- EFFECTIVE FIELD THEORIES -

VACUUM FLUCTUATIONS AND ENERGY DENSITY OF THE UNIVERSE

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COSMOLOGICAL CONSTANT PROBLEMS

1. **From QFT : Zero Point Energies contribution to CC**
2. **From QFT : Condensates contribution to CC**
3. **Why ρ_Λ has the tiny value we measure?**
4. **Why ρ_Λ and ρ^{mat} at present time coincide?**

... . Modified gravity, Voids, ...

We address the (QFT) Problems 1 & 2

Conclusion: ZPE & Condensates do not contribute to CC.

Where does this come from?

From the Effective Nature of Quantum Field Theories

VACUUM ENERGY & CC

QUESTION : **How do we usually manage with the**

Zero Point Energies contribution to CC?

ANSWER :

1. Lorentz Invariance $\Longrightarrow$ $T^{vac}_{\mu\nu} = \rho^{vac} g_{\mu\nu}$ $\quad (\; p^{vac} = -\rho^{vac} \;)$

Now take the example of : $\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2$

2. Vacuum Energy Density **($\Leftarrow$ Zero Point Energy) :**

$$\rho^{vac} = \frac{1}{V}\sum_{\vec{k}} \frac{1}{2}\sqrt{\vec{k}^2 + m^2} = \frac{1}{2}\int \frac{d^3\vec{k}}{(2\pi)^3}\sqrt{\vec{k}^2 + m^2} \simeq \frac{M_P^4}{16\,\pi^2}$$

3. Continuity Equation **:** $\dot{\rho} + 3\left(\frac{\dot{a}}{a}\right)(\rho + p) = 0$

$\rho^{vac} = Const. \simeq \frac{M_P^4}{16\,\pi^2}$ $\quad \geq 120$ **ord. of magn. problem ...**

.... BUT ....

EINSTEIN EQS. : ENERGY DENSITY & PRESSURE

... Let's start again from the beginning ...

$$G_{\mu\nu} - \lambda g_{\mu\nu} = 8\,\pi\, G \quad T_{\mu\nu}$$

Which $T_{\mu\nu}$ in the Ein. Eq. from our $\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2$?

Quantum Statistical Average (non diag. terms vanish):

$$T_{00} = \rho =<< \hat{T}_{00} >>= \frac{1}{V}\sum_{\vec{k}}\sum_n < n|\varrho|n > \; n_{\vec{k}}\,\omega_{\vec{k}} + \; \frac{1}{V}\sum_{\vec{k}}\frac{1}{2}\,\omega_{\vec{k}}$$

$$T_{ii} = p =<< \hat{T}_{ii} >>= \frac{1}{V}\sum_{\vec{k}}\sum_n < n|\varrho|n > \; n_{\vec{k}}\,\frac{(k^i)^2}{\omega_{\vec{k}}} + \; \frac{1}{V}\sum_{\vec{k}}\frac{(k^i)^2}{2\,\omega_{\vec{k}}}$$

$|n>$ **: generic element Fock space basis;**

ϱ **: density operator ;** $n_{\vec{k}} =< n|a^\dagger_{\vec{k}} a_{\vec{k}}|n >$ **;** $\omega_{\vec{k}} = \sqrt{\vec{k}^2 + m^2}$.

By performing the sum over n :

$$T_{00} = \rho = \int \frac{d^3\vec{k}}{(2\pi)^3}\,\omega_{\vec{k}}\, n_{BE}(\vec{k}^2) + \frac{1}{2}\int \frac{d^3\vec{k}}{(2\pi)^3}\,\omega_{\vec{k}} \equiv \rho^{mat} + \rho^{vac}$$

$$T_{ii} = p = \frac{1}{3}\int \frac{d^3\vec{k}}{(2\pi)^3}\,\frac{\vec{k}^2}{\omega_{\vec{k}}}\, n_{BE}(\vec{k}^2) + \frac{1}{3}\int \frac{d^3\vec{k}}{(2\pi)^3}\,\frac{\vec{k}^2}{2\,\omega_{\vec{k}}} \equiv p^{mat} + p^{vac}$$

$n_{BE}(\vec{k}^2)$ = **Bose-Einstein distribution**

In Blue : Gas of Rel. Part. (Matter/Radiation contribution)

In Red : The corresponding Vacuum contribution

Remark 1 : In the Einstein Eqs., ρ^{mat} & ρ^{vac}, as well as p^{mat} & p^{vac}, enter on an equal footing.

Remark 2 : UV cutoff needed to compute ρ^{vac} & p^{vac}

On the contrary, due to the presence of $n_{BE}(\vec{k}^2)$, ρ^{mat} & p_{mat} are finite

Computing ρ^{vac} & p^{vac} with an UV cutoff $\Lambda\,(= M_P)$

for our $\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2$ **we have :**

$$\rho^{vac} = \frac{1}{16\pi^2}\left[\Lambda(\Lambda^2+m^2)^{\frac{3}{2}} - \frac{\Lambda m^2(\Lambda^2+m^2)^{\frac{1}{2}}}{2} - \frac{m^4}{4}\ln\left(\frac{(\Lambda+(\Lambda^2+m^2)^{\frac{1}{2}})^2}{m^2}\right)\right]$$

$$p^{vac} = \frac{1}{16\pi^2}\left[\frac{\Lambda^3(\Lambda^2+m^2)^{\frac{1}{2}}}{3} - \frac{\Lambda m^2(\Lambda^2+m^2)^{\frac{1}{2}}}{2} + \frac{m^4}{4}\ln\left(\frac{(\Lambda+(\Lambda^2+m^2)^{\frac{1}{2}})^2}{m^2}\right)\right]$$

Note : For $\Lambda >> m \implies$ **RED dominant :**

$$\rho^{vac} \simeq \frac{\Lambda^4}{16\pi^2} \quad ; \quad p^{vac} \simeq \frac{1}{3}\,\frac{\Lambda^4}{16\pi^2}$$

QUESTION : How do we deal with the Divergences ?

Formal Point of View : **The Divergent Terms (which do not respect $T^{vac}_{\mu\nu} \propto g_{\mu\nu}$) have to be removed via Renormalization**

(This would "alleviate" the 120 order of magnitude problem ...However, we still have the condesates...)

Criticism (De Witt) :

- **For Zero Point Energies : Physical Meaning of Divergences rooted in the harmonic oscillator structure of a QFT.**
- **Lost if we cancel out these terms with a formal procedure such as normal ordering.**
- **Still, popular prescription for the automatic cancellation of these divergences : Dimensional Regularization**

Deeper Physical Point of View $\Longrightarrow$

Effective Field Theory Point of View

Lesson from Wilson RG & Effective Field Theories :

QFT = Effective Theory valid up to Λ ($\Lambda =$ "scale of new physics")

Hyerarchy of Field Theories up to M_P $\leftarrow$ String theory (?)

According to this view, the cutoff Λ is physical $\Longrightarrow$ we do not discard any term in $T_{\mu\nu}$

OK. Let's take this point of view

and go back to the Effective Field $T_{\mu\nu}$
that we have computed before

Effective Field $T_{\mu\nu}$

$$T_{00} = \rho^{vac} = \frac{1}{16\pi^2}\left[\Lambda(\Lambda^2+m^2)^{\frac{3}{2}} - \frac{\Lambda m^2(\Lambda^2+m^2)^{\frac{1}{2}}}{2} - \frac{m^4}{4}\ln\left(\frac{(\Lambda+(\Lambda^2+m^2)^{\frac{1}{2}})^2}{m^2}\right)\right]$$

$$T_{ii} = p^{vac} = \frac{1}{16\pi^2}\left[\frac{\Lambda^3(\Lambda^2+m^2)^{\frac{1}{2}}}{3} - \frac{\Lambda m^2(\Lambda^2+m^2)^{\frac{1}{2}}}{2} + \frac{m^4}{4}\ln\left(\frac{(\Lambda+(\Lambda^2+m^2)^{\frac{1}{2}})^2}{m^2}\right)\right]$$

For $\Lambda = M_P >> m$, **Red (= Vacuum contribution) is dominant :**

$$\rho^{vac} \simeq \frac{\Lambda^4}{16\pi^2} \quad ; \quad p^{vac} \simeq \frac{1}{3}\,\frac{\Lambda^4}{16\pi^2}$$

Then : $p^{vac} \simeq \frac{1}{3}\,\rho^{vac}$

As matter is relativistic : $p^{mat} \simeq \frac{1}{3}\,\rho^{mat}$

Therefore : $p = p^{vac} + p^{mat} \simeq \frac{\rho^{vac}+\rho^{mat}}{3} = \frac{\rho}{3}$

Consequences for the CC problem

- $\rho = \rho^{vac} + \rho^{mat}$ **follows evolution of relat. matter :**

$$\rho(t) \propto a(t)^{-4} \quad \Leftarrow \quad \dot{\rho} + 3\left(\frac{\dot{a}}{a}\right)(\rho + p) = 0$$

- **This Equation also holds for** ρ^{vac} **and** ρ^{mat} **separately**

$$\rho^{mat}(t) \propto a(t)^{-4} \quad \Leftarrow \quad \dot{\rho}^{mat} + 3\left(\frac{\dot{a}}{a}\right)(\rho^{mat} + p_{mat}) = 0$$

$$\rho^{vac}(t) \propto a(t)^{-4} \quad \Leftarrow \quad \dot{\rho}^{vac} + 3\left(\frac{\dot{a}}{a}\right)(\rho^{vac} + p^{vac}) = 0$$

In fact, when no matter is present, $\rho = \rho^{vac} \quad \Rightarrow$ **the continuity equation holds for** ρ^{vac} **alone. As the ZPEs do not interact directly with matter,** ρ^{mat} **satisfies the same equation.**

- **For relativistic matter :**

$$\rho^{mat}(t) = \frac{\pi^2}{30}\, T^4 \;\propto a^{-4}$$

- **But ρ^{vac} has the same scaling :**

$$\Rightarrow \;\; \rho^{vac}(t) = \frac{\rho^{vac}(t_P)}{\rho^{mat}(t_P)}\, \rho^{mat}(t) \qquad\qquad (t_P = \text{Planck time})$$

- **From $\dot{a}/a = -\dot{T}/T \quad \Rightarrow \quad$ Fried. Eq., $\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho$, becomes :**

$$\left(\frac{\dot{T}}{T}\right)^2 = \frac{8\pi G}{3}\left(1 + \frac{\rho^{vac}(t_P)}{\rho^{mat}(t_P)}\right)\rho^{mat}(t) = \frac{4\pi^3 G}{45}\left(1 + \frac{\rho^{vac}(t_P)}{\rho^{mat}(t_P)}\right) T^4$$

- **By integrating :** $\qquad T = \left(\frac{45}{16\pi^3\, K\, G}\right)^{\frac{1}{4}}\, t^{-\frac{1}{2}}$

where $K = 1 + \rho^{vac}(t_P)/\rho^{mat}(t_P)$. In the standard approach: $\rho^{vac}(t)$ is absent $\quad \Rightarrow \quad K = 1$

OUR SCENARIO

- **Assume that ϕ is a sort of "primordial field" (out of which "other fields" were born during the cosmic evolution), which describes physics at the scale $\Lambda = M_P$ at the Planck time t_P (beyond M_P...some UV completion).**

  At $\quad t = t_P \quad \rho^{vac}(t_P) = \frac{M_P^4}{16\pi^2} \quad , \rho_\phi^{mat}(t_P) = \frac{\pi^2}{30} T_P^4 \quad (G = M_P^{-2} = t_P^2)$

  Then, from $\quad T = \left(\frac{45}{16\pi^3 K G}\right)^{\frac{1}{4}} t^{-\frac{1}{2}} \quad \Rightarrow \quad \frac{\rho^{vac}(t_P)}{\rho_\phi^{mat}(t_P)} \simeq 0.27$

  As $\rho^{vac}(t)$ **scales as** $\rho_\phi^{mat}(t)$ **:** $\frac{\rho^{vac}(t)}{\rho_\phi^{mat}(t)} \simeq 0.27$ **at later times**

  This holds true until the primordial ϕ field decays into "other fields".

It is easy to estimate $\rho^{vac}(t_0)$ at present time t_0. In fact, $\rho^{vac}(t) \propto a(t)^{-4}$ always (= from t_P down to t_0). Therfore (t_{eq} is defined as : $\rho_{rel}(t_{eq}) = \rho_{nrel}(t_{eq})$):

- **From t_P to t_{eq} (radiation dominated era) :** $a(t) \propto t^{1/2} \;\Rightarrow$ $\rho^{vac}(t_{eq}) = \rho^{vac}(t_P) \left(\frac{t_P}{t_{eq}}\right)^2$
- **From t_{eq} to t_0 (matter dominated era) (neglect the harmless short vacuum dominance era) :** $a(t) \propto t^{2/3} \quad \Rightarrow$

$$\rho^{vac}(t_0) = \rho^{vac}(t_{eq}) \left(\frac{t_{eq}}{t_0}\right)^{\frac{8}{3}}$$

$\Rightarrow$ **at present time $\rho^{vac}(t_0)$ is :**

$$\rho^{vac}(t_0) = \rho^{vac}(t_P) \left(\frac{t_P}{t_0}\right)^2 \cdot \left(\frac{t_{eq}}{t_0}\right)^{\frac{2}{3}} = \rho^{vac}(t_P) \left(\frac{t_P}{t_0}\right)^2 \cdot \frac{a_{eq}}{a_0}$$

$$\rho^{vac}(t_0) = \rho^{vac}(t_P)\left(\frac{t_P}{t_0}\right)^2 \cdot \frac{a_{eq}}{a_0}$$

Inserting : $\rho^{vac}(t_P) = \frac{M_P^4}{16\pi^2}$ **,** $t_P = 5.39\,10^{-44}\,s$ **,** $t_0 \simeq 13.7\,Gy$ **and** $\frac{a_{eq}}{a_0} \simeq 1/3240 \;\Rightarrow$

$$\rho^{vac}(t_0) \simeq \left(1.71 \times 10^{-4}\,\mathrm{eV}\right)^4$$

Compare $\rho^{vac}(t_0)$ with $\rho_\gamma(t_0)$ measured at present time :

$$\rho_\gamma(t_0) \sim \left(2.11 \times 10^{-4}\,\mathrm{eV}\right)^4$$

$\Rightarrow$

$$\frac{\rho^{vac}(t_0)}{\rho_\gamma(t_0)} \sim 0.43$$

Predictions of this scenario

- **ZPEs red-shift as radiation**
- **ZPEs do not contribute to CC as $w = 1/3$ (not $w = -1$).**
- **It provides a mechanism for washing out the ZPEs.**
- **It predicts that ρ^{vac} negligible today. But ... at the BBN epoch? Could the presence of this new radiation component screw up well tested BBN predictions?**
- **Let's "go back" to the BBN epoch. The total amount of radiation during this epoch can be written as :**

$$\rho_{_R} = \left(1 + \tfrac{7}{8}\left(\tfrac{4}{11}\right)^{4/3} N_{\rm eff}\right)\rho^{\gamma}$$

where (neglecting the tiny out of equilibrium neutrino contributions) $N_{\rm eff} = 3 + \delta n$ **and** δn **accounts for extra d.o.f. (neutrino equivalent d.o.f.), i.e. :**

$$\rho_{_R} = \Big(1 + \tfrac{7}{8}\big(\tfrac{4}{11}\big)^{4/3}(3 + \delta n)\Big)\rho^{\gamma}$$

- **As** ρ^{vac} **in our model is:** $\rho^{vac}(t_0) \simeq 0.43\,\rho^{\gamma}(t_0)$

$\Rightarrow$ $$(\delta n)_{th} = 1.89$$

- **Consider now the analysis of BBN (Iocco, Mangano, Miele, Pisanti, Serpico, Phys. Rep. 472 (2009), 1) where a possible non-standard value of the relativistic energy content during BBN is retained as a free parameter**

$\Rightarrow$ $$(\delta n)_{exp} = 0.18^{+0.44}_{-0.41} \qquad \textbf{at 95 \% C.L.}$$

What do we learn from these results?

- **The experimental data suggest that our crude parametrization of physics at the Planck scale has to be refined (more realistic model needed)**

- **Not even touching the field content at t_P , a slight modification of the vacuum energy density at t_P ,**

$$M_P^4 \to T(t_P)^4 = \frac{45}{16\pi^3 G} t_P^{-2} \, ,$$

gives : $$(\delta n)_{th} = 0.17$$

(compare with $(\delta n)_{exp} = 0.18^{+0.44}_{-0.41}$ **)**

- **Another possibility : consider a boson/fermion model at t_P**

- **More generally :** $M_P \to \Lambda \quad , \quad t_P \to t_\Lambda$

As (see slide 14) $\frac{\rho^{vac}(t_\gamma)}{\rho_\gamma(t_\gamma)} = \frac{\rho^{vac}(t_0)}{\rho_\gamma(t_0)} = \frac{\rho^{vac}(t_\Lambda)}{\rho_\gamma(t_0)} \cdot \left(\frac{t_\Lambda}{t_0}\right)^2 \cdot (1+z_{eq})^{-1}$

(with $t_\gamma \sim t_{BBN}$ **and therefore after neutrino decoupling)**

$\Rightarrow$ $(\delta n)_{th}$ **as a function of** $\Lambda^4\, t_\Lambda^2$ $\Rightarrow$ **bound on** $\Lambda^4 t_\Lambda^2$

In fact, from $\rho_{_{rad}}(t_\gamma) = \Big(1 + \frac{7}{8}\big(\frac{4}{11}\big)^{4/3}(3+\delta n)\Big)\rho_\gamma(t_\gamma)$

comparing with the above expression we have:

$$(\delta n)_{th} = 3.13 \cdot 10^{-5} \cdot \frac{H_0^2}{T_0^4} \cdot \Lambda^4 t_\Lambda^2$$

where we have used :

$\rho^{vac}(t_\Lambda) = \frac{\Lambda^4}{16\pi^2}$; $t_0 = \frac{2}{3H_0}$; $\rho_\gamma(t_0) = \frac{\pi^2}{15}T_0^4$; $(1+z_{eq})^{-1} = \frac{1}{3048}$

Brief Summary up to now :

- **Cosmic evolution provides a mechanism to dilute (~ 0) the Zero Point Energy contribution to ρ ;**
- **Zero Point Energy does not contribute to CC ;**
- **At t_P, physics is described by some primordial quantum fields.**
- **Lower Energy Theories : born during cosmic evolution.**

Moreover note that :

- **Low Energy Fields : just a convenient way to parametrize physics at lower scales (NO NEW DOF).** $\Longrightarrow$

$\Rightarrow$ When computing Vacuum Energy : do not include Zero Point Energies of Lower Energy Effective Theories.
Otherwise : multiple counting of dof. Zero Point Energies of the original High Energy Theory give the whole contribution to Vacuum Energy.

ANOTHER IMPORTANT LESSON

Some Low Energy Theories have Condensates. They enter $T_{\mu\nu}$ as : $T_{\mu\nu} = \rho^{cond} g_{\mu\nu}$ $\Rightarrow$ should give large contributions to CC.

..... BUT .....

Effective Field Theory scenario : no such terms. Taking into account these terms would again result in a multiple counting of dof.

Let us elucidate these points with an example

Inspired to the analysis of top condensate models by Bardeen, Hill and Lindner

- **NJL high energy theory , defined at the scale Λ :**

$$Z = \int D\bar{\psi}\, D\psi \exp\left[i \int d^4 x \left(\bar{\psi}(i\gamma^\mu \partial_\mu - M)\psi + \frac{g^2}{2m_0^2}\bar{\psi}\psi\bar{\psi}\psi \right)\right]$$

- **Hubbard-Stratonovic $\Rightarrow$ auxiliary scalar field ϕ :**

$$Z = \frac{1}{\mathcal{N}} \int D\bar{\psi}\, D\psi\, D\,\phi \exp\left[i \int d^4 x \left(\bar{\psi}(i\gamma^\mu \partial_\mu - M)\psi - \frac{m_0^2}{2}\phi^2 + g\bar{\psi}\psi\phi \right)\right]$$

- **Normaliz. factor $\mathcal{N}$: ensures the equality of the two Eqs.**

- **Integrating high frequency modes from Λ to μ:**

$$Z = \frac{\mathcal{Q}}{\mathcal{N}} \int D\bar{\psi}_l \, D\psi_l \, D\,\phi_l \exp\left[i \int \, d^4\, x \left(\bar{\psi}_l (i\gamma^\mu \partial_\mu - M - \delta M)\psi_l + g\bar{\psi}_l\psi_l\phi_l + \frac{1}{2} Z_\phi \partial^\mu \phi_l \partial_\mu \phi_l - \frac{m_0^2 + \delta m_0^2}{2}\phi_l^2 - \frac{\lambda}{24}\phi_l^4 \right)\right]$$

ϕ_l and ψ_l : fields with Fourier components from 0 to μ.

- **Normally, the factor $\mathcal{Q}/\mathcal{N}$ is not (cannot be !) considered : no knowledge of the High Energy Theory ! No effect for evaluating Green's fcts. (scattering processes). However :**

If we compute the Vacuum Energy from this Effective Lagrangian, we end up with a result which differs from the one obtained from the "Fundamental" = "High Energy" NJL theory unless the factor $\mathcal{Q}/\mathcal{N}$ is taken into account.

Origin of the mismatch : erroneous counting of dof.

- **Condensates** – **The same argument applies when additional contributions to the vacuum energy come from the appearance of condensates such as, for instance, a vacuum expectation value for ϕ_l.**

Let me please repeat myself on this point :

The low energy theory is "just" a convenient way to describe physics at a lower scale in terms of a parametrization which is fit to that scale,

No new dof are created!

Curved space-time (FRW)

I have presented the model in a simple setting : Minkowski space-time.

We can consider our QFT in an expanding universe : a FRW background (which is of interest to us).

The "adiabatic basis" approach allows for : mode decomposition, definition of a Fock space at each time.

For the leading "divergences" in p^{vac} & ρ^{vac} again we have :

$$p^{vac} = \rho^{vac}/3 \quad \& \quad \rho^{vac} = a(t)^{-4}\Lambda^4$$

which are nothing but our results.

From the Effective Action

A common believe : when $T^{\mu\nu}$ is computed from the Effective Action, $T^{\mu\nu} = \frac{\delta \Gamma}{\delta g_{\mu\nu}}$, we necessarily get $w = -1$ even for the quartic divergent part of $T^{\mu\nu}$.

This gives me the opportunity to come back to a central point of my talk : The quartic divergences in $T^{\mu\nu}$, when a physical cut-off is used, come from summing up the ZPE up to the scale M_P.

In any other, formal, regularization, this very physical meaning of the quartic divergent term is lost.

Now remember : in the usual computation of Γ we use Schwinger-De Witt.

Consider the $1l$ contribution to the Effective Action :

$$\Gamma = -\frac{i}{2} Tr ln[-G_F]$$

When we use the poper time, we invert the $\int ds$ with the $\int d^4k$ integration (we perform the $\int d^4k$ first) (*).

Don't do that ... you'll get, of course, the physical result.

(*)

$$\int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2-m^2+i\epsilon} = -i \int \frac{d^4k}{(2\pi)^4} \int_0^\infty ds\, e^{is(k^2-m^2+i\epsilon)}$$

CONCLUSIONS

- **At t_P physics described by Effective Field Theory (-ies) with UV cutoff M_P $\Rightarrow$ Vacuum Energy Density ρ^{vac} undergoes a cosmic scaling : ρ^{vac} is negligible at present time t_0**
- **How does it come? For an Effective Field Theory : $T_{\mu\nu} = << \hat{T}_{\mu\nu} >>$ is such that $p^{vac} \sim \rho^{vac}/3$**
- **ρ^{vac} does not contribute to CC ($w^{vac} \sim 1/3$ while $w^{CC} \sim -1$)**
- **Condensates do not contribute to CC (otherwise : multiple counting of DOF)**