SPACETIME METRIC DEFORMATIONS

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Lavori

D. Pugliese, Deformazioni di metriche spazio-temporali, tesi di laurea quadriennale, relatori S. Capozziello e C. Stornaiolo

S. Capozziello e C. Stornaiolo, Space-Time deformations as extended conformal transformations

International Journal of Geometric Methods in Modern Physics 5, 185-196 (2008)

Introduction

- General Relativity
- Exact solutions: cosmology, black holes, solar system
- Approximate solutions: gravitational waves, gravitomagnetic effects, cosmological perturbations, post-newtonian parametrization

Spacetime metric

- The spacetime metric generalizes the notion of metric introduce with Special Relativity.
- In General Relativity the metric makes sense only as an infinitesimal distance between two events.
- From it all the properties of spacetime, geodesics, curvature can be obtained.
- Gravitational phenomena can be interpreted as due to the spacetime geometry. It takes the following form

Spacetime metric

$$ds^2 = g_{ab} dx^a dx^b$$

- Where the Einstein summation convention has been used
- $x^a = (ct, x, y, z)$ are the spacetime coordinates

FRIEDMANN-LEMAITRE-ROBERTSON-WALKER METRIC

- The FLRW metric is derived imposing the homogeneity and isotropy of spacetime it has the form

$$ds^2 = c^2 dt^2 - \frac{a^2(t)}{\left(1 - \frac{k}{4} r^2\right)^2}\left(dx^2 + dy^2 + dz^2\right)$$

- $a(t)$ satisfies the Einstein equations

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} = \frac{8\pi G}{3}\rho$$

$$2\frac{\ddot{a}}{a} = -\frac{8\pi G}{3}(\rho + 3P)$$

$$P = P(\rho)$$

SCHWARZSCHILD METRIC

- A spherical symmetric static field is described by the Schwarzschild metric, which rules with very good approximation the motions in the solar system.

$$ds^2 = \left(1 - \frac{2GM}{c^2 r}\right) c^2 dt^2 - \left(1 - \frac{2GM}{c^2 r}\right)^{-1} dr^2 - r^2 d\Omega^2$$

- The motion of the perihelion of Mercury's orbit was explained completely only applying this solution

Present problems

- Dark matter
- Dark energy: acceleration of the universe
- Pioneer anomaly
- Relation between the different scales
- Average problems in General Relativity
- Approximate symmetries

Our ignorance of the real structure of spacetime

- All these problems represent our ignorance of the real structure of spacetime
- Boundary conditions
- Initial conditions
- Approximate symmetries or complete lack of symmetry
- Unknown distribution of dark matter and dark energy (besides its nature)
- Alternative theories of gravity

Example: Dicke's problem

- Is the Sun oblate?
- If yes then part of the precession of 43 seconds of arc per century of the Mercury perihelion can be explained by classical tidal effects
- Paradoxically the exact explanation found in GR, would tell us that GR is wrong!
- But at the same time one should also consider the corrections to the Schwarzschild solution!
- The problem is still controversial, even if many evidences have been found against the oblateness of the Sun.
- A positive answer would imply to deform the Schwarzschild solution to take in account the lack of spherical symmetry
- But do not forget the Pioneer anomaly problem…..

How can a metric describe our ignorance ?

Given an exact metric we would like to "deform" it in order to capture the most important features of spacetime, not present in such a metric

Geometrical deformations in 2D: The surface of the earth

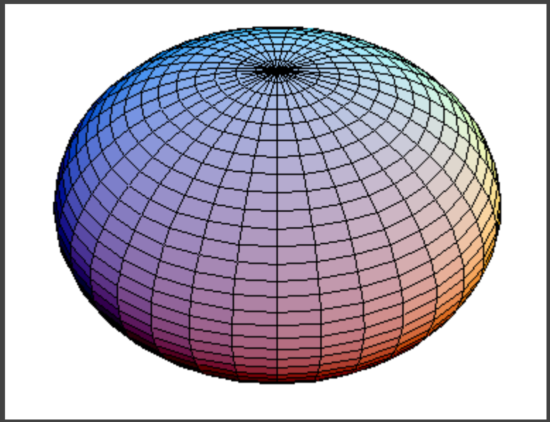
- Let us give an example of how we can deal with a deformed geometry

If we take the surface of the earth, we can consider it as a (rotating) sphere

But this is an approximation

The Earth is an oblate ellipsoid

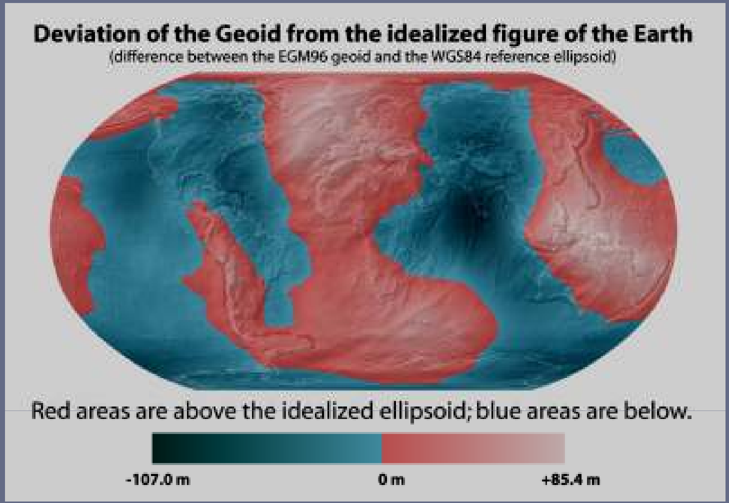
- Measurments indicate that it is better described by an oblate ellipsoid



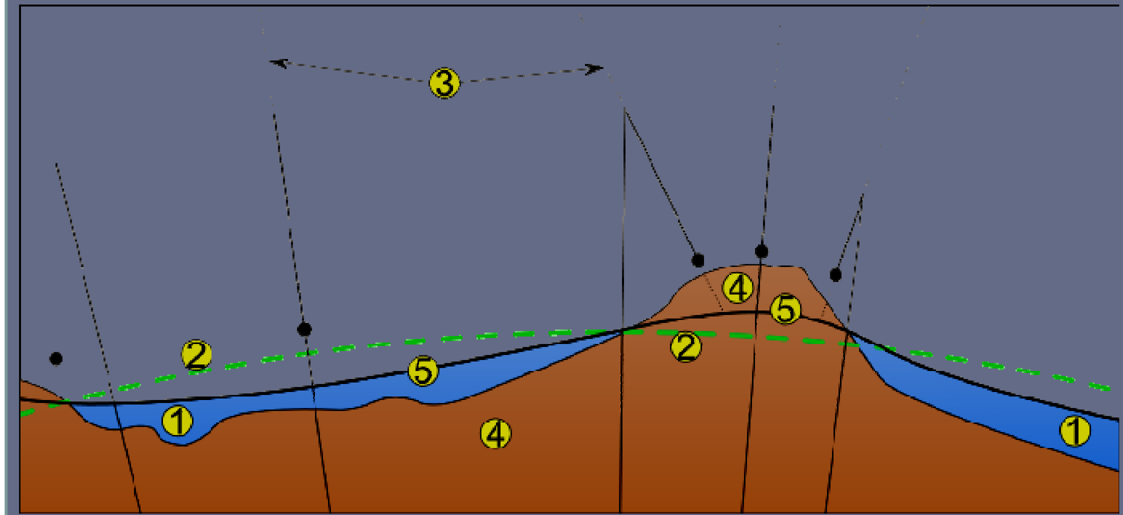
Mean radius 6,371.0 km
Equatorial radius 6,378.1 km
Polar radius 6,356.8 km
Flattening 0.0033528

- But this is still an approximation

The shape of the Geoid

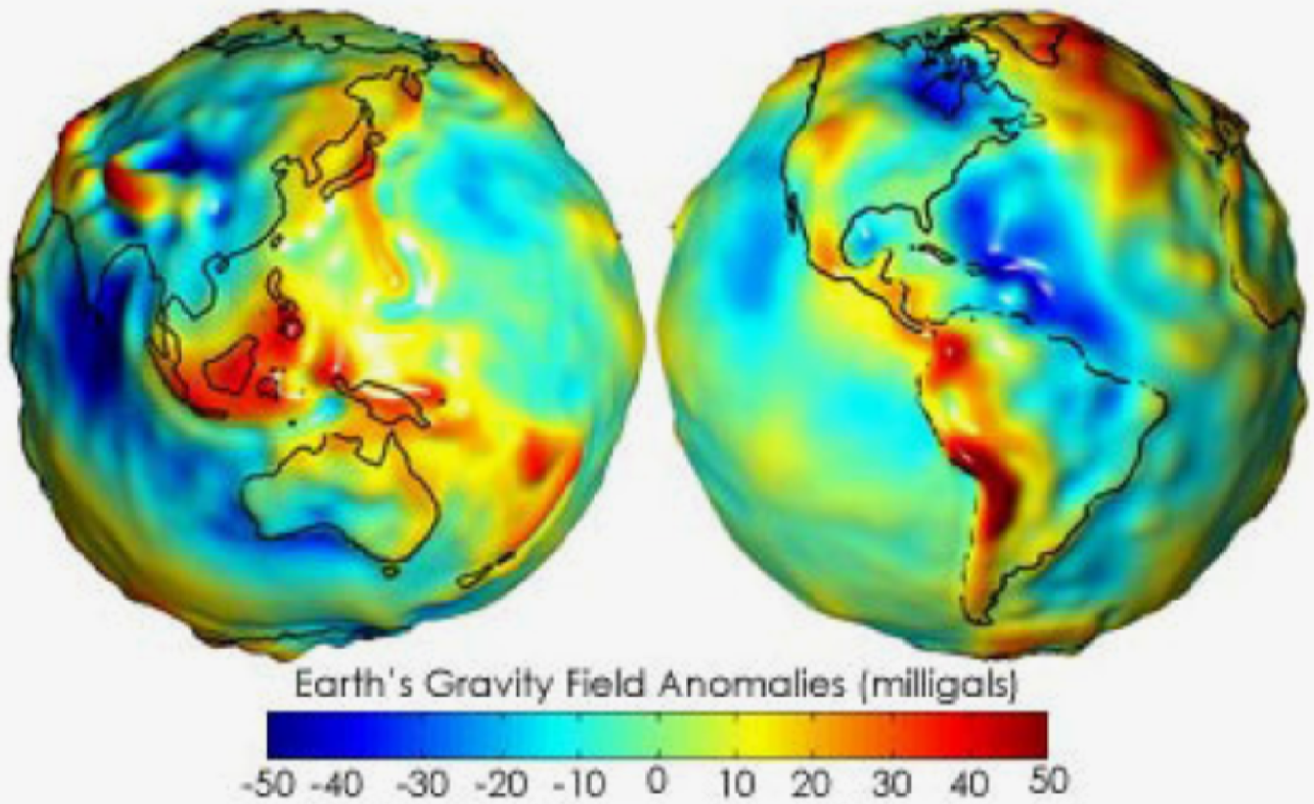


We can improve our measurements an find out that the shape of the earth is not *exactly* described by any regular solid. We call the geometrical solid representing the Earth a *geoid*

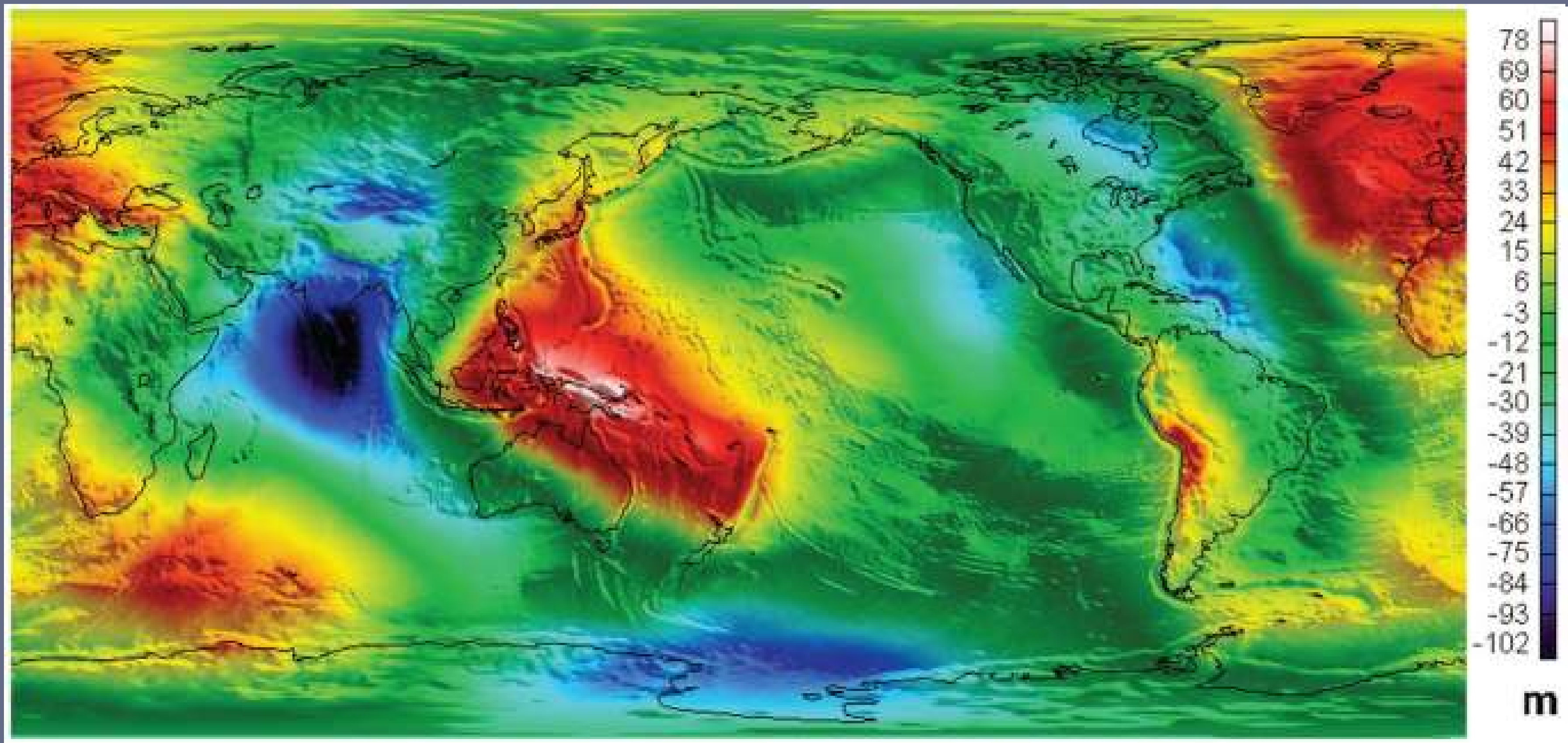


1. Ocean
2. Ellipsoid
3. Local plumb
4. Continent
5. Geoid

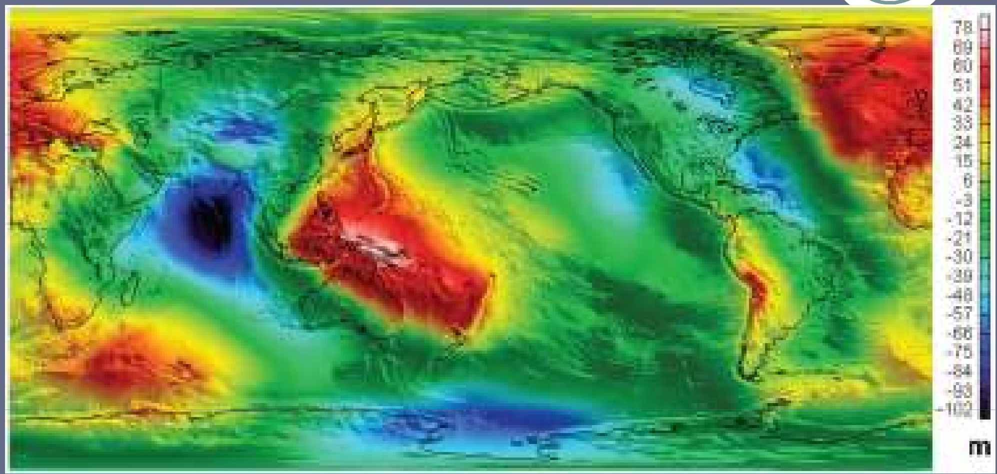
Geoid in 3D

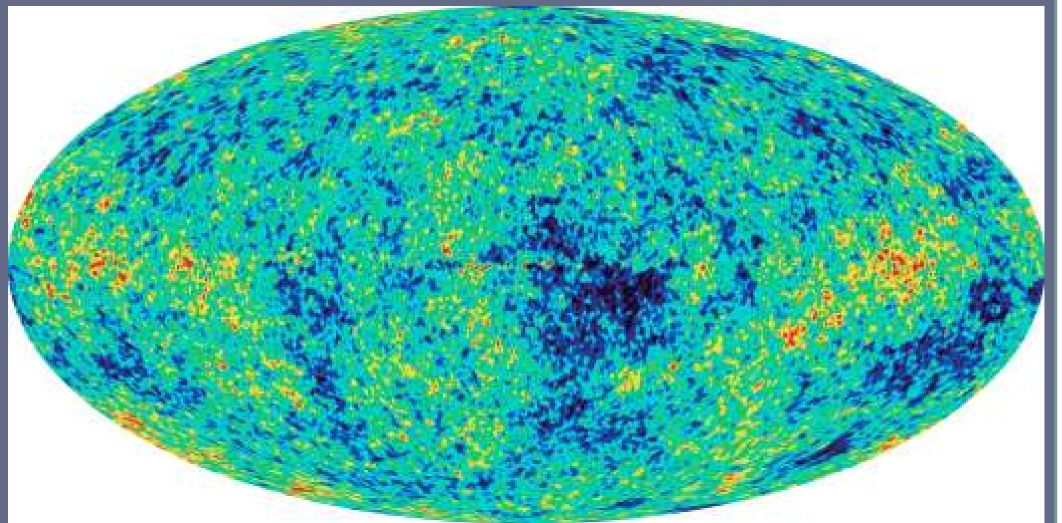

Earth's Gravity Field Anomalies (milligals)
-50 -40 -30 -20 -10 0 10 20 30 40 50

Another image of the geoid



Comparison between the deviations of the geoid from an idealized oblate ellipsoid and the deviations of the CMB from the homogeneity





Deformations in 2D

- It is well known that all the two dimensional metrics are related by conformal transformations, and are all locally conform to the flat metric

$$\tilde{g} = \Omega^2(x) g$$

A generalization?

- The question is if there exists **an intrinsic** and **covariant** way to relate similarly metrics in dimensions

$$n > 2$$

Riemann theorem

Riemann, G. F. B., (1953). Über die Hypothesen, welche der Geometrie zu Grunde liegen, Abhand. K. Ges. Wiss. Göttingen, **13**, 133, 1868; English translation by Clifford, W. K. *Nature* **8,** 14, 1873; reprinted and edited by Weyl, H., Springer, Berlin, 1920. Included in its *Gesammelte Mathematische Werke, wissenschaftlicher Nachlaβ und Nachträge*, eds. Weber, H., Dedekind, R., Teubner, B. G., Leipzig, 1892; 2d ed. Dover Publ., New York.

- In an n-dimensional manifold with metric the metric has

$$f = \frac{n(n-1)}{2}$$

degrees of freedom

Deformation in three dimensions

In 2002 Coll, Llosa and Soler (*General Relativity and Gravitation, Vol. 34, 269, 2002*) proved the theorem which states that any metric in a 3D space(time) is related to a constant curvature metric by the following relation

$$g_{ab} = \Omega^2 h_{ab} + \varepsilon \sigma_a \sigma_b$$

Generalization to an arbitrary number N of dimensions

- It was conjuctered by Coll (gravitation as a universal deformation law)

  and showed by Llosa and Soler (Class. Quantum Grav. **22 (2005) 893–908**) that a similar relation can be extended to an N-dimensional spacetime

$$\overline{g}_{\alpha\beta} = a g_{\alpha\beta} - \epsilon F^2_{\alpha\beta},$$

with $F^2_{\alpha\beta} := g^{\mu\nu} F_{\alpha\mu} F_{\nu\beta}$ *and* $|\epsilon| = 1$, *has constant curvature.*

Our definition of metric deformation

Let us see if we can generalize the preceding result possibly expressing the deformations in terms of scalar fields as for conformal transformations. What do we mean by metric deformation? Let us first consider the decomposition of a metric in tetrad vectors

$$g_{ab} = \eta_{AB} e^A e^B \quad g_{ab} = \eta_{AB} \Lambda^A_C(x) e^C \Lambda^B_D(x) e^D$$

$$\eta_{AB} \Lambda^A_C(x) \Lambda^B_D(x) = \eta_{CD}$$

[redacted]

$$g_{ab} = \eta_{AB} e^{A}{}_{a} e^{B}{}_{b}$$

$$g_{ab} = e^{0}{}_{a} e^{0}{}_{b} - e^{1}{}_{a} e^{1}{}_{b} - e^{2}{}_{a} e^{2}{}_{b} - e^{3}{}_{a} e^{3}{}_{b}$$

$$\eta_{AB} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

The deforming matrix

$$\tilde{g}_{ab} = \eta_{AB}\Phi^A_C(x)e^C{}_a\Phi^B_D(x)e^D{}_b$$

$$\Phi^A_B(x) \neq \Lambda^A_B(x)$$

[redacted]

$$\widetilde{e}^{A}{}_{a} = \Phi^{A}{}_{B} e^{B}{}_{a}$$

$$\widetilde{g}_{ab} = \eta_{AB} \widetilde{e}^{A}{}_{a} \widetilde{e}^{B}{}_{b}$$

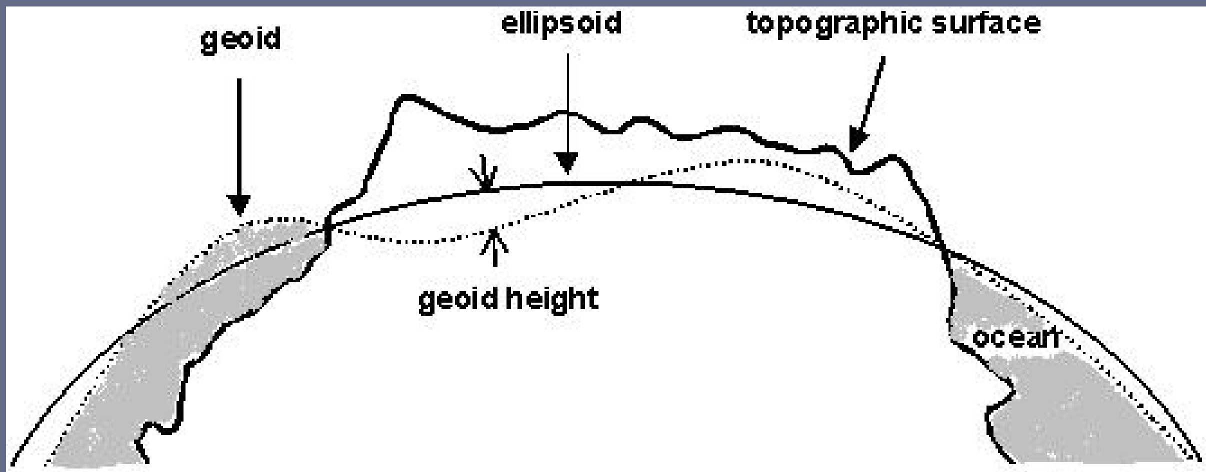
$$\widetilde{g}_{ab} = \widetilde{e}^{0}{}_{a} \widetilde{e}^{0}{}_{b} - \widetilde{e}^{1}{}_{a} \widetilde{e}^{1}{}_{b} - \widetilde{e}^{2}{}_{a} \widetilde{e}^{2}{}_{b} - \widetilde{e}^{3}{}_{a} \widetilde{e}^{3}{}_{b}$$

Role of the deformation matrices

- The deformation matrices are the corrections introduced in known exact metrics to consider a realistic spacetime
- The inverse problem is instead to find an average solution which is solution of the Einstein equations satisfying the given symmetries
- They are the unknown of our problem
- They can be obtained from phenomenolgy
- Or are solutions of a set of differential equations
- Let us give an example in terms of Newtonian gravity

Deformation of the gravitational potential

$$V = \frac{GM}{r}\left(1 + \sum_{n=2}^{n_{max}} \left(\frac{a}{r}\right)^n \sum_{m=0}^{n} \overline{P}_{nm}(\sin\phi)\left[\overline{C}_{nm}\cos m\lambda + \overline{S}_{nm}\sin m\lambda\right]\right),$$



Properties of the deforming matrices

- $\Phi^A_B(x)$ are matrices of scalar fields in spacetime,
- they are scalars with respect to coordinate transformations,
- they are defined within a Lorentz transformation. They define an equivalent class

$$\Phi^A_B(x) \cong \Lambda^A_C(x)\Phi^C_B(x)$$

Identity

The identity is obviously the Kronecker delta

$$\delta_A^B$$

but as seen before also the Lorentz matrices behave as identities, so we consider as identity the (infinite) the set

$$I = \left\{ \delta_A^B , \Lambda_A^B \right\}$$

Deformations and the Lorentz group

The deformation matrices form a right coset for the Lorentz group, i.e. any element is an equivalent class defined by the relation

$$\Phi_A^{\ B} \cong \Lambda_A^{\ C}\Phi_C^{\ B} \qquad \forall \Lambda_A^{\ B}$$

Conformal transformations

A particular class of deformations is given by

$$\Phi_B^A(x) = \Omega(x)\delta_B^A$$

which represent the conformal transformations

$$\widetilde{g}_{ab} = \eta_{AB}\Phi_C^A e^C \Phi_D^B e^D = \Omega^2(x) g_{ab}$$

This is one of the first examples of deformations known from literature. For this reason we can consider deformations as an extension of conformal transformations.

More precise definition of deformation

If the metric tensors of two spaces M and $\widetilde{M}$ are related by the relation

$$\Phi : g = \eta_{AB} e^A e^B \rightarrow \widetilde{g} = \eta_{AB} \Phi^A_C \Phi^B_D e^C e^D$$

we say that $\widetilde{M}$ is the deformation of M

(cfr. L.P. Eisenhart, Riemannian Geometry, pag. 89)

Other properties of the deforming matrices

- They are not necessarily real
- They are not necessarily continuos (may associate spacetime with different topologies)
- They are not coordinate transformations (one should transform correspondingly all the covariant and contravariant tensors), i.e. they are not diffeomorphisms of a spacetime M to itself
- They can be composed to give successive deformations
- They may be singular in some point, if we expect that they lead to a solution of the Einstein equations

Second deforming matrix

$$\tilde{g}_{ab} = \underbrace{\eta_{AB}\Phi^{A}_{C}(x)\Phi^{B}_{D}(x)}_{\mathcal{G}_{AB}(x)} e^{C} e^{D}$$

[redacted]

We note simply that General Relativity and all the other metric theories can be re-expressed in terms of tetrads (tetrad-gravity) as variables

The matrix $g_{AB}(x)$ can be an alternative approach in terms of variables. We shall call this matrix the **second deforming matrix** and $\Phi_A^{\;B}(x)$ the **first deforming matrix.**

A **third** approach to define spacetime deformations

$\Phi_A^{\ B}(x)$ can be written as a spacetime tensor by contracting it with the tetrad $\Phi_a^{\ b} = \Phi_A^{\ B}(x) e_a^{\ A} e_B^{\ b}$ and using the identity $g_{cd} e_C^c e_D^d = \eta_{CD}$ then a metric can be written as

$$\tilde{g}_{ab} = g_{cd} \Phi_a^c \Phi_b^d$$

[redacted]

- By lowering its index with a Minkowski matrix we can decompose the first deforming matrix

$$\eta_{AC}\Phi^{C}_{B} = \Phi_{AB} = \Phi_{(AB)} + \Phi_{[AB]}$$

$$\Phi_{AB} = \Omega\eta_{AB} + \Theta_{AB} + \omega_{AB}$$

$$\Phi^{B}_{A} = \Omega\delta^{B}_{A} + \Theta^{B}_{A} + \omega^{B}_{A}$$

The deforming functions and the degree of freedom

- Ω is the trace of the first deforming matrix
- Θ_A^B is the symmetric part

  ω_A^B is the antisymmetric part
- Riemann theorem: a metric in n-dimensional spacetime has $(n-1)n/2$ degrees of freedom
- the components of the first deforming matrix are redundant

Expansion of the second deforming matrix

Substituting in the expression for deformation

the second deforming matrix takes the form

$$\mathscr{g}_{AB} = \Omega^2 \eta_{AB} + 2\Omega\Theta_{AB} + \eta_{CD}\Theta^C_A\Theta^D_B$$
$$+\eta_{CD}\left(\Theta^C_A\omega^D_B + \omega^C_A\Theta^D_B\right) + \eta_{CD}\omega^C_A\omega^D_B$$

Inserting the tetrad vectors to obtain the metric it follows that (next slide)

Tensorial definition of the deformations

Reconstructing a deformed metric leads to

$$\tilde{g}_{ab} = \Omega^2 g_{ab} + \gamma_{ab}$$

This is the *most general* relation between two metrics.

This is the third way to define a deformation

Deforming the contravariant metric

To complete the definition of a deformation we need to define the deformation of the corresponding contravariant tensor

$$\tilde{g}^{ab}$$

$$\tilde{g}^{ab} = \eta^{AB} \left(\Phi^{-1}\right)_A^C \left(\Phi^{-1}\right)_B^D e_C^a e_D^b$$

A third way to define the controvariant metric deformed

In tensorial terms we can procede as we did for the covariant tensor. We can make the ansatz

$$\tilde{g}^{ab} = \alpha^2 g^{ab} + \lambda^{ab}$$

If $\lambda^{ab} = 0$, then $\alpha = \Omega^{-1}$, and we can suppose this condition true even for $\lambda^{ab} \neq 0$

Third way to define the contravariant metric deformations

Regarding λ_{ab}, it is related to γ_{ab} by the following relation,

$$\lambda_a^{\ b} = -\Omega^{-4}\left(\delta + \Omega^{-2}\gamma\right)^{-1\,c}{}_{a}\,\gamma_c^{\ b}$$

where indices are raised and lowered with the undeformed metrics.

Deformed connections

We are now able to define the connections

where $$\widetilde{\Gamma}^{c}_{ab} = \Gamma^{c}_{ab} + C^{c}_{ab}$$

is a tensor

$$C^{c}_{ab} = 2\tilde{g}_{d(a}\nabla_{b)}\Omega^2 - g_{ab}\tilde{g}^{cd}\nabla_d\Omega^2 + \tfrac{1}{2}\tilde{g}^{cd}\left(\nabla_a\gamma_{db} + \nabla_b\gamma_{ad} - \nabla_d\gamma_{ab}\right)$$

Deformed geodesic equations

- The deformed connection suggests that in a deformed spacetime the geodesic equations correspond to a deviation of the geodesic motion in the undeformed spacetime

$$\frac{d^2x^\lambda}{ds^2}+\widetilde{\Gamma}^\lambda_{\mu\nu}\frac{dx^\mu}{ds}\frac{dx^\nu}{ds}=0\Rightarrow\frac{d^2x^\lambda}{ds^2}+\Gamma^\lambda_{\mu\nu}\frac{dx^\mu}{ds}\frac{dx^\nu}{ds}=-C^\lambda_{\mu\nu}\frac{dx^\mu}{ds}\frac{dx^\nu}{ds}$$

Deformed geodesic motion

- Therefore the deformed geodesic motion takes into account of our ''ignorance'' of the real spacetime metric, independently of the origin of the deformation
- Examples: Pioneer anomaly, unexpected lensing properties, dark matter distribution, deviation from the supposed spacetime symmetries, different metric theories, and so on

The Pioneer anomaly

- **Pioneer anomaly**
- **From Wikipedia, the free encyclopedia**
- Unsolved problems in physics: *What causes the apparent residual sunward acceleration of the Pioneer spacecraft?*
- The **Pioneer anomaly** or **Pioneer effect** is the observed deviation from expectations of the trajectories of various unmanned spacecraft visiting the outer solar system, notably Pioneer 10 and Pioneer 11. Both spacecraft are escaping from the solar system, and are slowing down under the influence of the Sun's gravity. Upon very close examination, however, they are slowing down slightly more than expected from influence of all known sources. The effect can be modelled as a slight additional acceleration towards the Sun.
- At present, there is no universally accepted explanation for this phenomenon; while it is possible that the explanation will be mundane—such as thrust from gas leakage—the possibility of entirely new physics is also being considered.

Deformed Curvature tensors

Finally we can define how the curvature tensors are deformed

$$\widetilde{R}_{abc}{}^{d} = R_{abc}{}^{d} + \nabla_b C^d{}_{ac} - \nabla_a C^d{}_{bc} + C^e{}_{ac} C^d{}_{be} - C^e{}_{bc} C^d{}_{ae}$$

$$\widetilde{R}_{ab} = R_{ab} + \nabla_d C^d{}_{ab} - \nabla_a C^d{}_{db} + C^e{}_{ab} C^d{}_{de} - C^e{}_{db} C^d{}_{ae}$$

$$\widetilde{R} = \widetilde{g}^{ab} \widetilde{R}_{ab} = \widetilde{g}^{ab} R_{ab} + \widetilde{g}^{ab} \left[\nabla_d C^d{}_{ab} - \nabla_a C^d{}_{db} + C^e{}_{ab} C^d{}_{de} - C^e{}_{db} C^d{}_{ae} \right]$$

Einstein equations for the deformed spacetime in the vacuum

The equations in the vacuum take the form

$$\tilde{R}_{ab} = 0 \Rightarrow R_{ab} + \nabla_d C^d{}_{ab} - \nabla_a C^d{}_{db}$$

$$+ C^e{}_{ab} C^d{}_{de} - C^e{}_{db} C^d{}_{ae} = 0$$

The deformed Einstein equations in presence of deformed matter sources

In presence of matter sources the equations for the deformed metric are of the form

$$\widetilde{R}_{ab} = \widetilde{T}_{ab} - \tfrac{1}{2}\widetilde{g}_{ab}\widetilde{T}$$

$$\widetilde{R}_{ab} = R_{ab} + \nabla_d C^d{}_{ab} - \nabla_a C^d{}_{db} + C^e{}_{ab} C^d{}_{de} - C^e{}_{db} C^d{}_{ae}$$

$$\nabla_d C^d{}_{ab} - \nabla_a C^d{}_{db} + C^e{}_{ab} C^d{}_{de} - C^e{}_{db} C^d{}_{ae} = \widetilde{T}_{ab} - \tfrac{1}{2}\widetilde{g}_{ab}\widetilde{T} - \left(T_{ab} - \tfrac{1}{2} T g_{ab}\right)$$

Standard gravitational theories

- History of gravitational theory
- Newtonian gravity (NG)
  - Classical mechanics
- General relativity (GR)
  - History
  - Mathematics
  - Resources
  - Tests
  - Twistors

Alternatives to GR

- Classical theories of gravitation
- **Conformal gravity**
- Scalar theories
  - Nordström
  - Yilmaz
- Scalar-tensor theories
  - Brans–Dicke
  - Self-creation cosmology
- Bimetric theories
- f(R) theories

Other alternatives

- Einstein–Cartan
  - Cartan connection
- Whitehead
- Nonsymmetric gravitation
- Scalar-tensor-vector
- Tensor-vector-scalar
- f(R) theories with torsion

Unified field theories

- Teleparallelism
- Geometrodynamics
- Quantum gravity
  - Semiclassical gravity
  - Discrete Lorentzian QG
  - Euclidean QG
  - Induced gravity
  - Causal sets
  - Loop quantum gravity
  - Wheeler–deWitt eqn

Theory of everything

- Supergravity
- M-theory
- Omega Point quantum gravity TOE
- Superstrings
- String theory
  - String theory topics

Other

- Higher-dimensional GR
  - Kaluza–Klein
  - DGP model
- Alternatives to NG
  - Aristotle
  - Mechanical explanations
    - Fatio–Le Sage
  - MOND
- Unclassified
  - Composite gravity
  - Massive gravity
- Electrogravitics
  - Gravitomagnetism
  - Anti-gravity
  - Levitation

Conformal transformations

- Conformal transformations have often been thought as a mathematical device to find solutions of the Einstein equations
- They were used to find a relation between the solutions of alternative gravitational theories (e.g. Brans-Dicke, f(R) theories) and Einstein's general relativity

Deformation and bimetric theories

- **Bimetric theory** refers to a class of modified theories of gravity in which two metric tensors are used instead of one. Often the second metric is introduced at high energies, with the implication that the speed of light may be energy dependent.
- In general relativity, it is assumed that the distance between two points in spacetime is given by the metric tensor. Einstein's field equations are then used to calculate the form of the metric based on the distribution of energy.

Bimetric theories (N. Rosen)

- The first bimetric theory was introduced by Nathan Rosen (remember EPR) in the early '40s
- Gravitation is interpreted as a physical field described by a tensor (physical metric) on a geometrical background with a geometrical metric
- The decomposition is like found by us with

$$\Omega^2 = 1 \qquad\qquad g_{ab} = \eta_{ab} + \gamma_{ab}$$

- The field equations for the physical metric are precisely the ones written previously

Einstein equations for the deformed in physical metric of the bimetric theory in the vacuum

The equations in the vacuum take the form

$$\widetilde{R}_{ab} = 0 \Rightarrow R_{ab} + \nabla_d C^d{}_{ab} - \nabla_a C^d{}_{db}$$
$$+ C^e{}_{ab} C^d{}_{de} - C^e{}_{db} C^d{}_{ae} = 0$$

Kerr-Schild deformation

$$\tilde{g} = \eta + 2H\ell \otimes \ell,$$

where η is the Minkowski metric and ℓ is a null 1-form.

The Kerr–Schild Ansatz was soon generalized to the case in which the base metric is not *flat* [9, 10, 11, 12]. Thus, two metrics $\tilde{g}$ and g are linked by a *generalized Kerr–Schild relation* if there exist a function H and a null 1-form ℓ such that

$$\tilde{g} = g + 2H\ell \otimes \ell.$$

Kerr-Schild metrics

- Eddington: Schwarzschild metric (1924)
- Trautman: Gravitational waves transport information (1962)
- Kerr: rotating spherical body (rotating black hole) (1963)
- Newmann and Raina complex coordinate transformation from a Schwarzschild to a Kerr metric (1966)
- Kerr and Schild: the Kerr-Schild ansatz
- Coll, Hildebrandt and Senovilla: ''Kerr-Schild symmetries'' (2000)

Eddington metric

- Eddington found that the Schwarzschild metric in a coordinate system can be written in the following form

$$ds^2 = du^2 + 2dudr - r^2 d\Omega^2 - \frac{2GM}{c^2 r} du^2$$

$$u = ct - r - \frac{2GM}{c^2} \ln\left(r - \frac{2GM}{c^2} \right)$$

Propagation of information by Gravitational Waves

- Trautman showed that gravitational waves propagate information.
- He studied the metric

$$g_{ab} = \hat{g}_{ab} + F(\sigma)\ell_a(x)\ell_b(x)$$

$$g^{ab} = \hat{g}^{ab} - F(\sigma)\ell^a(x)\ell^b(x)$$

The Kerr-Schild deformation matrix

- We can reformulate Kerr-Schild's ansatz introducing a suitable first deformation matrix

$$\begin{pmatrix} 1+V & -V & 0 & 0 \\ +V & 1-V & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$V = -1 \pm \sqrt{1+H}$$

Applications: Gravitational waves and perturbations

- The gravitational waves solutions are considered as from linearizing the metric, perturbing the Minkowski spacetime,

$$g_{ab} \cong \eta_{ab} + h_{ab}$$

with the condition $|h_{ab}| << 1$ and restricting the coordinate transformations to the Lorentz ones.

Gravitational waves

- Imposing the gauge conditions

$$\nabla^a h_{ab} = 0 \qquad h^a_a = 0$$

the linearized Einstein equations are

$$\partial_a \partial^a h_{bc} = 0$$

Gravitational waves

- In our approach the approximation is given by the conditions

$$\Phi^{A}{}_{B}(x) \cong \delta_{A}^{B} + \varphi^{A}{}_{B}(x) \qquad \left|\varphi^{A}{}_{B}(x)\right| << 1$$

- This conditions are covariant for three reasons: 1) we are using scalar fields; 2) this objects are adimensional; 3) they are subject only to Lorentz transformations; 4) They are tensors with respect to Lorentz transformations of rank (1,1)

Gravitational Waves

- It follows that

$$\left(\Phi^{-1}\right)^{A}_{B} = \delta^{A}_{B} - \varphi^{A}{}_{B}$$

- The metric decomposition we have found does not need to be a linearization
- However applying the usual approach to gravitational waves we have to consider the following equations in the vacuum

Gravitational waves and perturbations

$$\widetilde{R}_{ab} = R_{ab} + \nabla_d C^d{}_{ab} - \nabla_a C^d{}_{db} + C^e{}_{ab} C^d{}_{de} - C^e{}_{db} C^d{}_{ae} = 0$$

$$\nabla_d C^d{}_{ab} - \nabla_a C^d{}_{db} + C^e{}_{ab} C^d{}_{de} - C^e{}_{db} C^d{}_{ae} = 0$$

$$\nabla_d C^d{}_{ab} - \nabla_a C^d{}_{db} = 0$$

$$\Downarrow$$

$$\nabla_a \nabla^a \gamma_{bc} - 2 R_b{}^a{}_c{}^d \gamma_{ad} = 0$$

The equations for the scalar potentials

$$\partial^d \partial_d \varphi^C{}_A \varphi_{CB} + 2\, \partial_d \varphi^C{}_A \partial^d \varphi_{CB} + \varphi^C{}_A \partial^d \partial_d \varphi_{CB} = 0.$$

The above gauge conditions are now

$$\varphi_{AB} \varphi^{BA} = 0$$

and

$$e^d_D \left[\partial_d \varphi_{CA} \varphi^C_B + \varphi_{CA} \partial_d \varphi^C_B \right] = 0.$$

Gravitational waves and perturbations

- We have used the gauge conditions which are no more coordinate conditions.
- They appear as restrictions on the deforming matrices.
- The linearized Einstein equations can be translated in equations on the deforming matrices .

Discussion and Conclusions

- We have presented the deformation of spacetime metrics as the corrections one has to introduce in the metric in order to deal with our ignorance of the fine spacetime structure.
- Conceptually this can simplify many calculations, but also suggests us more ambitious goals
- To this aim we have introduced the in 4 dimensional spacetime 6 independent scalar gravitational potentials
- The conditions to be imposed on these potentials are not restrictive
- As a result we showed that we can consider the gravitational theory as a theory of these potential given in a background geometry
- We showed that this scalar field are suitable for a covariant definition of perturbations and gravitational waves
- But also to study the relation between GR and alternative gravitational theories
- They can connect spacetime with different topologies
- We found that the can suggest a way to show the production of the area entropy in Black Holes
- It could be interesting to consider the quantization of these potentials in order to find a quantum gravity theory
- A great deal of work in the future years!!!!

Thanks for coming!