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Cosmography and Large Scale Structure by Higher Order Gravity: New Results

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What about the Universe?

DATA	RESULTS
Supernovae Hubble Diagram	Accelerating expansion
Galaxy power spectrum and BAO	Low matter content
Cosmic Microwave Background	Spatially flat universe

	SOLUTIONS WITHIN GENERAL RELATIVITY	SHORTCOMINGS
Flatness $\Omega_k = -0.046^{+0.0066}_{-0.0067}$	Inflationary Epoch	—
Acceleration $q_0 < 0$	Cosmological constant (ΛCDM) Quintessence	120 orders of difference What kind?
Low matter content $\Omega_b = 0.0462 \pm 0.0015$	Dark Energy $\Omega_{de} \approx 0.72$ Dark Matter $\Omega_m \approx 0.25$	Undetected components Coincidence problem Fine tuning problems

Extended Theories of Gravity

SOLUTIONS OUTSIDE GENERAL RELATIVITY

MOTIVATIONS:

- General Relativity tested only up to Solar System
- Effective actions from fundamental field theories
- Inflationary models
- No need of dark components

REQUIREMENTS:

- Reproducing Newtonian Dynamics in Solar System
- Flat Rotation Curves of Spiral Galaxies by Baryonic constituents
- Reproducing Large Scale Structure (Galaxy Clusters scale)
- Successful Fit of SNeIa + CMB + BAO data
- Accelerated Hubble fluid and Dark Energy phenomenology

f(R) Theories of Gravity : Résumé

From Extended Theories of Gravity ⇒ f(R) gravity

- **Gravity action :**

$$\mathcal{A} = \int \sqrt{-g}\,\left[f(R) + \mathcal{L}_M\right] d^4x$$

- **Field equations :**

$$f'(R)R_{\alpha\beta} - \frac{1}{2}f(R)g_{\alpha\beta} - f'(R)^{;\alpha\beta}\left(g_{\alpha\mu}g_{\beta\nu} - g_{\alpha\beta}g_{\mu\nu}\right) = \tilde{T}^{M}_{\alpha\beta}$$

- **Cosmological equations :**

$$H^2 = \frac{1}{3}\left[\frac{\rho_m}{f'(R)} + \rho_{\mathrm{curv}}\right]$$

1st Friedmann eq.

$$2\,\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2 = -(p_{curv} + p_m)$$

2nd Friedmann eq.

- **Curvature Fluid :**

$$\rho_{\mathrm{curv}} = \frac{1}{f'(R)}\left\{\frac{1}{2}\left[f(R) - Rf'(R)\right] - 3H\dot{R}f''(R)\right\}.$$

$$w_{\mathrm{curv}} = -1 + \frac{\ddot{R}f''(R) + \dot{R}\left[\dot{R}f'''(R) - Hf''(R)\right]}{\left[f(R) - Rf'(R)\right]/2 - 3H\dot{R}f''(R)}$$

Constraining Extended Theories of Gravity by Cosmography

- e.g. **Constraining f(R)-gravity by Cosmography**

  Capozziello, S., Cardone, V., Salzano, V., PRD 78 (2008) 063504

- Constraining f(R)– gravity by Clusters of Galaxies

  Capozziello, S., De Filippis, E., **Salzano**, **V.**, MNRAS 394 (2009) 947

Cosmography

GR based models vs f(R) gravity

↓

Agreement with Data…

} How can we discriminate?

- No a priori dynamical model = **Model Independent** Approach;
- Robertson – Walker metric;
- Expansion series of the scale factor with respect to cosmic time:

$$\frac{a(t)}{a(t_0)} = 1 + H_0(t-t_0) - \frac{q_0}{2}H_0^2(t-t_0)^2 + \frac{j_0}{3!}H_0^3(t-t_0)^3 + \frac{s_0}{4!}H_0^4(t-t_0)^4 + \frac{l_0}{5!}H_0^5(t-t_0)^5 + O[(t-t_0)^6$$

$$q(t) = -\frac{1}{a}\frac{d^2a}{dt^2}\frac{1}{H^2}$$

Deceleration

$$j(t) = \frac{1}{a}\frac{d^3a}{dt^3}\frac{1}{H^3}$$

Jerk

$$s(t) = \frac{1}{a}\frac{d^4a}{dt^4}\frac{1}{H^4}$$

Snap

$$l(t) = \frac{1}{a}\frac{d^5a}{dt^5}\frac{1}{H^5}$$

Lerk

Expansion up to fifth order :
- error on $d_L(z)$ less than 10% up to z = 1
- error on $\mu(z)$ less than 3% up to z = 2

Cosmography by f(R): How many parameters…

- Definition: $H(t) = \frac{1}{a}\frac{da}{dt}, \quad q(t) = -\frac{1}{a}\frac{d^2a}{dt^2}\frac{1}{H^2}, \quad j(t) = \frac{1}{a}\frac{d^3a}{dt^3}\frac{1}{H^3}, \quad s(t) = \frac{1}{a}\frac{d^4a}{dt^4}\frac{1}{H^4}, \quad l(t) = \frac{1}{a}\frac{d^5a}{dt^5}\frac{1}{H^5}$

- Derivatives of H(t):

$$\dot{H} = -H^2(1+q)$$

$$\ddot{H} = H^3(j+3q+2)$$

$$d^3H/dt^3 = H^4\left[s-4j-3q(q+4)-6\right]$$

$$d^4H/dt^4 = H^5\left[l-5s+10(q+2)j+30(q+2)q+24\right]$$

- Derivatives of scalar curvature:

$$R = -6(\dot{H}+2H^2)$$

$$R_0 = -6H_0^2(1-q_0)$$

$$\dot{R}_0 = -6H_0^3(j_0-q_0-2)$$

$$\ddot{R}_0 = -6H_0^4\left(s_0+q_0^2+8q_0+6\right)$$

$$d^3R_0/dt^3 = -6H_0^5\left[l_0-s_0+2(q_0+4)j_0-6(3q_0+8)q_0-24\right]$$

Cosmography by f(R): What equations…?

- 1st Friedmann eq. :

$$H_0^2 = \frac{H_0^2\Omega_M}{f'(R_0)} + \frac{f(R_0) - R_0 f'(R_0) - 6H_0\dot{R}_0 f''(R_0)}{6f'(R_0)},$$

- 2nd Friedmann eq. :

$$-\dot{H}_0 = \frac{3H_0^2\Omega_M}{2f'(R_0)} + \frac{\dot{R}_0^2 f'''(R_0) + (\ddot{R}_0 - H_0\dot{R}_0)f''(R_0)}{2f'(R_0)}$$

- Derivative of 2nd Friedmann eq. :

$$\ddot{H} = \frac{\dot{R}^2 f'''(R) + \left(\ddot{R} - H\dot{R}\right) f''(R) + 3H_0^2\Omega_M a^{-3}}{2\left[\dot{R} f''(R)\right]^{-1} [f'(R)]^2} - \frac{\dot{R}^3 f^{(iv)}(R) + \left(3\dot{R}\ddot{R} - H\dot{R}^2\right) f'''(R)}{2f'(R)} - \frac{\left(d^3R/dt^3 - H\ddot{R} + \dot{H}\dot{R}\right) f''(R) - 9H_0^2\Omega_M H a^{-3}}{2f'(R)}$$

- Constraint from gravitational constant:

$$H^2 = \frac{8\pi G}{3f'(R)}[\rho_m + \rho_{curv} f'(R)]$$

$$G_{eff}(z=0) = G \rightarrow f'(R_0) = 1.$$

f(R) equations and Cosmographic Parameters

- Final solutions:

$$\frac{f(R_0)}{6H_0^2} = -\frac{\mathcal{P}_0(q_0, j_0, s_0, l_0)\Omega_M + \mathcal{Q}_0(q_0, j_0, s_0, l_0)}{\mathcal{R}(q_0, j_0, s_0, l_0)}$$

$$f'(R_0) = 1$$

$$\frac{f''(R_0)}{(6H_0^2)^{-1}} = -\frac{\mathcal{P}_2(q_0, j_0, s_0)\Omega_M + \mathcal{Q}_2(q_0, j_0, s_0)}{\mathcal{R}(q_0, j_0, s_0, l_0)}$$

$$\frac{f'''(R_0)}{(6H_0^2)^{-2}} = -\frac{\mathcal{P}_3(q_0, j_0, s_0, l_0)\Omega_M + \mathcal{Q}_3(q_0, j_0, s_0, l_0)}{(j_0 - q_0 - 2)\mathcal{R}(q_0, j_0, s_0, l_0)}$$

- Taylor expand *f(R)* in series of *R* up to third order (higher not necessary)

- Linear equations in f(R) and derivatives

- Ω_M is model dependent:

$$\Omega_M = 0.041$$
$$\Omega_M = 0.250$$

f(R) derivatives and CPL models

"Precision cosmology" $\nLeftrightarrow$ Values of cosmographic parameters?

Cosmographic parameters $\Leftrightarrow$ Dark energy parameters = equivalent f(R)

CPL approach:

(Chevallier, Polarski, Linder)

$$w = w_0 + w_a(1-a) = w_0 + w_a z(1+z)^{-1}$$

Cosmographic parameters:

$$q_0 = \frac{1}{2} + \frac{3}{2}(1-\Omega_M)w_0$$

$$j_0 = 1 + \frac{3}{2}(1-\Omega_M)\left[3w_0(1+w_0)+w_a\right]$$

$$\begin{aligned} s_0 &= -\frac{7}{2} - \frac{33}{4}(1-\Omega_M)w_a - \frac{9}{4}(1-\Omega_M)\left[9+(7-\Omega_M)w_a\right]w_0 + \\ &- \frac{9}{4}(1-\Omega_M)(16-3\Omega_M)w_0^2 - \frac{27}{4}(1-\Omega_M)(3-\Omega_M)w_0^3 \end{aligned}$$

$$\begin{aligned} l_0 &= \frac{35}{2} + \frac{1-\Omega_M}{4}\left[213+(7-\Omega_M)w_a\right]w_a + \frac{(1-\Omega_M)}{4}\left[489+9(82-21\Omega_M)w_a\right]w_0 + \\ &+ \frac{9}{2}(1-\Omega_M)\left[67-21\Omega_M+\frac{3}{2}(23-11\Omega_M)w_a\right]w_0^2 + \frac{27}{4}(1-\Omega_M)(47-24\Omega_M)w_0^3 + \\ &+ \frac{81}{2}(1-\Omega_M)(3-2\Omega_M)w_0^4 \end{aligned}$$

CPL Cosmography and f(R): the ΛCDM Model

ΛCDM model: $(w_0, w_a) = (-1, 0)$

$$q_0 = \frac{1}{2} - \frac{3}{2}\Omega_M; \quad j_0 = 1; \quad s_0 = 1 - \frac{9}{2}\Omega_M; \quad l_0 = 1 + 3\Omega_M + \frac{27}{2}\Omega_M^2$$

$$f(R_0) = R_0 + 2\Lambda, \qquad f''(R_0) = f'''(R_0) = 0,$$

ΛCDM fits well many data → cosmographic values strictly depend on Ω_M

$$q_0 = q_0^\Lambda \times (1 + \varepsilon_q), \qquad j_0 = j_0^\Lambda \times (1 + \varepsilon_j),$$
$$s_0 = s_0^\Lambda \times (1 + \varepsilon_s), \qquad l_0 = l_0^\Lambda \times (1 + \varepsilon_l),$$

$$\eta_{20} = f''(R_0)/f(R_0) \times H_0^4$$

$$\eta_{30} = f'''(R_0)/f(R_0) \times H_0^6$$

$$\eta_{20} = \frac{64 - 6\Omega_M(9\Omega_M + 8)}{[3(9\Omega_M + 74)\Omega_M - 556]\,\Omega_M^2 + 16} \times \frac{\varepsilon}{27}$$

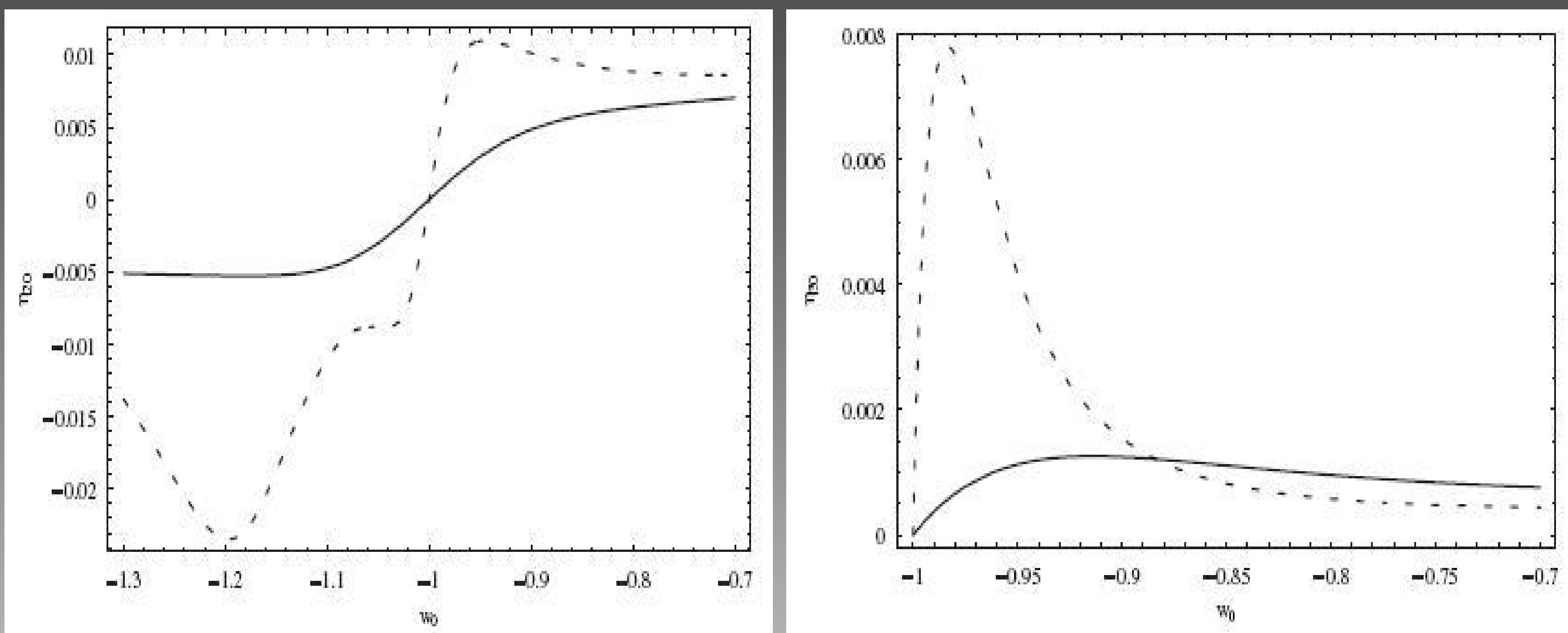
$$\eta_{30} = \frac{6\,[(81\Omega_M - 110)\Omega_M + 40]\,\Omega_M + 16}{[3(9\Omega_M + 74)\Omega_M - 556]\,\Omega_M^2 + 16} \times \frac{\varepsilon}{243\Omega_M^2}$$

$$\begin{cases} \eta_{20} \simeq 0.15 \times \varepsilon & \text{for } \Omega_M = 0.041 \\ \eta_{20} \simeq -0.12 \times \varepsilon & \text{for } \Omega_M = 0.250 \end{cases}$$

$$\begin{cases} \eta_{30} \simeq 4 \times \varepsilon & \text{for } \Omega_M = 0.041 \\ \eta_{30} \simeq -0.18 \times \varepsilon & \text{for } \Omega_M = 0.250 \end{cases}$$

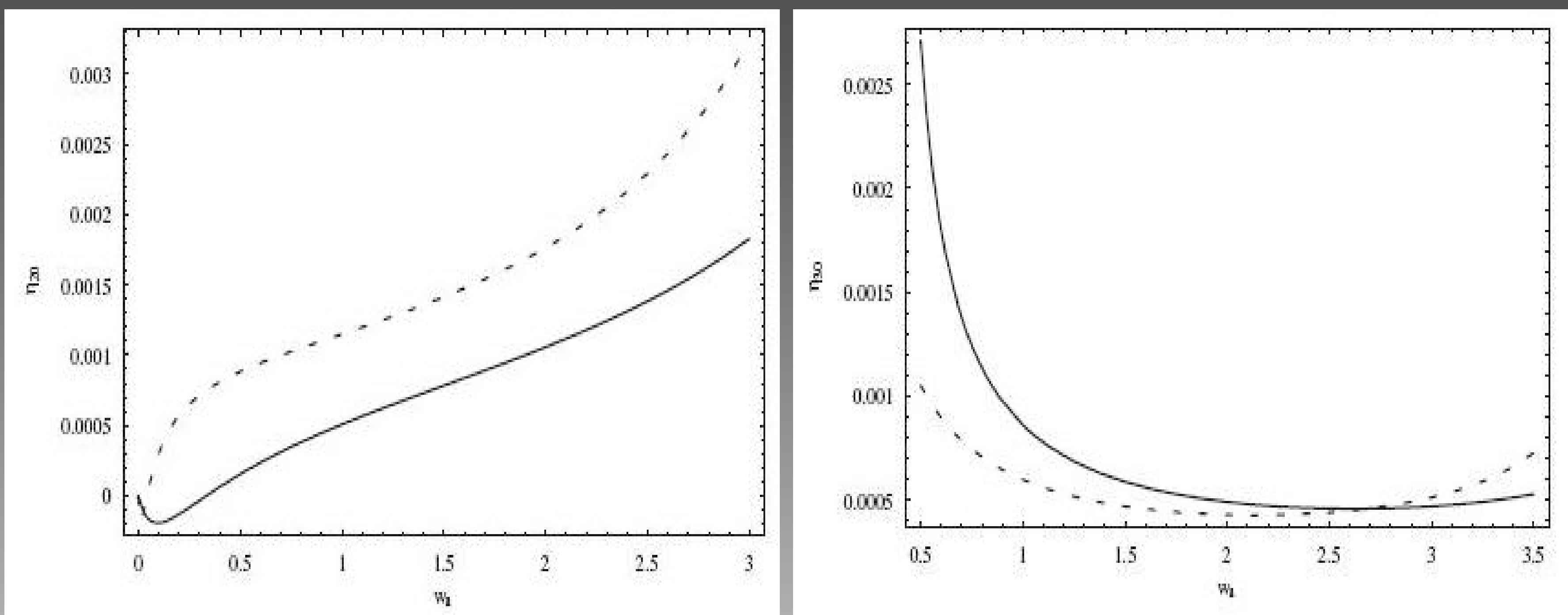
CPL Cosmography and f(R): constant EoS case

- Constant EoS: $w_a = 0$
- Beware of divergences in the *f(R)* derivatives
- Small deviations from GR
- Large deviations for baryonic dominated universe



CPL Cosmography and f(R) : varying EoS case

- General case: $w_a \neq 0;\; w_0 = -1$
- Beware of divergences in the *f(R)* derivatives
- Small deviations from GR
- Large deviations for baryonic dominated universe



Constraining f(R) models by Cosmography

- Procedure:
  1. Estimate (q(0), j(0), s(0), l(0)) observationally
  2. Compute f(R0), f ’(R0), f “ (R0), f ’”(R0)
  3. Solve for f(R) parameters from derivatives
  4. Constraint f(R) models

- e.g. Double Power-Law:

$$f(R) = R(1 + \alpha R^{n} + \beta R^{-m})$$

$$\begin{cases} f(R_0) = R_0(1 + \alpha R_0^{n} + \beta R_0^{-m}) \\ f'(R_0) = 1 + \alpha(n+1)R_0^{n} - \beta(m-1)R_0^{-m} \\ f''(R_0) = \alpha n(n+1)R_0^{n-1} + \beta m(m-1)R_0^{-(1+m)} \\ f'''(R_0) = \alpha n(n+1)(n-1)R_0^{n-2} \\ \qquad - \beta m(m+1)(m-1)R_0^{-(2+m)}. \end{cases}$$

$$\begin{cases} \alpha = \frac{1-m}{n+m}\left(1 - \frac{\phi_0}{R_0}\right)R_0^{-n} \\ \beta = -\frac{1+n}{n+m}\left(1 - \frac{\phi_0}{R_0}\right)R_0^{m}, \end{cases}$$

$$\begin{cases} \alpha = \frac{\phi_2 R_0^{1-n}[1+m+(\phi_3/\phi_2)R_0]}{n(n+1)(n+m)} \\ \beta = \frac{\phi_2 R_0^{1+n}[1-n+(\phi_3/\phi_2)R_0]}{m(1-m)(n+m)}. \end{cases}$$

$$\begin{cases} \frac{n(n+1)(1-m)(1-\phi_0/R_0)}{\phi_2 R_0[1+m+(\phi_3/\phi_2)R_0]} = 1 \\ \frac{m(n+1)(m-1)(1-\phi_0/R_0)}{\phi_2 R_0[1-n+(\phi_3/\phi_2)R_0]} = 1. \end{cases}$$

$$m = -[1 - n + (\phi_3/\phi_2)R_0]$$

$$n = \frac{1}{2}\left[1 + \frac{\phi_3}{\phi_2}R_0 \pm \frac{\sqrt{\mathcal{N}(\phi_0, \phi_2, \phi_3)}}{\phi_2 R_0(1 + \phi_0/R_0)}\right]$$

Cosmography and data

- Cosmographic parameter from SNeIa: What we have to expect from data

$$q_0 = -0.90 \pm 0.65, \qquad j_0 = 2.7 \pm 6.7,$$
$$s_0 = 36.5 \pm 52.9, \qquad l_0 = 142.7 \pm 320.$$

- Fisher information matrix method:

$$F_{ij} = \left\langle \frac{\partial^2 L}{\partial \theta_i \partial \theta_j} \right\rangle$$

- FM ingredients :

$$\chi^2(H_0, \mathbf{p}) = \sum_{n=1}^{\mathcal{N}_{SNeIa}} \left[\frac{\mu_{obs}(z_i) - \mu_{th}(z_n, H_0, \mathbf{p})}{\sigma_i(z_i)} \right]^2$$

$$d_L(z) = \mathcal{D}_L^0 z + \mathcal{D}_L^1 \; z^2 + \mathcal{D}_L^2 \; z^3 + \mathcal{D}_L^3 \; z^4 + \mathcal{D}_L^4 \; z^5$$

$$\sigma(z) = \sqrt{\sigma_{\rm sys}^2 + \left(\frac{z}{z_{\rm max}}\right)^2 \sigma_m^2}$$

- Estimating error on g:

$$\sigma_g^2 = \left| \frac{\partial g}{\partial \Omega_M} \right|^2 \sigma_M^2 + \sum_{i=1}^{i=4} \left| \frac{\partial g}{\partial p_i} \right|^2 \sigma_{p_i}^2 + \sum_{i \neq j} 2 \frac{\partial g}{\partial p_i} \frac{\partial g}{\partial p_j} C_{ij}$$

- Survey: Davis (2007)

$\sigma_M / \Omega_M = 10\%$; $\sigma_{sys} = 0.15$

$N_{SNeIa} = 2000$; $\sigma_m = 0.33$

$z_{max} = 1.7$

$\sigma_1 = 0.38$

$\sigma_2 = 5.4$

$\sigma_3 = 28.1$

$\sigma_4 = 74.0$

$\sigma_{20} = 0.04$

$\sigma_{30} = 0.04$

- Snap like survey:

$\sigma_M / \Omega_M = 1\%$; $\sigma_{sys} = 0.15$

$N_{SNeIa} = 2000$; $\sigma_m = 0.02$

$z_{max} = 1.7$

$\sigma_1 = 0.08$

$\sigma_2 = 1.0$

$\sigma_3 = 4.8$

$\sigma_4 = 13.7$

$\sigma_{20} = 0.007$

$\sigma_{30} = 0.008$

- Ideal PanSTARRS survey:

$\sigma_M / \Omega_M = 0.1\%$; $\sigma_{sys} = 0.15$

$N_{SNeIa} = 60000$; $\sigma_m = 0.02$

$z_{max} = 1.7$

$\sigma_1 = 0.02$

$\sigma_2 = 0.2$

$\sigma_3 = 0.9$

$\sigma_4 = 2.7$

$\sigma_{20} = 0.0015$

$\sigma_{30} = 0.0016$

Constraining Extended Theories of Gravity by Large Scale Structure

- Constraining f(R) -gravity by Cosmography

  Capozziello, S., Cardone, V., **Salzano, V.**, PRD 78 (2008) 063504

- e.g. **Constraining f(R) - by Clusters of Galaxies**

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f(R) gravity motivations

- Gravity action : $$\mathcal{A} = \int d^4x\sqrt{-g}\,[f(R) + \mathcal{X}\mathcal{L}_m]$$
- General requirement: Taylor expandable Lagrangian $$f(R) \sim a_1 R + a_2 R^2 + \ldots$$

Point like potential:

$$\phi(r) = -\frac{3GM}{4a_1 r}\left(1 + \frac{1}{3}e^{-\frac{r}{L}}\right)$$

Interaction length:

$$L \equiv L(a_1, a_2) = \left(-\frac{6a_2}{a_1}\right)^{1/2}$$

Purpose: Fit clusters mass profiles → **Build in a Self consistent theory**

If: $$\mathcal{L} = f(R, \Box R, ..\Box^k R, ..\Box^n R)\sqrt{-g}$$ → Effective actions from quantum field theory on curved space-time

$$\phi(r) = -\frac{GM}{r}\left[1 + \sum_{k=1}^{n} \alpha_k e^{-r/L_k}\right]$$

If:
- $r \gg L$: Gravitational coupling G
- $r \ll L$: Gravitational coupling $G(1+\alpha_1)$

Our potential: $$\phi(r) = -\frac{3GM}{4a_1 r}\left(1 + \frac{1}{3}e^{-\frac{r}{L}}\right) \Leftrightarrow \phi(r) = -\frac{GM}{r}\left[1 + \alpha_1 e^{-r/L_1}\right]$$

Clusters of galaxies dynamics

Cluster model: spherical mass distribution in hydrostatic equilibrium

- Boltzmann equation: $$-\frac{d\Phi}{dr} = \frac{kT(r)}{\mu m_p r}\left[\frac{d\ln\rho_{gas}(r)}{d\ln r} + \frac{d\ln T(r)}{d\ln r}\right]$$

- Newton *classical* approach: $$\begin{cases} \phi(r) = -\frac{GM}{r} \\ \rho_{cl,EC}(r) = \rho_{dark} + \rho_{gas}(r) + \rho_{gal}(r) + \rho_{CDgal}(r) \end{cases}$$

- f(R) approach: $$\begin{cases} \phi(r) = -\frac{3GM}{4a_1 r}\left(1 + \frac{1}{3}e^{-\frac{r}{L}}\right) \\ \rho_{cl,EC}(r) = \rho_{gas}(r) + \rho_{gal}(r) + \rho_{CDgal}(r) \end{cases}$$

- Rearranging the Boltzmann equation:

$$\phi_N(r) = -\frac{3GM}{4a_1 r}$$

$$\phi_C(r) = -\frac{GM}{4a_1}\frac{e^{-\frac{r}{L}}}{r}$$

$$\begin{cases} M_{bar,th}(r) = \frac{4a_1}{3}\left[-\frac{kT(r)}{\mu m_p G}r\left(\frac{d\ln\rho_{gas}(r)}{d\ln r} + \frac{d\ln T(r)}{d\ln r}\right)\right] - \frac{4a_1}{3G}r^2\frac{d\Phi_C}{dr}(r) \\ M_{bar,obs}(r) = M_{gas}(r) + M_{gal}(r) + M_{CDgal}(r) \end{cases}$$

Fitting Mass Profiles

DATA:

- Sample: 12 clusters from Chandra (Vikhlinin 2005, 2006)
- Temperature profile from spectroscopy
- Gas density: modified beta-model

$$n_p n_e = n_0^2 \cdot \frac{(r/r_c)^{-\alpha}}{(1+r^2/r_c^2)^{3\beta-\alpha/2}} \cdot \frac{1}{(1+r^\gamma/r_s^\gamma)^{\epsilon/\gamma}} + \frac{n_{02}^2}{(1+r^2/r_{c2}^2)^{3\beta_2}}$$

- Galaxy density:

$$\rho_{gal}(r) = \begin{cases} \rho_{gal,1} \cdot \left[1+\left(\frac{r}{R_c}\right)^2\right]^{-\frac{3}{2}} & r < R_c \\ \rho_{gal,2} \cdot \left[1+\left(\frac{r}{R_c}\right)^2\right]^{-\frac{2.6}{2}} & r > R_c \end{cases}$$

$$\rho_{CDgal} = \frac{\rho_{0,J}}{\left(\frac{r}{r_c}\right)^2 \left(1+\frac{r}{r_c}\right)^2}$$

Table 1. Column 1: Cluster name. Column2: Richness. Column 2: cluster total mass. Column 3: gas mass. Column 4: galaxy mass. Column 5: cD-galaxy mass. All mass values are estimated at $r = r_{max}$. Column 6: ratio of total galaxy mass to gas mass. Column 7: minimum radius. Column 8: maximum radius.

name	R	$M_{cl,N}$ ($M_\odot$)	M_{gas} ($M_\odot$)	M_{gal} ($M_\odot$)	M_{cDgal} ($M_\odot$)	$\frac{gal}{gas}$	r_{min} (kpc)	r_{max} (kpc)
A133	0	$4.35874 \cdot 10^{14}$	$2.73866 \cdot 10^{13}$	$5.20269 \cdot 10^{12}$	$1.10568 \cdot 10^{12}$	0.23	86	1060
A262	0	$4.45081 \cdot 10^{13}$	$2.76659 \cdot 10^{12}$	$1.71305 \cdot 10^{11}$	$5.16382 \cdot 10^{12}$	0.25	61	316
A383	2	$2.79785 \cdot 10^{14}$	$2.82467 \cdot 10^{13}$	$5.88048 \cdot 10^{12}$	$1.09217 \cdot 10^{12}$	0.25	52	751
A478	2	$8.51832 \cdot 10^{14}$	$1.05583 \cdot 10^{14}$	$2.15567 \cdot 10^{13}$	$1.67513 \cdot 10^{12}$	0.22	59	1580
A907	1	$4.87657 \cdot 10^{14}$	$6.38070 \cdot 10^{13}$	$1.34129 \cdot 10^{13}$	$1.66533 \cdot 10^{12}$	0.24	563	1226
A1413	3	$1.09598 \cdot 10^{15}$	$9.32466 \cdot 10^{13}$	$2.30728 \cdot 10^{13}$	$1.67345 \cdot 10^{12}$	0.26	57	1506
A1795	2	$1.24313 \cdot 10^{14}$	$1.00530 \cdot 10^{13}$	$4.23211 \cdot 10^{12}$	$1.93957 \cdot 10^{12}$	0.11	79	1151
A1991	1	$1.24313 \cdot 10^{14}$	$1.00530 \cdot 10^{13}$	$1.24608 \cdot 10^{12}$	$1.08241 \cdot 10^{12}$	0.23	55	618
A2029	2	$8.92392 \cdot 10^{14}$	$1.24129 \cdot 10^{14}$	$3.21543 \cdot 10^{13}$	$1.11921 \cdot 10^{12}$	0.27	62	1771
A2390	1	$2.09710 \cdot 10^{15}$	$2.15726 \cdot 10^{14}$	$4.91580 \cdot 10^{13}$	$1.12141 \cdot 10^{12}$	0.23	83	1984
MKW4	-	$4.69503 \cdot 10^{13}$	$2.83207 \cdot 10^{12}$	$1.71153 \cdot 10^{11}$	$5.29855 \cdot 10^{11}$	0.25	60	434
RXJ1159	-	$8.97997 \cdot 10^{13}$	$4.33256 \cdot 10^{12}$	$7.34414 \cdot 10^{11}$	$5.38799 \cdot 10^{11}$	0.29	64	568

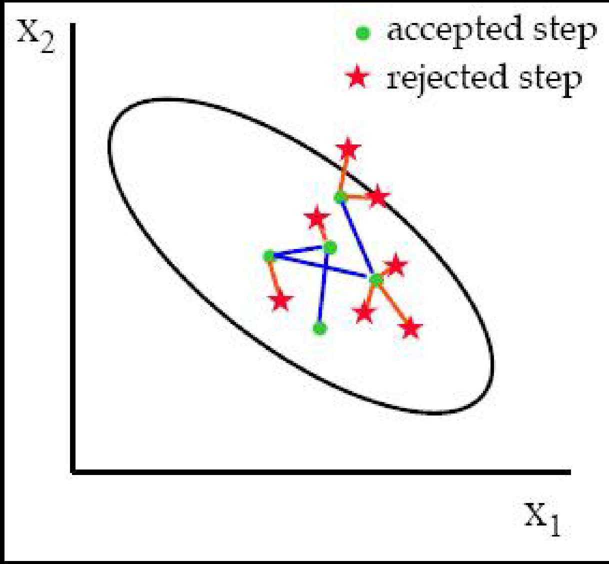
Fitting Mass Profiles

METHOD:

- Minimization of chi-square: $$\chi^2 = \frac{1}{N - n_p - 1} \cdot \sum_{i=1}^{N} \frac{(M_{bar,obs} - M_{bar,theo})^2}{M_{bar,theo}}$$

- Markov Chain Monte Carlo:

$$\alpha(\mathrm{p},\mathrm{p}') = min\left\{1, \frac{L(\mathrm{d}|\mathrm{p}')P(\mathrm{p}')q(\mathrm{p}',\mathrm{p})}{L(\mathrm{d}|\mathrm{p})P(\mathrm{p})q(\mathrm{p},\mathrm{p}')}\right\}$$



Reject min < 1:
- new point out of prior
- new point with greater chi-square

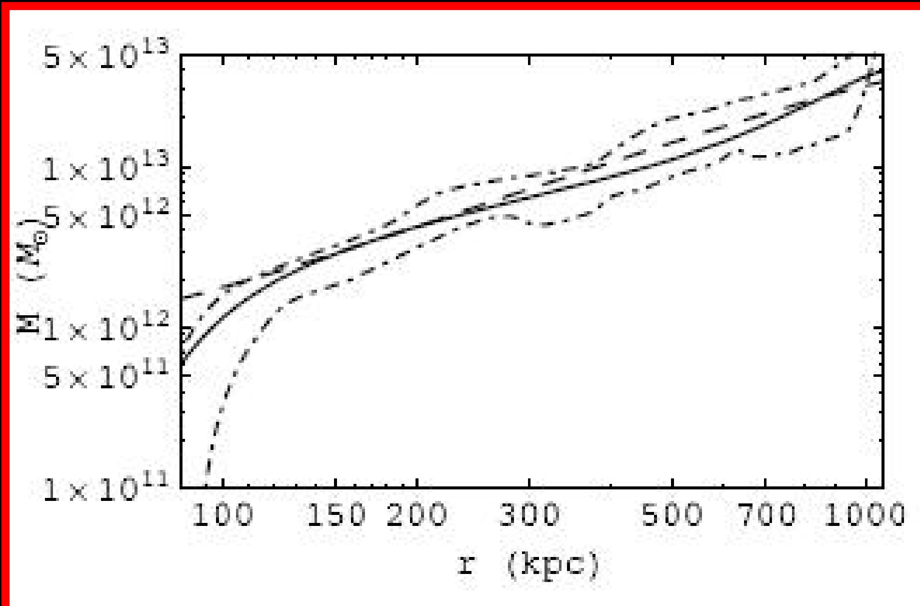
Accept min = 1: new point in prior and less chi-square

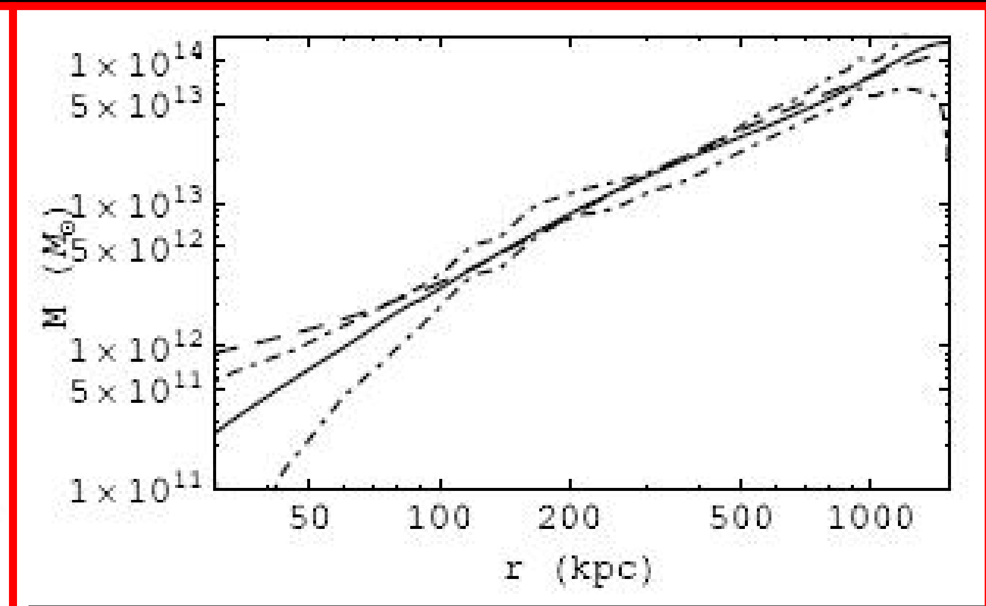
Sample of accepted points → Sampling from underlying probability distribution

- Power spectrum test convergence:

Discrete power spectrum from samples → Convergence = flat spectrum

Results: gravitational length





- Differences between theoretical and observed fit ***less than 5%***
- ***Typical scale*** in [100; 150] kpc range where is a turning-point:
  - Break in the hydrostatic equilibrium
  - Limits in the expansion series of f(R): $R - R_0 << \frac{a_1}{a_2}$ in the range [19;200] kpc

  Proper gravitational scale (as for galaxies, see Capozziello et al MNRAS 2007)

  - Similar issues in Metric-Skew-Tensor-Gravity (Brownstein, 2006): we have better and more detailed approach

Results

name	a_1	$[a_1 - 1\sigma,\ a_1 + 1\sigma]$	a_2 (kpc^2)	$[a_2 - 1\sigma,\ a_2 + 1\sigma]$ (kpc^2)	L (kpc)	$[L - 1\sigma,\ L + 1\sigma]$ (kpc)
A133	0.085	[0.078, 0.091]	$-4.98 \cdot 10^3$	$[-2.38 \cdot 10^4,\ -1.38 \cdot 10^3]$	591.78	[323.34, 1259.50]
A262	0.065	[0.061, 0.071]	-10.63	$[-57.65,\ -3.17]$	31.40	[17.28, 71.10]
A383	0.099	[0.093, 0.108]	$-9.01 \cdot 10^2$	$[-4.10 \cdot 10^3,\ -3.14 \cdot 10^2]$	234.13	[142.10, 478.06]
A478	0.117	[0.114, 0.122]	$-4.61 \cdot 10^3$	$[-1.01 \cdot 10^4,\ -2.51 \cdot 10^3]$	484.83	[363.29, 707.73]
A907	0.129	[0.125, 0.136]	$-5.77 \cdot 10^3$	$[-1.54 \cdot 10^4,\ -2.83 \cdot 10^3]$	517.30	[368.84, 825.00]
A1413	0.115	[0.110, 0.119]	$-9.45 \cdot 10^4$	$[-4.26 \cdot 10^5,\ -3.46 \cdot 10^4]$	2224.57	[1365.40, 4681.21]
A1795	0.093	[0.084, 0.103]	$-1.54 \cdot 10^3$	$[-1.01 \cdot 10^4,\ -2.49 \cdot 10^2]$	315.44	[133.31, 769.17]
A1991	0.074	[0.072, 0.081]	-50.69	$[-3.42 \cdot 10^2,\ -13]$	64.00	[32.63, 159.40]
A2029	0.129	[0.123, 0.134]	$-2.10 \cdot 10^4$	$[-7.95 \cdot 10^4,\ -8.44 \cdot 10^3]$	988.85	[637.71, 1890.07]
A2390	0.149	[0.146, 0.152]	$-1.40 \cdot 10^6$	$[-5.71 \cdot 10^6,\ -4.46 \cdot 10^5]$	7490.80	[4245.74, 15715.60]
MKW4	0.054	[0.049, 0.060]	-23.63	$[-1.15 \cdot 10^2,\ -8.13]$	51.31	[30.44, 110.68]
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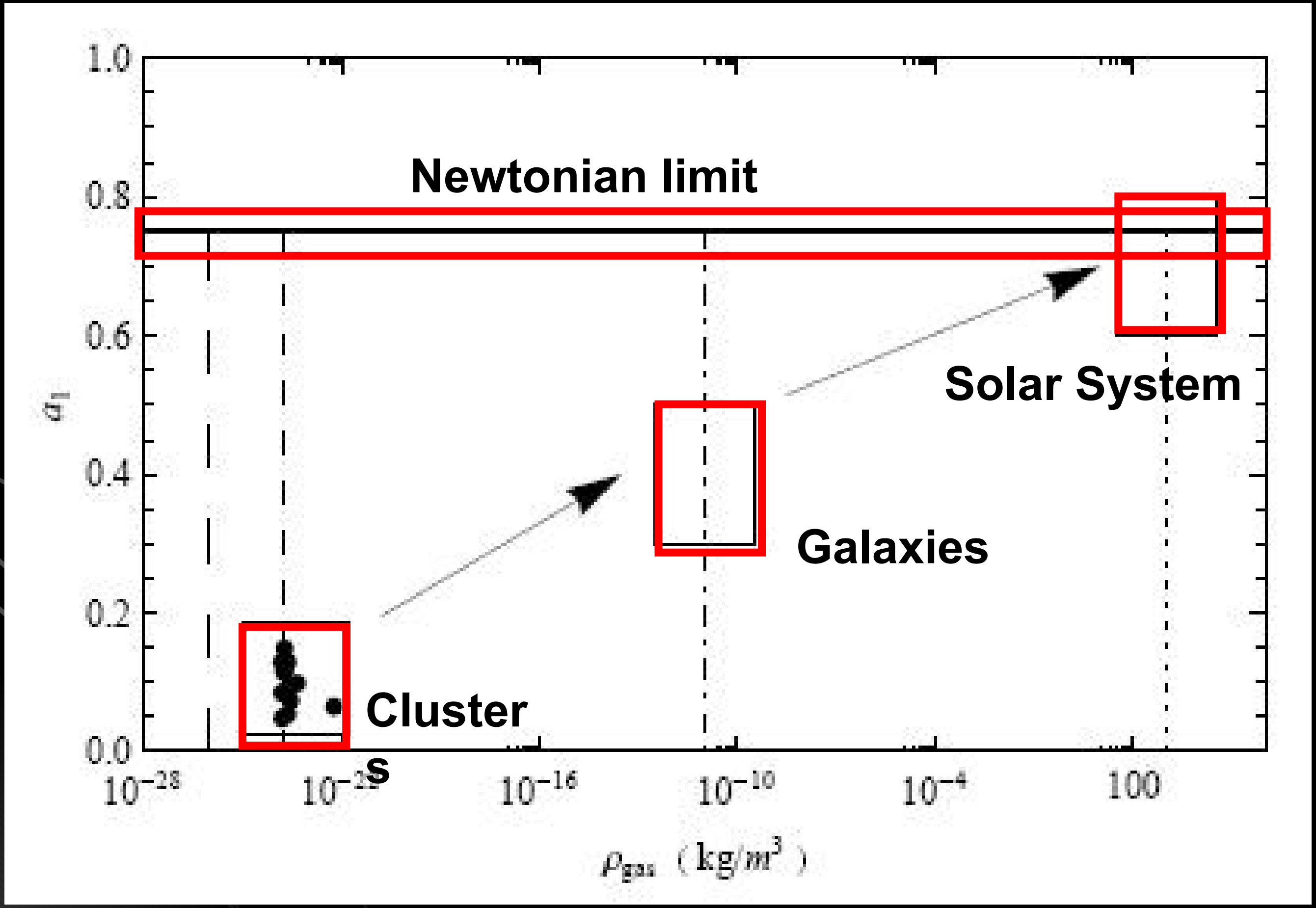
Results: expectations

- First derivative, a_1 : very well constrained → It scales with the system size

- Newtonian limit: $\phi(r) = -\frac{3GM}{4a_1 r}\left(1 + \frac{1}{3}e^{-\frac{r}{L}}\right)$ → $a_1 \rightarrow 3/4$



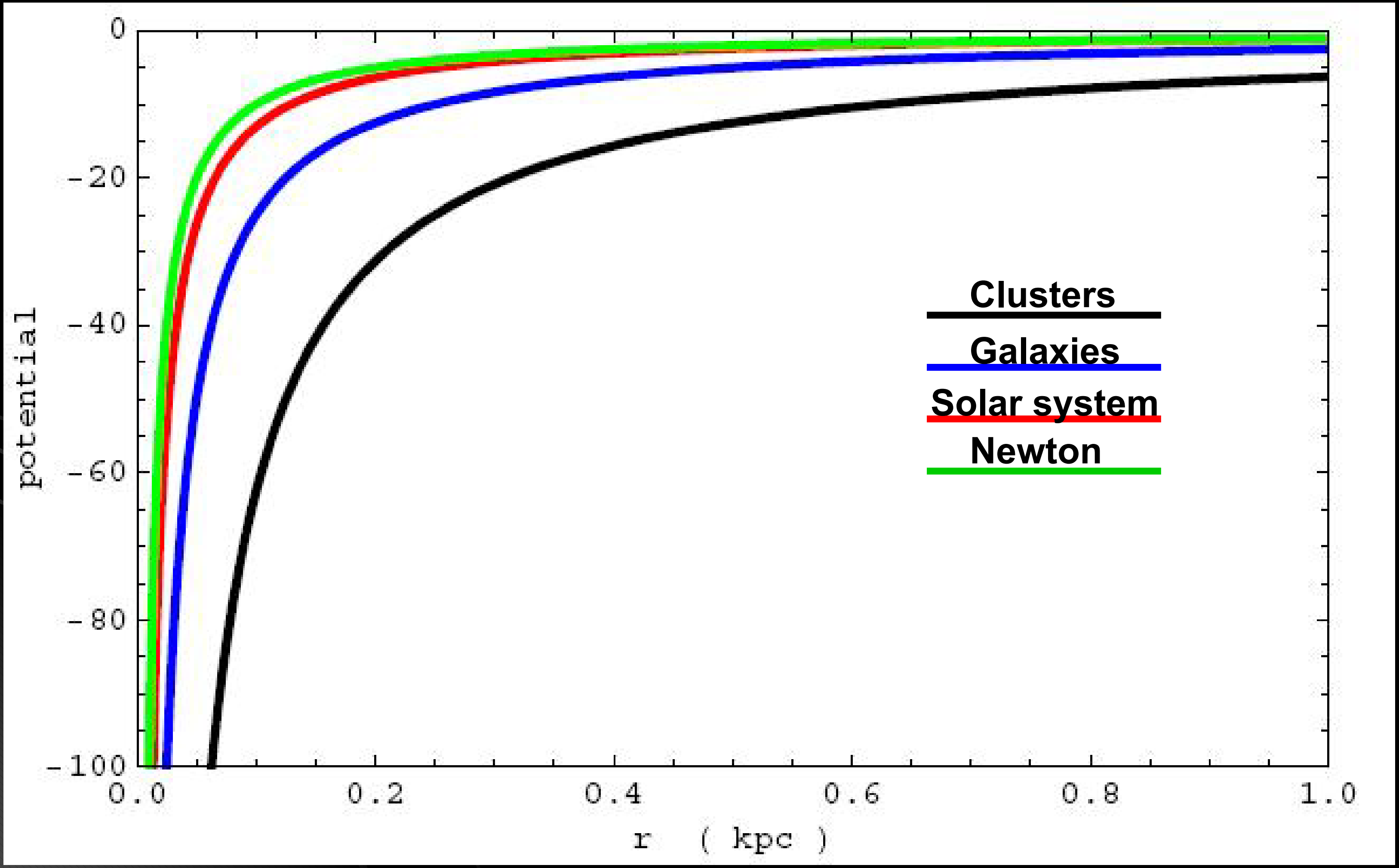
Point like potential:

Cluster of Galaxies: a_1 = 0.16 - L = 1000 kpc

Galaxies: a_1 = 0.4 - L = 100 kpc

Solar System: a_1 = 0.75 - L = 1 kpc

Newton Limit: a_1 = 0.75 - L = 0 kpc



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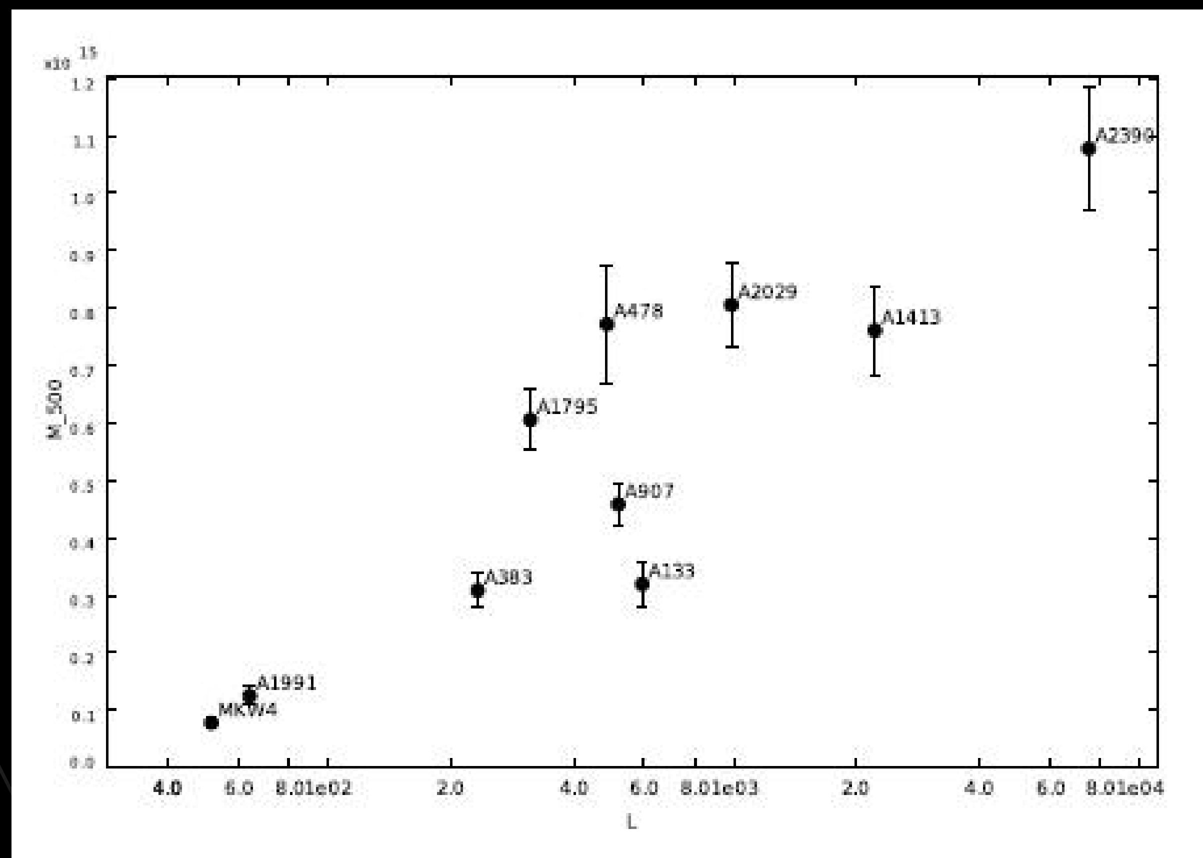
Results: expectations

- Gravitational length: $L \equiv L(a_1, a_2) = \left(-\frac{6a_2}{a_1}\right)^{1/2}$ ⇔ **Strong characterization of Gravitational potential**

- Mean length:

$$\begin{aligned} <L>_\rho &= 318 \text{ kpc} & <\mathrm{a}_2>_\rho = -3.40 \cdot 10^4 \\ <L>_M &= 2738 \text{ kpc} & <\mathrm{a}_2>_M = -4.15 \cdot 10^5 \end{aligned}$$

- Strongly related to virial mass (the same for gas mass):



- Strongly related to average temperature:

Results: expectations

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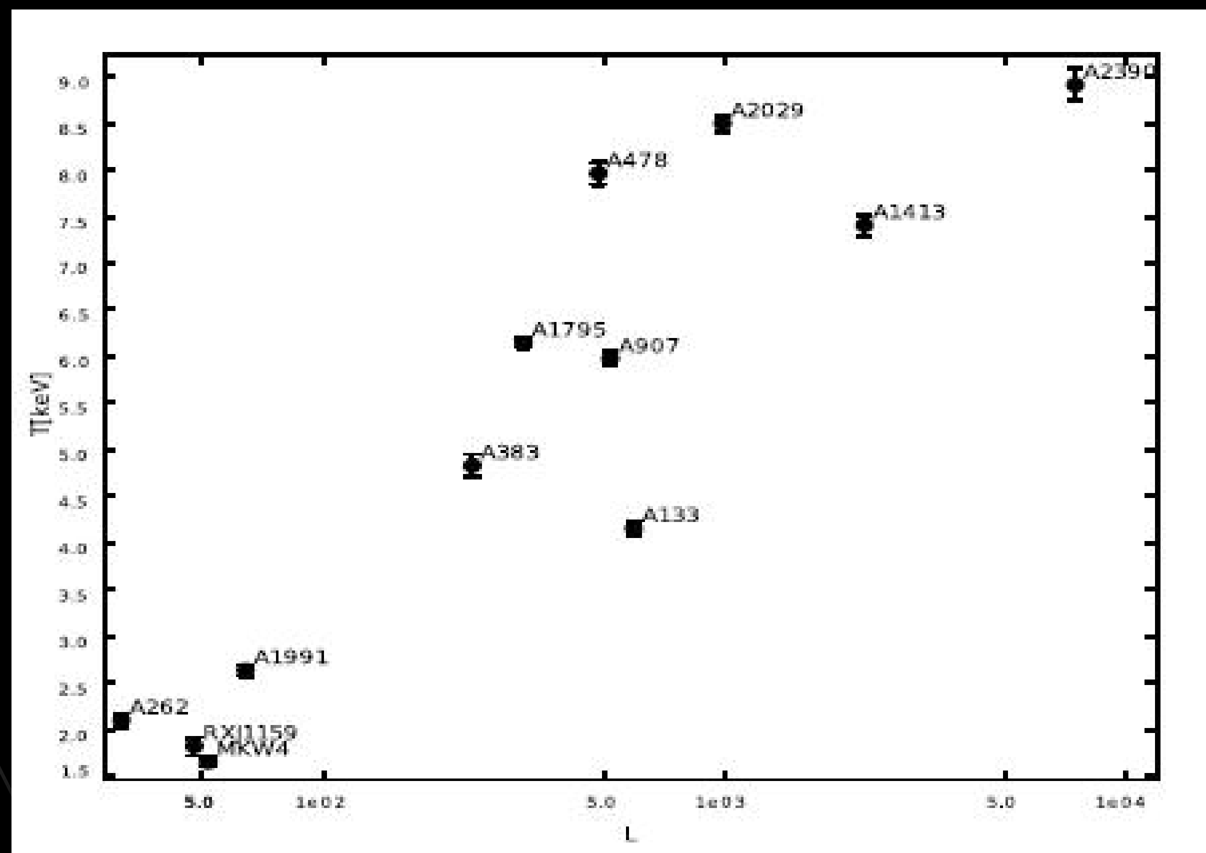

Conclusions

- Cosmography: model independent approach to f(R) -gravity
- Cosmographic parameters to constraint f(R) - gravity models
- Cosmography to "discriminate" between Dark Energy and f(R)

Perspectives:
- Montecarlo simulations to assess precision on cosmography
- Combine different datasets to strengthen the constraints
- Introduce theoretically motivated priors on cosmography

- Fitting Large Scale Structure with f(R) gravity (Clusters of Galaxies)
- Well motivated f(R) models (in agreement with observations)
- f(R) parameters strongly characterize gravitational systems

Perspectives:
- Extending to any self-gravitating systems
- Recover Newtonian limit and evade Solar System tests
- Understand physical meaning or dependency of parameters