Phase diagram of a Y-shaped Josephson junction network

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Main idea

(Bosonic) Y-Shaped network of Josephson junction chains (YJJN) $\Rightarrow$ Relevant instanton operators arising at the strongly coupled fixed point of the phase diagram $\Rightarrow$ Finite-coupling fixed point (FFP) in the phase diagram;

Application: YJJN working near the FFP $\Rightarrow$ Frustration of decoherence in the two-level quantum system (2LQS) emerging at the FFP;

Technology: renormalization group+boundary conformal field theory.

Plan of the talk:

1. Review of Y-shaped devices, either fermionic, or bosonic;

2. Building a bosonic Y-shaped network with a FFP with Josephson junction arrays;

3. Emergence of a FFP in the phase diagram;

4. Current's patter in the YJJN as a phase probe;

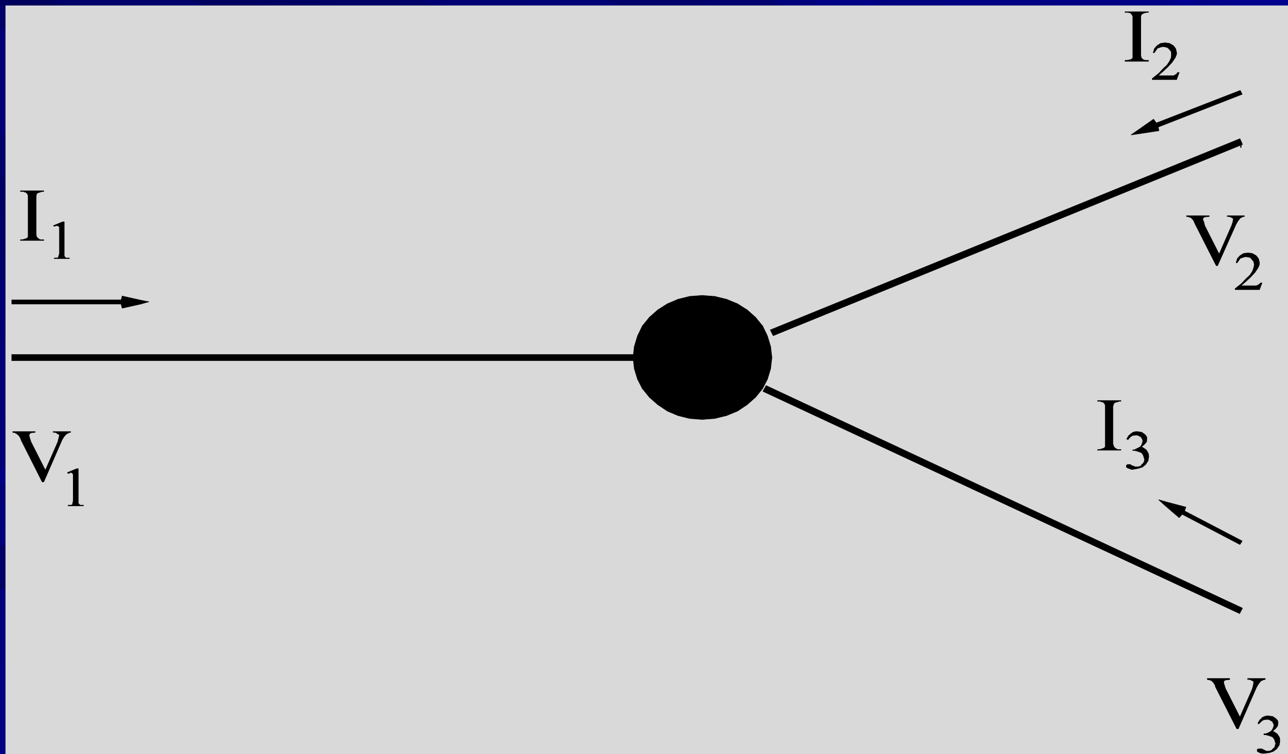
5. Spectral density and frustration of decoherence in the YJJN working near the FFP;

6. Conclusions, possible applications, perspectives.

1.Review of previously studied Y-shaped devices;

Y-junction of (spinless) fermionic quantum wires

(M. Oshikawa, C. Chamon, I. Affleck, J. Stat. Mech. P02008 (2006)).



"Bulk" Hamiltonian

$$H_f = -iv \int dx \sum_{j=1,2,3} \left[\psi_{R,j} \frac{\partial \psi_{R,j}}{\partial x} - \psi_{L,j} \frac{\partial \psi_{L,j}}{\partial x} \right]$$

"Boundary" Hamiltonian

$$H_B = -i\Gamma e^{i\varphi/3} \sum_{j=1,2,3} \psi_j^+ \psi_{j+1} + h.c.$$

Probing the boundary parameters: conductance tensor

$$I_j = \sum_k G_{jk} V_k$$

$$G_{jk} = \frac{G_S}{2}(3\delta_{jk} - 1) + \frac{G_A}{2} \varepsilon_{ijk}$$

G_A measures time-reversal breaking

When the bulk is noninteracting, the boundary interaction is a combination of marginal operators: in this case, there is a manifold of fixed point, characterized by different values of the boundary parameters Γ,φ. The behavior of the system at the junction is fully described in terms of the single-electron S matrix. The model is exactly solvable.

$$\psi_{L,j} = S_{jj}\psi_{R,j} + \sum_{k \neq j} S_{jk}\psi_{R,k}$$

Switching on an interaction in the bulk -> Luttinger liquid formalism

Bosonic bulk Hamiltonian

$$H_b = \frac{g}{4\pi} \sum_{j=1,2,3} \int dx \left[\frac{1}{u} \left(\frac{\partial \Phi_j}{\partial t} \right)^2 + u \left(\frac{\partial \Phi_j}{\partial x} \right)^2 \right]$$

Bosonic boundary Hamiltonian

$$H_B = i\Gamma e^{i\varphi/3} \sum_{a=1,2,3} \eta_a e^{i[\vec{K}_a \bullet \vec{\Phi}(0)]} + h.c.$$

Bosonization recipe (for fermions)

$$\psi_j(x,t) = e^{ik_f x}\psi_{R,j}(x,t) + e^{-ik_f x}\psi_{L,j}(x,t)$$

$$\psi_j(0) = \overline{\eta}_j e^{-i\Phi_j(0)}, \left\{\overline{\eta}_j, \overline{\eta}_k\right\} = \delta_{j,k}, \left(\overline{\eta}_j\right)^+ = \overline{\eta}_j, (\eta_a = \varepsilon_{ajk}\overline{\eta}_j\overline{\eta}_k)$$

$$\vec{\Phi}(0) = \left(\frac{1}{\sqrt{2}}[\Phi_1(0) - \Phi_2(0)], \frac{1}{\sqrt{6}}[\Phi_1(0) + \Phi_2(0) - 2\Phi_3(0)]\right), \vec{K}_{1,2,3} = \begin{pmatrix} \left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right) \\ \left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right) \\ (1,0) \end{pmatrix}$$

For interacting wires ($g\neq 1$) it is not possible to account for the boundary conditions by means of linear relations between the fields (in bosonic coordinates, $\Phi_j,\partial\Phi_j$), except at the RG fixed point, where the bc's become conformal.

Usually, in fermionc systems one gets $g<1$ (i.e., repulsive interation). In this case, only the "trivial" fixed point $\partial\Phi_j=0$ is table. However, also in view of different applications of the same formalism, it is worth studying the whole phase diagram of the system, in the space of the parameters Γ,φ.

Mapping onto the Dissipative Hofstadter model on a triangular lattice->Phase diagram

(C. Chamon, M. Oshikawa, I. Affleck, PRL 91, 206403 (2003))

$$S_0[\vec{X}] = \int \frac{d\omega}{(2\pi)^2} \left[\alpha |\omega| \delta_{\mu\nu} + \beta \omega \varepsilon_{\mu\nu} \right] X^*_\mu(\omega) X_\nu(\omega)$$

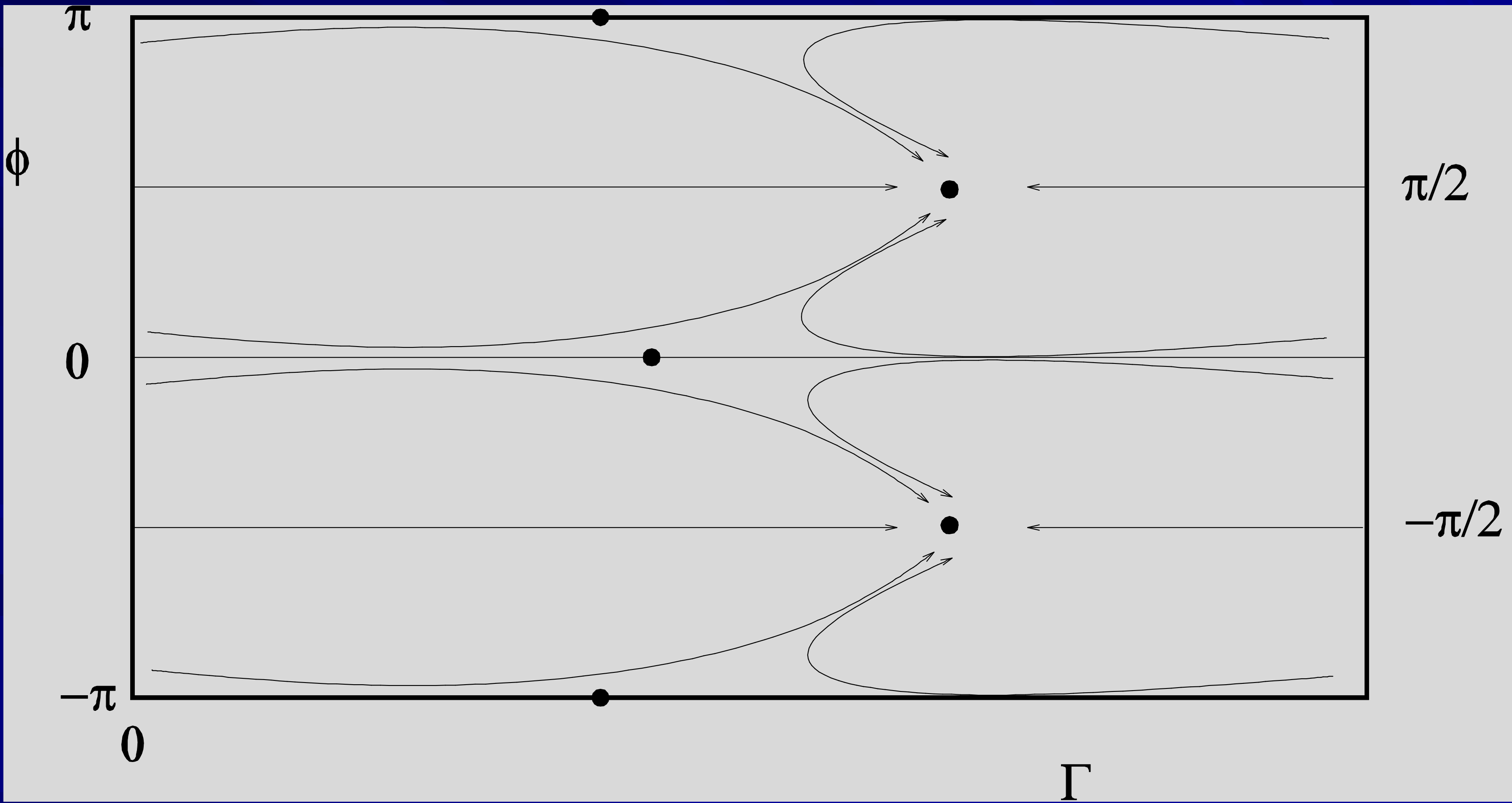
$$S_V[\vec{X}] = -V e^{i\delta/3} \int d\tau \sum_{a=1,2,3} e^{i\vec{K}_a \bullet \vec{X}(\tau)} + h.c.$$

Parameter correspondance

$$\frac{\alpha}{\alpha^2+\beta^2} = \frac{1}{g}, \frac{\sqrt{3}\pi}{2}\frac{\beta}{\alpha^2+\beta^2} + \delta = \varphi(\mathrm{mod}.2\pi), \frac{\sqrt{3}\pi}{2}\frac{\beta}{\alpha^2+\beta^2} - \delta = \pi - \varphi(\mathrm{mod}.2\pi)$$

Klein factors-> δ≠0 even when φ=0->nontrivial phase diagram (when g>1), with finite coupling fixed points, even without time-reversal breaking (i.e., φ=0, mod.π).

(Displayed 1<g<3)



Y-junction of (bosonic) one-dimensional atomic condensates

(A. Tokuno, M. Oshikawa, E. Demler, PRL 100, 140402 (2008)).

$$H_f = \sum_{j=1,2,3} \int dx \left[\frac{\hbar^2}{2m} \left(\partial \Psi_j^{+} \partial \Psi_j \right) + \frac{U}{2} \left(\Psi_j^{+} \Psi_j \right)^2 \right]$$

$$H_B = -\Gamma \sum_{j=1,2,3} \Psi_j^{+}(0) \Psi_{j+1}(0) + h.c.$$

Bosonization recipe (for bosons)

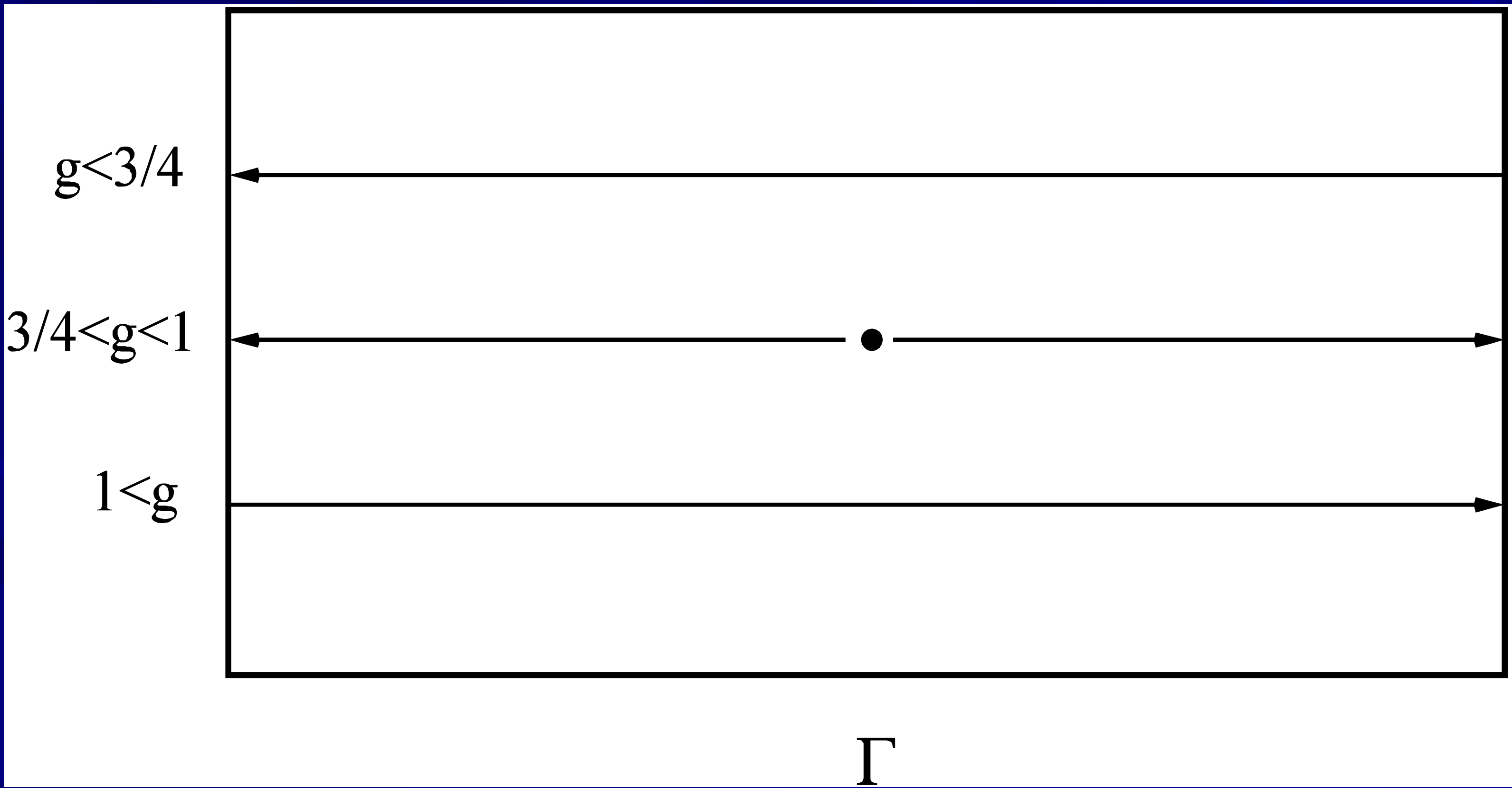
$$\Psi_j^+(x) = \left[\rho_0 + \frac{1}{\pi}\partial\theta\right]^{\frac{1}{2}} e^{-i\phi_j(x)}$$

$$H_f = \frac{v}{2\pi}\int dx \sum_{j=1,2,3}\left[g(\partial\phi_j)^2 + \frac{1}{g}(\partial\theta_j)^2\right]$$

Now g can take any value

Correspondance with the DHM and phase diagram

$$\frac{\alpha}{\alpha^2+\beta^2}=\frac{1}{g},\beta=\delta=0$$



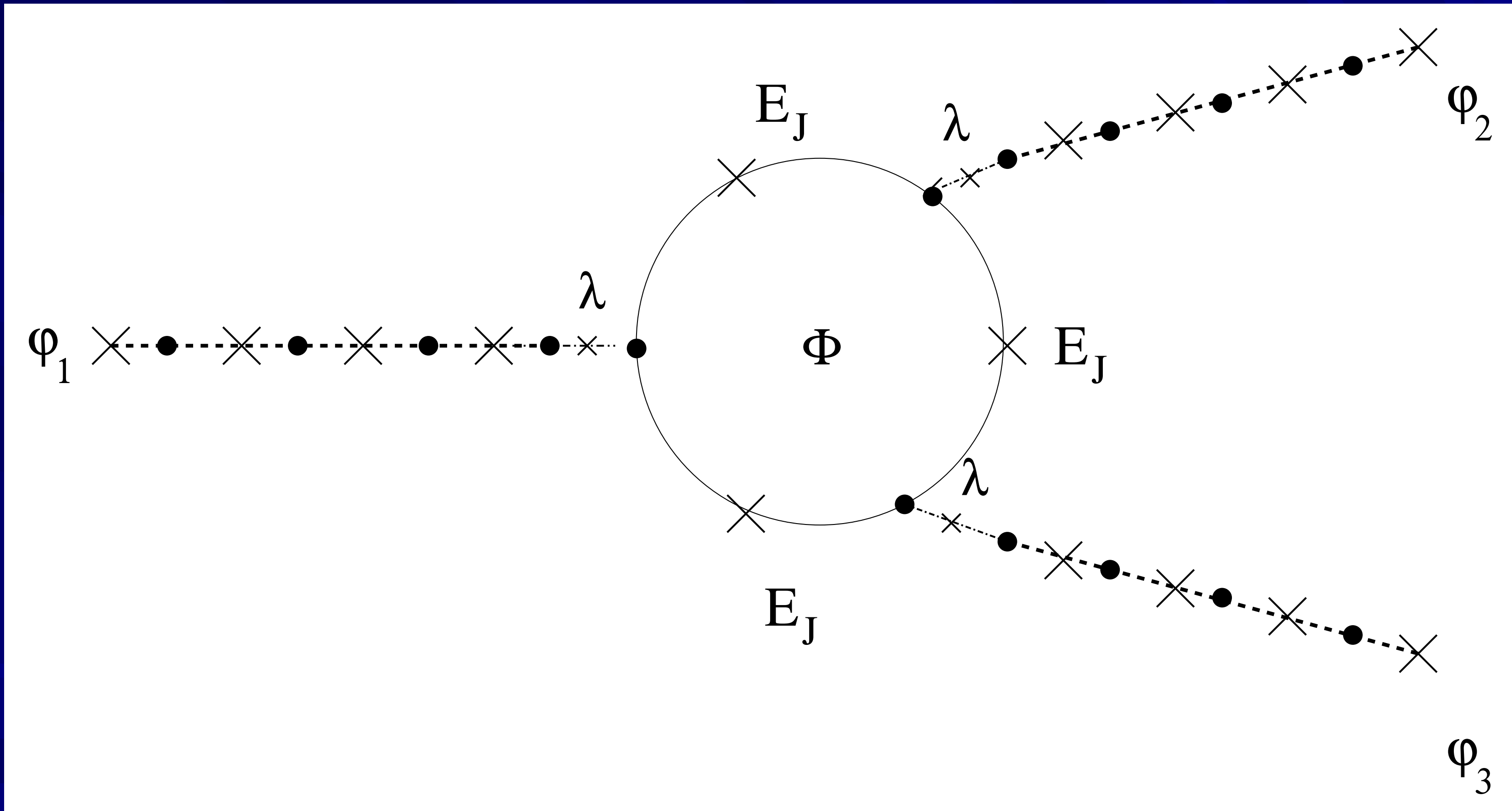
Other Y-shaped devices

* **Spinful fermionic junctions** (C. Y. Hou, C. Chamon, PRB 77, 155422 (2008)).

* **Anyonic wires** (B. Bellazzini, P. Calabrese, M. Mintchev, PRB 79, 085122 (2009)).

* **Josephson junction networks** (D.G., P. Sodano, New Jour. Phys. 10, 093023 (2008), NPB 811, 395 (2009)).

2. Building a bosonic Y-shaped network with a FFP with Josephson junction arrays;



(Circular) central region Hamiltonian

$$H_\Delta = \frac{E_C}{2}\sum_{i=1}^{3}\left[-i\frac{\partial}{\partial \phi_i^{(0)}} - e^* W_g\right]^2 - \frac{E_J}{2}\sum_{i=1}^{3}\left[e^{i(\phi_i^{(0)} - \phi_{i+1}^{(0)} + \varphi/3)} + h.c.\right]$$

$E_C >> E_J \Rightarrow$ Effective (3)-spin-1/2 Hamiltonian

$$[S_i^{(0)}]^z = -i\frac{\partial}{\partial \phi_i^{(0)}} - N - \frac{1}{2}$$

$$[S_i^{(0)}]^+ = e^{i\phi_i^{(0)}}$$

$$e^* W_g = N + h + \frac{1}{2}$$

$$H_\Delta = -h\sum_{i=1}^{3}[S_i^{(0)}]^z - \frac{E_J}{2}\sum_{i=1}^{3}\left\{[S_i^{(0)}]^+[S_{i+1}^{(0)}]^- e^{i\varphi/3} + h.c.\right\}$$

Low-energy eigenstates (h>E$_J$)

$$|\uparrow\uparrow\uparrow\rangle \qquad \varepsilon_0 = -\tfrac{3}{2}h$$

$$\frac{1}{\sqrt{3}}[|\uparrow\uparrow\downarrow\rangle + |\uparrow\downarrow\uparrow\rangle + |\downarrow\uparrow\uparrow\rangle] \qquad \varepsilon_{11} = -\tfrac{1}{2}h - \frac{\tau}{2}\cos(\frac{\varphi}{3})$$

$$\frac{1}{\sqrt{3}}[|\uparrow\uparrow\downarrow\rangle - e^{-i\frac{\pi}{3}}|\uparrow\downarrow\uparrow\rangle - e^{i\frac{\pi}{3}}|\downarrow\uparrow\uparrow\rangle] \qquad \varepsilon_{12} = -\tfrac{1}{2}h - \frac{\tau}{2}\cos(\frac{\varphi-\pi}{3})$$

$$\frac{1}{\sqrt{3}}[|\uparrow\uparrow\downarrow\rangle - e^{i\frac{\pi}{3}}|\uparrow\downarrow\uparrow\rangle - e^{-i\frac{\pi}{3}}|\downarrow\uparrow\uparrow\rangle] \qquad \varepsilon_{13} = -\tfrac{1}{2}h - \frac{\tau}{2}\cos(\frac{\varphi+\pi}{3})$$

<u>**Only these states will be kept in the effective theory**</u>

Connection to the leads at the "inner boundaries"

$$H_T = -\lambda \sum_{i=1}^{3} \cos[\phi_i^{(0)} - \phi_i^{(1)}]$$

"Weak tunneling" limit: $\lambda << h, E_J \Rightarrow$ Schrieffer-Wolff transformation $\Rightarrow$ Boundary interaction term

$$H_B = -E_W \sum_{i=1}^{3} [e^{i(\phi_i^{(1)} - \phi_{i+1}^{(1)})} e^{i\gamma} + h.c.]$$

$$E_W \approx \frac{\lambda^2 E_J}{24 h^2} \sqrt{[\cos^2(\frac{\Phi}{3}) + 9 \sin^2(\frac{\Phi}{3})}$$

$$\gamma = \arctan[3 \tan(\frac{\Phi}{3})]$$

The leads: Effective field theory of a JJ-chain

(L. I. Glazman and A. I. Larkin, PRL 79, 3736 (1997);

D.G., P. Sodano, NPB 711, 480 (2005))

$$H_0 = \sum_{k=1}^{3} \left\{ \frac{E_C}{2} \sum_j \left[-i \frac{\partial}{\partial \phi_j^{(k)}} - N \right]^2 - E_J \sum_j \cos\left[\phi_j^{(k)} - \phi_{j+1}^{(k)} \right] + \left(E_Z - \frac{3}{16} \frac{E_J^2}{E_C} \right) \sum_j n_j^{(k)} n_{j+1}^{(k)}\right.$$

(N=n+1/2)

Mapping onto spin chain+Jordan-Wigner fermions+Bosonization ⇒Luttinger liquid (LL) effective Hamiltonian

$$H_{LL} = \sum_{k=1}^{3} \left\{ \frac{g}{4\pi} \int_0^L u \left[\left(\frac{\partial \Phi^{(k)}}{\partial x} \right)^2 + \frac{1}{u^2} \left(\frac{\partial \Phi^{(k)}}{\partial t} \right)^2 \right] dx \right\}$$

LL parameters

$$g = \sqrt{\frac{v_F + g_2 - g_4}{v_F + g_2 + g_4}}$$

$$u = \sqrt{(v_F + g_2)^2 - g_4^{\ 2}}$$

$$g_2 = g_4 = 4\pi a \Delta [1 - \cos(2k_F a)]$$

$$(\Delta = E^z - \frac{3}{16}\frac{J^2}{E_C})$$

Contacts at the outer boundary with massive superconductors at fixed phase: will matter later on!

Boundary Hamiltonian

$$H_B = -E_W \sum_{a=1,2,3} \exp[i(\vec{K}_a \bullet \vec{\Phi}(0) + \gamma)] + h.c.$$

Boundary conditions

$$\frac{ug}{2\pi}\frac{\partial\vec{\Phi}(0)}{\partial x} - 2\overline{E}_W \sum_{a=1,2,3} \vec{K}_a \sin[\vec{K}_a \bullet \vec{\Phi}(0) + \gamma] = 0$$

$$\Phi_1(L) = \varphi_1 - \varphi_2 + 2\pi n_{12}$$

$$\Phi_2(L) = \frac{\phi_1 + \phi_2 - 2\phi_3}{\sqrt{3}} + 2\pi n_{13}$$

Weakly coupled FP

$$\frac{\partial \vec{\chi}}{\partial x} = 0$$

Strongly coupled FP

Minimum of H_{Bou}

3. Phase diagram and emergence of a FFP;

Weakly coupled fixed point

$$\Phi_i(x,t) = \xi_i + \sqrt{\frac{2}{g}} \sum_n \cos[\frac{\pi}{L}(n+\frac{1}{2})x] \frac{\alpha_i(n)}{n+\frac{1}{2}} e^{i\frac{\pi}{L}(n+\frac{1}{2})ut}$$

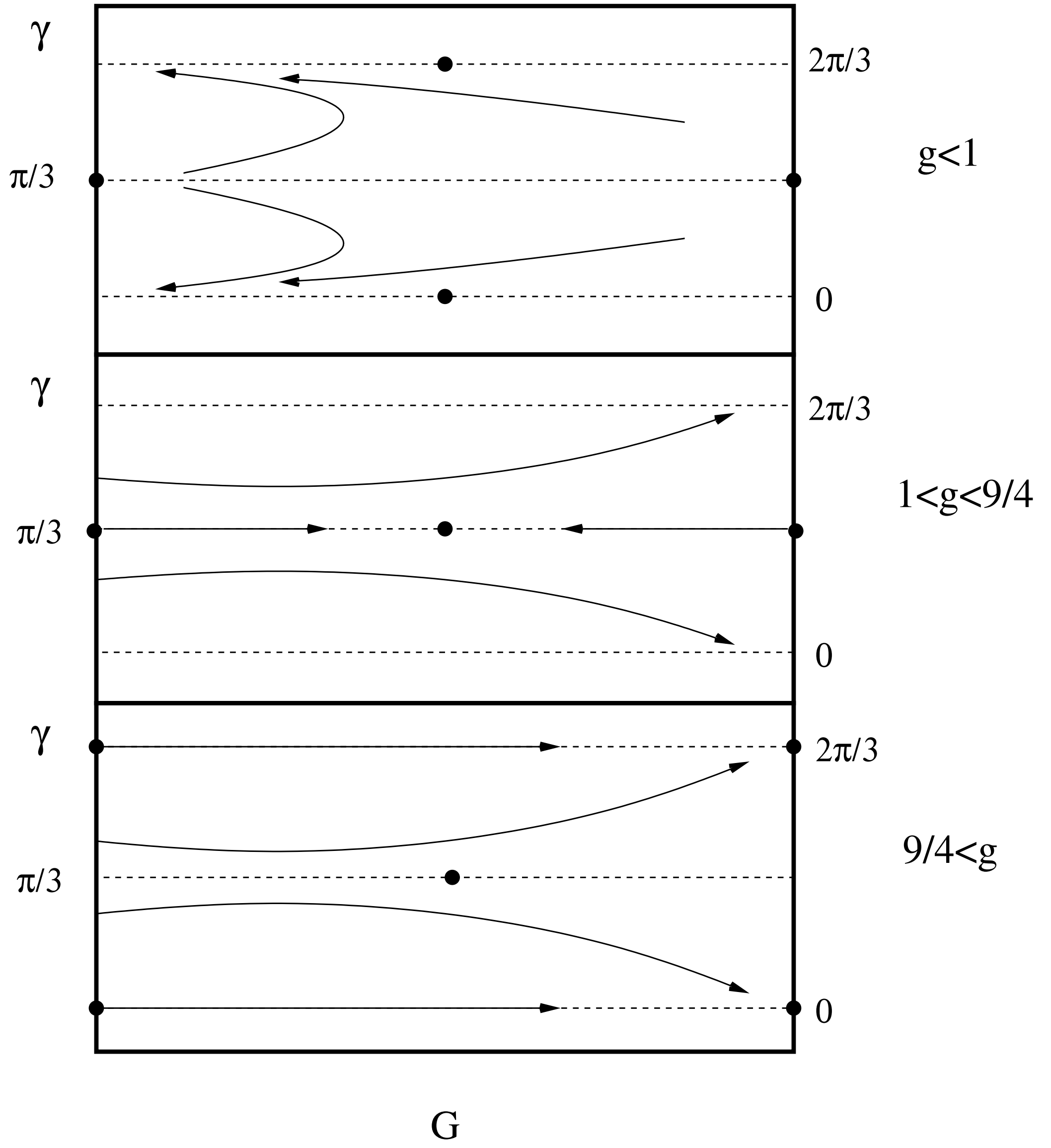
O.P.E. between boundary vertices:

$$: e^{i\vec{\alpha}_i \bullet \vec{\chi}(\tau)} :: e^{i\vec{\alpha}_j \bullet \vec{\chi}(\tau')} :\approx [\tau - \tau']^{-2/g} : e^{-i\vec{\alpha}_k \bullet \vec{\chi}(\tau')} :$$

Second-order renormalization group equations

$$\frac{d[G(L)e^{i\gamma}]}{d\ln(L/L_0)} = [1-\frac{1}{g}]G(L)e^{i\gamma} + 2G^2(L)e^{-2i\gamma}, (G(L) = L\overline{E}_w)$$

Phase diagram



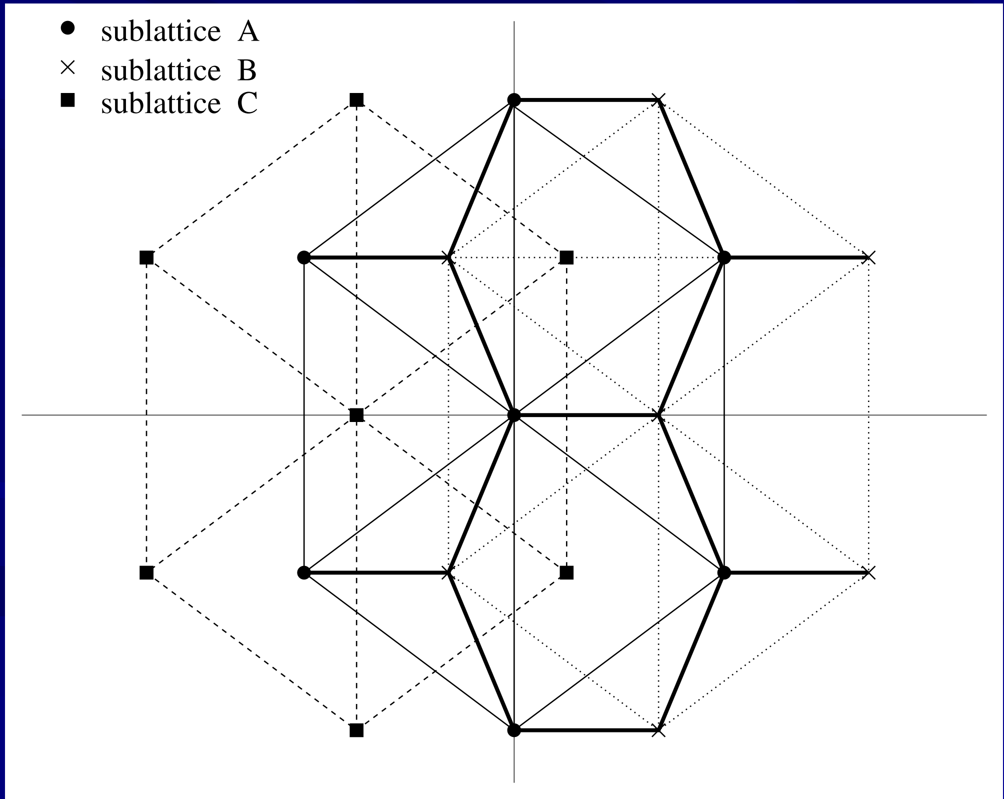
*** g<1:stable fixed point at G=γ=0; fixed lines at γ=0,π/3,2π/3.**

***1<g<9/4:strongly coupled fixed point for γ≠π/3; finite coupling fixed point for γ=π/3.**

***9/4<g: strongly coupled stable FP**

Strongly coupled fixed point

G->∞ ⇒Dirichlet boundary conditions at the inner boundary, as well. ($\Phi_1(0)$, $\Phi_2(0)$) span a triangular lattice, depending on the value of γ



For γ=π/3 the minima lie on a honeycomb lattice (merging of two triangular sublattices)

Mode expansion of the plasmon fields at the SFP

$$\Phi_i(x,t) = \xi_i + \sqrt{\frac{2}{g}}\left[-P_i\frac{\pi x}{L} - \sum_n \sin[\frac{\pi}{L}nx]\frac{\alpha_i(n)}{n}e^{i\frac{\pi}{L}nut}\right]$$

Dual fields

$$\psi_i(x,t) = \sqrt{2g}\left\{\theta_i + \frac{\pi v t}{L}P_i + \frac{\pi x}{L} + i\sum_n \cos[\frac{\pi}{L}nx]\frac{\alpha_i(n)}{n}e^{-i\frac{\pi}{L}nut}\right\}$$

For γ≠kπ+ π /3 the minima span only one of the three sublattices : in this case, the leading boundary perturbation is given by a combination of "long" V-instantons.

$$H_S = -Y\sum_{i=1}^{3}\overline{V}_i(0) + h.c.$$

$$\overline{V}_j(\tau) =: \exp\left[\pm i2\sqrt{\frac{2}{3}}\vec{\rho}_j \bullet \vec{\psi}(\tau)\right]:$$

$$\vec{\rho}_1 = (0,1); \vec{\rho}_2 = (\frac{\sqrt{3}}{2}, -\frac{1}{2}); \vec{\rho}_3 = (-\frac{\sqrt{3}}{2}, -\frac{1}{2})$$

The "V-instanton" operators have conformal dimension $h_S(g)=4g/3$: for $3/4<g<1$ (and for γ≠kπ+π/3) both the weakly coupled and the strongly coupled fixed point is stable (repulsive FFP).

Emergence of a stable finite coupling fixed point

For γ=kπ+ π /3 two triangular sublattices become degenerate in energy: they merge to form a honeycomb lattice. In this case, the leading boundary perturbation is given by a combination of "short" W-instanton.

$$H_F = -\varsigma \sum_{i=1}^{3} \tau^{+} \overline{W}_i(0) + h.c.$$

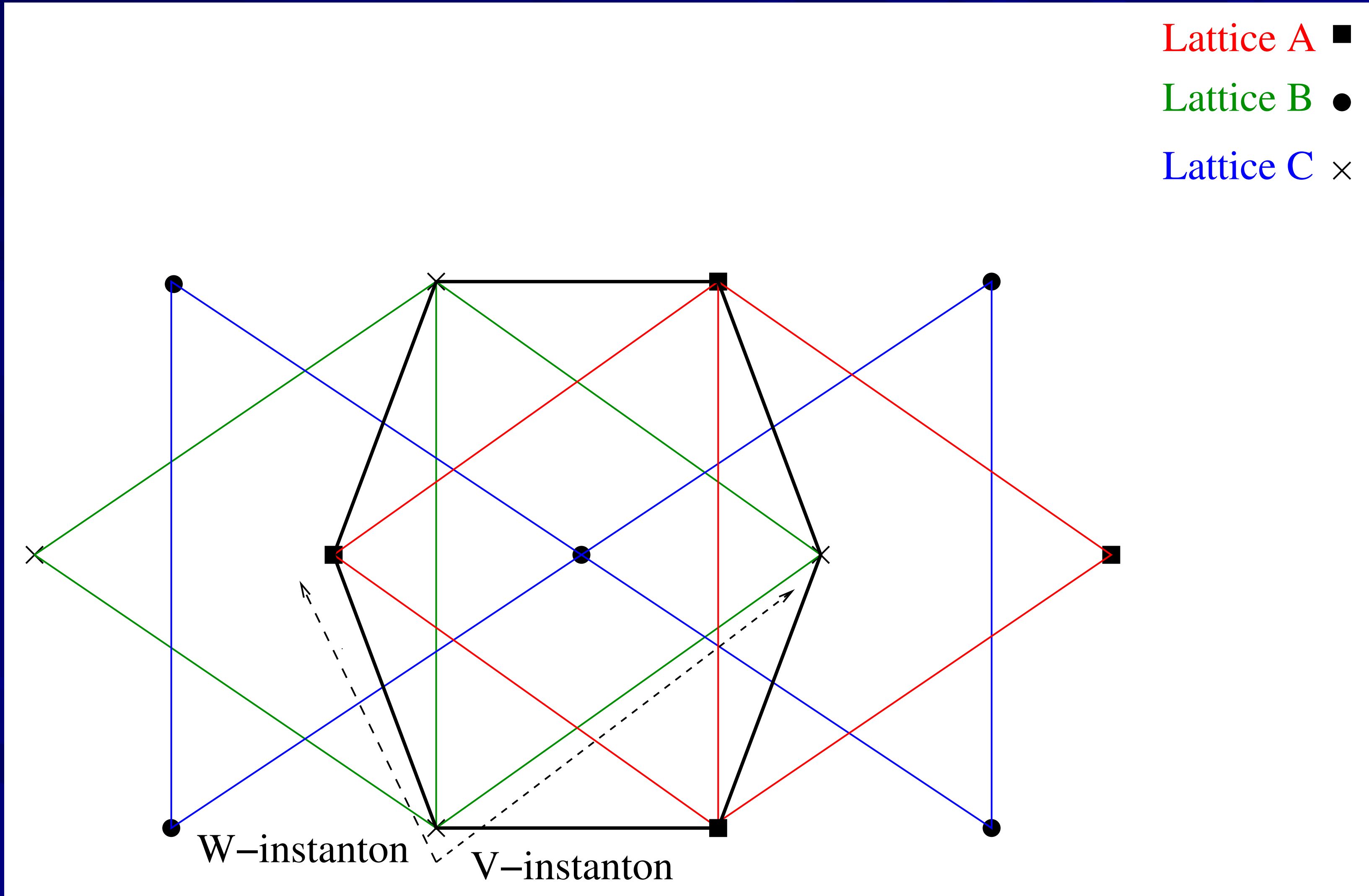
$$\overline{W}_j(\tau) =: \exp\left[\pm i \frac{2}{3} \vec{\alpha}_j \bullet \vec{\psi}(\tau) \right]:$$

τ^+,τ^- are effective isospin operators, connecting sites on inequivalent sublattices

Perturbative renormalization group equation for the running coupling strength

$$\frac{d\zeta}{d\ln(L/L_0)} = \left[1 - \frac{4g}{9}\right]\zeta - 2\zeta^3$$

The "W-instanton" operators have conformal dimension $h_F(g)=4g/9$: for $1<g<9/4$ neither the weakly coupled, or the strongly coupled fixed point is stable: the IR behavior of the system is driven by an attractive FFP


Lattice A
Lattice B
Lattice C
W−instanton
V−instanton

4. Current's patter in the YJJN as a phase probe;

Current: logarithmic derivatives of the partition function Z

$$I_1 = \frac{e^*}{g\beta}\left[\frac{1}{\sqrt{2}}\frac{\partial \ln Z}{\partial \beta_1} + \frac{1}{\sqrt{6}}\frac{\partial \ln Z}{\partial \beta_2}\right]$$

$$I_2 = \frac{e^*}{g\beta}\left[-\frac{1}{\sqrt{2}}\frac{\partial \ln Z}{\partial \beta_1} + \frac{1}{\sqrt{6}}\frac{\partial \ln Z}{\partial \beta_2}\right]$$

$$I_3 = -\frac{e^*}{g\beta}\sqrt{\frac{2}{3}}\frac{\partial \ln Z}{\partial \beta_2}$$

$$\beta_1 = \frac{\varphi_1 - \varphi_2}{\sqrt{2}}$$

$$\beta_2 = \frac{\varphi_1 + \varphi_2 - 2\varphi_3}{\sqrt{6}}$$

Weakly coupled fixed point

Perturbative calculation: the result is the "typical" sinusoidal behavior, as a function of the applied phase differences

$$I_1 = \frac{2e^* G}{gL}\left[\sin(\vec{\alpha}_1 \bullet \vec{\beta} + \gamma) - \sin(\vec{\alpha}_3 \bullet \vec{\beta} + \gamma)\right]$$

$$I_2 = \frac{2e^* G}{gL}\left[\sin(\vec{\alpha}_2 \bullet \vec{\beta} + \gamma) - \sin(\vec{\alpha}_1 \bullet \vec{\beta} + \gamma)\right]$$

$$I_3 = \frac{2e^* G}{gL}\left[\sin(\vec{\alpha}_3 \bullet \vec{\beta} + \gamma) - \sin(\vec{\alpha}_2 \bullet \vec{\beta} + \gamma)\right]$$

Strongly coupled fixed point

Zero-mode contribution to the energy eigenvalues

$$E = E[n_{12}, n_{13}] + E_{osc}$$

$$E[n_{12}, n_{13}] = \frac{\pi v g}{L}\left\{\left[n_{12} + \frac{\beta_1}{2\pi} + \varepsilon_l\right]^2 + \left[n_{13} + \frac{n_{12}}{2} + \frac{\sqrt{3}}{2}\frac{\beta_2}{2\pi}\right]^2\right\}$$

$$\varepsilon_A = 0, \varepsilon_B = 1, \varepsilon_C = -1$$

On a finite-size system this breaks the degeneracy between the minima of the boundary potential (labelled by the n's)

Tuning two eigenstates of the zero-mode operator near by a degeneracy $\Rightarrow$ effective two-level quantum device

For instance: setting

$$-\frac{1}{6} < \frac{\beta_1}{2\pi} < \frac{1}{6}$$

$$\frac{\beta_2}{2\pi} = -\frac{1}{\sqrt{3}} + \delta$$

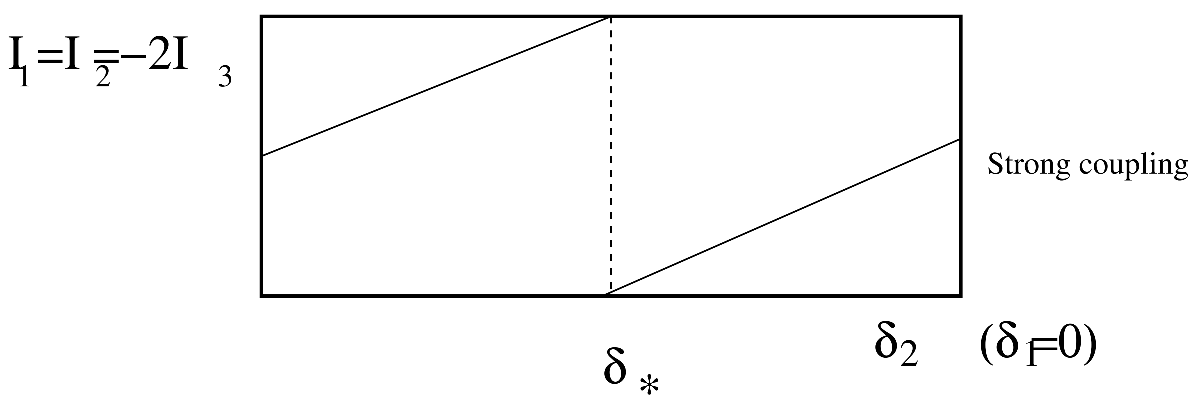
$$\left(\left(\frac{\delta}{\pi}\right) << 1\right)$$

The following two states define an effective 2LQD

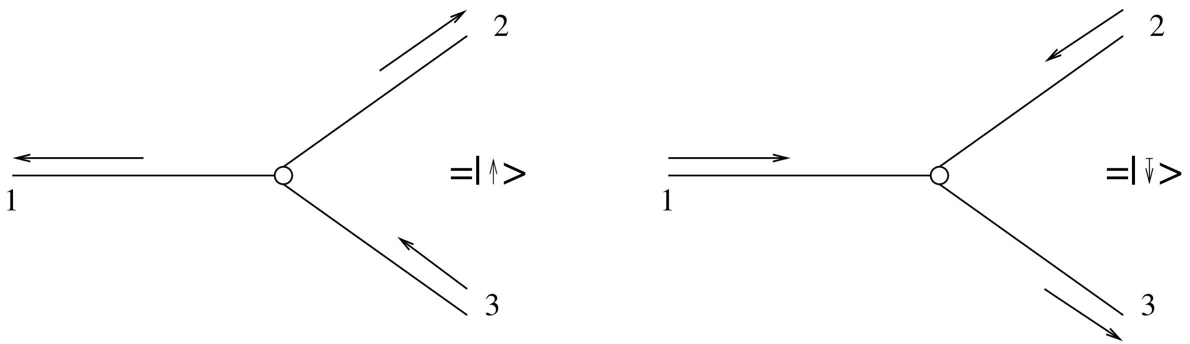
$$|\,0,0>_A;|\,0,1>_A \equiv |\uparrow>,|\downarrow>$$

Operating the system as a quantum switch

a)



b)



δ_2 measures the detuning off the degeneracy: acting on this parameter one makes the system "switch" between the two states

5. Spectral density and frustration of decoherence in the YJJN working near the FFP;

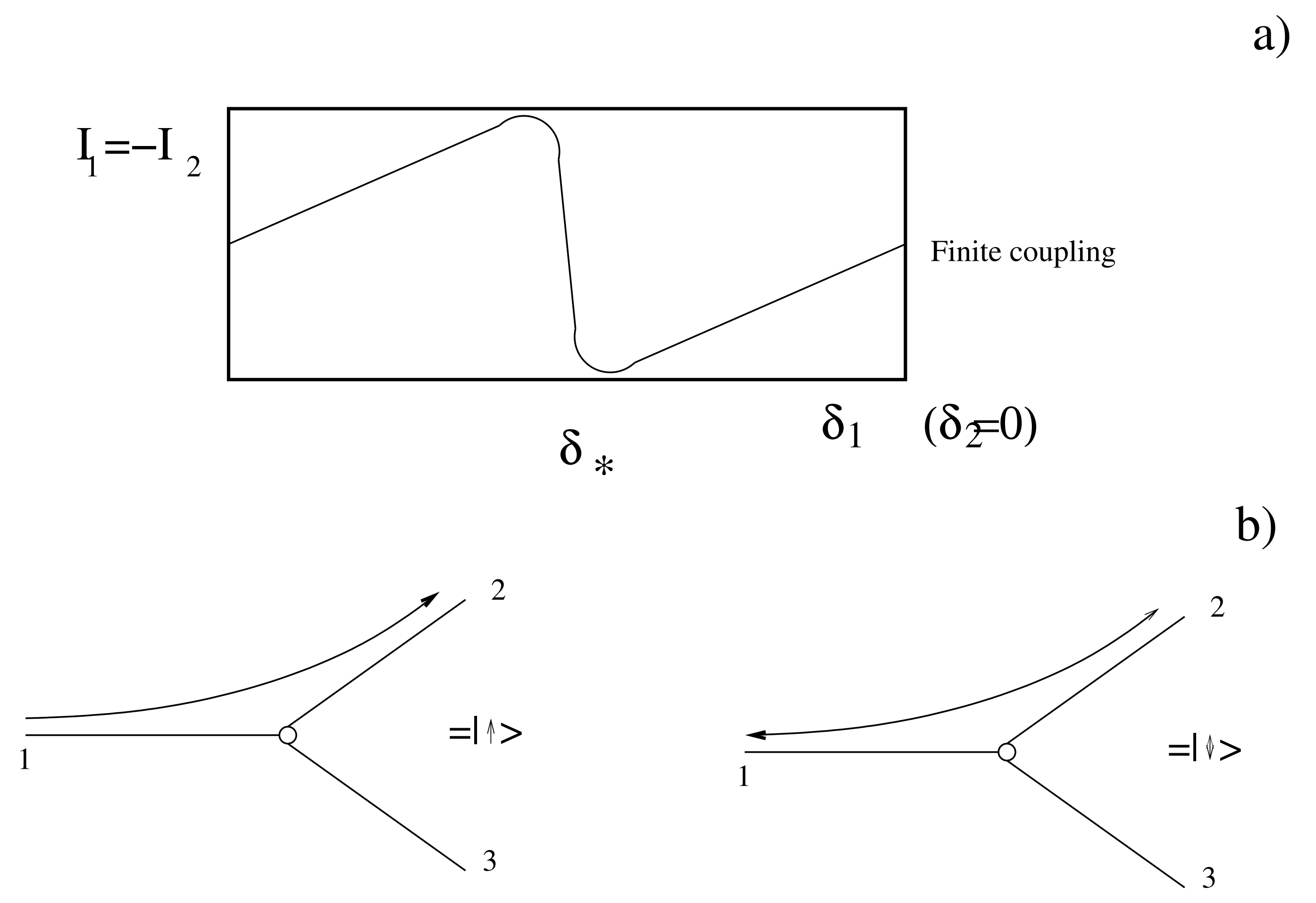
The current pattern near the FFP

Though it is possible to set up a self-consistent formalism to formally derive the current pattern near the FFP, a closed-formula can be given only for g=9/4-ε, with ε<<1. In this case, one may set the parameters as

$$\alpha = \frac{g}{6\pi}$$

$$\beta_1 \approx \beta_1 * + \delta = -\frac{\pi}{3} + \delta$$

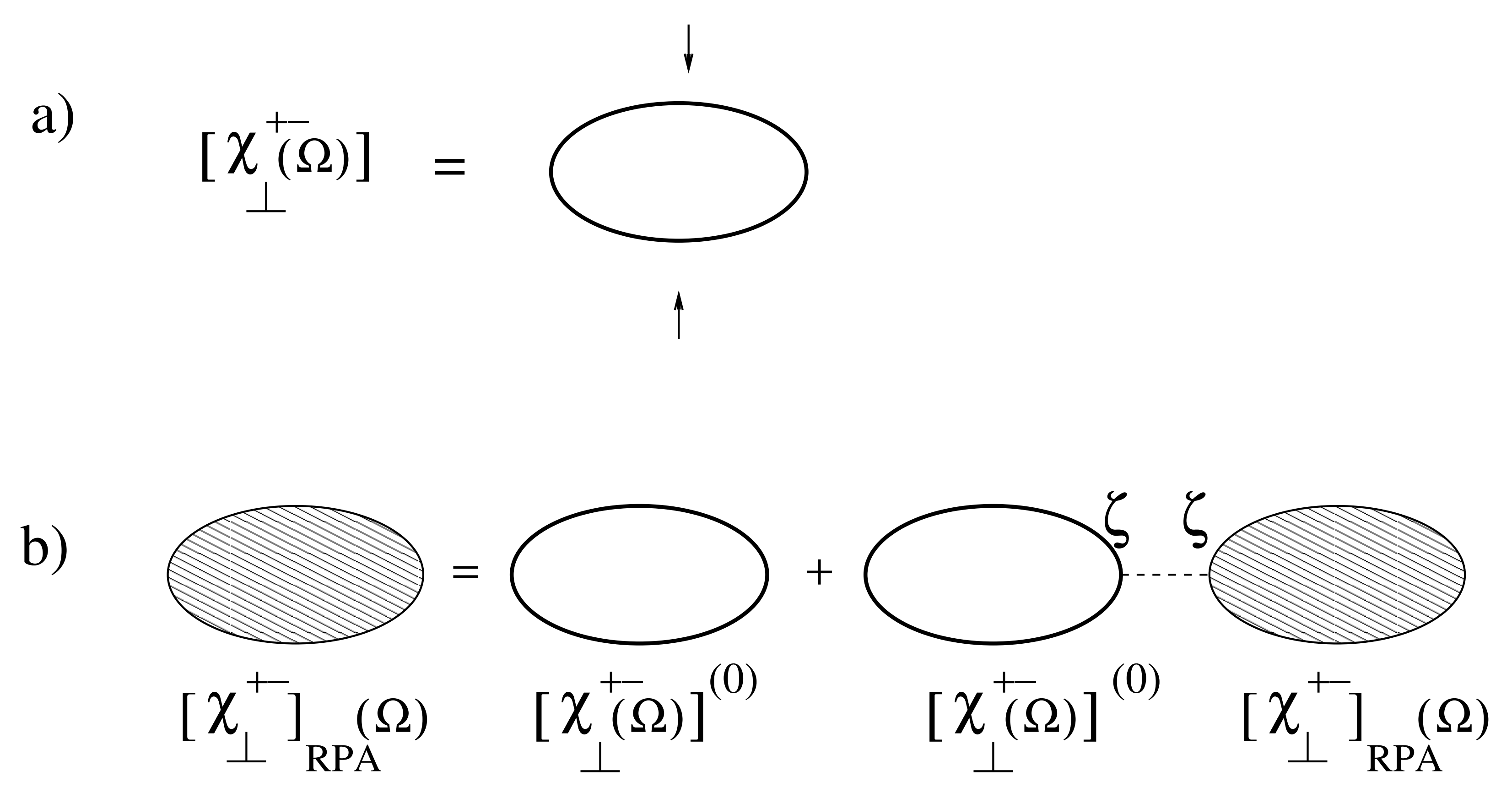
$$\zeta_* \approx \varepsilon^{\frac{1}{2}}$$



Again, this is a smoothened sawtooth-like behavior but, now, it is associated to a stable FP

We relate the decoherence to the entanglement of the system with the plasmon bath ⇒ spectral density of states of the effective 2LQD, X"(Ω)/Ω

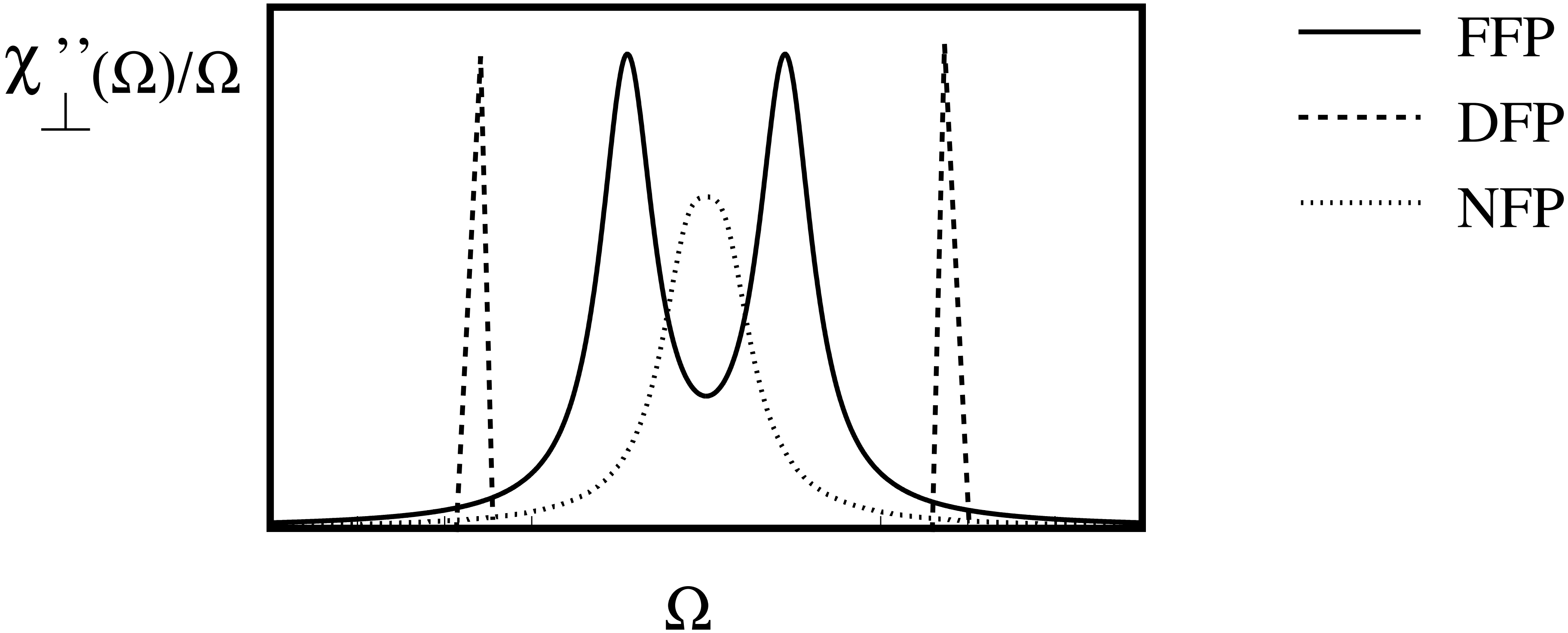
(E. Novais et al., Phys. Rev. B 72, 014417 (2005))



$$\chi^{RPA}{}_{\perp}(\Omega) = \frac{1}{\Omega - \Delta_*(\vec{\beta}) - \varsigma^2 \Gamma[-1 - \frac{9}{8}\varepsilon][-\Omega]^{1+\frac{9}{8}\varepsilon}}$$

$$\Delta_*(\vec{\beta}) = \sqrt{[(E(\vec{\beta}))^2 + (\varsigma_* / L)^2]}$$

Using the RPA approximation sketched above yields



<u>Near the SFP</u>: no entanglement between the 2LQD and the bath, but no quantum tunneling between the states either (no energy renormalization);

<u>Near the WFP</u>: full entanglement between the 2LQD and the bath (full decoherence);

<u>Near the FFP</u>: consistent (and robust) tunnel splitting of the two states, with an accettable level of (frustrated) decoherence

6. Conclusions

1. A Y-shaped JJ-network may exhibit a FFP in its phase diagram;

2. At the FFP an effective 2-level quantum devices emerges, with enhanced quantum coherence;

3. Relevant issues for a practical realization: stabilizing γ, applying the external phases, controlling other sources of noise…