String Inspired Kaluza Klein Theories

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Talk based on:

P. Di Vecchia, A. Liccardo, F. Pezzella and R.M., Kähler Metrics: String vs Field Theoretical Approach.
arXiv:0901.4458 [hep-th]

P. Di Vecchia, A. Liccardo, F. Pezzella and R.M., Kahler Metrics and Yukawa Couplings in Magnetized Brane Models.
To be published on JHEP.
arXiv:0810.5509[hep-th]

P. Di Vecchia, A. Liccardo, F. Pezzella, I. Pesando and R.M.,
Wrapped magnetized branes: two alternative descriptions?
JHEP 0711:100,2007.
arxiv:0719.4149.

Plan of Talk

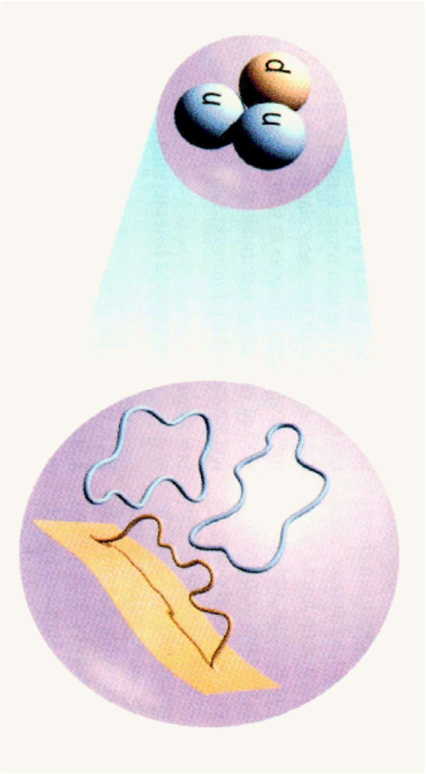
1. Introduction and Motivations.
2. Magnetized Branes in Bottom Up Approach.
3. Kähler metrics.
4. Yukawa couplings.
5. Conclusions.

Introduction and Motivations

Why Strings?

- The fundamental objects of string theories are open and closed strings of finite length $\ell_s = 2\pi\sqrt{\alpha'}$

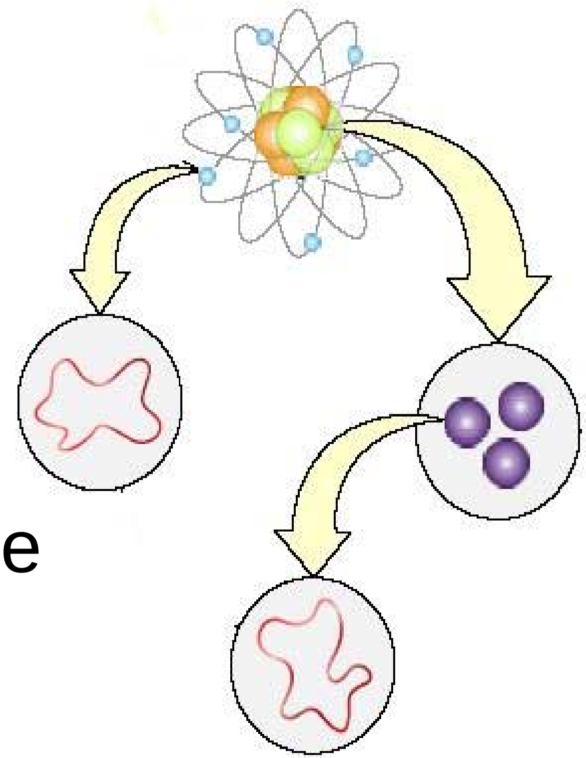
- The elementary particles are the excitation modes of the elementary strings and these modes contain the Graviton and the Vector Bosons of the known gauge interactions.



These are the ingredients for an unified theory of all forces and particles.

- o The String length acts as an ultraviolet (UV) cut-off which makes all the loop integrals finite.

- o In the limit $\ell_s \to 0$ the strings become points and the perturbative string theory reduces to a perturbative gauge theory unified with an extension of the General relativity.



- o Furthermore, all the UV-divergences, related to the point particle description, of the field theory are recovered.

String Theory is an extension of the Field Theory.

The local (“gauge”) symmetry of the theory is kept at quantum level only in the critical dimensions d=10.

Emergent space-time dimensions

There exist five consistent perturbative theories of Superstring: Type IIA, Type IIB, Type I, Heterotic $SO(32).$ and Heterotic $E_8 \times E_8$

They are inequivalent in the perturbative regime.

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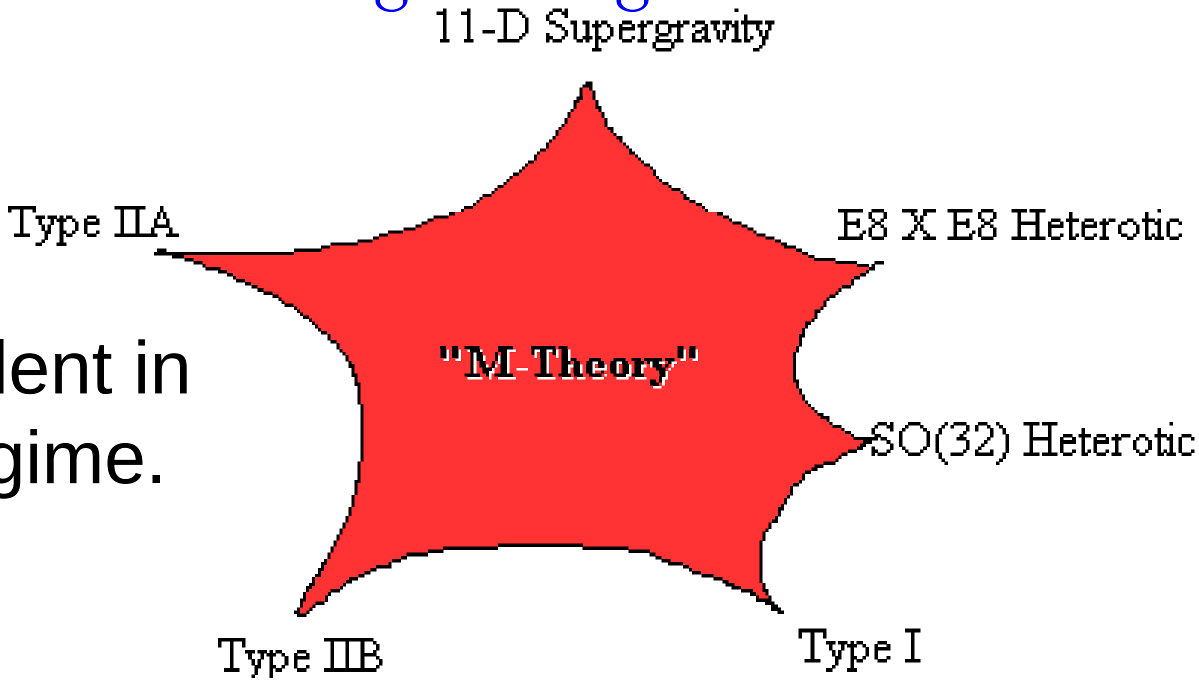


Dp –branes

1. Type II string theories are theories of closed strings at the perturbative level.

2. But at the non-perturbative level there are additional states that are in general p-dimensional.

3. They are called Dp branes: generalization in string theory of the solitons in field theory.

Where do they come from?

The spectrum of massless states of the Type II theories

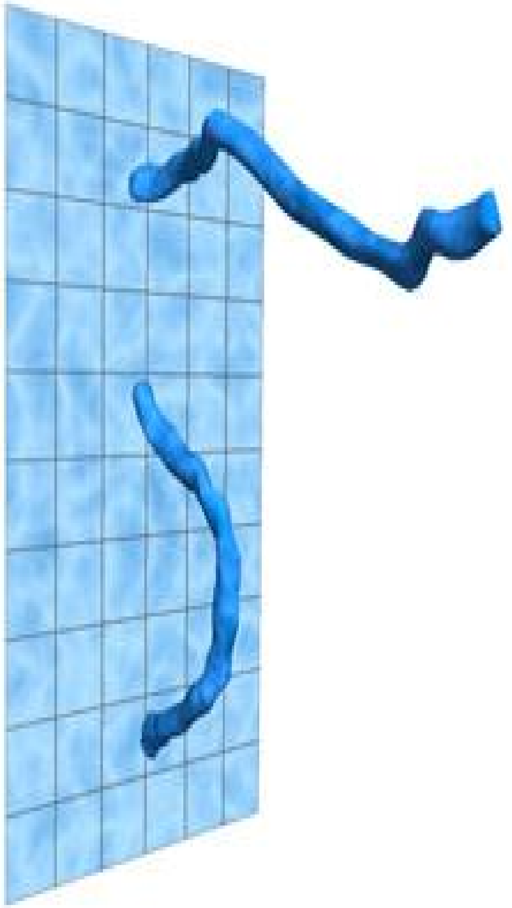
$G_{\mu\nu}$	$B_{\mu\nu}$	ϕ_{10}	NS-NS sector
Metric	Kalb-Ramond	Dilaton	
C_0, C_2	C_4, C_6	C_8	RR sector IIB
C_1, C_3	C_5	C_7	RR sector IIA

$$C_p \equiv C_{\mu_1\mu_2\ldots\mu_p}$$

RR p-forms

The RR p-forms are the generalization of the electromagnetic potential A_μ .

As the electromagnetic potential is coupled to point-like particles, the p-forms are coupled to p-dimensional objects called Dp-branes.



$$\int A_\mu dx^\mu \quad \Longrightarrow \int C_{\mu_1\mu_2\dots\mu_{p+1}} d\sigma^{\mu_1\mu_2\dots\mu_{p+1}}$$

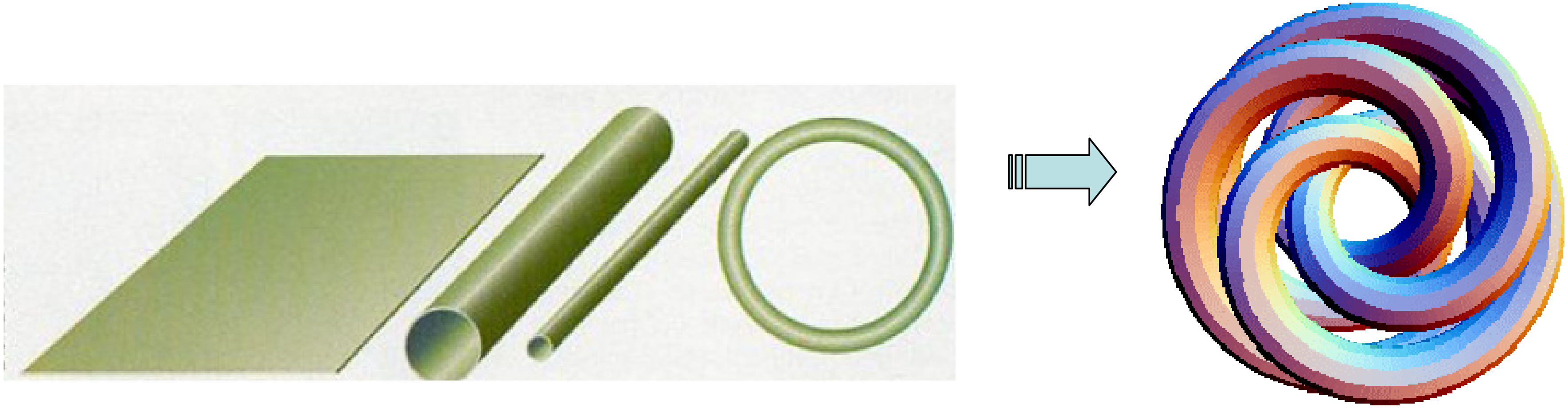
A stack of N Dp-branes has in its world-volume a U(N)=SU(N)xU(1) gauge theory described by a p+1 dimensional action:

$$S = -\tau_p \int d^{p+1}x \sqrt{-\det\left(G_{\alpha\beta} + \ell_s^2\, F_{\alpha\beta}\right)} + \mu_p \int \left[C \wedge e^{\ell_s^2\, F}\right]_{p+1} + \dots$$

tension

charge

Dimensional reduction to d=4, in the spirit of Kaluza-Klein theories (Compactification).



The compactification introduces in the model light scalars called moduli fields. Their expectation values parameterize the size and the shape of the compact manifold, and may also determine the parameters (masses or gauge couplings) of the effective d=4 lagrangian.

String models are not predictive without determining the expectation values of such moduli.

Moduli stabilization

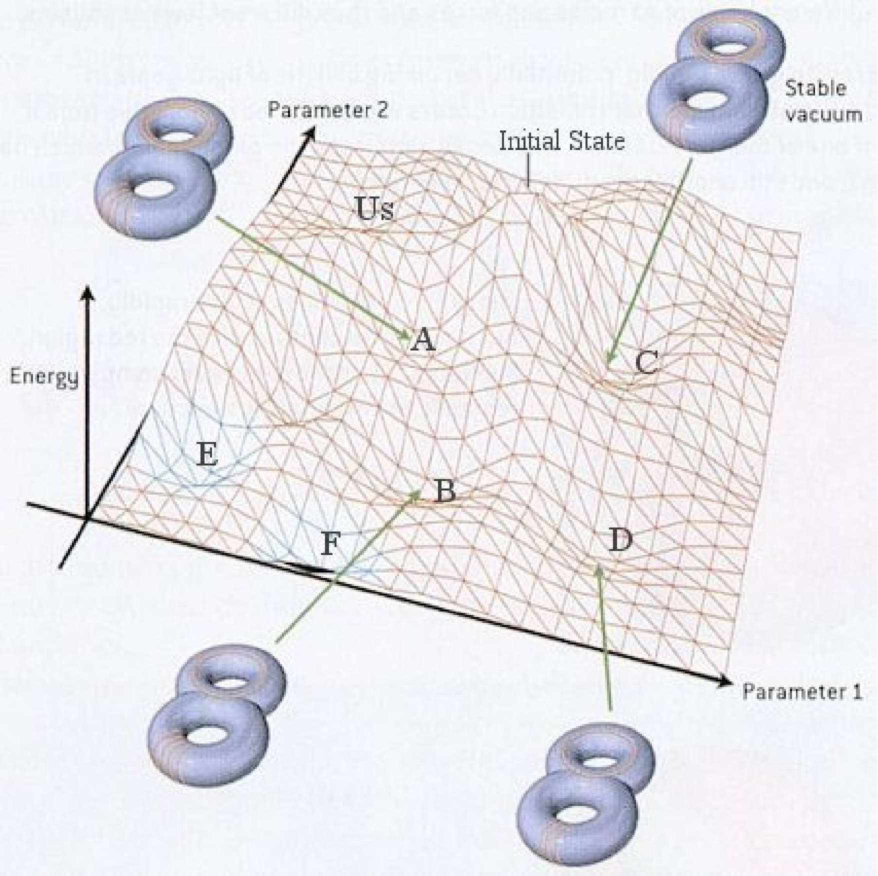
o A moduli potential is generated by allowing non trivial expectation values to he compact components of the RR-field strength.

o The moduli are frozen to the minimum of the potential.

Flux Compactification.

Still a large number of string vacua.

How can be fixed the vacua in which we live?



Can we see stringy effects in experiments?

o If $\ell_s E < 1$ then one will see only the limiting theory.

o If $\ell_s E \sim 1$, then we can start to see stringy effects.

o The string scale ans also the size of the extra-dimensions were supposed to be of the order of the Planck-length:

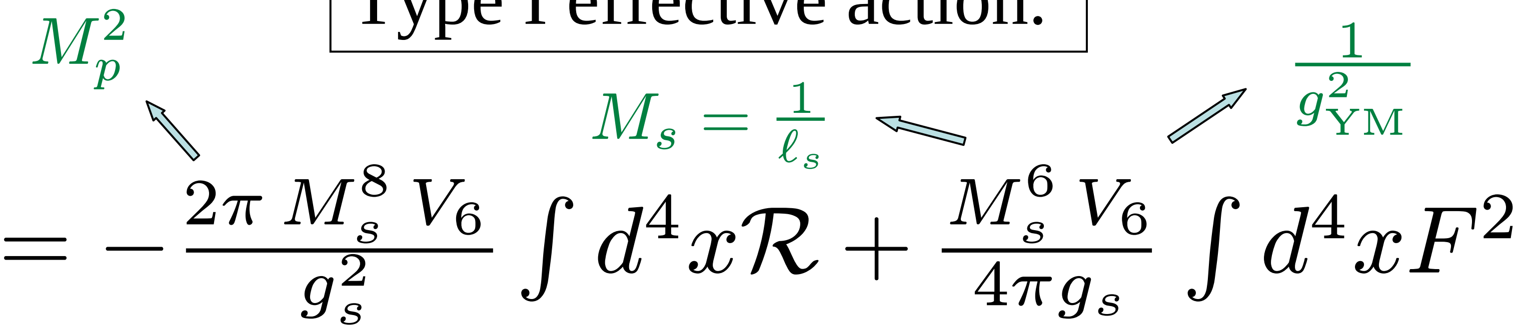
$$\frac{1}{\ell_s} \sim M_P \qquad , \qquad R \sim \ell_s$$

Too small to be observed in present and future experiments.

One need a very good control of the theory to be able to extrapolate to low-energy.

After the discovery of the D-branes, the distinct origin of gauge and gravitational interaction, has determined a different relation between the Planck and string scale.

Type I effective action.



M_p^2

$M_s = \frac{1}{\ell_s}$

$\frac{1}{g^2_{\rm YM}}$

$$S = -\frac{2\pi\, M_s^8\, V_6}{g_s^2} \int d^4x\mathcal{R} + \frac{M_s^6\, V_6}{4\pi g_s} \int d^4xF^2$$

Choosing n internal dimensions to have a common radius R_I and the remaining 6-n of the string size, one has:

$$(2\pi R_I)^{-n} M_s^{6-n} \sim g^4_{YM} M_p^2 \quad \Longrightarrow \quad R = 1/(M_s^2 R_I)$$

T-duality

$$\Longrightarrow \quad M_p^2 \sim R^n M_s^{n+2} \frac{1}{g^4_{\rm YM}}$$

Keeping small g_{YM}, M_s can be chosen smaller then M_P at the expense of introducing extra large dimensions.

Antoniadis, Arkani-Hamed, Dimopoulos, Dvali, 1998

Magnetized Branes in Bottom Up Approach

Look for local configurations of D-branes with world-volume theories resembling the Standard Model as much as possible.
(Very restrictive)

- Search for the gauge group SU(3) x SU(2) x U(1).
- Presence of Chiral quark-lepton generations.

Embed the local D-brane configuration in a larger global model, where the six transverse dimensions are compactified.
(Big arbitrariety)

Starting from a 10-dim string theory with suitable configuration of D-branes and given a certain compactification, how to compute the four dimensional low-energy effective action that should be compared with experiments?

In principle, the low-energy four dimensional effective Lagrangian will depend on the microscopic data specifying the D-brane configurations and the geometry of the compact space.

If not immediately interested in string corrections one may start from the Supegravity lagrangian which contains only the lowest string excitations ($\ell_s \to 0$ approximation).

Any N=1 supergravity Lagrangian in d=4 with only chiral and vector multiplets and no more than two derivatives, depends only on the Kähler potential K (real function of the chiral fields), the gauge kinetic function f_a and the superpotential W (holomorphic functions of the fields):

$$\mathcal{L} = -\frac{1}{2\kappa^2}\mathcal{R} - Z_{IJ}\mathcal{D}_m\bar{\Phi}^I\mathcal{D}^m\Phi^J - V(\Phi,\,\bar{\Phi}) - \sum_a\left[\mathrm{Re}f_a(\Phi)\frac{1}{4}F_a^2 + \mathrm{Im}f_a(\Phi)\frac{1}{4}F_a^2\right]$$

$$Z_{IJ} = \partial_{\Phi^I}\partial_{\bar{\Phi}^J}Z$$

Scalar potential determined by W

Kähler metric spanned by the complex scalars Φ

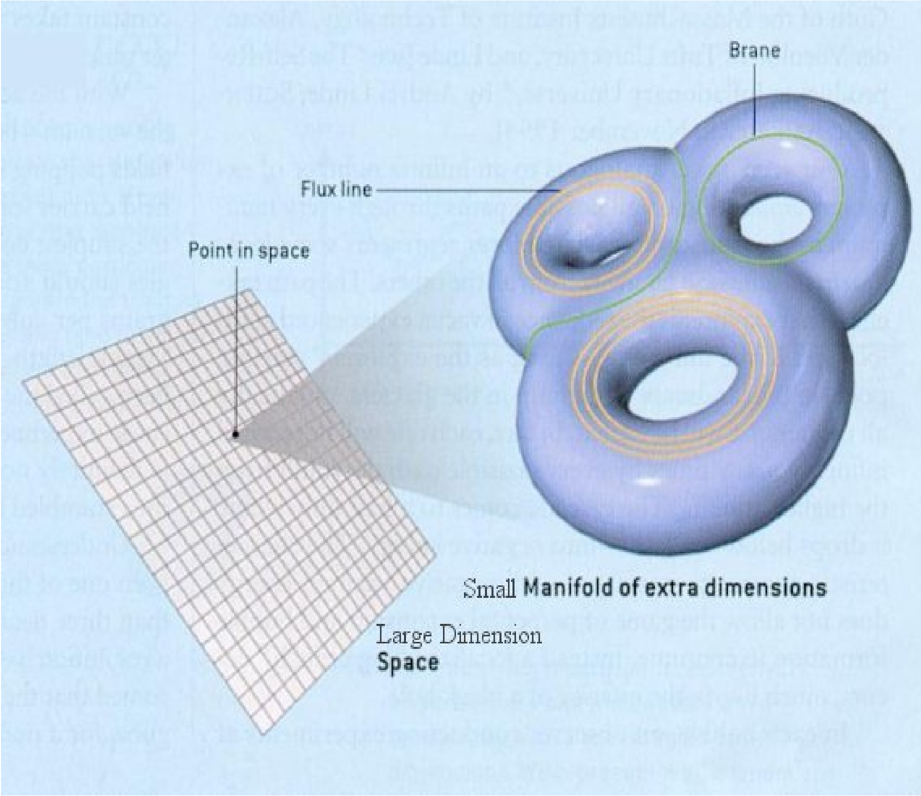
The parameters of the low-energy effective action can be explicitly determined in the framework of magnetized D9-branes compactified on a three torus $T^2 \times T^2 \times T^2$.

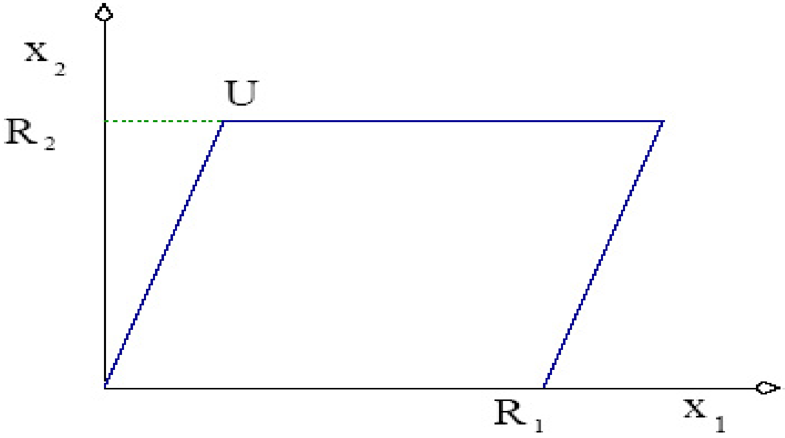
Magnetized branes are D-branes having a constant magnetic field turned on along the compact directions.

SET-UP

World-volume theory (in the limit $\ell_s \to 0$) of a stack of M magnetized D9-branes in the background:

$$\mathcal{R}^{1,3} \times T^2 \times T^2 \times T^2$$





The torus is defined through the identification:

$$x_i \equiv x_i + 2\pi R$$

$$i = 1, 2$$

Kähler modulus: $\mathcal{T}_2^{(r)} = \sqrt{\det G^{(r)}}$

The torus metric:

$$G_{ij}^{(x^1,x^2)} = \frac{\mathcal{T}_2}{U_2} \begin{pmatrix} 1 & U_1 \\ U_1 & |U|^2 \end{pmatrix}$$

Complex structure: $U^{(r)} = U_1^{(r)} + iU_2^{(r)}$

Introducing “flat” dimensionless coordinates:

$$z = \frac{x^1 + U\, x^2}{2\pi R} \quad \Longrightarrow \quad \left\{ \begin{array}{l} z \equiv z + 1 \\ z \equiv z + U \end{array} \right.$$

The low–energy limit of the world-volume action of a stack of M D9-branes

$$S = \frac{1}{g^2}\int d^{10}x \,\mathrm{Tr}\left(-\frac{1}{4}F_{MN}F^{MN} + \frac{i}{2}\bar{\lambda}\Gamma^M\lambda\right)$$

$g^2 = 4\pi g_s (2\pi \ell_s)^6$

Ten dimensional Majorana–Weyl fermion

- Kaluza Klein Reduction from d=10 to d=4.
- Gauge Group and Chiral fermions of the SM.

Turn on a background gauge field in the six compact dimensions and along the Cartan subalgebra of U(M).

Field theory description of Magnetized branes.

Separate the generators of the gauge group U(M) into those of the Cartan subalgebra U_a and those outside e_{ab} .

$$A_M = B_M^a U_a + W_M^{ab} e_{ab} \quad \lambda = \chi^a U_a + \Psi^{ab} e_{ab}$$

Separate the ten dimensional coordinate X^M into a four dimensional non-compact x^μ and a six-dimensional compact variables y^i .

Perform a Kaluza-Klein reduction by expanding around the background fields

$$B_M^a(x^\mu, y^i) = \langle B_M^a \rangle (y^i) + \delta B_M^a(x^\mu, y^i)$$
$$W_M^{ab}(x^\mu, y^i) = 0 + \Phi_M^{ab}(x^\mu, y^i)$$

Four dimensional Lorentz invariance is kept by allowing a background value $\langle B_i^a \rangle (y^i)$ only along the compact directions.

The presence of differents backgrounds values along the cartan subalgebra breaks the original U(M) symmetry into $(U(1))^M$.

Is some of the background values are equal then the original gauge group U(M) is broken in the product of non-abelian subgroups.

In terms of D- branes this corresponds to generate M stack with different magnetizations.

Rewrite the original action in terms of the fields

$$\Phi_i^{ab}(x^\mu, y^i) \qquad \delta B_M(x^\mu, y^i) \qquad \chi^a(x^\mu, y^i) \qquad \Psi^{ab}(x^\mu, y^i)$$

We focus only on the terms containing the Kähler metric and the Yukawa couplins.

The quadratic term for the field Φ_i^{ab}

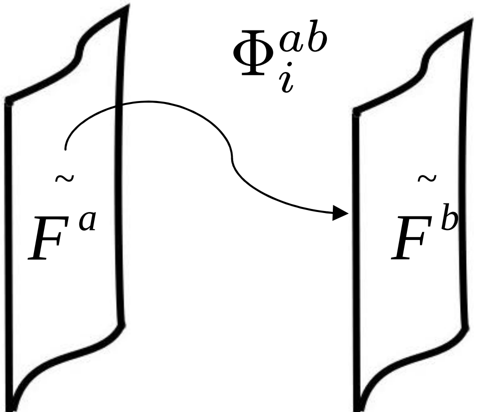
$$S_2^{(\Phi)} = \frac{1}{2g^2}\int d^4x\sqrt{G_4}\int d^6x\sqrt{G_6}\,\Phi^{jba}\left[G^i_j\left(D_\mu D^\mu + \tilde{D}^k\tilde{D}_k\right) + 2i\langle\tilde{F}^j_i\rangle^{ab}\right]\Phi_i^{ab}$$

$$D_\mu\Phi_j^{ab} = \partial_\mu\Phi_j^{ab} - i(B_\mu^a - B_\mu^b)\Phi_j^{ab}$$

Scalar fields transforming in the bifundamental representation of the gauge group U(1) x U(1).

$$\tilde{F}^{ab} = \tilde{F}^a - \tilde{F}^b$$

Field strenght obtained from the field B.



They correspond to open strings stretched between two branes with different magnetization F.

Analogously for the δB_i^a

$$S_2^{(\delta B)} = \frac{1}{2g^2}\int d^4x\sqrt{G_4}\int d^6y\sqrt{G_6}\,\delta B_i^a\left(\partial_j\partial^j + D_\mu D^\mu\right)\delta B^{ai}$$

and for the fermions

$$S_2^{(\Psi)} = \frac{i}{2g^2}\int d^4x\sqrt{G_4}\int d^6y\sqrt{G_6}\,\bar{\Psi}^{ba}\left(\Gamma^\mu D_\mu + \Gamma^i\tilde{D}_i\right)\Psi^{ab}$$

Yukawa couplings

$$S_3^{(\Phi)} = \frac{1}{2g^2}\int d^4x\sqrt{G_4}\int d^6y\sqrt{G_6}\left(\bar{\Psi}^{ca}\Gamma^i\Phi_i^{ab}\Psi^{bc} - \bar{\Psi}^{ca}\Gamma^i\Phi_i^{bc}\Psi^{ab}\right)$$

For bifundamental scalar fields.

$$S_3^{(\delta B)} = \frac{1}{2g^2}\int d^4x\sqrt{G_4}\int d^6y\sqrt{G_6}\,\bar{\Psi}^{ab}(\ \delta\not{B}^b - \ \delta\not{B}^a)\Psi^{ba}$$

For adjoint scalars.

Kaluza-Klein reduction:

$$\Phi_i^{ab}(X) = N_\phi \sum_n \varphi^{ab}_{n,i}(x^\mu)\phi^{ab}_n(y^i) \quad ; \quad \Psi^{ab}(X) = N_\psi \sum_n \psi^{ab}_n(x^\mu) \otimes \eta^{ab}_n(y^i)$$

Moduli dependent normalization factors

The spectrum of the Kaluza-Klein states and their wave-functions along the compact direction are obtained by solving the eigenvalue equations for the six-dimensional Laplace and Dirac operators:

$$-\tilde{D}_k \tilde{D}^k \phi^{ab}_n = m^2_n \phi^{ab}_n \quad , \quad i\gamma^i_{(6)} \tilde{D}_i \eta^{ab}_n = \lambda_n \, \eta^{ab}_n$$

Decomposition of the ten-dimensional Dirac-matrices

$$\Gamma^\mu = \gamma^\mu_{(4)} \otimes I_{(6)} \quad , \quad \Gamma^i = \gamma^5_{(4)} \otimes \gamma^i_{(6)}$$

On the torus $T^2\times T^2\times T^2$ and for the bi-fundamental scalars:

$$-\tilde{D}_k\tilde{D}^k\phi_n^{ab} = \sum_{s=1}^{3}\frac{2\pi|I_s|}{\mathcal{T}_2^{(s)}}\left(2N_s+1\right)\phi_n^{ab} = \frac{\hat{m}_n^2}{(2\pi R)^2}\phi_n^{ab}$$

$$N_s = a_s^\dagger\, a_s$$

$$\tilde{D}_{z^s}\equiv\left(\partial_{z^s}-\frac{\pi|I_s|\bar{z}^s}{2U_2^{(s)}}\right)=i\sqrt{\frac{\pi|I_s|}{U_2^{(s)}}}a_s^\dagger \quad ; \quad \tilde{D}_{\bar{z}^s}\equiv\left(\partial_{\bar{z}^s}+\frac{\pi|I_s|z^s}{2U_2^{(s)}}\right)=i\sqrt{\frac{\pi|I_s|}{U_2^{(s)}}}a_s$$

$$[a_s,\, a_s^\dagger]=1$$

Creation and annihilation operators.

The groud state is determined by solving the equation:

$$a_s\phi_0^{ab}=0 \quad\Rightarrow\quad \tilde{D}_{z^s}\phi_0^{ab}=0$$

together with ⇒

The boundary conditions are the identification, up to a gauge transformation, of the fields when translate along the one-cycles of the torus.

Background gauge field

$$B^a_{z^r,\,\bar z^r}(z^r+1,\ \bar z^r+1) \equiv B^a_{z^r,\,\bar z^r}(z^r,\ \bar z^r) + \partial_{z^r,\,\bar z^r}\chi^a_{1,r}$$

$$B^a_{z^r;\,\bar z^r}(z^r+U^{(r)},\ \bar z^r+\bar U^{(r)}) \equiv B^a_{z^r;\,\bar z^r}(z^r,\ \bar z^r) +\ \partial_{z^r;\,\bar z^r}\chi^a_{2,r}$$

In the gauge: $B_{z^r} = -\frac{1}{2}F^r_{z^r\bar z^r}\bar z^r$

r =1,2,3 labels the three tori

$$B_{z^r}(z^r+1,\ \bar z^r+1) = B_{z^r}(z^r,\ \bar z^r) - \frac{1}{2}F^r_{z^r\bar z^r}$$

$$B_{z^r}(z^r+U^{(r)},\ \bar z^r+\bar U^{(r)}) = B_{z^r}(z^r,\ \bar z^r) - \frac{1}{2}F^r_{z^r\bar z^r}\bar U^{(r)}$$

Gauge Bundle

The expression of F^r can be obtained from the observation that the first Chern-class is an integer I_r .

$$\int \frac{F^r}{2\pi} = I_r \Longrightarrow F^r_{z^r \bar{z}^r} = -\frac{\pi I_r}{i U_2^{(r)}}$$

First Chern-class

$$\chi_{1,r} = \frac{\pi I_r}{\mathrm{Im} U^{(r)}} \mathrm{Im}(z^r) \quad ; \quad \chi_{2,r} = \frac{\pi I_r}{\mathrm{Im} U^{(r)}} \mathrm{Im}(\bar{U}^{(r)} \, z^r)$$

The boundary conditions for the scalars transforming in the bi-fundamental representation of the gauge group, are:

$$\chi^{ab} = \chi^a - \chi^b$$

$$\phi^{ab}(z^r + 1, \bar{z}^r + 1) = e^{i\, \chi^{ab}_{1,r}(z^r,\, \bar{z}^r)} \phi^{ab}(z^r,\, \bar{z}^r)$$

$$\phi^{ab}(z^r + U^{(r)}, \bar{z}^r + \bar{U}^{(r)}) = e^{i\, \chi^{ab}_{2,r}(z^r,\, \bar{z}^r)} \phi^{ab}(z^r,\, \bar{z}^r)$$

$$r = 1, 2, 3$$

The solution is:

$$\phi_0^{ab} = \prod_{r=1}^{3} \phi^{ab;n^r}_{r,signI_r}$$

$$\phi^{ab;n_r}_{r,+} = e^{\pi i I_r z_r \frac{\mathrm{Im} z_r}{\mathrm{Im} U^{(r)}}} \Theta \left[\begin{array}{c} \frac{2n_r}{I_r} \\ 0 \end{array} \right] (I_r z_r | I_r U^{(r)}) \quad \text{for } I_r > 0$$

$$\phi^{ab;n_r}_{r,-} = e^{i\pi |I_r| \bar{z}_r \frac{\mathrm{Im} \bar{z}_r}{\mathrm{Im} U^{(r)}}} \Theta \left[\begin{array}{c} \frac{-2n_r}{I_r} \\ 0 \end{array} \right] (I_r \bar{z}_r | I_r \bar{U}^{(r)}) \quad \text{for } I_r < 0$$

$$n_r = 1 \ldots |I|_r - 1 \Longrightarrow$$

The vacuum state is degenerate.

(Landau Levels)

The wave-function for the fermions in the bi-fundamental representation of the gauge group has the same structure as the one of the scalars. The one for the adjoint scalars is the identity.

Kähler Metrics

The Kinetic term for the bi-fundamental scalars:

$$\begin{aligned} S_2^{(\Phi)} &= \frac{1}{2g^2}\int d^4x\sqrt{G_4}\sum_n\prod_{r=1}^{3}\left[(2\pi R)^2\int d^2z_r\sqrt{G^r}\right]\phi_n^{ba}\phi_n^{ab} \\ &\times\left\{\sum_{r=1}^{3}N_{\varphi_r}^2\left[\varphi_{nr}^{ba,z}(x)\left[D_\mu D^\mu - m_n^2 + \frac{4\pi I_r}{(2\pi R)^2\mathcal{T}_2^{(r)}}\right]\varphi_{nrz}^{ab}(x)\right]\right. \\ &\left.+\sum_{r=1}^{3}N_{\varphi_r}^2\left[\varphi_{nr}^{ba,\bar z}(x)\left[D_\mu D^\mu - m_n^2 - \frac{4\pi I_r}{(2\pi R)^2\mathcal{T}_2^{(r)}}\right]\varphi_{nr\bar z}^{ab}(x)\right]\right\} \end{aligned}$$

Two towers of Kaluza-Klein states

$$(M_{n,r}^{\pm})^2 = \frac{1}{(2\pi R)^2}\left[\sum_{s=1}^{3}\frac{2\pi|I_s|}{\mathcal{T}_2^{(s)}}(2N_s+1)\pm\frac{4\pi I_r}{\mathcal{T}_2^{(r)}}\right]$$

In agreement with the field theory limit of the string.

Massless state condition:

$$N_s = 0 \qquad \frac{1}{2}\sum_{s=1}^{3}\frac{|I_s|}{\mathcal{T}_2^{(s)}} - \frac{|I_r|}{\mathcal{T}_2^{(r)}} = 0$$

restauring $\mathcal{N}$=1supersymmetry, because this scalar is in the same chiral multiplet with the fermion.

Keeping only the massless contribution in $S_2^{(\Phi)}$ one gets, in the Einsten frame:

$$S_2^{(\phi)} = -\int d^4x\sqrt{G_4}\; Z(m,\bar{m})\,(D_\mu\bar{\varphi}(x))(D^\mu\varphi(x)) + \ldots$$

with:

$$Z(m,\bar{m}) = \frac{4\pi \mathrm{e}^{2\phi_4}}{2g^2} N_\varphi^2 \prod_{s=1}^{3}\left[(2\pi R)^2\int d^2 z_s\sqrt{G^{(z_s,\bar{z}_s)}}\right]\phi_0^{ba}\phi_0^{ab}$$

The integral over the six-dimensional compact space may be explicitely performed:

$$Z = \frac{N_\varphi^2}{2 s_{2_{N_\varphi}}^{1/4}} \prod_{r=1}^{3} \left[\frac{1}{(2u_2^{(r)})^{1/2} (t_2^{(r)})^{1/4}} \left(\frac{T_2^{(r)}}{|I_r|_{N_\varphi}} \right)^{1/2} \right]$$

$$s_2 = \mathrm{e}^{-\phi_{10}} \prod_{r=1}^{3} T_2^{(r)}$$

$$u_2^{(r)} = U_2^{(r)}$$

$$t_2^{(r)} = \mathrm{e}^{-\phi_{10}} T_2^{(r)}$$

Kähler modulus in string units: $T_2 = \mathcal{T}_2 (\frac{R}{\ell_s})^2$

We have determined how the Kähler metric depends on the moduli apart from the normalization factor N_φ .

N_φ is fixed from the holomorphicity of the superpotential.

Yukawa Couplings

The Kähler metric, for the fermions transforming in the bi-fundamental representation of the gauge group, is obtained from the fermions kinetic term.

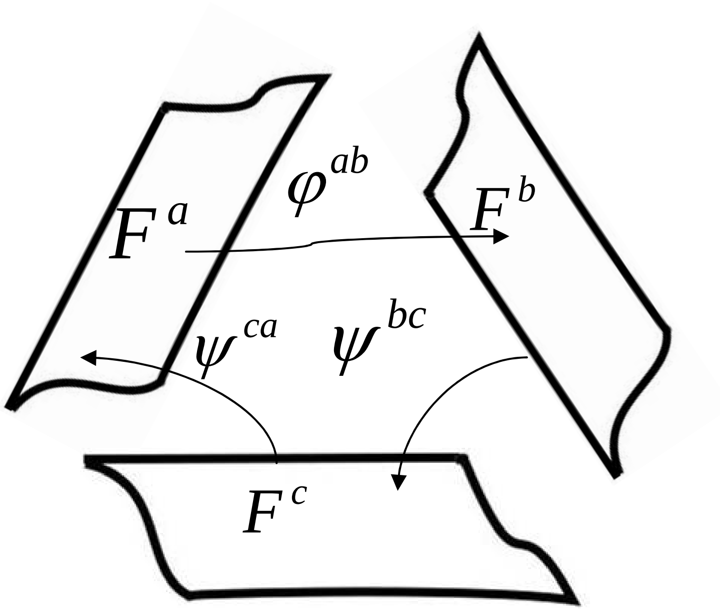
$$S_2^{(\Psi)} = \frac{i}{2}\int d^4x Z(m,\bar{m})\sqrt{G_4}\bar{\psi}^{ba}\,\gamma^{\mu}_{(4)} D_{\mu}\,\psi^{ab} \ .$$

$$Z = \frac{\mathrm{e}^{2\phi_4}}{g^2} N_{\psi}^2 \int d^6y \sqrt{G_6} (\eta^{ab})^{\dagger}\,\eta^{ab} = \frac{\mathrm{e}^{\phi_4}}{4\pi} N_{\psi}^2 \prod_{r=1}^{3} \left[\left(\frac{T_2^{(r)}}{|I_r|} \right)^{1/2} \left(\frac{1}{2U_2^{(r)}} \right)^{1/2} \right]$$

It, as a consequence of $\mathcal{N}$=1 SUSY, coincides with the one of the scalars transforming in the same representation of the gauge group.

Yukawa Couplings

The Yukawa couplings are the trilinear couplings of the superpotential W.



$$Y_{ijk} = e^{K/2} W_{ijk}$$

Kähler potential

Kähler modulus in string units: $T_2 = \mathcal{T}_2 (\frac{R}{\ell_s})^2$

$$K = -\log s_2 - \sum_{r=1}^{3} \log t_2^{(r)} - \sum_{r=1}^{3} \log u_2^{(r)} \left\{ \begin{array}{c} s_2 = \mathrm{e}^{-\phi_{10}} \prod_{r=1}^{3} T_2^{(r)} \\ t_2^{(r)} = \mathrm{e}^{-\phi_{10}} T_2^{(r)} \\ u_2 = U_2^{(r)} \end{array} \right.$$

Antoniadis, Bacas, Fabre Partouche, Taylor, 1996

These can be computed from the trilinear couplings in .

$$S_3^{(\Phi)} = \int d^4x\sqrt{G_4}\,\bar{\psi}^{ca}\,\gamma^5_{(4)}\varphi_1^{ab}\,\psi^{bc}Y^E$$

$$Y^E = \frac{\mathrm{e}^{K/2}}{\sqrt{8\pi}}\sigma N_\varphi N_\psi N_\psi \prod_{r=1}^{3}\left\{\boxed{\frac{(T_2^{(r)})^{1/2}}{\left(2|I_r^{ab}|\chi_r^{ab}|I_r^{bc}|\chi_r^{bc}|I_r^{ca}|\chi_r^{ca}\right)^{1/2}}}\right.$$

$$\left.\times\,\Theta\left[\begin{array}{c} 2\left(\frac{n'}{I_r^{ca}}+\frac{m'}{I_r^{bc}}+\frac{l'}{I_r^{ab}}\right)\\ 0\end{array}\right](0|-I_r^{ab}I_r^{bc}I_r^{ca}U_f^{(r)})\right\}$$

$$\boxed{\chi_r = \frac{(1+sign(I_r\,I_r))}{2}}$$

$$\boxed{U_f^{(r)} = \left\{\begin{array}{lll} U^{(r)} & \text{for} & sign(I^{ca}I^{bc}I^{ab})<0\\ \bar{U}^{(r)} & \text{for} & sign(I^{ca}I^{bc}I^{ab})>0\end{array}\right.}$$

Because of the terms depending on the magnetizations, the Yukawa is not an holomorphic function of the moduli unless we choose the normalization factors N_φ and N_ψ to eliminate such dependence.

$$N_{\varphi_1}^{ab} = \left(\frac{|I_1^{ab}|}{T_2^{(1)}} \right)^{1/2} \quad ; \quad N_{\psi}^{ca} = \left(\frac{|I_2^{ca}|}{T_2^{(2)}} \right)^{1/2} \quad ; \quad N_{\psi}^{bc} = \left(\frac{|I_3^{bc}|}{T_2^{(3)}} \right)^{1/2}$$

Using these normalization factors in the expression of the Kähler metrics determined from the kinetic terms, one gets:

$$Z_{ab}^{chiral} = \frac{1}{2 s_2^{1/4}} \prod_{r=1}^{3} \left[\frac{1}{(2u_2^{(r)})^{1/2} (t_2^{(r)})^{1/4}} \right] \left(\frac{\nu_1^{ab}}{\pi \nu_2^{ab} \nu_3^{ab}} \right)^{1/2}$$

$$\nu_r = \frac{I_r}{T_2^{(r)}}$$

It is in agreement with the field theory limit of the corresponding expression obtained in string theory by computing three and four point amplitudes or by means Instanton calculus !!!

Conclusions.

We have discussed in a simple set-up of toroidal compactification how to obtain the effective four-dimensional action for chiral fields.

How to get in a unique way the Standard Model or one of its extensions?

Instanton effects.

Different kinds of compactifications.

Still a lot of work must be done….