \rightarrow \quad \mathbf{v}_i^2 = w_i^2 - 1, \ \mathbf{v}_f^2 = w_f^2 - 1, \ \mathbf{v}_i.\mathbf{v}_f = w_i w_f - w_{if}$

Simple derivation of Bjorken and Uraltsev SR

Differentiating the Sum Rules $\left[\frac{d^{p+q}(L_{Hadrons}-R_{OPE})}{dw_{if}^{p}dw^{q}}\right]_{w_{if}=w=1} = 0$

Vector ($\Gamma_i = \not{v}_i, \Gamma_f = \not{v}_f$), Axial ($\Gamma_i = \not{v}_i\gamma_5, \Gamma_f = \not{v}_f\gamma_5$) currents

Vector + Axial SR : $\xi^{(L)}(1) =$
$\frac{1}{4}(-1)^L L! \sum_n \left[\frac{L+1}{2L+1}4[\tau_{L+1/2}^{(L)(n)}(1)]^2 + [\tau_{L-1/2}^{(L-1)(n)}(1)]^2 + [\tau_{L-1/2}^{(L)(n)}(1)\right]^2$

$L = 1 \;\rightarrow\;$ Bjorken SR $\qquad \rho^2 = \frac{1}{4} + \sum_n \left[|\tau_{1/2}^{(n)}(1)|^2 + 2|\tau_{3/2}^{(n)}(1)|^2\right]$

Axial SR : $\qquad \xi^{(L)}(1) = (-1)^L L! \sum_n [\tau_{L+1/2}^{(L)(n)}(1)]^2$

Combining both :

$$\sum_n \left[\frac{L}{2L+1}[\tau_{L+1/2}^{(L)(n)}(1)]^2 - \frac{1}{4}[\tau_{L-1/2}^{(L)(n)}(1)]^2\right] = \sum_n \frac{1}{4}[\tau_{L-1/2}^{(L-1)(n)}(1)]^2$$

$L = 1 \;\rightarrow\;$ Uraltsev SR $\qquad \sum_n \left[|\tau_{3/2}^{(n)}(1)|^2 - |\tau_{1/2}^{(n)}(1)|^2\right] = \frac{1}{4}$

Generalizations for higher derivatives

Combination of Vector and Axial Sum Rules

$$(-1)^L \xi^{(L)}(1) = L!\,\frac{2L+1}{4L} \sum_n \left[[\tau_{L-1/2}^{(L-1)(n)}(1)]^2 + [\tau_{L-1/2}^{(L)(n)}(1)]^2 \right]$$

Slope (L = 1)

$$\rho^2 = -\xi'(1) = \frac{3}{4}[1 + [\tau_{1/2}^{1)(n)}(1)]^2] \;\rightarrow\; \rho^2 \geqslant \frac{3}{4}$$

Curvature (L = 2)

$$\sigma^2 = \xi''(1) = \frac{5}{4} \sum_n \left[[\tau_{3/2}^{(1)(n)}(1)]^2 + [\tau_{3/2}^{(2)(n)}(1)]^2 \right]$$

$$\geqslant \frac{5}{4} \sum_n [\tau_{3/2}^{(1)(n)}(1)]^2 = \frac{5}{4}\rho^2 \geqslant \frac{15}{16}$$

L-th derivative

$$(-1)^L \xi^{(L)}(1) \geqslant \frac{2L+1}{4} (-1)^{L-1} \xi^{(L-1)}(1) \geqslant \frac{(2L+1)!!}{2^{2L}}$$

Improved bound on the curvature

$$\left[\frac{d^{p+q}L^V_{Hadrons}}{dw^p_{if}dw^q}\right]_{w_{if}=w=1} = \left[\frac{d^{p+q}R^V_{OPE}}{dw^p_{if}dw^q}\right]_{w_{if}=w=1} = 0 \qquad \text{(p+q = 0,1,2)}$$

$$\left[\frac{d^{p+q}L^A_{Hadrons}}{dw^p_{if}dw^q}\right]_{w_{if}=w=1} = \left[\frac{d^{p+q}R^A_{OPE}}{dw^p_{if}dw^q}\right]_{w_{if}=w=1} = 0 \qquad \text{(p+q = 0,1,2,3)}$$

4 linearly independent equations for the curvature $\sigma^2 = \xi''(1)$

Eliminating the non-positive quantities :

$\rho^2 - \frac{5}{4}\sigma^2 + \sum_n [\tau_{3/2}^{(1)(n)}(1)]^2 = 0 \quad \rightarrow \quad \sigma^2 \geqslant \frac{5}{4}\rho^2$ (see above)

$\frac{4}{3}\rho^2 + (\rho^2)^2 - \frac{5}{3}\sigma^2 + \sum_{n\neq 0}[\xi'^{(n)}(1)]^2 = 0$ ($\frac{1}{2}^-$ excited states)

$\rightarrow \qquad \sigma^2 \geqslant \frac{1}{5}\left[4\rho^2 + 3(\rho^2)^2\right]$ new improved bound

since $\rho^2 \geqslant \frac{3}{4}$ both bounds imply $\sigma^2 \geqslant \frac{15}{16}$

term $\frac{3}{5}(\rho^2)^2$ dominant in non-relativistic limit for the light quark

The Isgur-Wise function in the BPS limit

Matrix elements of dimension 5 operators in HQET

$\mu_\pi^2 = -\frac{1}{2m_B} < \overline{B}|\overline{h}_v(iD)^2 h_v|\overline{B} >$ kinetic operator

$\mu_G^2 = \frac{1}{2m_B} < \overline{B}|\frac{g_s}{2}\overline{h}_v \sigma_{\alpha\beta} G^{\alpha\beta} h_v|\overline{B} >$ chromomagnetic operator

Sum Rules in terms of $\frac{1}{2}^- \to \frac{1}{2}^+, \frac{3}{2}^+$ IW functions $\tau_j^{(n)}$ and level spacings $\Delta E_j^{(n)}$ (Bigi et al., 1995) :

$$\mu_\pi^2 = 6 \sum_n [\Delta E_{3/2}^{(n)}]^2 [\tau_{3/2}^{(n)}(1)]^2 + 3 \sum_n [\Delta E_{1/2}^{(n)}]^2 [\tau_{1/2}^{(n)}(1)]^2$$

$$\mu_G^2 = 6 \sum_n [\Delta E_{3/2}^{(n)}]^2 [\tau_{3/2}^{(n)}(1)]^2 - 6 \sum_n [\Delta E_{1/2}^{(n)}]^2 [\tau_{1/2}^{(n)}(1)]^2$$

Inequality $\mu_\pi^2 \geqslant \mu_G^2$ (expt. $\mu_\pi^2 \cong 0.40\ GeV^2$, $\mu_G^2 \cong 0.35\ GeV^2$)

BPS limit of HQET

$\mu_\pi^2 = \mu_G^2 \quad \rightarrow \quad \tau_{1/2}^{(n)}(1) = 0 \qquad$ (Uraltsev, 2001)

Limit of HQET $\quad (\vec{\sigma}.i\vec{D})h_v|\overline{B}> = 0$ (small components in $\overline{B} \rightarrow 0$)

Covariant form $\quad \gamma_5 i\not{D} h_v|\overline{B}> = 0 \quad$ (eq. of motion $\;(iD.v)h_v = 0$)

$\gamma_5 i\not{D}\gamma_5 i\not{D} = -\left[(iD)^2 + \frac{g_s}{2}\sigma_{\alpha\beta}G^{\alpha\beta}\right] \quad \rightarrow \quad \mu_\pi^2 = \mu_G^2$

Leading and subleading matrix elements $\quad \left(\frac{1}{2}^-, 0^-\right) \rightarrow \left(\frac{1}{2}^+, 0^+\right)$

$$< D(0^+)(v')|\overline{h}_{v'}^{(c)}\Gamma h_v^{(b)}|\overline{B}(v)> = 2\tau_{1/2}(w)\, Tr\left[P'_+\Gamma P_+(-\gamma_5)\right]$$

$$< D(0^+)(v')|\overline{h}_{v'}^{(c)}\Gamma i\overrightarrow{\mathcal{D}}_\lambda h_v^{(b)}|\overline{B}(v)> = Tr\left[S_\lambda^{(b)}P'_+\Gamma P_+(-\gamma_5)\right]$$

$$< D(0^+)(v')|\overline{h}_{v'}^{(c)} i\overleftarrow{\mathcal{D}}_\lambda \Gamma h_v^{(b)}|\overline{B}(v)> = Tr\left[S_\lambda^{(c)}P'_+\Gamma P_+(-\gamma_5)\right]$$

$$S_\lambda^{(Q)} = \zeta_1^{(Q)}v_\lambda + \zeta_2^{(Q)}v'_\lambda + \zeta_3^{(Q)}\gamma_\lambda$$

Shape of the Isgur-Wise function in the BPS limit of HQET

Eq. of motion + translational invariance : $\zeta_3^{(b)(n)}(1) = -\Delta E_{1/2}^{(n)} \tau_{1/2}^{(1)(n)}(1)$

$i\partial_\lambda < D(0^+)(v')|\overline{h}_{v'}^{(c)} \Gamma h_v^{(b)}|\overline{B}(v) > = (\overline{\Lambda} v_\lambda - \overline{\Lambda}^* v'_\lambda)\ 2\tau_{1/2}(w) Tr[P'_+ \Gamma P_+ (-\gamma_5)]$

BPS $< D(0^+)(v')|\overline{h}_{v'}^{(c)} \Gamma i\overrightarrow{\mathcal{D}}_\lambda h_v^{(b)}|\overline{B}(v) > = 0 \ \rightarrow \ \zeta_3^{(b)(n)}(1) = 0$

$\rightarrow \ \tau_{1/2}^{(1)(n)}(1) = 0 \ \rightarrow \ \rho^2 = \frac{3}{4}$ (from Bjorken + Uraltsev SR)

BPS with two derivatives $\rightarrow \ \tau_{3/2}^{(2)(n)}(1) = 0 \ \rightarrow \ \sigma^2 = \frac{15}{16}$

To generalize need to demonstrate $\tau_{L-1/2}^{(L)(n)}(1) = 0$

By induction : $\tau_{1/2}^{(1)(n)}(1) = \tau_{3/2}^{(2)(n)}(1) = 0$, assume $\tau_{L-3/2}^{(L-1)(n)}(1) = 0$

Vector and Axial SR $\rightarrow \ \tau_{L-1/2}^{(L)(n)}(1) = 0 \ \rightarrow (-1)^L \xi^{(L)}(1) = \frac{(2L+1)!!}{2^{2L}}$

Therefore BPS implies the explicit form $\xi(w) = \left(\frac{2}{w+1}\right)^{3/2}$

Radiative corrections

Two types of radiative corrections : (1) within HQET
(2) Wilson coefficients to make the matching with QCD

Modified sum rule (Dorsten 2003)
μ-dependence in OPE side and cut-off Δ in hadronic sum

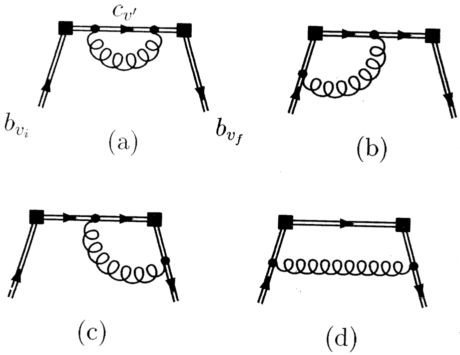
$$\sum_{X_c} W_\Delta(E_M - E_{X_c}) < \overline{B}_f|J_f(0)|X_c >< X_c|J_i(0)|\overline{B}_i >$$
$$= 2\xi(w_{if})\,[1 + \alpha_s(\mu)F(w_i, w_f, w_{if})]\; Tr\left[P_{f+}\not{v}_f(\gamma_5)P'_+\not{v}_i(\gamma_5)P_{i+}\right]$$

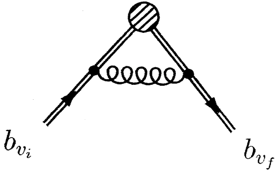
Universal function $F(w_i, w_f, w_{if}) \to F(1, w, w) = F(w, 1, w) = 0$

Modified bound due to radiative corrections within HQET

$$\sigma^2(\mu) > \frac{3}{5}\left[\rho^2(\mu)\right]^2 + \frac{4}{5}\rho^2(\mu)\left[1 + \frac{20\alpha_s(\mu)}{27\pi}\right] - \frac{148\alpha_s(\mu)}{675\pi} \qquad (\Delta = 2\mu)$$

Curvature of physical axial form factor $\sigma^2_{A_1} > 0.94 - 0.07_p - 0.2_{np}$


$c_{v'}$
b_{v_i}
(a)
b_{v_f}
(b)
(c)
(d)


b_{v_i}
b_{v_f}

The case of baryons $\Lambda_b(v_i) \to \Lambda_c^{(n)}(v') \to \Lambda_b(v_f)$

$\Lambda_b\ :\ (j^P, J^P) = \left(0^+, \frac{1}{2}^+\right)$
$\Lambda_c^{(n)}\ :$ tower $(j^P, J^P), J = j, j = L, P = (-1)^L$

Sum rule

$$\xi_\Lambda(w_{if}) = \sum_n \sum_{L\geq 0}\ \tau_L^{(n)}(w_i)^* \tau_L^{(n)}(w_f)$$
$$\sum_{0\leq k\leq L/2} C_{L,k}\ (w_i^2-1)^k (w_f^2-1)^k (w_i w_f - w_{if})^{L-2k}$$

IW functions $\tau_L(w)\ :\ 0^+ \to L^P, P = (-1)^L$

One finds the constraints on the derivatives :

$$\rho_\Lambda^2 = -\xi'_\Lambda(1) \geq 0 \qquad\qquad \xi''_\Lambda(1) \geq \frac{3}{5}[\rho_\Lambda^2 + (\rho_\Lambda^2)^2]$$

Isgur-Wise functions and the Lorentz group

Matrix element of a current between heavy hadrons **factorizes** into a trivial **heavy quark current matrix element**
and a **light cloud overlap** (that contains the long distance physics)

$$< H'(v')|J^{Q'Q}(q)|H(v) > =$$

$$< Q'(v'), \pm\tfrac{1}{2}|J^{Q'Q}(q)|Q(v), \pm\tfrac{1}{2} >< v', j', M'|v, j, M >$$

The light cloud follows the heavy quark with the same four-velocity

Isgur-Wise functions : light cloud overlaps $\xi(v.v') = < v'|v >$

Factorization valid only in absence of **hard radiative corrections**

Light cloud Hilbert space

Can demonstrate that the light cloud states form a Hilbert space

on which acts a unitary representation of the Lorentz group
$\Lambda \to U(\Lambda)$ $\qquad U(\Lambda)|v, j, \epsilon > = |\Lambda v, j, \Lambda\epsilon >$

$|v, j, \epsilon > = \sum_M (\Lambda^{-1}\epsilon)_M \; U(\Lambda)|v_0, j, M >$

$\Lambda v_0 = v \qquad v_0 = (1, 0, 0, 0) \qquad \Lambda^{-1}\epsilon :$ polarization vector at rest

This fundamental formula defines, in the Hilbert space $\mathcal{H}$
of a unitary representation of $SL(2, C)$ the states $|v, j, \epsilon >$
whose scalar products define the IW functions
in terms of $|v_0, j, M >$ which occur as $SU(2)$ multiplets
in the restriction to $SU(2)$ of the $SL(2, C)$ representation

Choose the simpler case of baryons with $\mathbf{j = 0}$

Baryons $\Lambda_b(v)$, $\Lambda_c(v)$ ($S_{qq} = 0, L = 0$ in quark model language)

Then, the Isgur-Wise function writes

$$\xi(v.v') = < U(B_{v'})\phi_0 | U(B_v)\phi_0 >$$

$|\phi_0>$ represents the light cloud at rest and B_v, $B_{v'}$ are boosts

$$\xi(w) = < \phi_0 | U(\Lambda)\phi_0 > \qquad \Lambda v_0 = v \qquad v^0 = w$$

Λ is for instance the boost along Oz

$$\Lambda_\tau = \begin{pmatrix} e^{\tau/2} & 0 \\ 0 & e^{-\tau/2} \end{pmatrix} \qquad w = ch(\tau)$$

Method completely general, for any j and any transition $j \to j'$

Decomposition into irreducible representations

The unitary representation $U(\Lambda)$ is in general reducible

Useful to decompose it into irreducible representations $U_\chi(\Lambda)$

Hilbert space $\mathcal{H}$ made of functions $\qquad \psi : \chi \in X \to \psi_\chi \in \mathcal{H}_\chi$

Scalar product in $\mathcal{H}$

$$< \psi' | \psi > = \int_X < \psi'_\chi | \psi_\chi > d\mu(\chi)$$

$\chi \in X$: irreducible unitary representation
$d\mu(\chi)$: a positive measure

$$(U(\Lambda)\,\psi)_\chi = U_\chi(\Lambda)\psi_\chi \qquad\qquad \psi_\chi \in \mathcal{H}_\chi$$

$\mathcal{H}_\chi$: Hilbert space of χ on which acts $U_\chi(\Lambda)$

Integral formula for the Isgur-Wise function

Notation $\qquad \xi_\chi(w) = < \phi_{0,\chi} | U_\chi(\Lambda) \phi_{0,\chi} >$

irreducible Isgur-Wise function corresponding to irreducible χ

General form of the IW function : $\qquad \xi(w) = \int_{X_0} \xi_\chi(w) \; d\nu(\chi)$

Isgur-Wise function as a mean value of irreducible IW functions with respect to some *positive* normalized measure ν

$\int_{X_0} d\nu(\chi) = 1$

$X_0 \subset X$ irreducible representations of $SL(2, C)$
containing a non-zero $SU(2)$ scalar subspace ($j = 0$ case)

Irreducible IW function $\xi_\chi(w)$ when ν is a δ function

Irreducible unitary representations of the Lorentz group

Naïmark (1962)

Principal series $\qquad \chi = (p, n, \rho)$

$n \in Z$ and $\rho \in R \qquad (n = 0, \rho \geq 0; n > 0, \rho \in R)$

Hilbert space $\mathcal{H}_{p,n,\rho}$

$$< \phi' | \phi > = \int \overline{\phi'(z)}\ \phi(z)\ d^2z \qquad d^2z = d(Rez)d(Imz)$$

Unitary operator $U_{p,n,\rho}(\Lambda)$

$$(U_{p,n,\rho}(\Lambda)\phi)(z) = \left(\frac{\alpha - \gamma z}{|\alpha - \gamma z|}\right)^n |\alpha - \gamma z|^{2i\rho - 2}\ \phi\left(\frac{\delta z - \beta}{\alpha - \gamma z}\right)$$

$$\Lambda = \left(\begin{array}{cc} \alpha & \beta \\ \gamma & \delta \end{array} \right) \qquad \alpha\delta - \beta\gamma = 1 \qquad (\alpha, \beta, \gamma, \delta) \in C$$

Supplementary series $\quad \chi = (s, \rho)$

$\rho \in R \qquad (0 < \rho < 1)$

Hilbert space $\mathcal{H}_{s,\rho}$

$$< \phi' | \phi > = \int \overline{\phi'(z_1)}\ |z_1 - z_2|^{2\rho - 2}\ \phi(z_2)\ d^2 z_1 d^2 z_2$$

(non-standard scalar product)

Unitary operator $U_{s,\rho}(\Lambda)$

$$(U_{s,\rho}(\Lambda)\phi)(z) = |\alpha - \gamma z|^{-2\rho - 2}\ \phi\left(\frac{\delta z - \beta}{\alpha - \gamma z}\right)$$

Trivial representation $\quad \chi = t$

Hilbert space $\mathcal{H}_t = C$

$$< \phi' | \phi > = \overline{\phi'(z)}\phi(z)$$

Unitary operator $\qquad U_t(\Lambda) = 1$

Decomposition under the rotation group

Need restriction to $SU(2)$ of unitary representations χ of $SL(2,C)$

For a χ there is an orthonormal basis $\phi^{\chi}_{j,M}$ of $\mathcal{H}_{\chi}$ adapted to $SU(2)$

Particularizing to $j=0$: all types of representations contribute

$$\phi^{p,0,\rho}_{0,0}(z)=\frac{1}{\sqrt{\pi}}(1+|z|^2)^{i\rho-1} \qquad (\chi=(p,0,\rho),\ \rho\geq 0)$$

$$\phi^{s,\rho}_{0,0}(z)=\frac{\sqrt{\rho}}{\pi}(1+|z|^2)^{-\rho-1} \qquad (\chi=(s,\rho),\ 0<\rho<1)$$

$$\phi^{t}_{0,0}(z)=1 \qquad (\chi=t)$$

For $j\neq 0$ enters also the matrix element

$$D^{j}_{M',M}(R)=<j,M'|U_j(R)|j,M> \qquad R\in SU(2)$$

Irreducible IW functions in the case j = 0

Need $\xi_\chi(w) = < \phi^\chi_{0,0} | U_\chi(\Lambda_\tau) \phi^\chi_{0,0} >$ $\quad (\Lambda_\tau$: boost, $w = ch(\tau))$

Transformed elements $U_\chi(\Lambda_\tau)\phi^\chi_{0,0}$

$$\left(U_{p,0,\rho}(\Lambda_\tau)\phi^{p,0,\rho}_{0,0} \right)(z) = \frac{1}{\sqrt{\pi}} (e^\tau + e^{-\tau}|z|^2)^{i\rho - 1}$$

$$\left(U_{s,\rho}(\Lambda_\tau)\phi^{s,\rho}_{0,0} \right)(z) = \frac{\sqrt{\rho}}{\sqrt{\pi}} (e^\tau + e^{-\tau}|z|^2)^{-\rho - 1}$$

$$U_t(\Lambda_\tau)\phi^t_{0,0} = 1$$

Using the scalar products for each class of representations

$$\xi_{p,0,\rho}(w) = \frac{sin(\rho\tau)}{\rho \ sh(\tau)}$$

$$\xi_{s,\rho}(w) = \frac{sh(\rho\tau)}{\rho \ sh(\tau)} \qquad (0 < \rho < 1)$$

$$\xi_t(w) = 1$$

Integral formula for the IW function in the case $j = 0$

$$\xi(w) = \int_{[0,\infty[} \frac{sin(\rho\tau)}{\rho\ sh(\tau)}\ d\nu_p(\rho) + \int_{]0,1[} \frac{sh(\rho\tau)}{\rho\ sh(\tau)}\ d\nu_s(\rho) + \nu_t$$

ν_p and ν_s are positive measures and ν_t a real number ≥ 0

$$\int_{[0,\infty[} d\nu_p(\rho) + \int_{]0,1[} d\nu_s(\rho) + \nu_t = 1$$

One-parameter family $\qquad \xi_x(w) = \frac{sh(\tau\sqrt{1-x})}{sh(\tau)\sqrt{1-x}} = \frac{sin(\tau\sqrt{x-1})}{sh(\tau)\sqrt{x-1}}$

covers all irreducible representations $\rightarrow$ simplifies integral formula

$$\xi(w) = \int_{[0,\infty[} \xi_x(w)\ d\nu(x) \quad \left(\nu \text{ positive measure } \int_{[0,\infty[} d\nu(x) = 1\right)$$

$$\begin{array}{lllll}
\xi_{p,0,\rho}(w) = \xi_x(w) & x = 1+\rho^2, \rho \in [0,\infty[& \Leftrightarrow & x \in [1,\infty[\\
\xi_{s,\rho}(w) = \xi_x(w) & x = 1-\rho^2, \rho \in]0,1[& \Leftrightarrow & x \in]0,1[\\
\xi_t(w) = \xi_x(w) & x = 0 & & x \in \{0\}
\end{array}$$

$\rightarrow$ a transparent deduction of constraints on the derivatives $\xi^{(n)}(1)$

Constraints on the derivatives of the Isgur-Wise function

Derivative $\xi^{(k)}(1)$: *expectation value* of a polynomial of degree k

$$\xi^{(k)}(1) = (-1)^k\ 2^k \frac{k!}{(2k+1)!} < \prod_{i=1}^{k}(x + i^2 - 1) >$$

In terms of moments $\qquad \mu_n = < x^n >$

$$\xi(1) = \mu_0 = 1$$

$$\xi'(1) = -\frac{1}{3}\ \mu_1$$

$$\xi''(1) = \frac{1}{15}\ (3\mu_1 + \mu_2)$$

...

Moments μ_k in terms of derivatives $\xi(1)$, $\xi'(1)$, ... $\xi^{(k)}(1)$

$$\mu_0 = \xi(1) = 1$$

$$\mu_1 = -3\ \xi'(1)$$

$$\mu_2 = 3\,[3\ \xi'(1) + 5\ \xi''(1)]$$

...

Constraints on moments of a variable with positive values

$det\left[(\mu_{i+j})_{0\leq i,j\leq n}\right] \geq 0$

$det\left[(\mu_{i+j+1})_{0\leq i,j\leq n}\right] \geq 0$

Lower moments

$\mu_1 \geq 0$

$\mu_2 \geq \mu_1^2$

...

That imply for the derivatives of the Isgur-Wise function

$\rho_\Lambda^2 \geq 0$

$\xi''(1) \geq \frac{3}{5}\rho_\Lambda^2(1+\rho_\Lambda^2)$

...

Same results as with the Sum Rule approach

The Isgur-Wise function is a function of positive type

For any N and any complex numbers a_i and velocities v_i

$\sum_{i,j=1}^{N} a_i^* a_j \; \xi(v_i.v_j) \geq 0$ or, in a covariant form

$\int \frac{d^3\vec{v}}{v^0} \frac{d^3\vec{v}\,'}{v\,'^0} \; \psi(v')^* \; \xi(v.v') \; \psi(v) \geq 0$ for any $\psi(v)$

From the Sum Rule $(w_i = v_i.v', w_j = v_j.v', w_{ij} = v_i.v_j)$

$$\xi(w_{ij}) = \sum_n \sum_L \tau_L^{(n)}(w_i)^* \tau_L^{(n)}(w_j)$$
$$\sum_{0 \leq k \leq L/2} C_{L,k} \; (w_i^2 - 1)^k (w_j^2 - 1)^k (w_i w_j - w_{ij})^{L-2k}$$

Legendre polynomial. Use rest frame $v' = (1,0,0,0)$

$$\sum_{i,j=1}^{N} a_i^* a_j \; \xi(v_i.v_j) = 4\pi \sum_{i,j=1}^{N} \sum_n \sum_L \frac{2^L (L!)^2}{(2L+1)!} \sum_{m=-L}^{m=+L}$$
$$\left[a_i \; \tau_L^{(n)}\left(\sqrt{1+\vec{v}_i^{\,2}}\right) \mathcal{Y}_L^m(\vec{v}_i) \right]^* \left[a_j \; \tau_L^{(n)}\left(\sqrt{1+\vec{v}_j^{\,2}}\right) \mathcal{Y}_L^m(\vec{v}_j) \right] \geq 0$$

One example : application to the exponential form

$\xi(w) = exp\left[-c(w-1)\right]$

$I = \int \frac{d^3\vec{v}}{v^0}\frac{d^3\vec{v}\,'}{v\,'^0}\ \phi(|\vec{v}\,'|)^*\ exp\left[-c((v.v')-1)\right]\ \phi(|\vec{v}|)$

$= 16\pi^3\ \frac{e^c}{c}\int_{-\infty}^{\infty} K_{i\rho}(c)\ |\tilde{f}(\rho)|^2\ d\rho$

$f(\eta) = sh(\eta)\ \phi(sh(\eta))$
$K_\nu(z) = \frac{1}{2}\int_{-\infty}^{\infty} exp[-z\ ch(t)]\ e^{\nu t}\ dt$ Macdonald function

Whatever the slope $c > 0$, $K_{i\rho}(c)$ *takes negative values*

Asymptotic formula

$K_{i\rho}(c) \sim \sqrt{\frac{2\pi}{\rho}}\ e^{-\rho\pi/2}\ cos\left[\rho\left(log\left(\frac{2\rho}{c}\right)-1\right)-\frac{\pi}{4}\right] \qquad (\rho >> c)$

Therefore there a function $\psi(v)$ for which the integral $I < 0$

The exponential form is inconsistent with the Sum Rules

Sum Rule and Lorentz group approaches are equivalent

- The Lorentz group approach implies that $\xi(w)$ is of positive type

$\xi(w) = < U(B_{v'})\psi_0 | U(B_v)\psi_0 >$ $\qquad (B_v$: boost $v_0 \rightarrow v)$

$\sum_{i,j=1}^{N} a_i^* a_j \ \xi(v_i.v_j) = \| \sum_{j=1}^{N} a_j U(B_{v_j})\psi_0 \|^2 \geq 0$

- The Sum Rule approach implies the Lorentz group approach

A function $f(\Lambda)$ on the group $SL(2,C)$ is of positive type when

$\sum_{i,j=1}^{N} a_i^* a_j \ f(\Lambda_i^{-1}\Lambda_j) \geq 0$ $\qquad (N \geq 1$, complex a_i, $\Lambda_i \in SL(2,C))$

Theorem (Dixmier) : for any function $f(\Lambda)$ of positive type exists a unitary representation $U(\Lambda)$ of $SL(2,C)$ in a Hilbert space $\mathcal{H}$ and an element $\phi_0 \in \mathcal{H} \rightarrow f(\Lambda) = < \phi_0 | U(\Lambda)\phi_0 >$

Definition of $f(\Lambda_i^{-1}\Lambda_j) = \xi(v_i.v_j) = \xi(v_0.\Lambda_i^{-1}\Lambda_j v_0)$

Consistency test for any Ansatz of the Isgur-Wise function

We have the integral representation

$$\xi(w) = \int_{[0,\infty[} \frac{sin(\rho\tau)}{\rho\ \ sh(\tau)}\ d\nu_p(\rho) + \int_{]0,1[} \frac{sh(\rho\tau)}{\rho\ \ sh(\tau)}\ d\nu_s(\rho) + \nu_t$$

ν_p and ν_s are positive measures and ν_t a real number ≥ 0 satisfying

$$\int_{[0,\infty[} d\nu_p(\rho) + \int_{]0,1[} d\nu_s(\rho) + \nu_t = 1$$

One can invert this formula by Fourier transforming and check if a given Ansatz for $\xi(w)$ satisfies it with *positive measures*

Example of the exponential (only the principal series contributes)

$$exp[-c(w-1)] = \frac{2}{\pi}\frac{e^c}{c}\int_0^\infty \rho^2\ K_{i\rho}(c)\ \frac{sin(\rho\tau)}{\rho\ \ sh(\tau)}\ d\rho \qquad (w = ch(\tau))$$

Inconsistent : $K_{i\rho}(c)$ can be negative $\rightarrow d\nu_p(\rho)$ is not positive

Other phenomenological examples

Example 1 (both principal and suplementary series contribute)

$$\xi(w) = \left(\frac{2}{1+w}\right)^{2c} = \frac{4^{2c}}{\pi} \int_0^\infty \rho^2 \; \frac{|\Gamma(2c+i\rho-1)|^2}{\Gamma(4c-1)} \; \frac{sin(\rho\tau)}{\rho \; sh(\tau)} \; d\rho$$

$$+ \; \theta(1-2c) \; (1-2c) \; 2^{4c} \; \frac{sh((1-2c)\tau)}{(1-2c) \; sh(\tau)}$$

valid for any slope $c \geqslant \frac{1}{4}$

Example 2 (only the principal series contributes)

$$\xi(w) = \frac{1}{\left[1+\frac{c}{2}(w-1)\right]^2} = \frac{8}{c^2} \int_0^\infty \frac{\rho^2}{sh(\pi\rho)} \; \frac{sh(\gamma\rho)}{sh(\gamma)} \; \frac{sin(\rho\tau)}{\rho \; sh(\tau)} \; d\rho$$

$(cos\gamma = \frac{2}{c} - 1)$ valid for any slope $c \geqslant 1$

New rigorous results for non-perturbative physics in HQET

• Decomposing into irreducible representations a unitary representation of the Lorentz group → integral formula for the Isgur-Wise function with positive measures

• Explicitly given for $j = 0$ ($\Lambda_b \to \Lambda_c \ell \nu$)

• Derivatives of the IW function given in terms of moments of a positive variable → inequalities between the derivatives

• Sum Rules → IW function is a function of positive type

• Application : exponential form of the IW function is inconsistent

• Equivalence between Sum Rule and Lorentz group approaches

• Consistency test for any Ansatz of the IW function

• Application to phenomenological examples

• Can be generalized for any j ($j = \frac{1}{2}$ for mesons $\overline{B}_d \to D^{(*)} \ell \nu$)

Lorentz group in our approach vs. Poincaré group

One can ask the question about which is the relation between the Lorentz group used in our approach and the Poincaré group

• Naïmark : we use the Lorentz group (no translations),
more precisely the orthochronous proper Lorentz group,
more precisely its connected recovering to get half-integer spin
(parity must also be included)

• Wigner : Poincaré group (translations included)
$\rightarrow$ classification of massive and massless particles

• These are two quite different kinds of problems