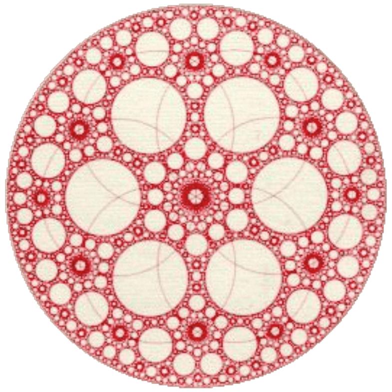


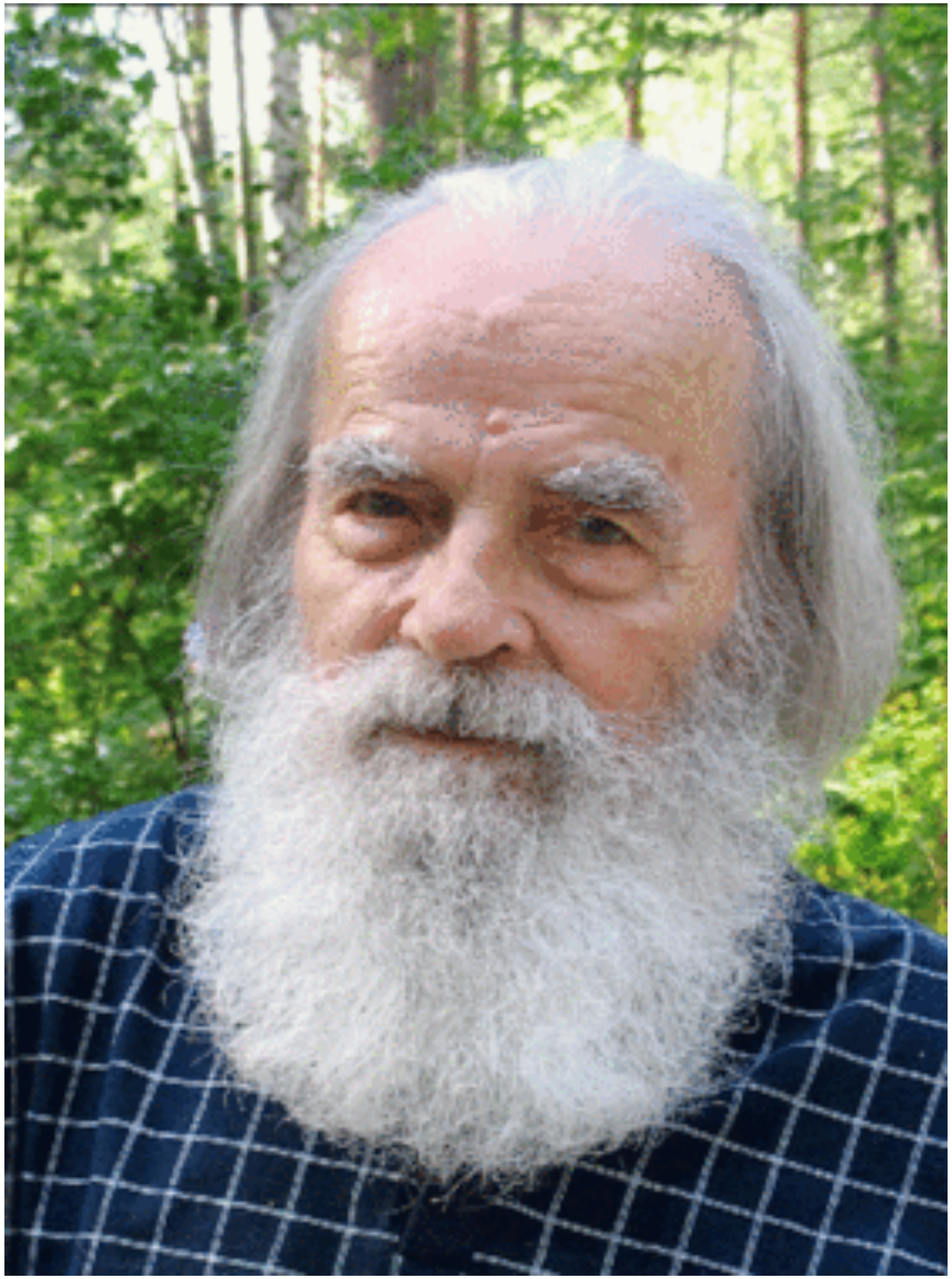

Roberto Artuso


# Caos e diffusione (normale e anomala)


Lucia Cavallasca, Giampaolo Cristadoro, Predrag Cvitanović, Per Dahlqvist, Gregor Tanner

Damiano Belluzzo, Fausto Borgonovi, Giulio Casati, Shmuel Fishman, Italo Guarneri, Laura Rebuzzini, Michele Rusconi, Dima Shepelyansky


Napoli, 29 maggio 2008



Boris Chirikov

Ed Lorenz

# microscopic dynamics / macroscopic transport

microscopic dynamics / macroscopic transport

↓

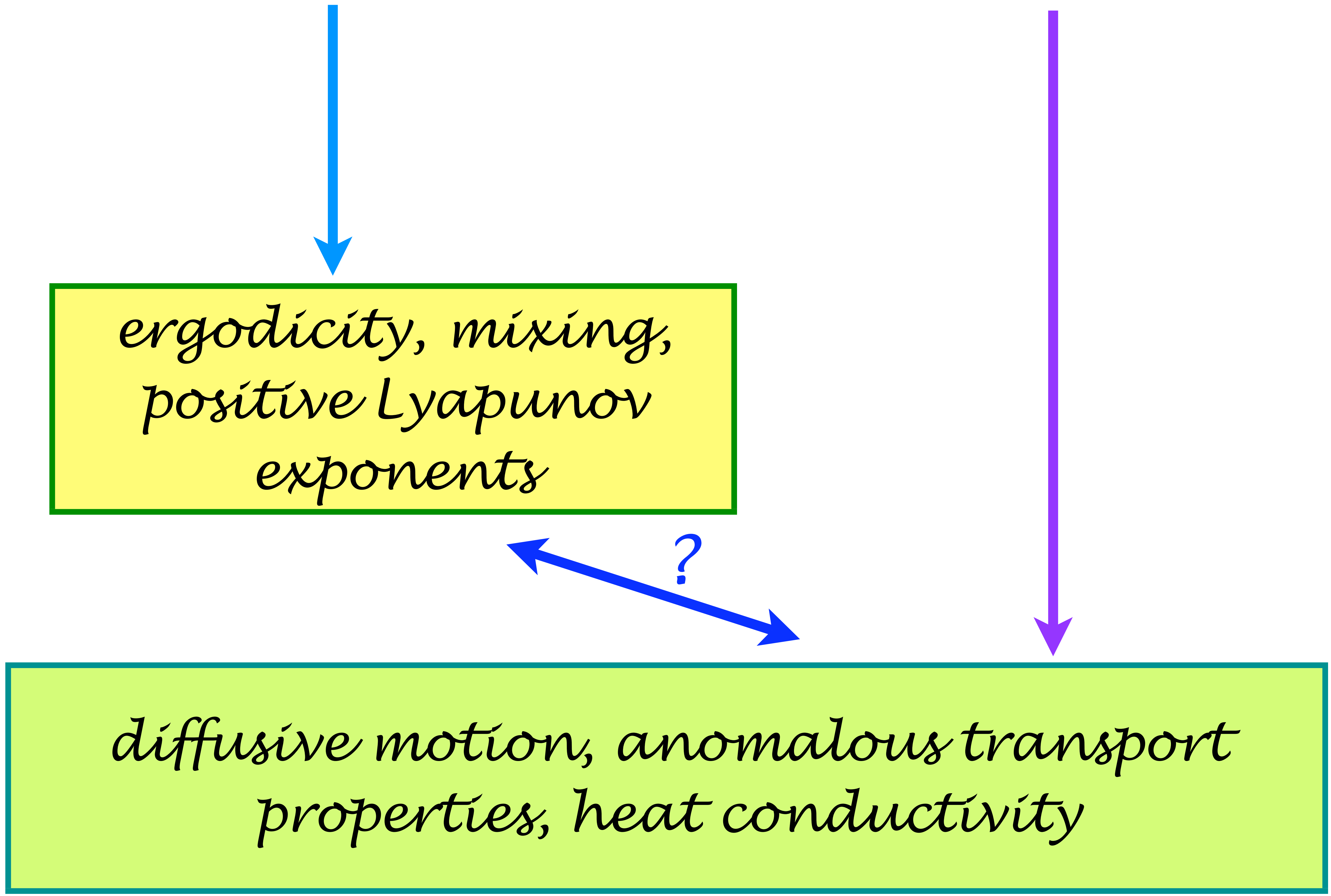
ergodicity, mixing, positive Lyapunov exponents

## microscopic dynamics / macroscopic transport

ergodicity, mixing, positive Lyapunov exponents

diffusive motion, anomalous transport properties, heat conductivity

# microscopic dynamics / macroscopic transport



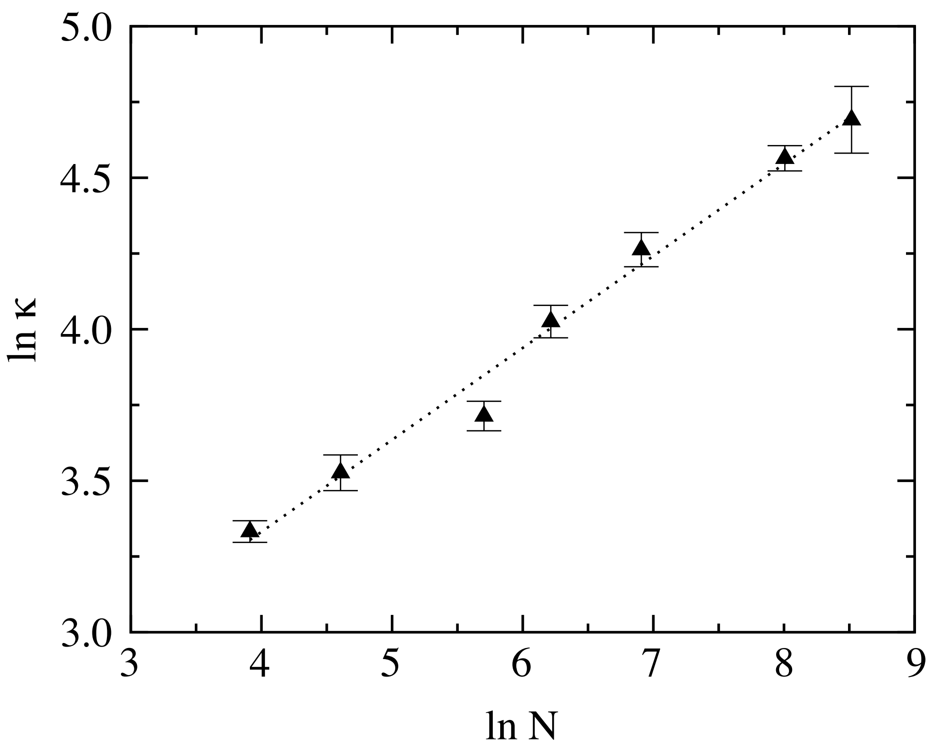
266.

# STUDIES OF NON LINEAR PROBLEMS

E. FERMI, J. PASTA, and S. ULAM
Document LA-1940 (May 1955).

ABSTRACT.

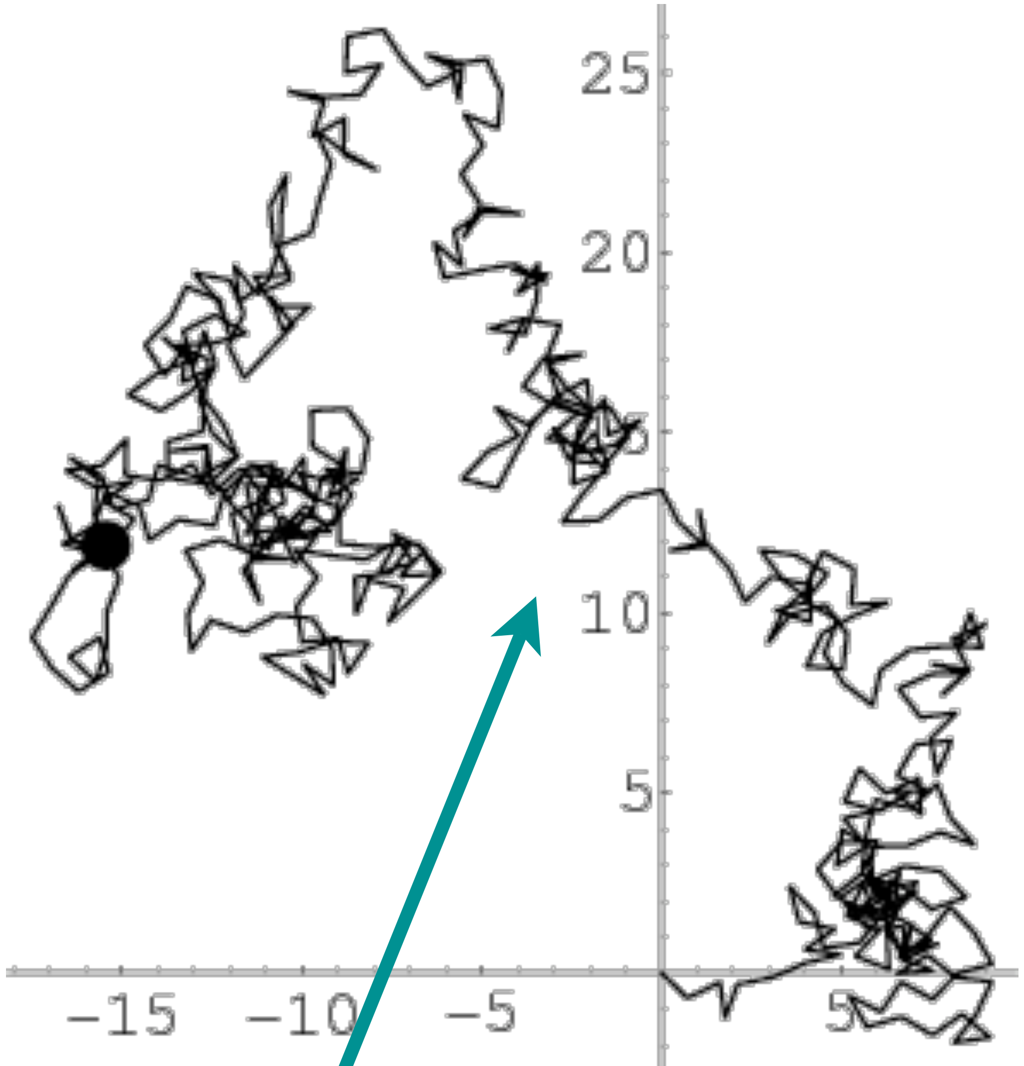
A one-dimensional dynamical system of 64 particles with forces between neighbors containing nonlinear terms has been studied on the Los Alamos computer MANIAC I. The nonlinear terms considered are quadratic, cubic, and broken linear types. The results are analyzed into Fourier components and plotted as a function of time.

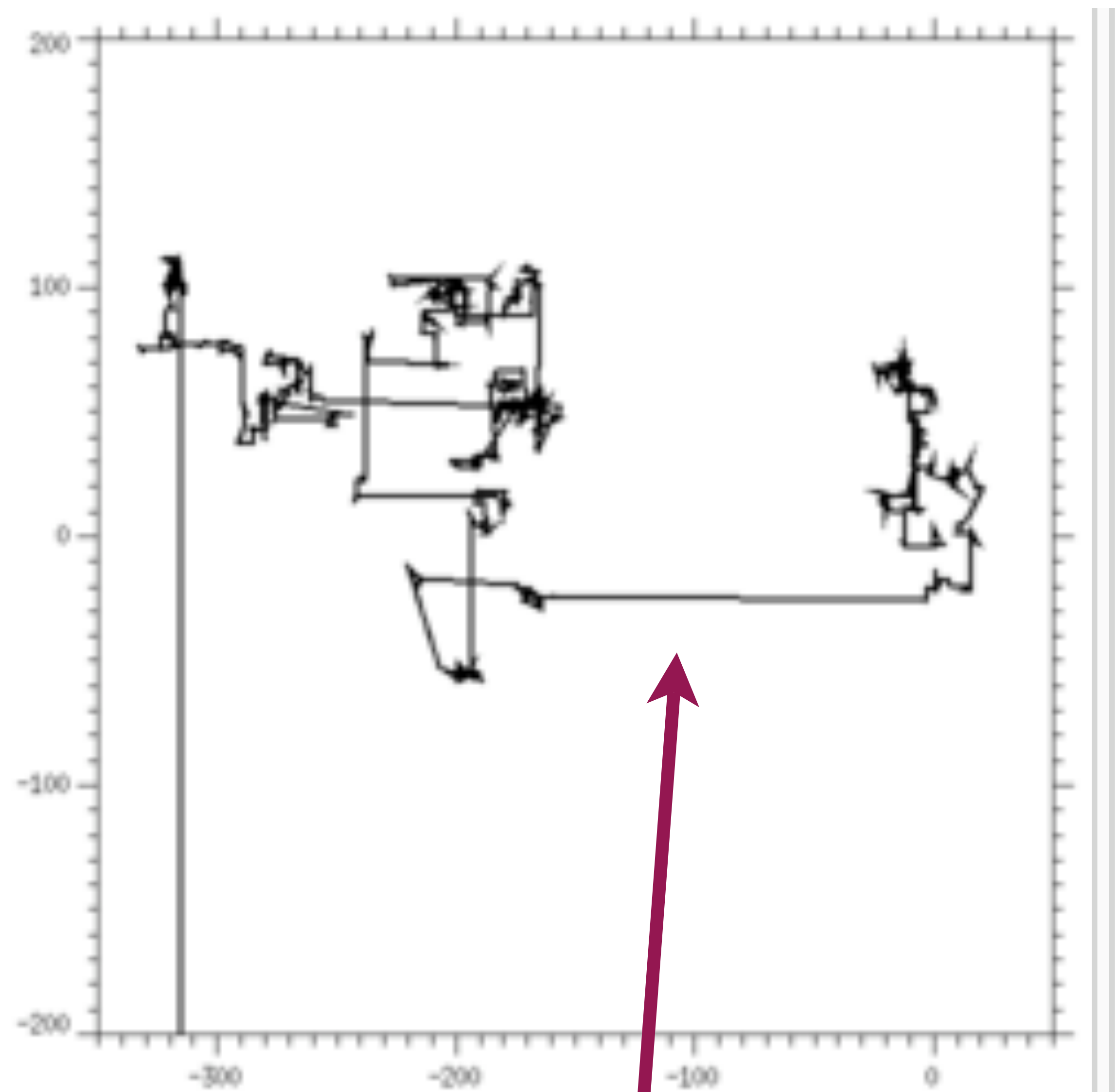



*diverging heat conductivity*

*Lepri, Livi, Politi*

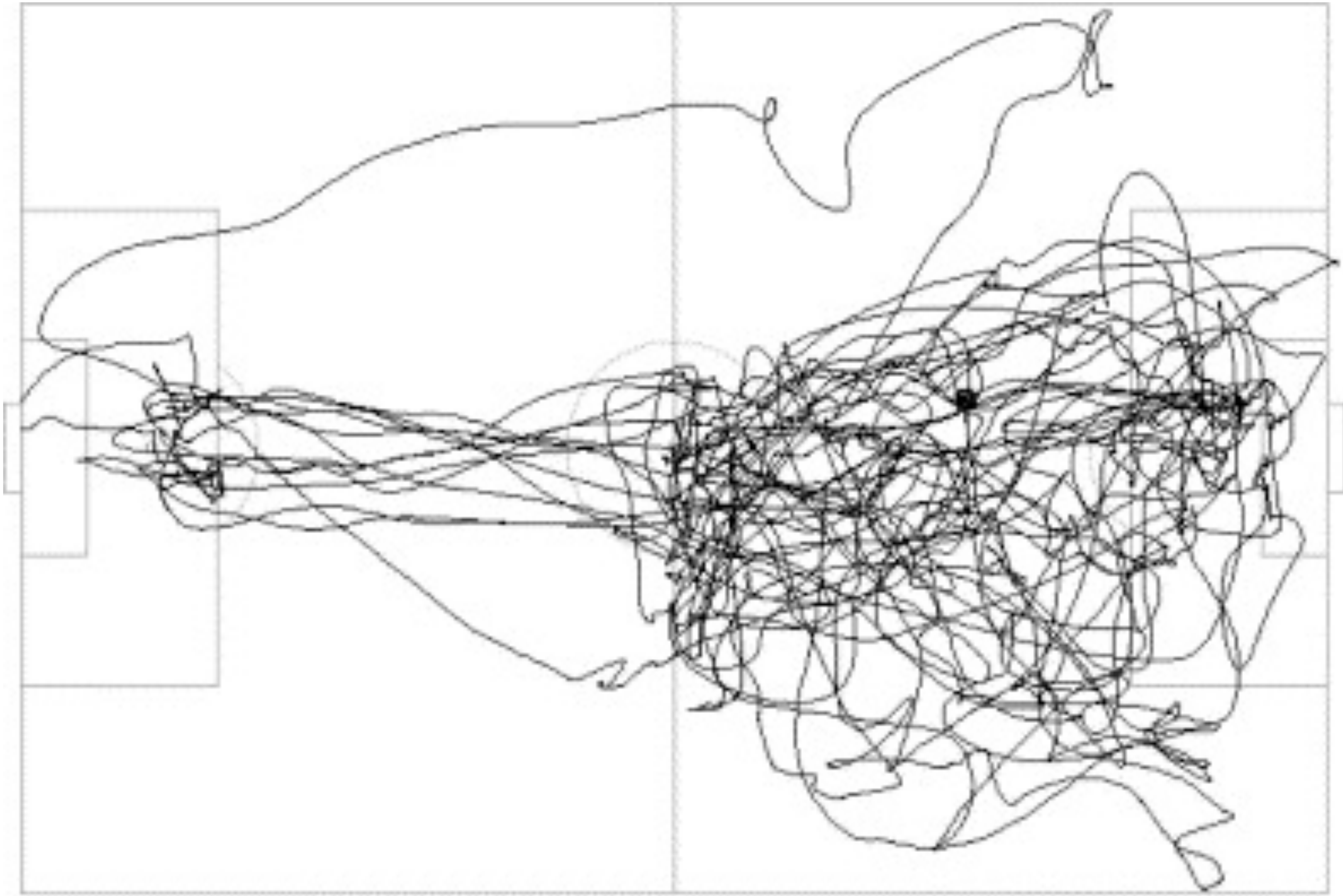
# stochastic transport




random walk (fixed increments)

Levy flight (increments from a power law distribution)



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Computer Vision and Image Understanding 101 (2006) 122–135

Computer Vision and Image Understanding

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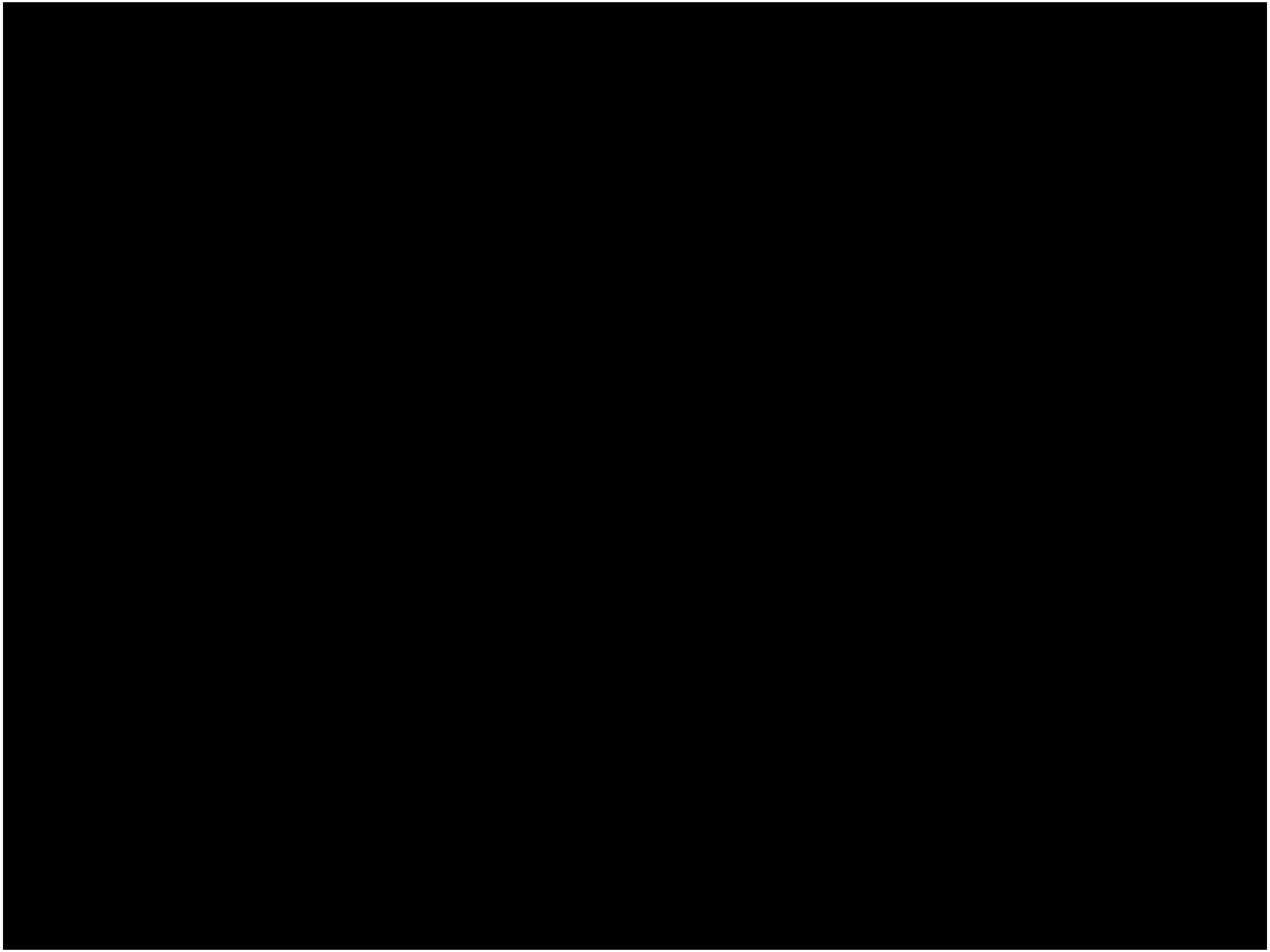
# Tracking soccer players aiming their kinematical motion analysis

Pascual J. Figueroa [a], Neucimar J. Leite [a,*], Ricardo M.L. Barros [b]

Epidemics Brockmann, Hufnagel, Geisel

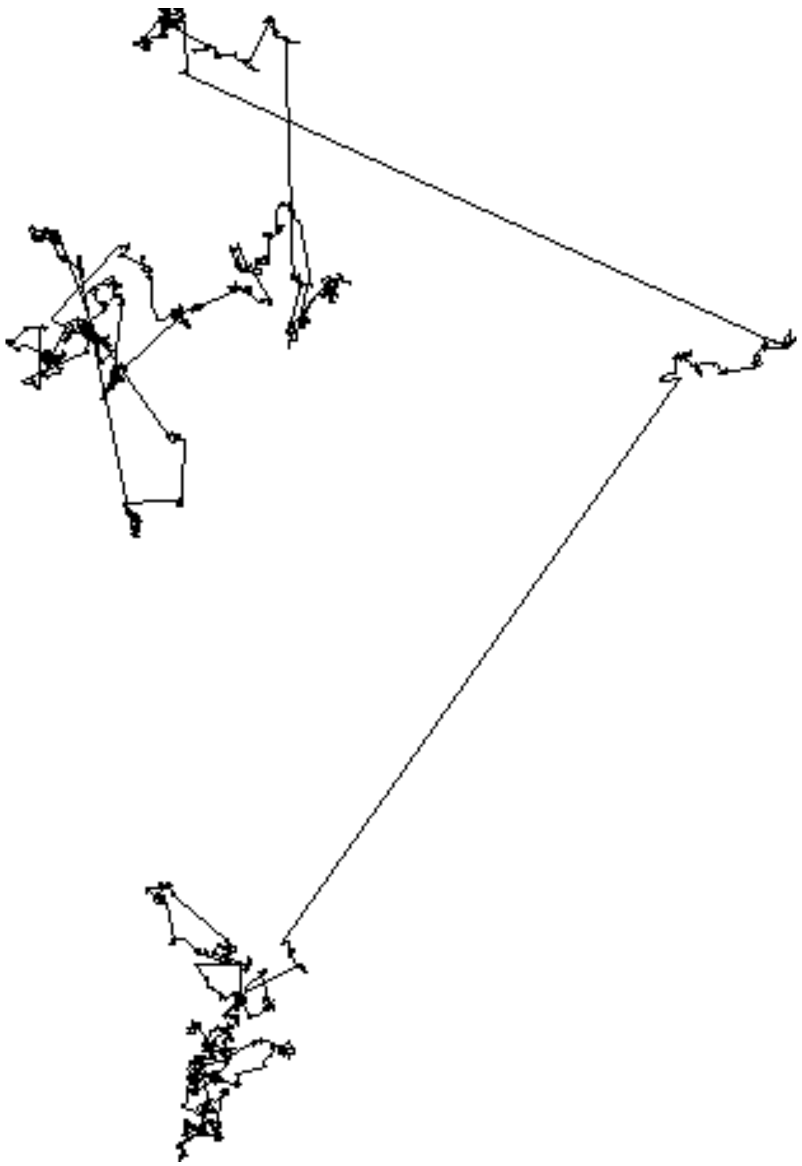



I might be moving to Montana soon .. Frank Zappa
Bighorn Canyon
Copyright© 2003 Ultimate Press

# tracers motion in chaotic fluids

## Weeks, Urbach, Solomon, Swinney

random walk

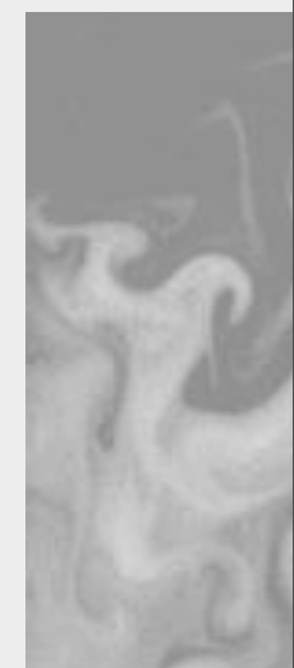


Levy flight

## Navier-Stokes Equation

Waves follow our boat as we meander across the lake, and turbulent air currents follow our flight in a modern jet. Mathematicians and physicists believe that an explanation for and the prediction of both the breeze and the turbulence can be found through an understanding of solutions to the Navier-Stokes equations. Although these equations were written down in the 19th Century, our understanding of them remains minimal. The challenge is to make substantial progress toward a mathematical theory which will unlock the secrets hidden in the Navier-Stokes equations.


- The Mille
- Official P Charles F
- Lecture b

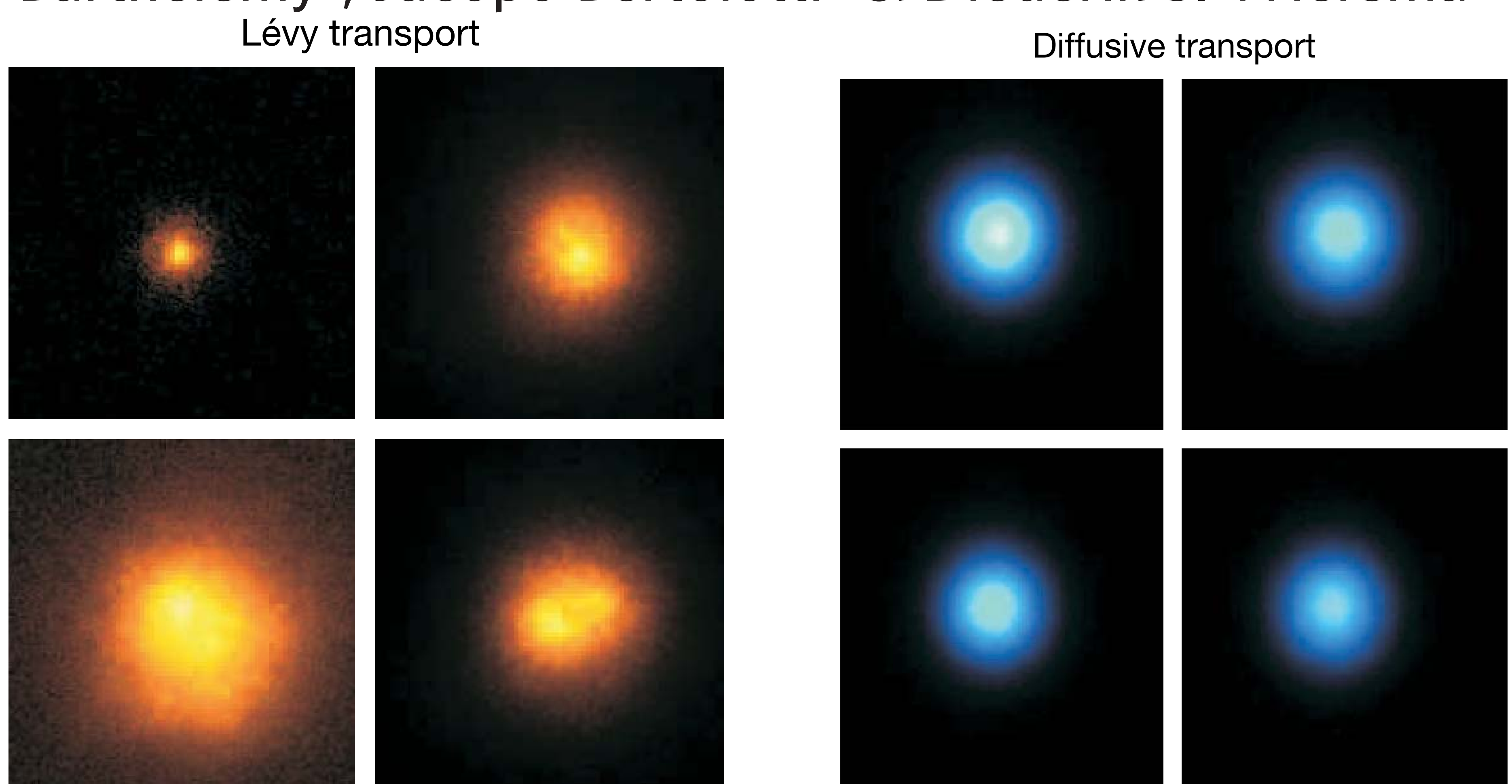
*good answer is worth 1 million $*

Vol 453|22 May 2008|**doi:10.1038/nature06948**

*heavy tailed distribution of scattering centers*

# A Lévy flight for light

Pierre Barthelemy[1], Jacopo Bertolotti[1] & Diederik S. Wiersma[1]



# Ratchet effect - Parrondo paradox

# Ratchet effect - Parrondo paradox

# Ratchet effect - Parrondo paradox

Game A - simple coin
winning probability 1/2 -ε

# Ratchet effect - Parrondo paradox

Game A - simple coin
winning probability 1/2 -ϵ

Game B - if the capital is a multiple of 3

winning probability 1/10 -ϵ
- if not
winning probability 3/4 -ϵ

# Ratchet effect - Parrondo paradox

Game A - simple coin
winning probability 1/2 -ϵ

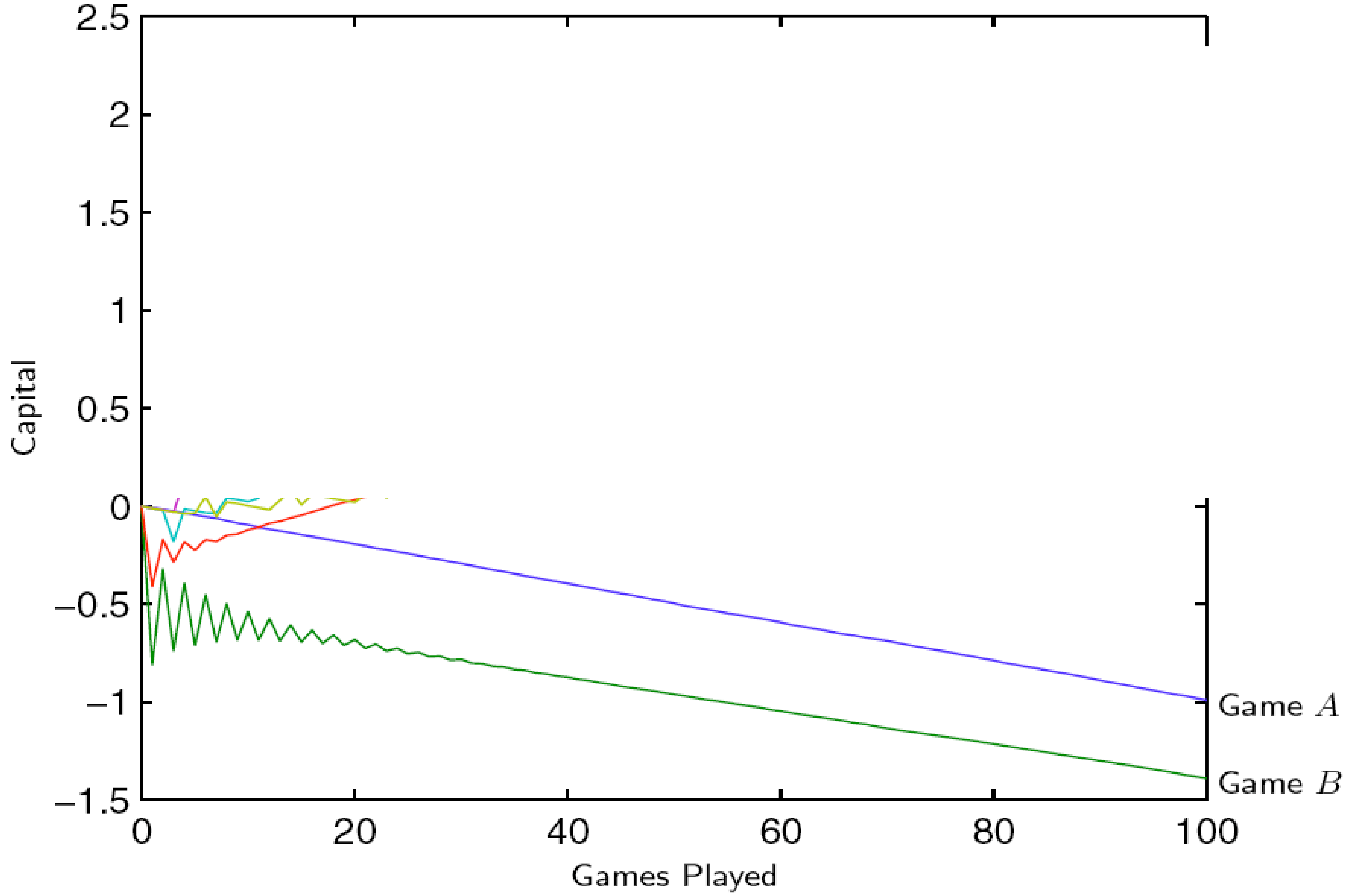
Game B - if the capital is a multiple of 3

winning probability 1/10 -ϵ
- if not
winning probability 3/4 -ϵ

If ϵ>0 both games are unfair

# *the paradox!*

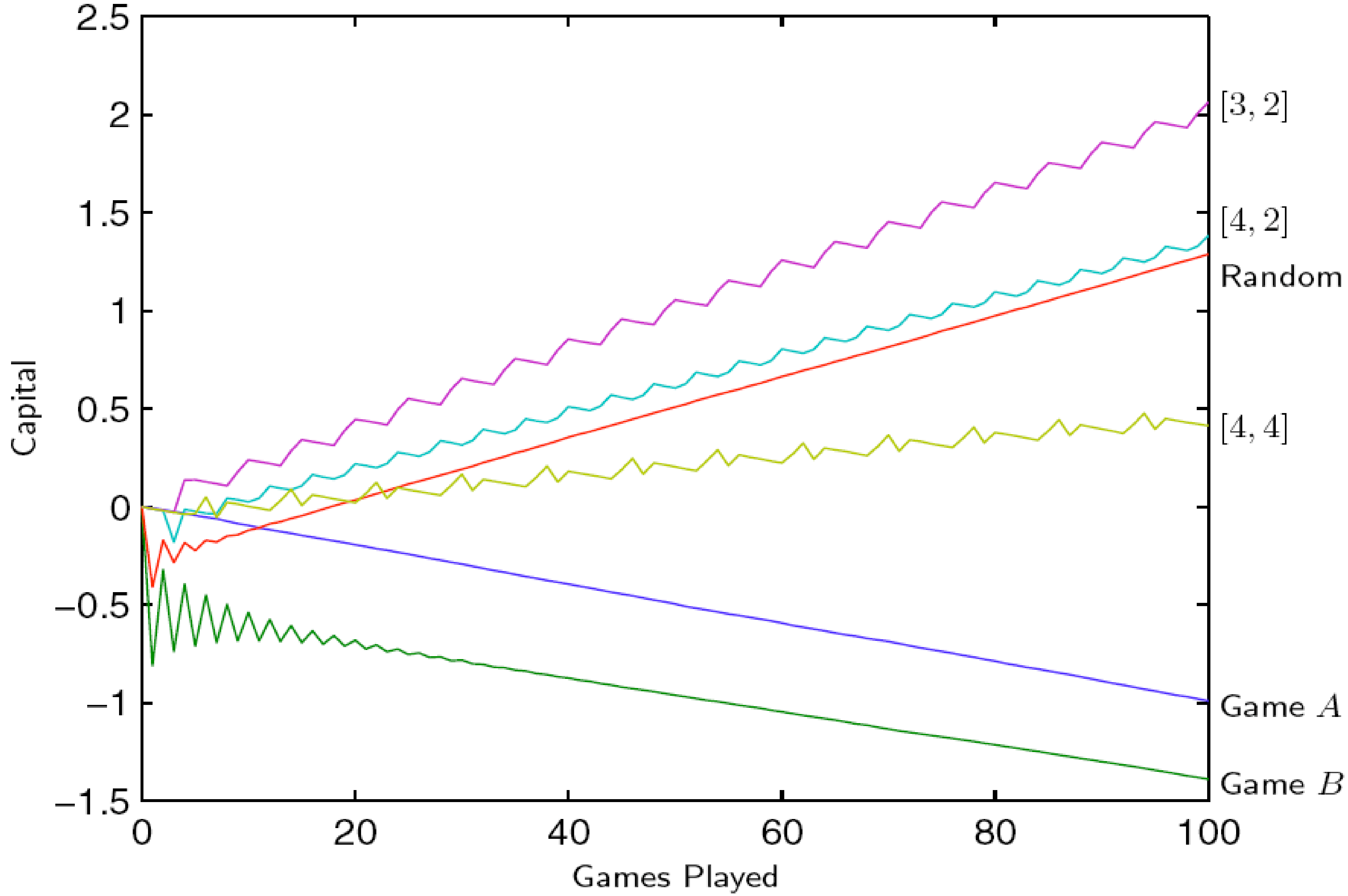
# *the paradox!*



2.5
2
1.5
1
0.5
0
−0.5
−1
−1.5
Capital
0
20
40
60
80
100
Games Played
Game $A$
Game $B$

# *the paradox!*

# *the paradox!*



2.5
2
1.5
1
0.5
0
−0.5
−1
−1.5
Capital
0
20
40
60
80
100
Games Played
[3, 2]
[4, 2]
Random
[4, 4]
Game $A$
Game $B$

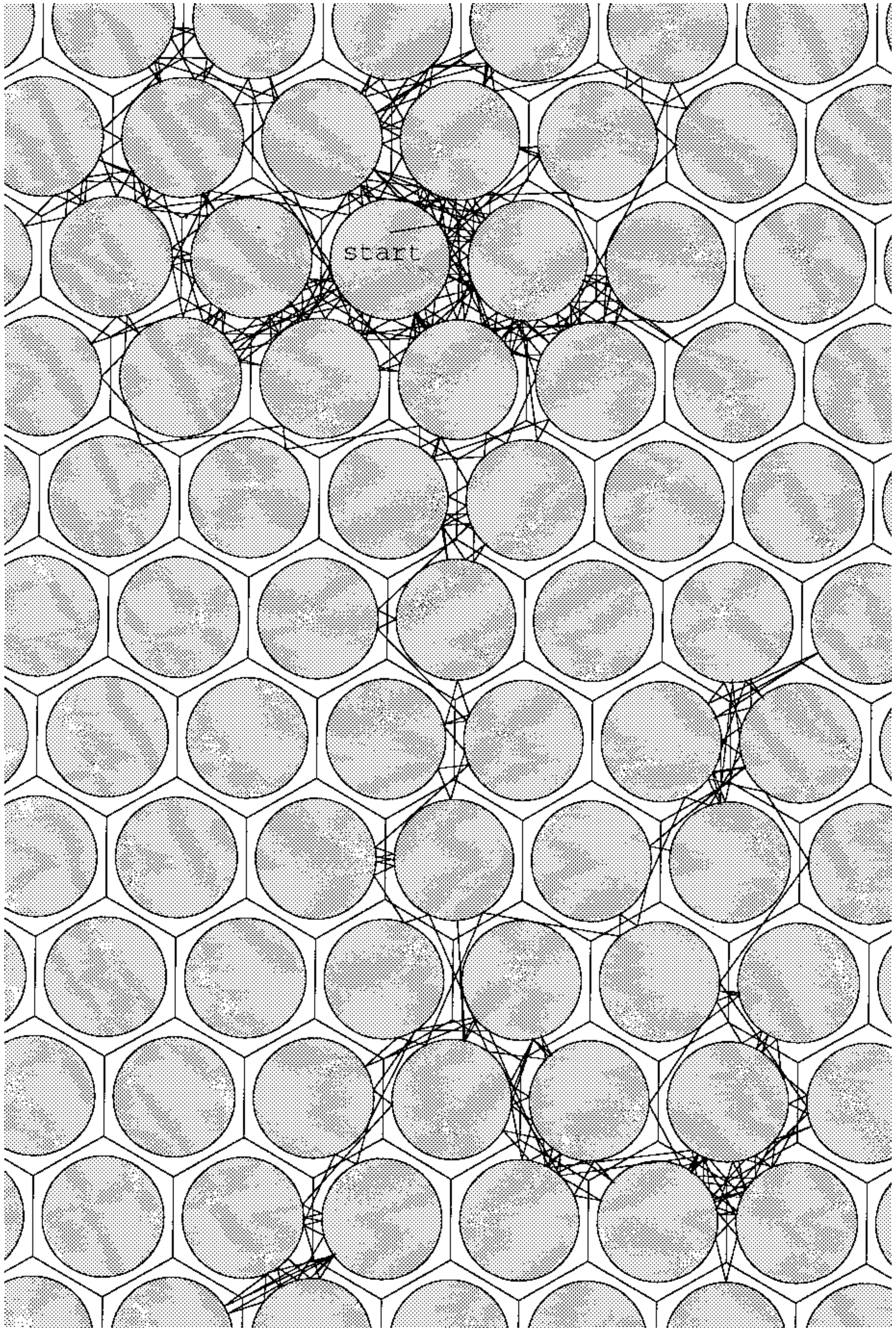
examples of deterministic transport

normal vs anomalous

the full spectrum

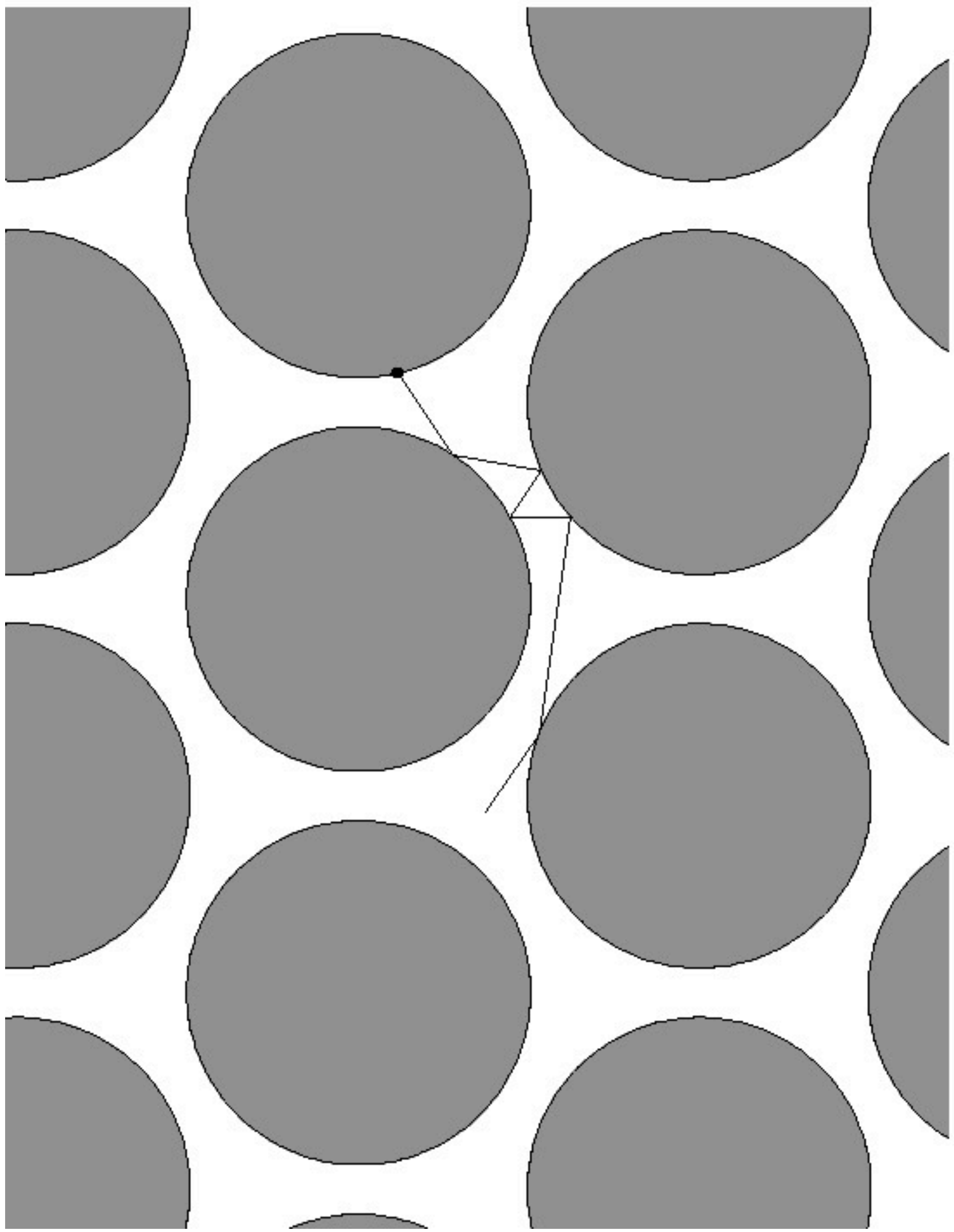
theoretical tools

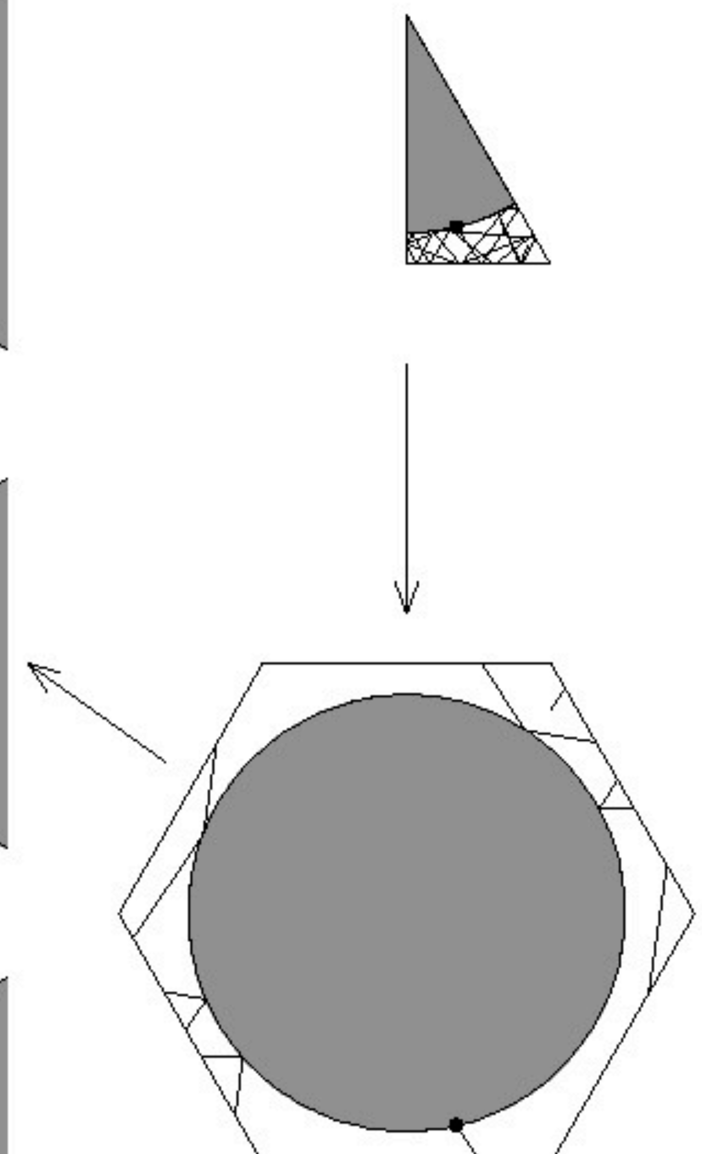
# *purely deterministic motion*



# space-periodic systems

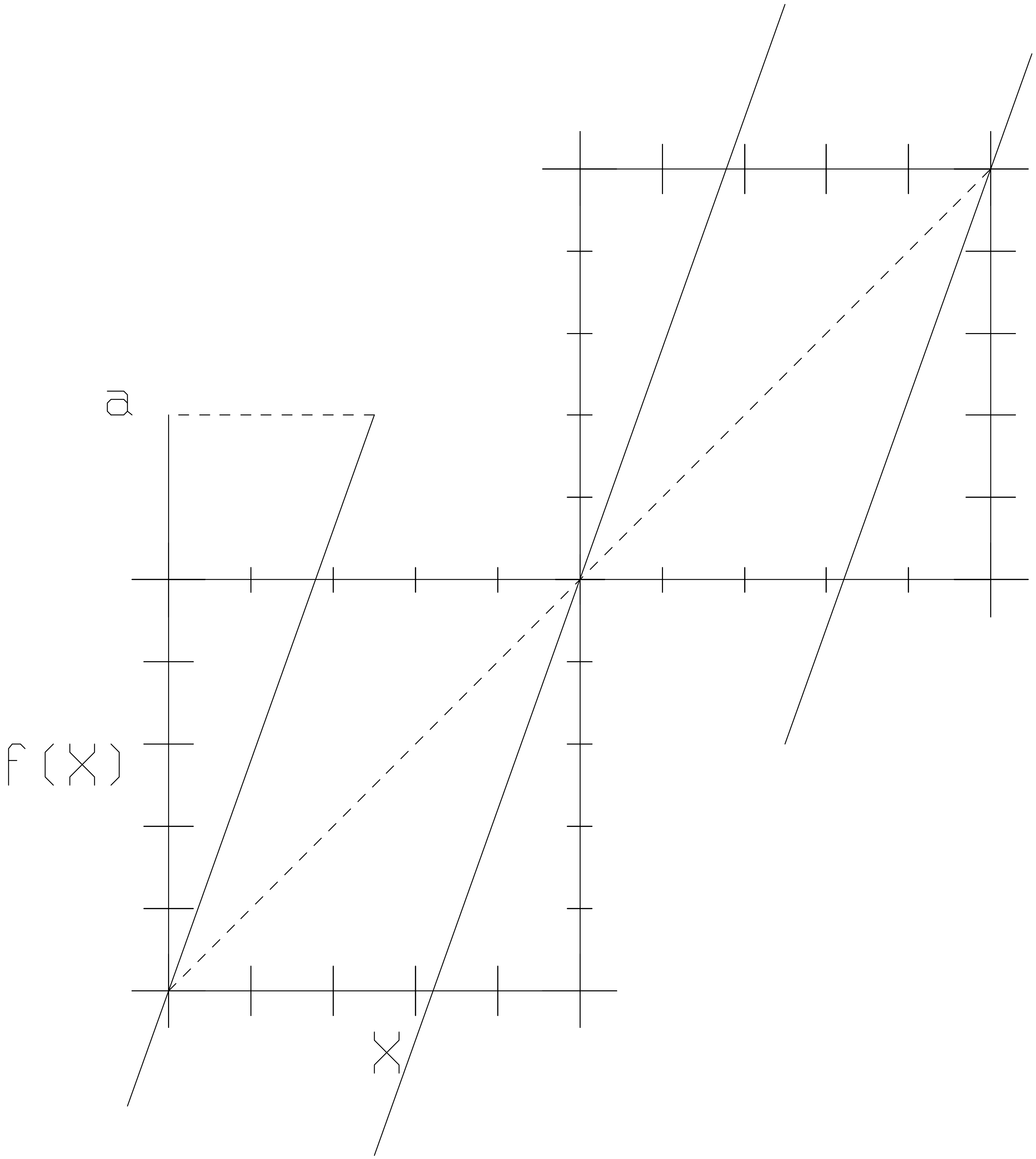
Lorentz gas



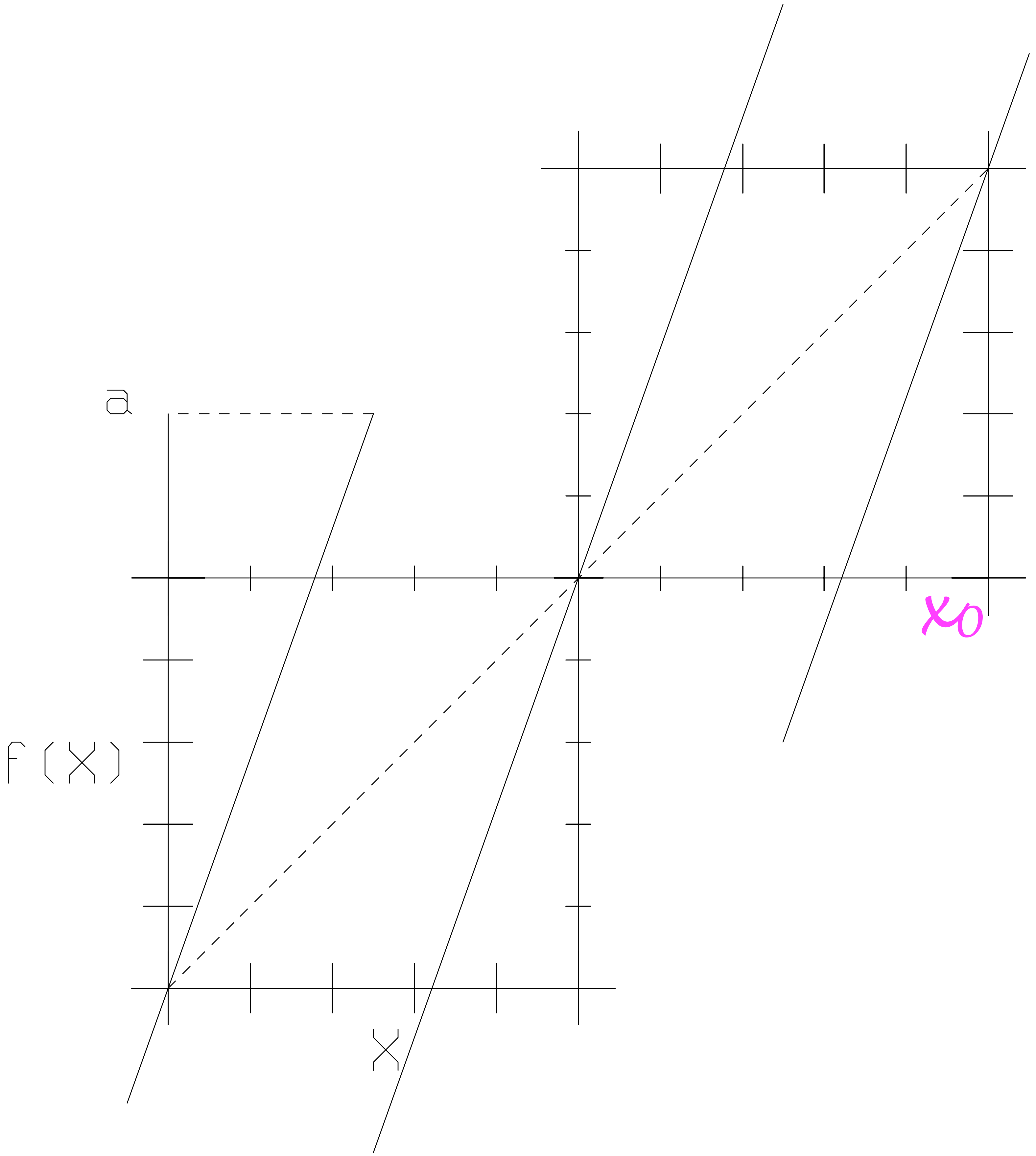


Sinai billiard

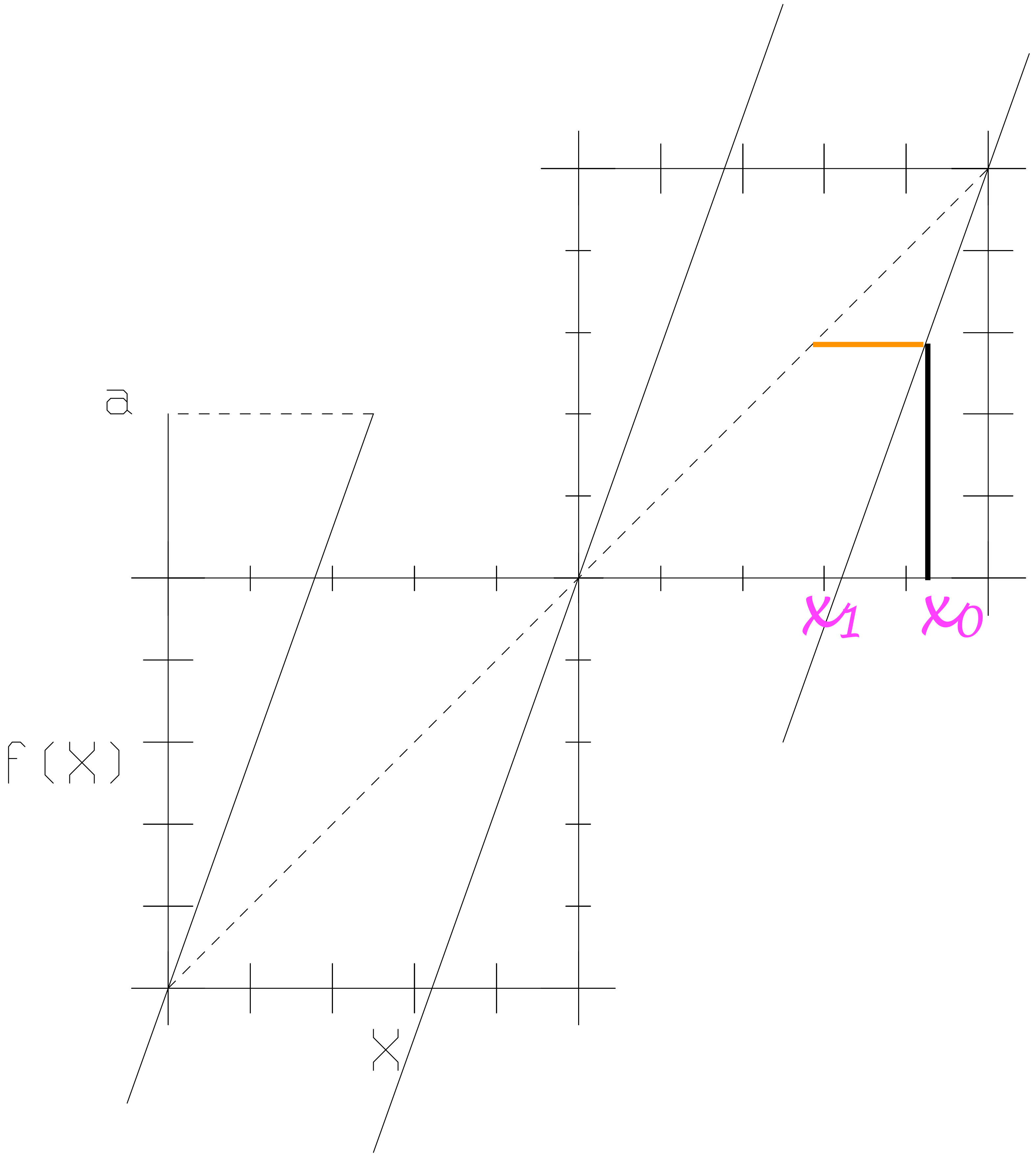
# the simplest example: 1d maps



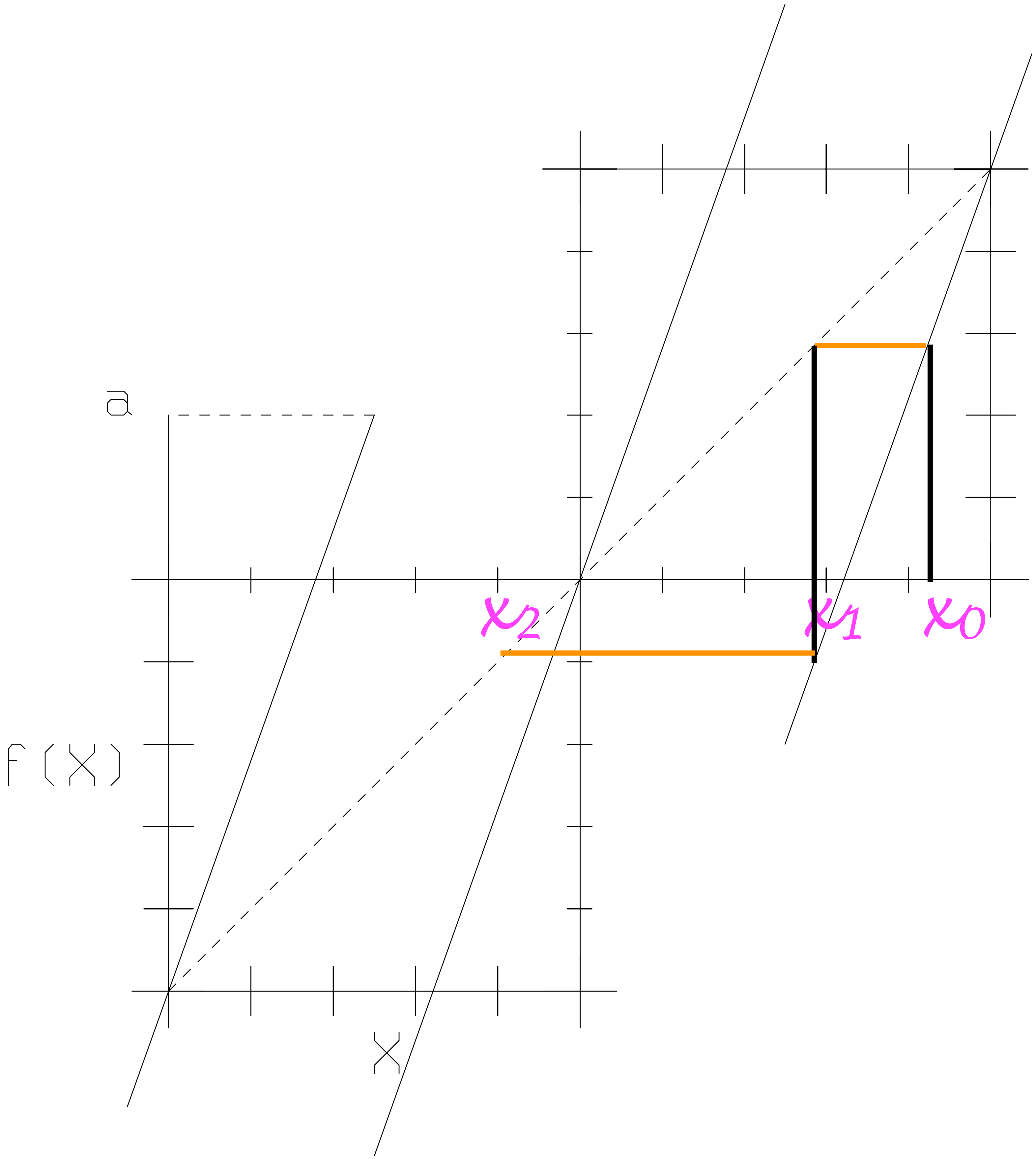
# the simplest example: 1d maps



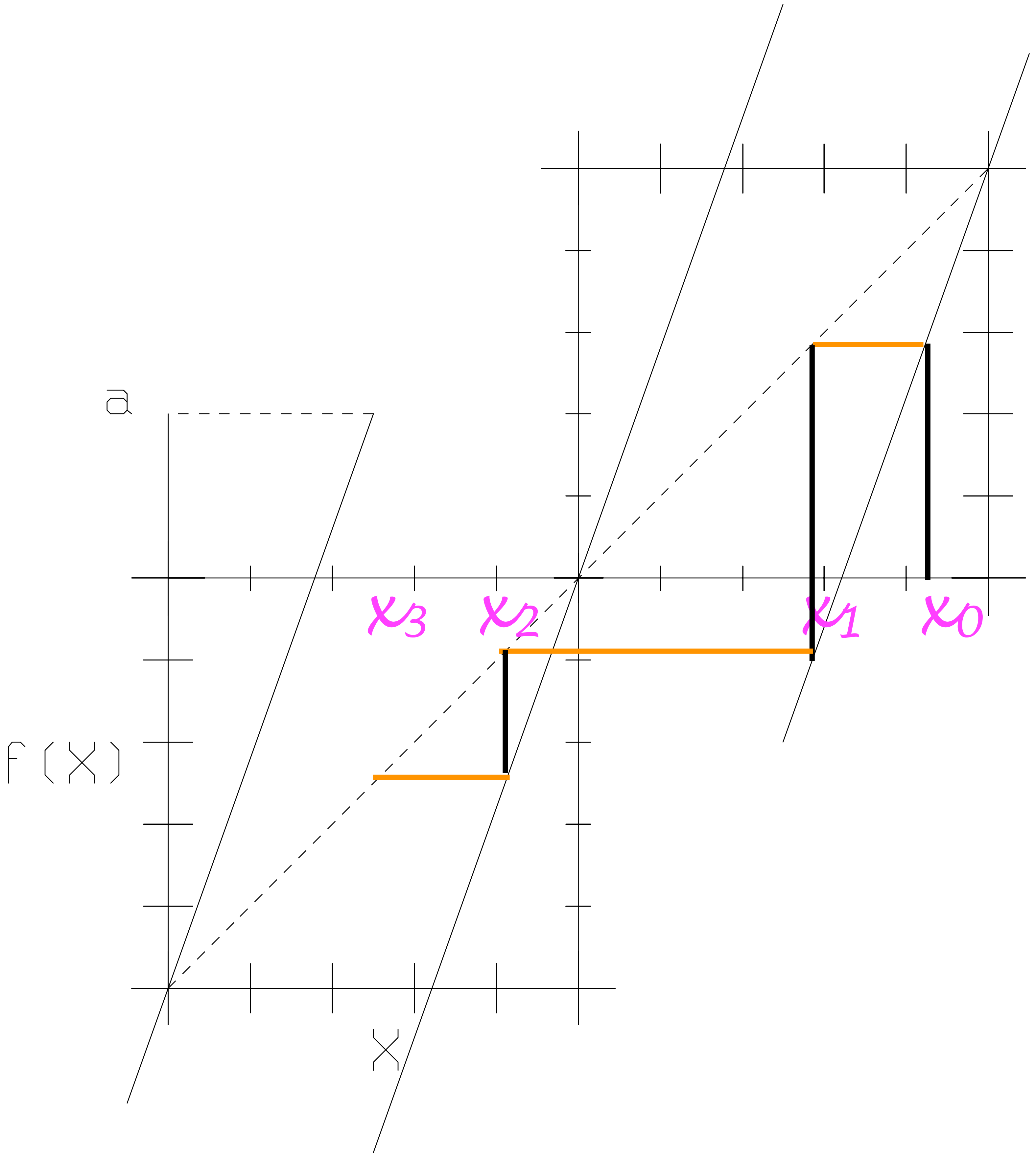
# the simplest example: 1d maps



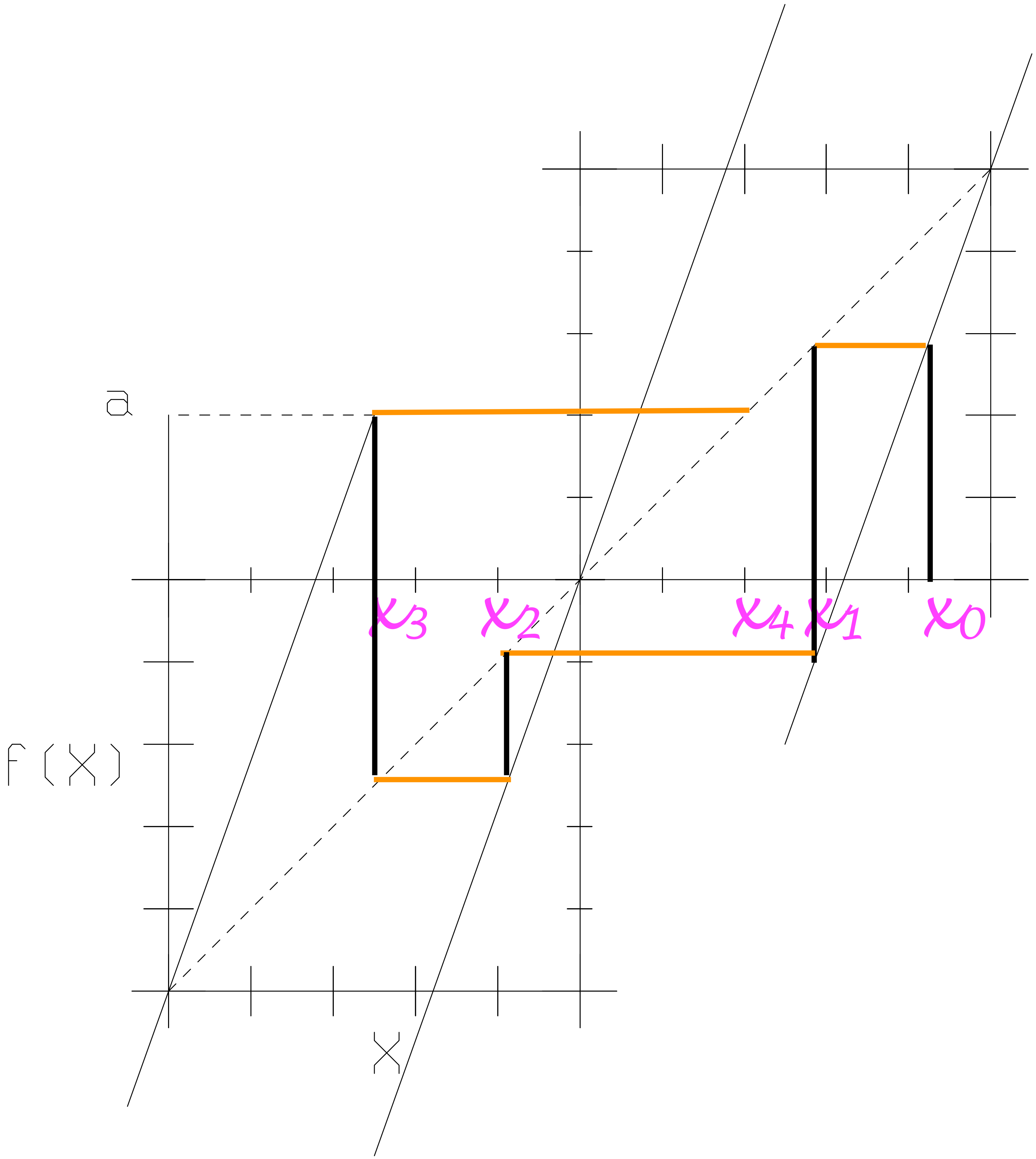
# the simplest example: 1d maps



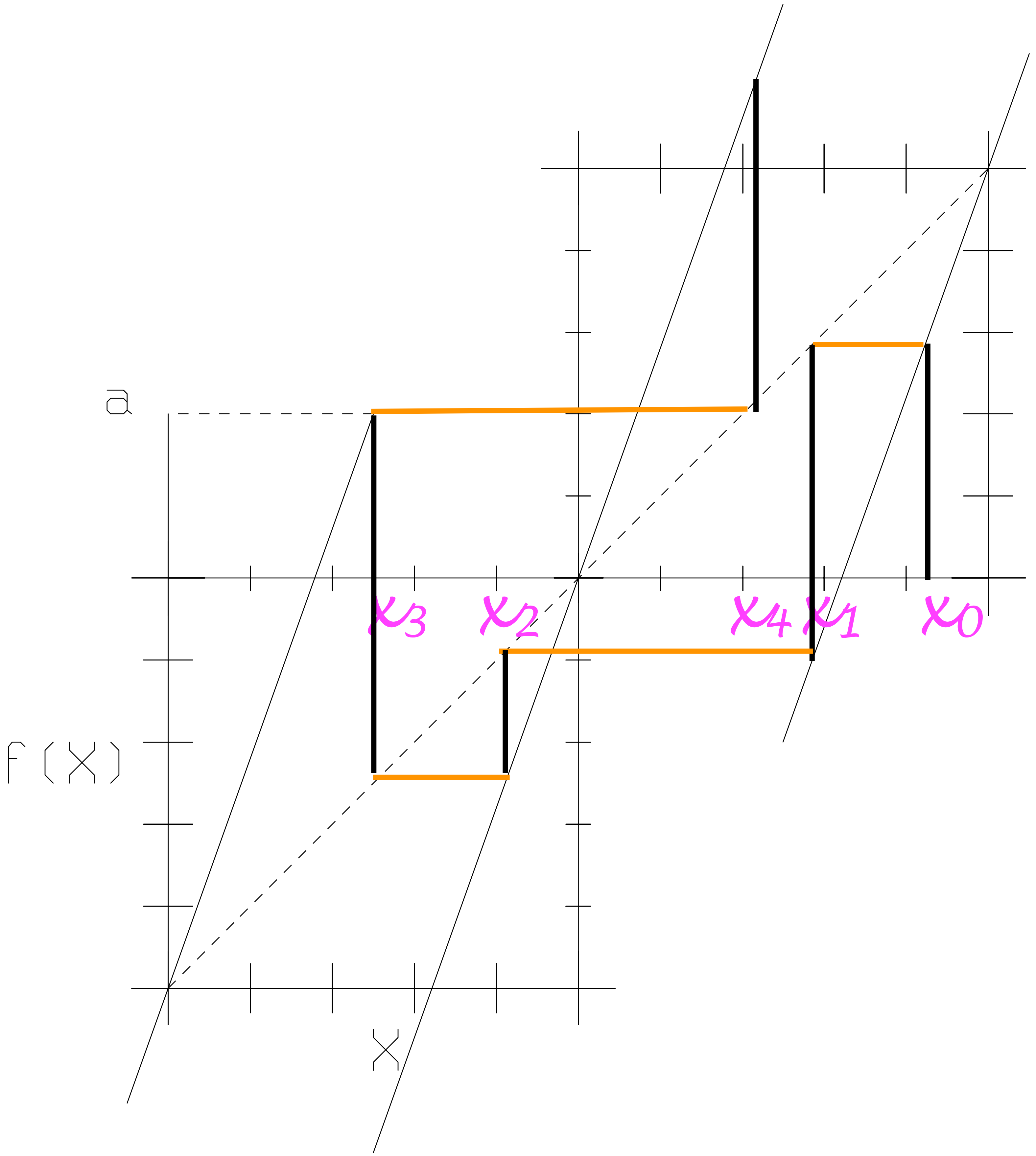
# the simplest example: 1d maps



# the simplest example: 1d maps



# the simplest example: 1d maps



Microscopic instability: Lyapunov exponent

Transport properties: diffusion constant

Microscopic instability: Lyapunov exponent

Transport properties: diffusion constant

$$\delta x(t) \sim \delta x(0) e^{\lambda t}$$

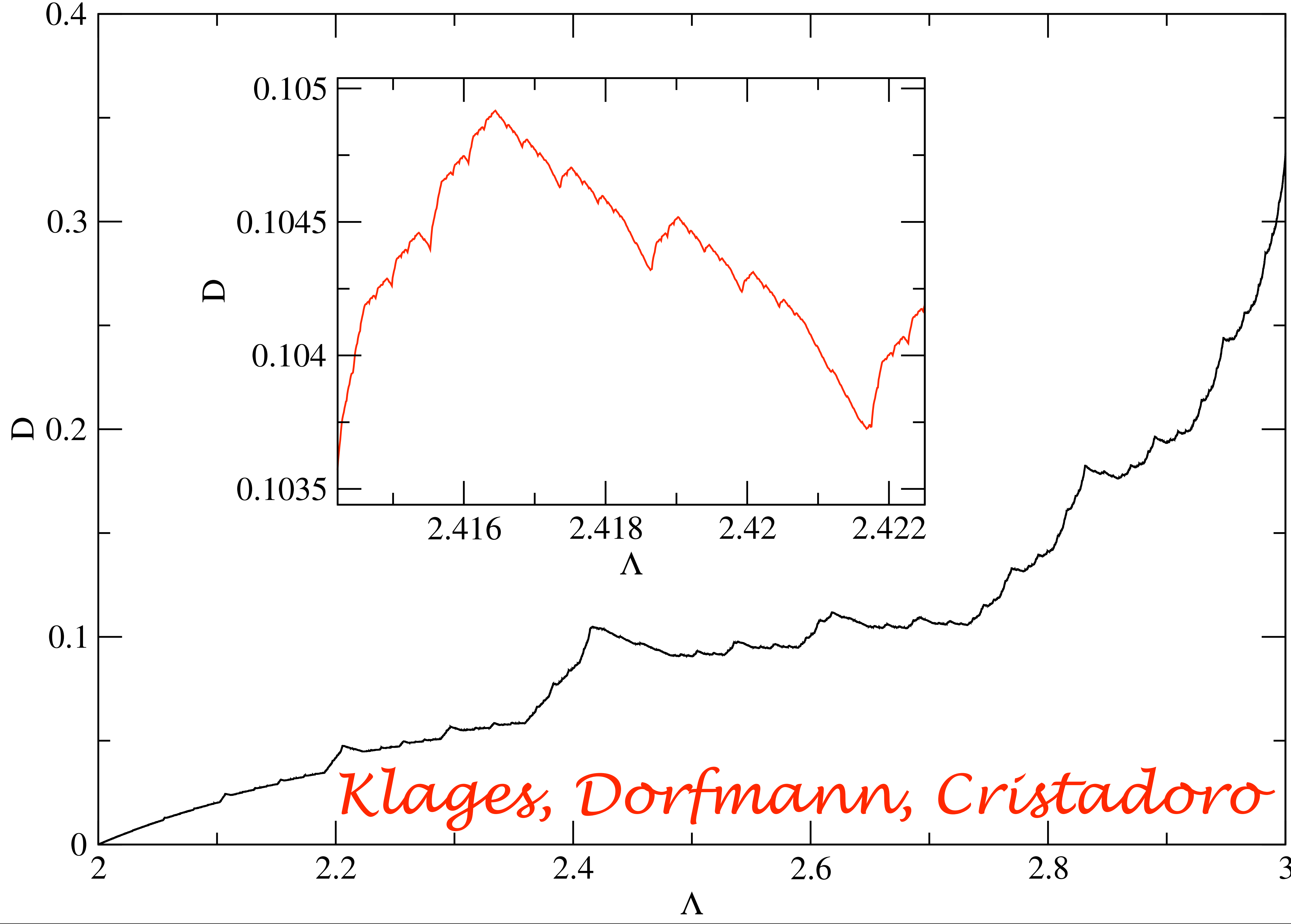
$$\langle (x_t - x_0)^2 \rangle \sim D \cdot t$$

Microscopic instability: Lyapunov exponent

Transport properties: diffusion constant

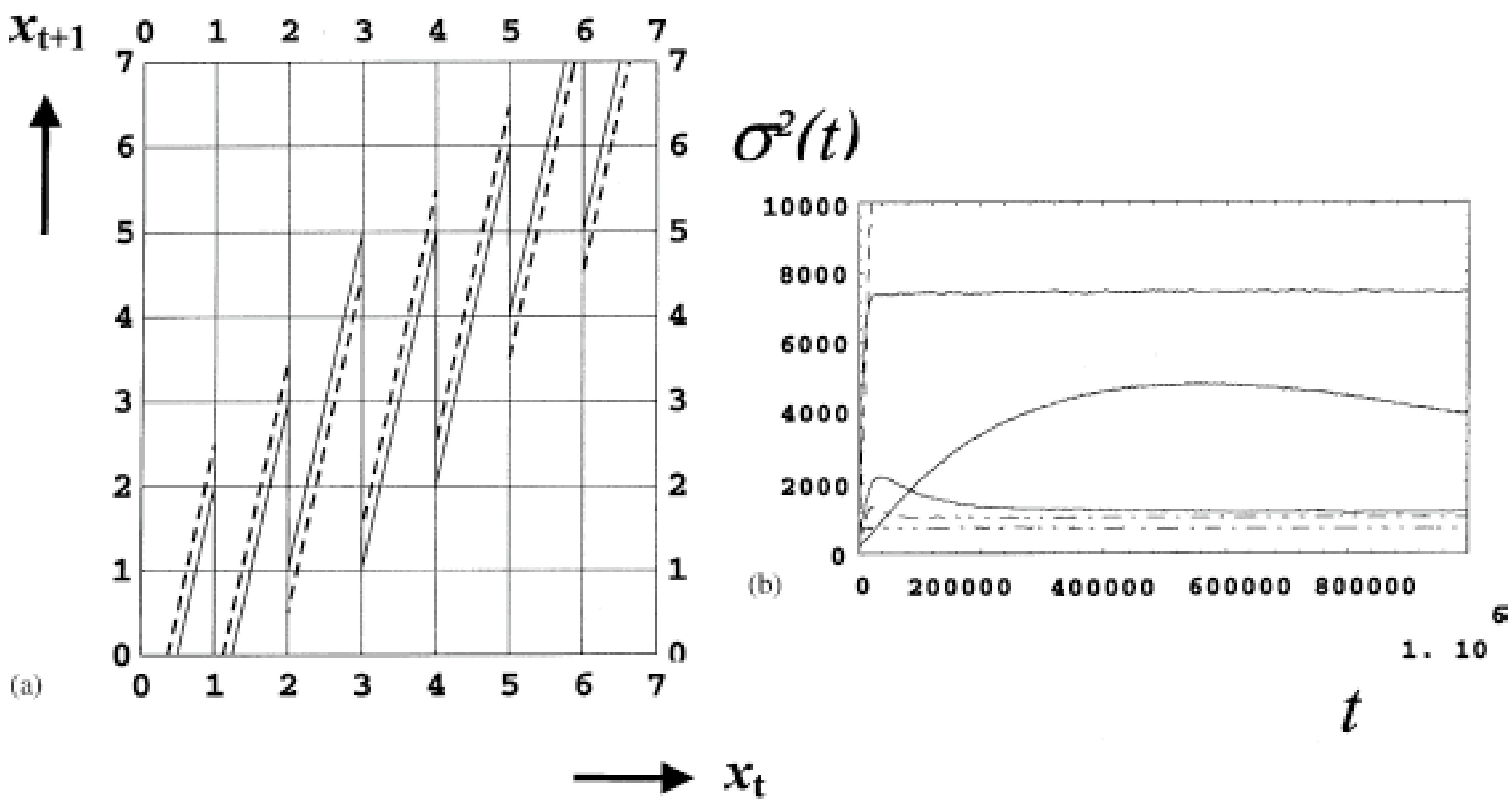
Microscopic instability: Lyapunov exponent

Transport properties: diffusion constant

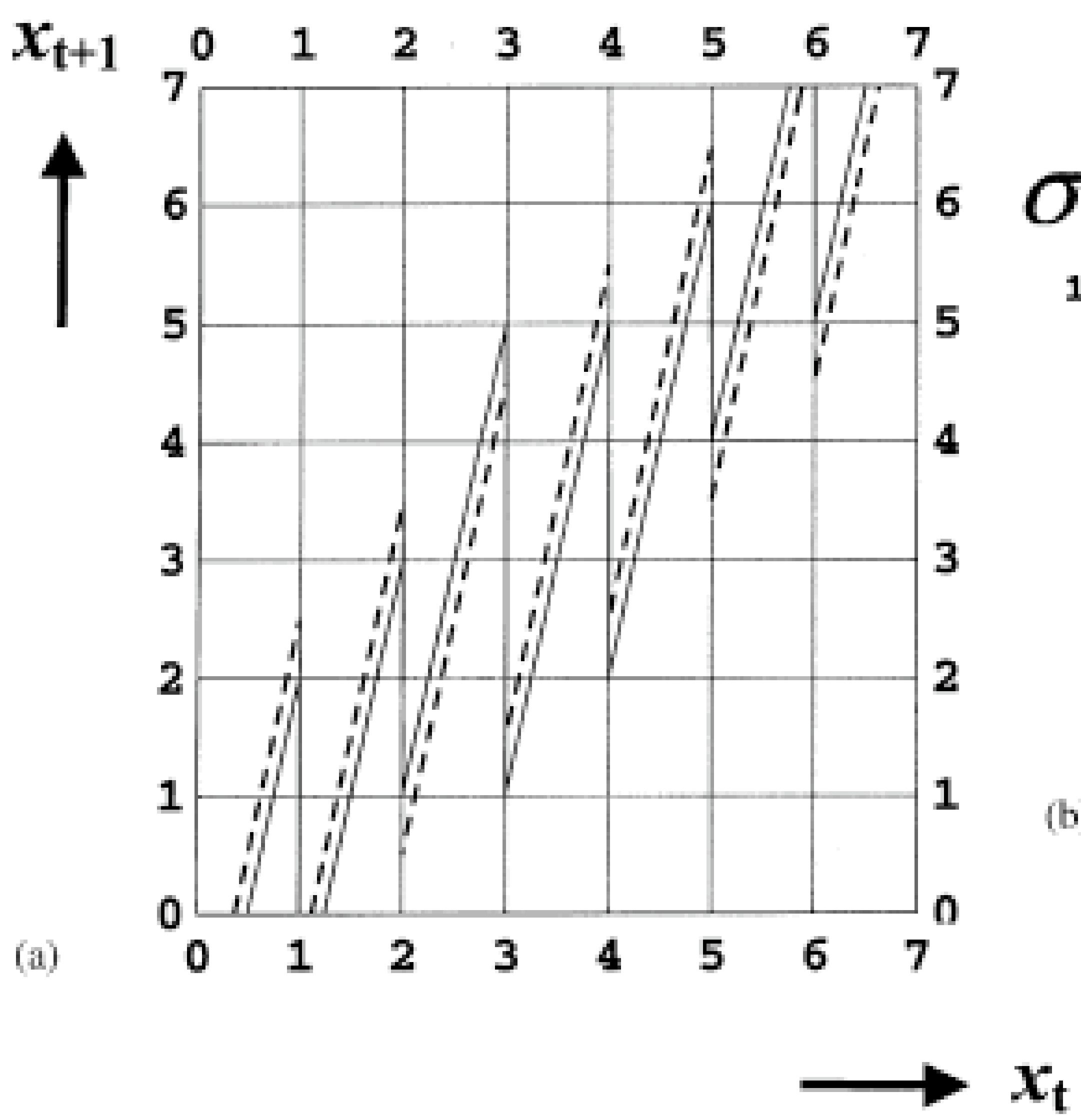


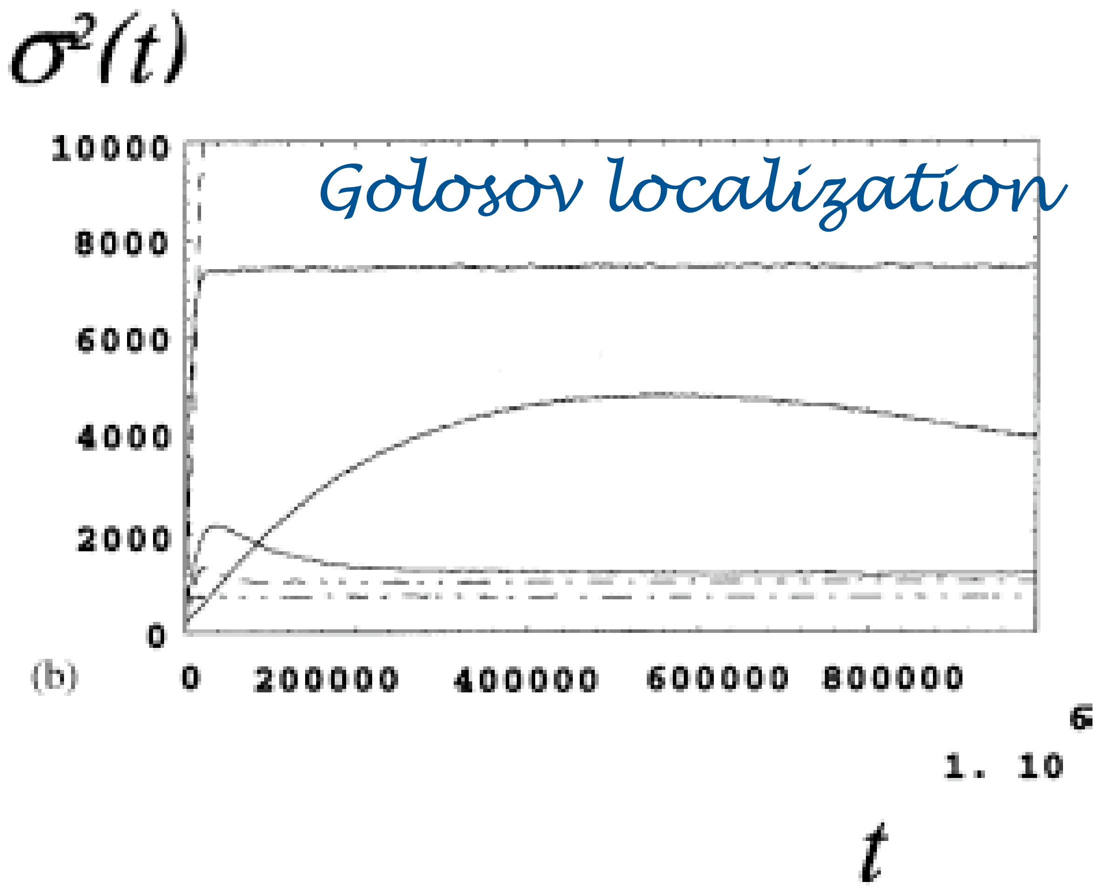
# Radons quenched disorder

# Radons quenched disorder



# Radons quenched disorder



# Sanders, Larralde

# Sanders, Larralde

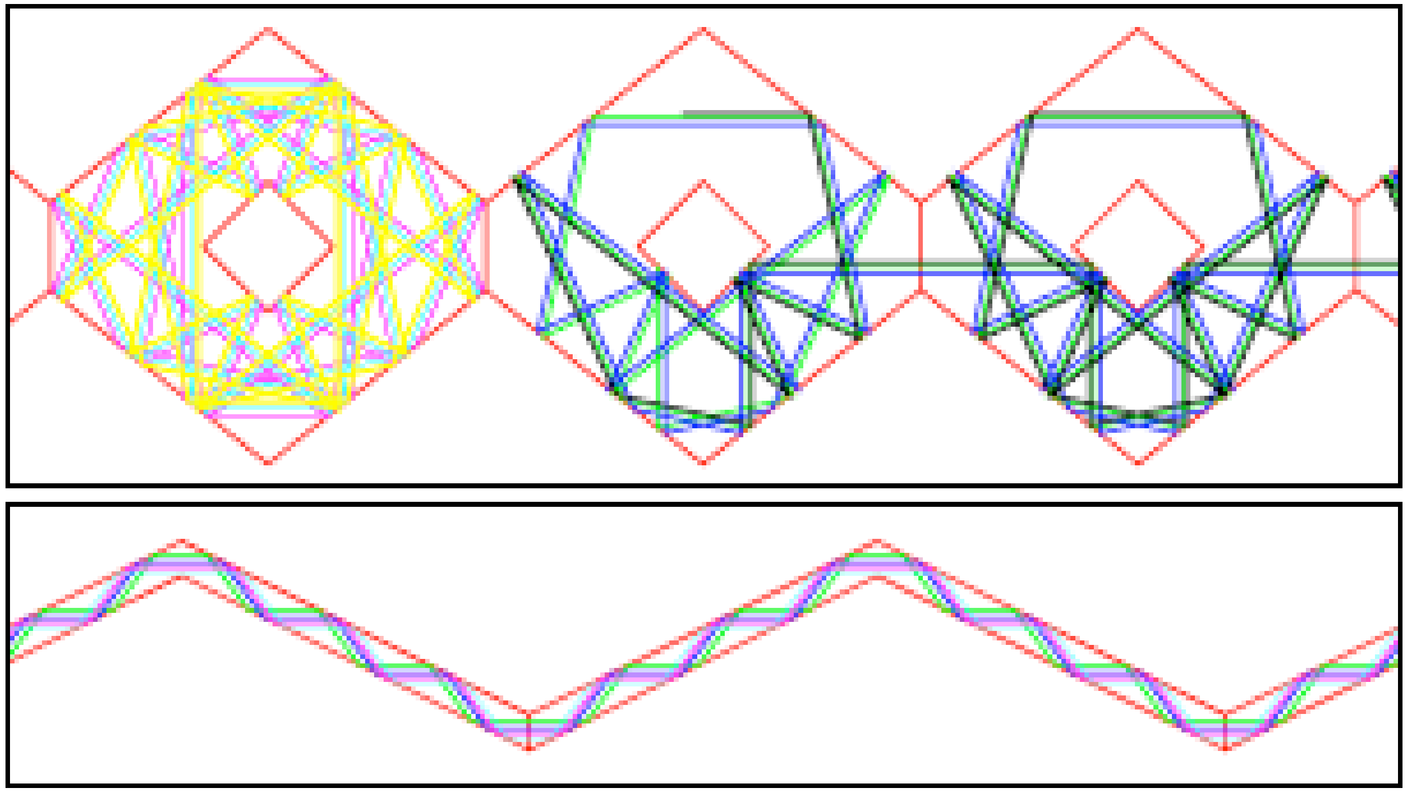
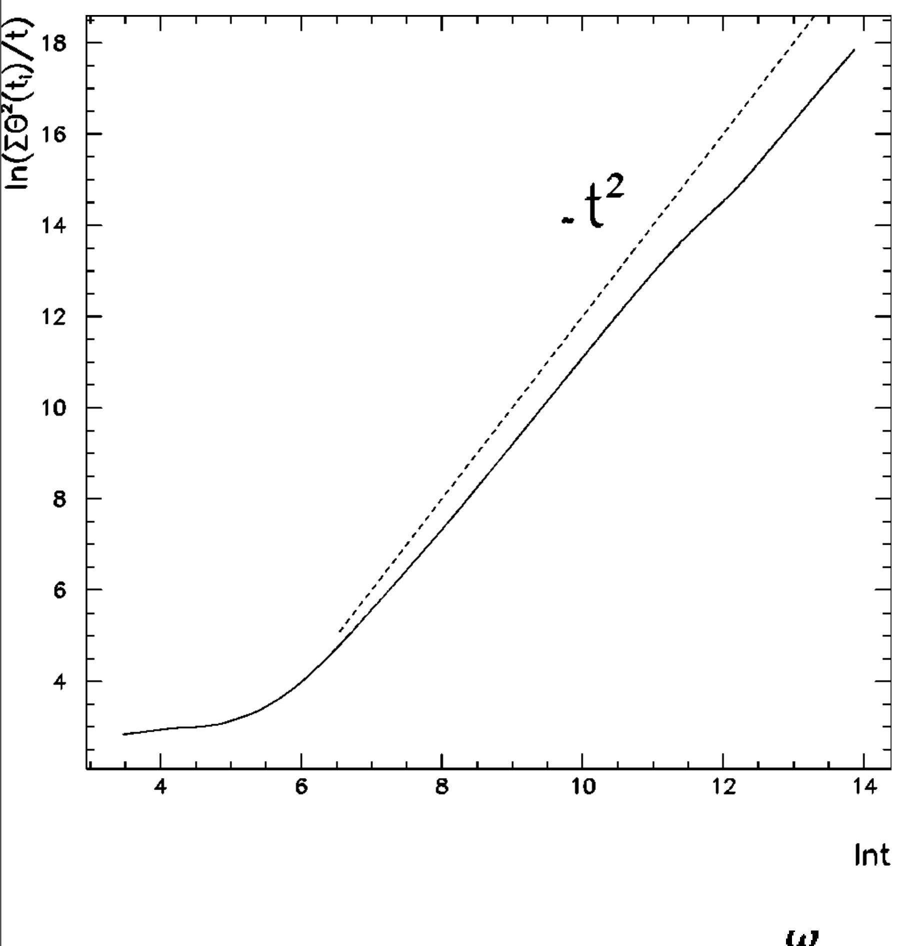
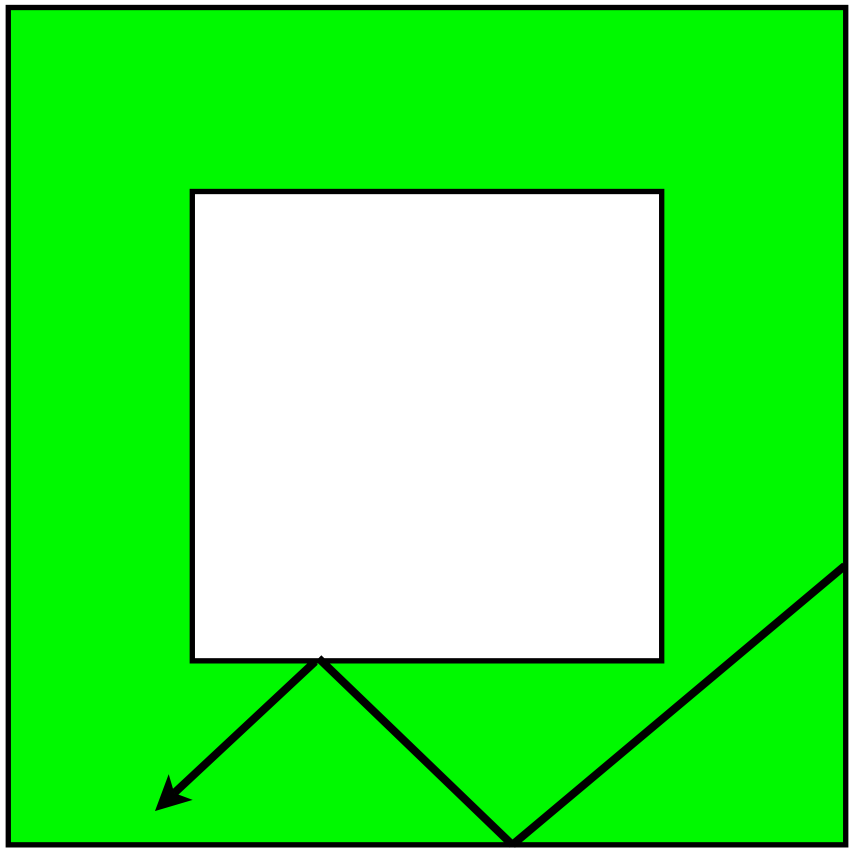
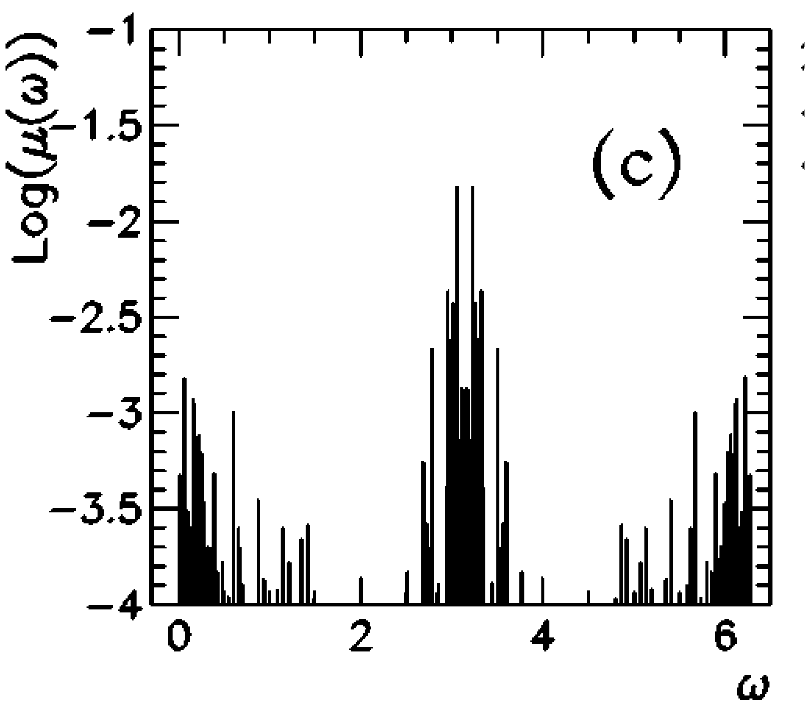


FIG. 13: (Color online) Families of trapped and propagating periodic orbits in parallel systems.

# *non-trivial transport without chaos*





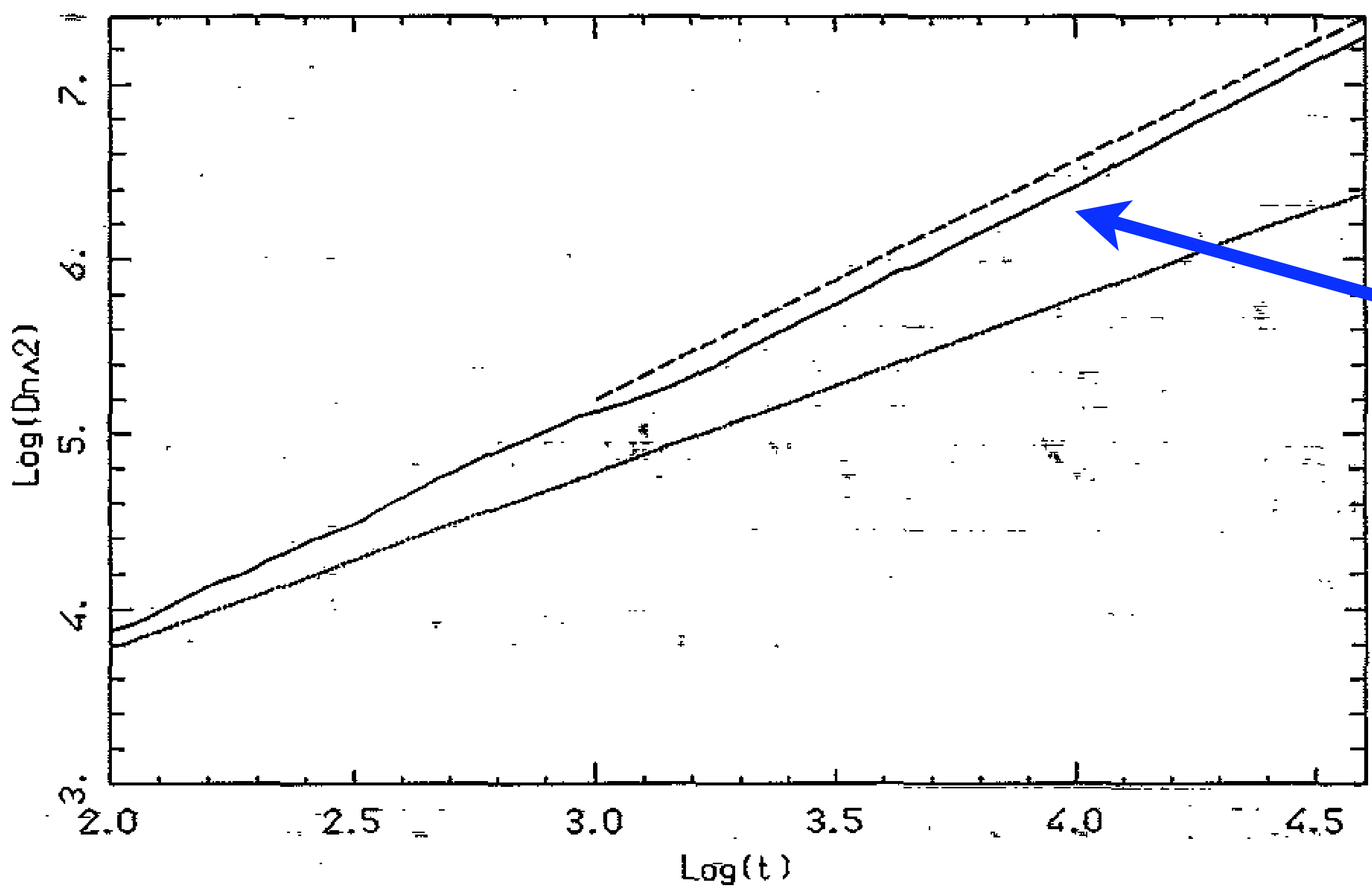

*anomalous transport, singular continuous spectrum*

*RA, Guarneri, Rebuzzini*

# *quantum interlude*

$$p_{n+1} = p_n + K\sin(x_n)$$
$$x_{x+1} = x_n - L\sin(p_{n+1})$$

*kicked Harper map*

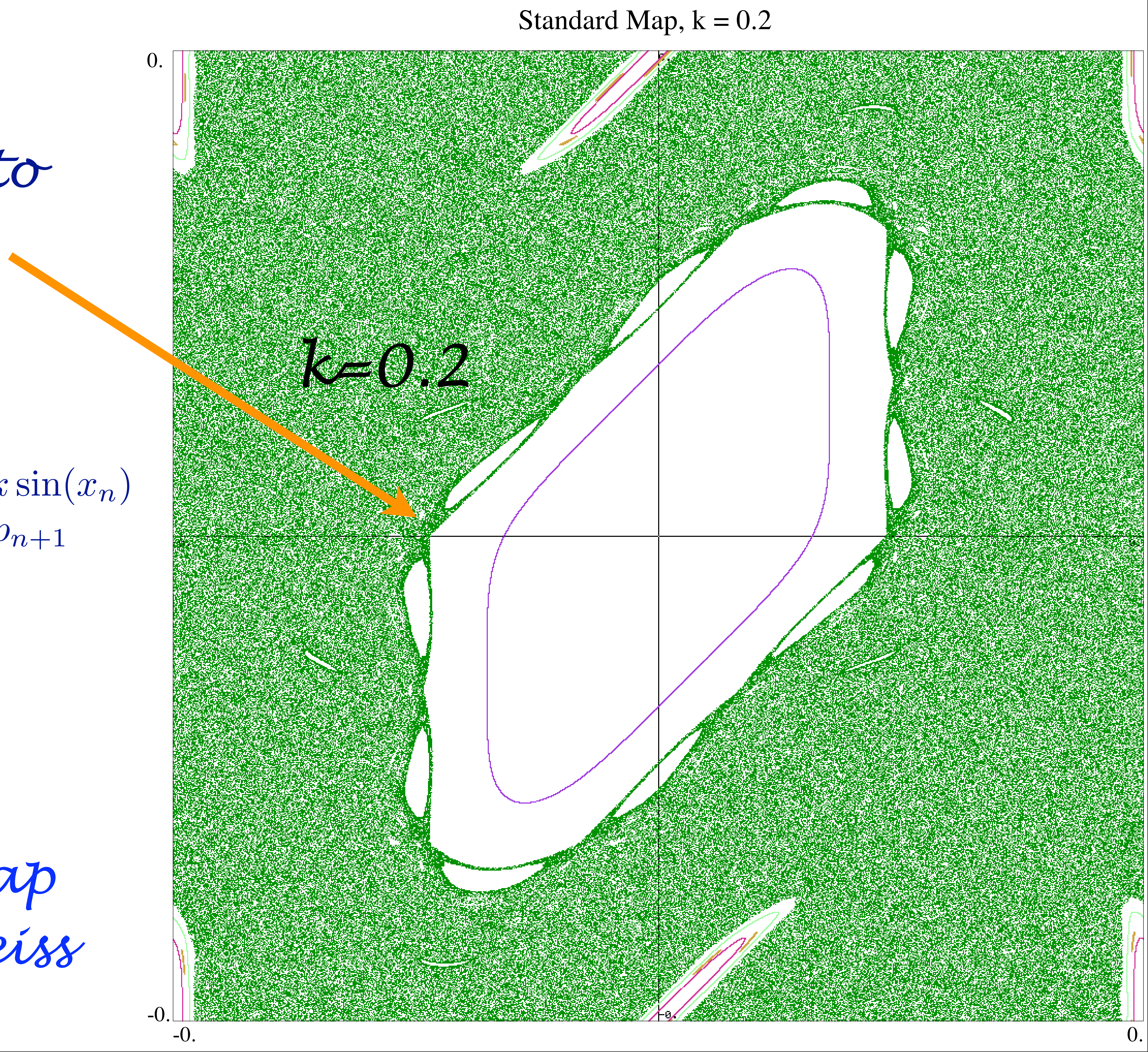



*superdiffusion*

FIG. 4. Asymptotic dependence of $\log\langle\Delta n_t^2\rangle$ on $\log t$ for $K=L=5$, $\hbar=2\pi/(18+\rho_{GM})$ (solid line): The dashed line has a slope $2D_H=1.36$, while the dotted line has slope 1 (case of normal diffusion).

*RA, Casati, Shepelyansky*

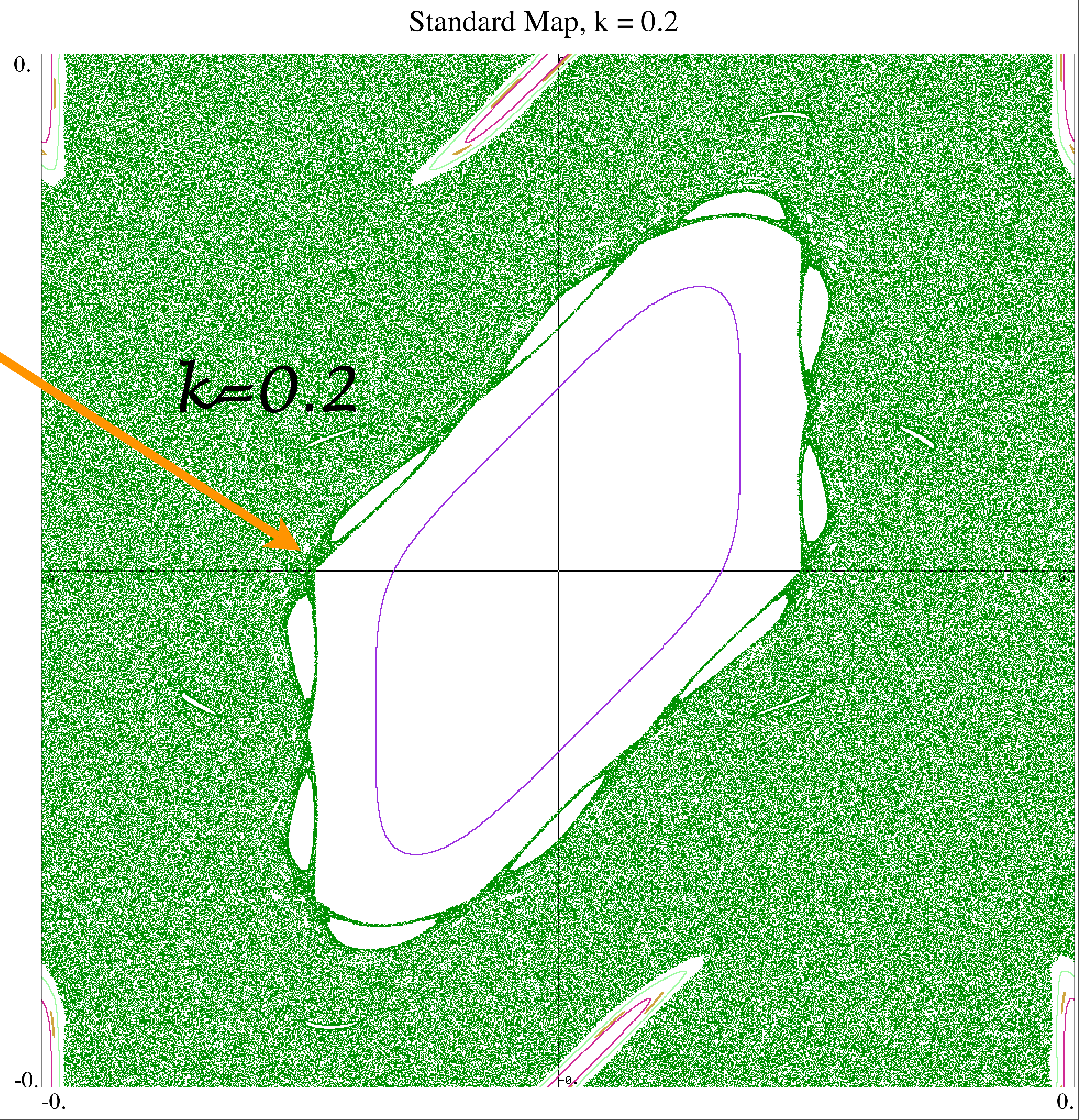
# Sticking to regular regions




$$
\begin{aligned}
p_{n+1} &= p_n + k\sin(x_n) \\
x_{x+1} &= x_n + p_{n+1}
\end{aligned}
$$

StdMap
J.D.Meiss

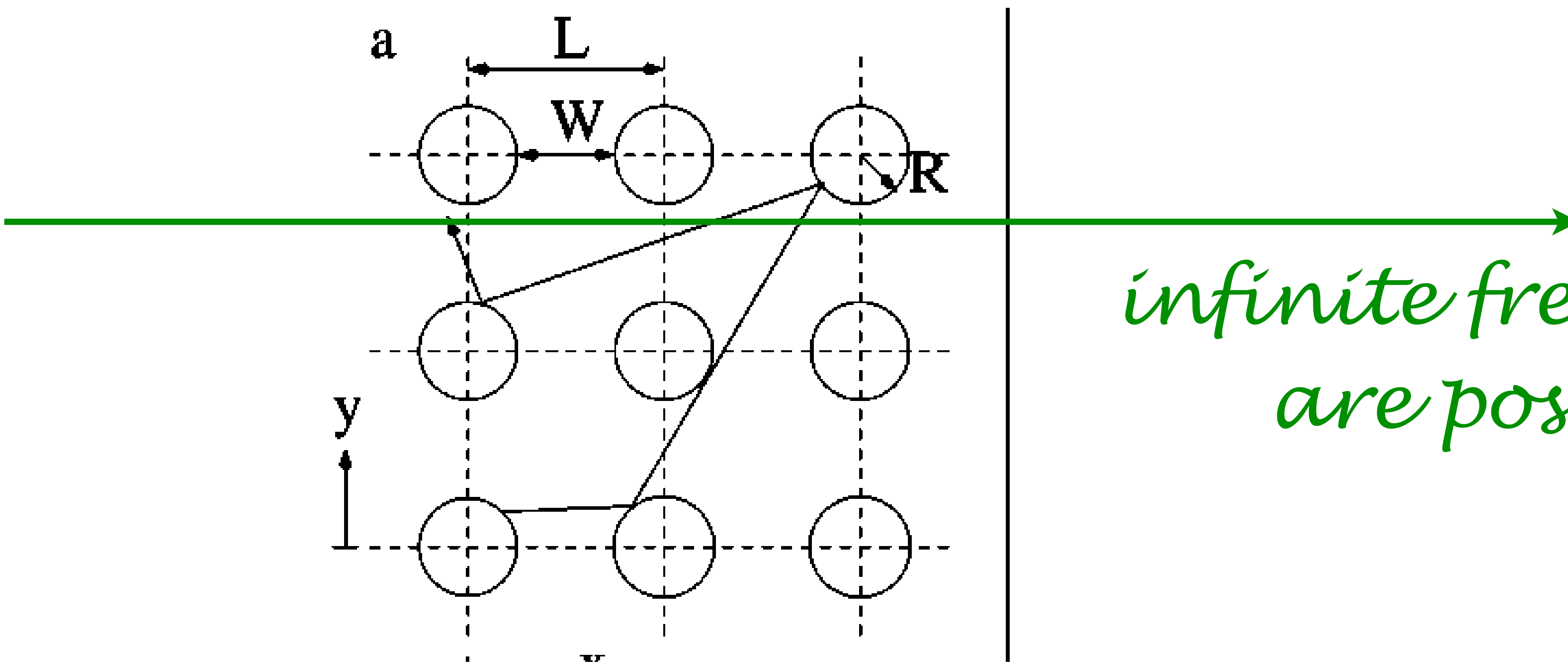
Sticking to regular regions




$$\begin{aligned} p_{n+1} &= p_n + k\sin(x_n) \\ x_{x+1} &= x_n + p_{n+1} \end{aligned}$$

StdMap
J.D.Meiss

# Lorentz gas with infinite horizon




infinite free flights
are possible

$$\langle (\overrightarrow{x}_t - \overrightarrow{x}_0)^2 \rangle \sim t \cdot \ln t$$

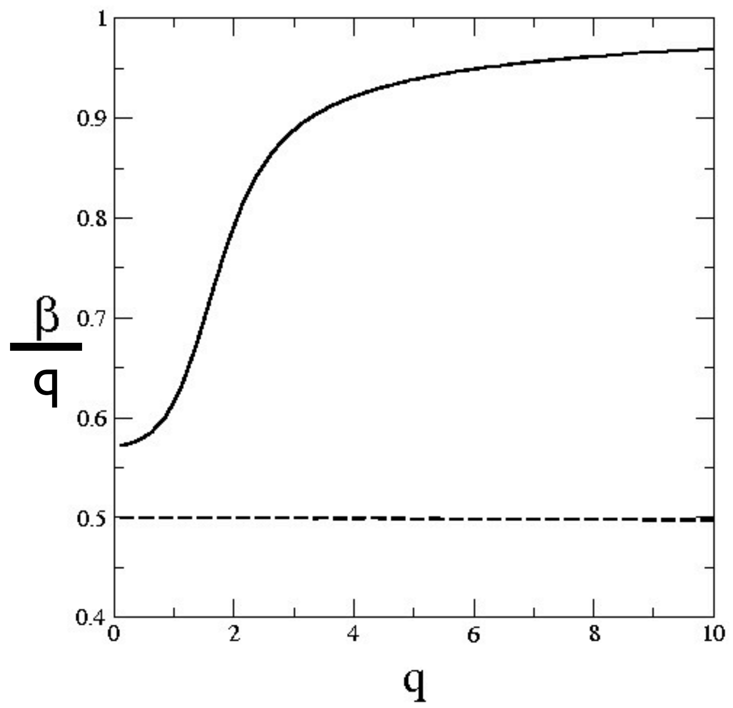
logarithmic divergence of D

# Transport exponents

## Anomalous behavior in moments' spectrum

$$\langle |x_t - x_0|^q \rangle \sim t^{\beta(q)}$$

Normal, gaussian transport yields $\beta(q)=q/2$

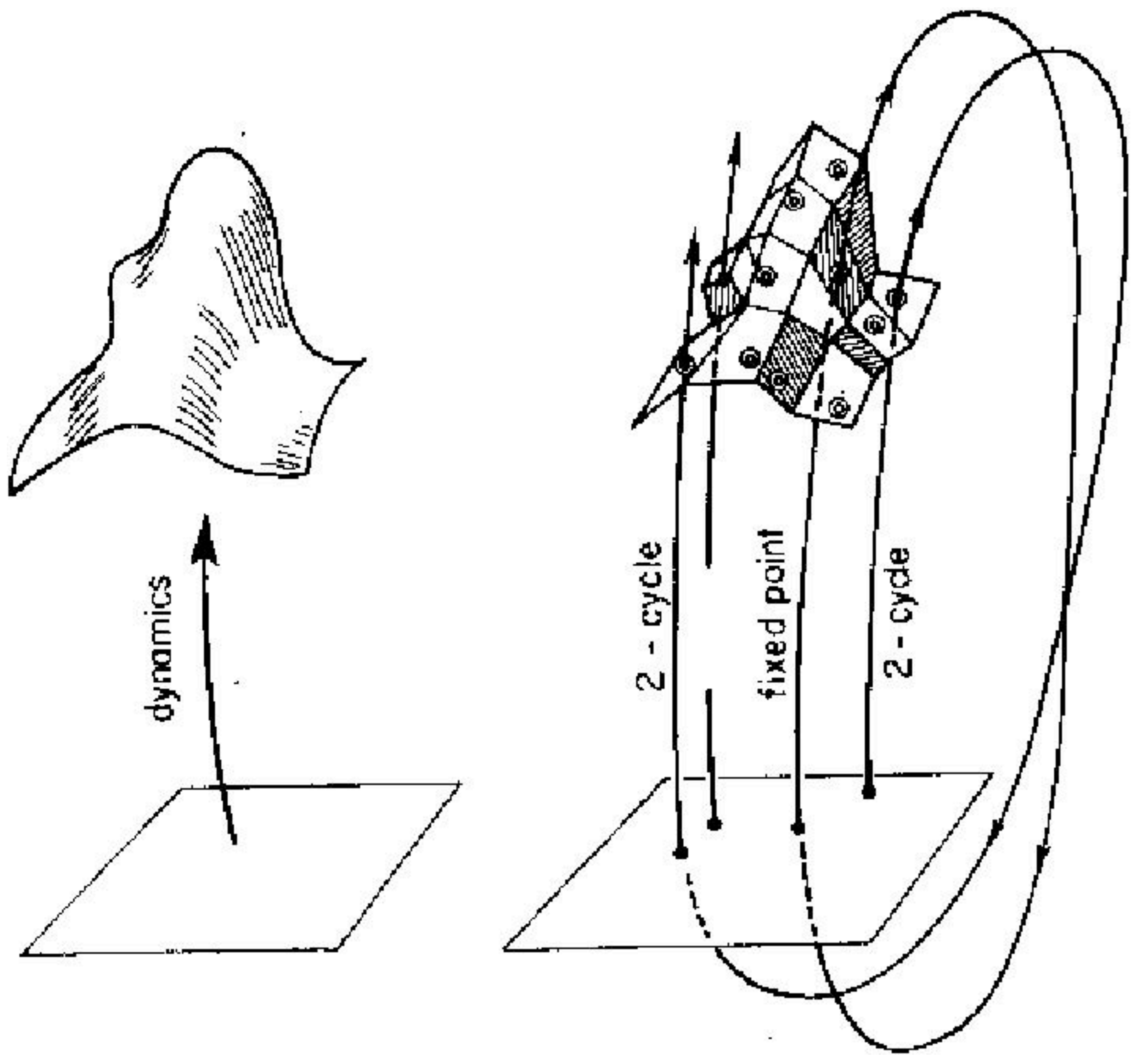



Different parameter values
for the standard map

Castiglione, Mazzino, Muratore, Vulpiani

# The approach

- Transfer matrix - Perron Frobenius operator
- Employ periodic orbits (families of them)



# Chaos: Classical and Quantum

## Part I: Deterministic Chaos

Predrag Cvitanović, Roberto Artuso, Ronnie Mainieri, Gregor Tanner and Gábor Vattay
formerly of CATS

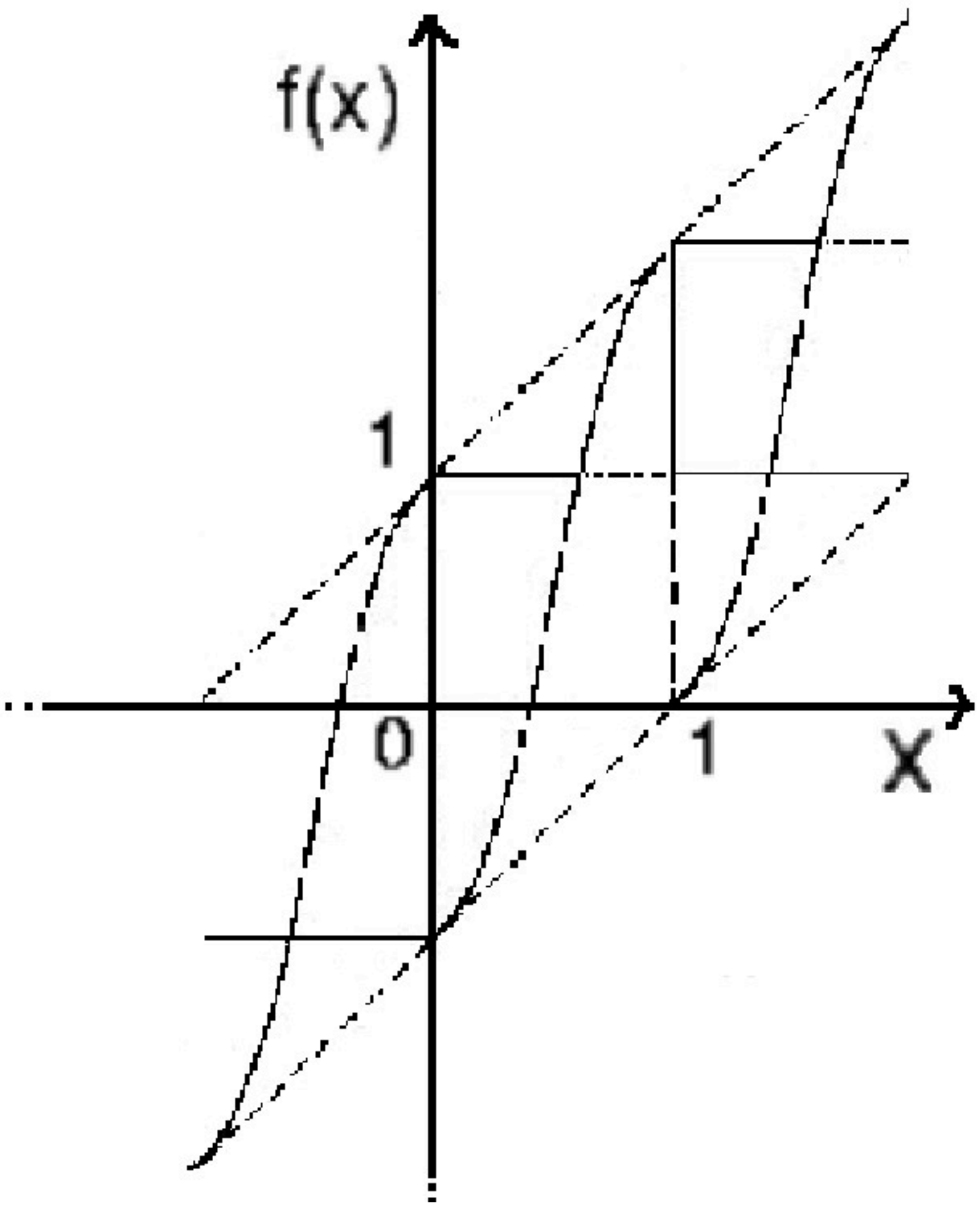
www.chaosbook.org

# Unbounded vs torus dynamics

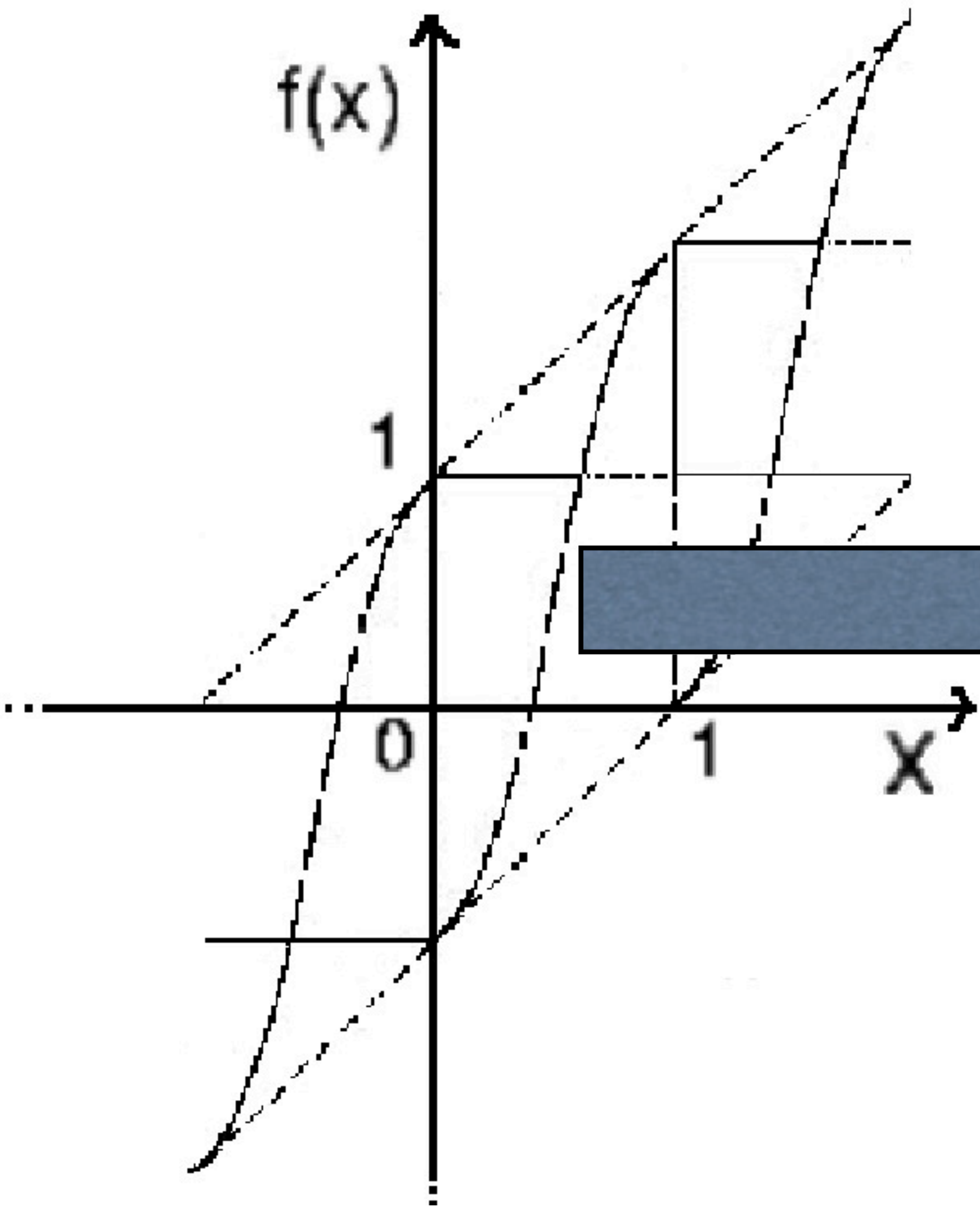
forse qualcosa di meglio

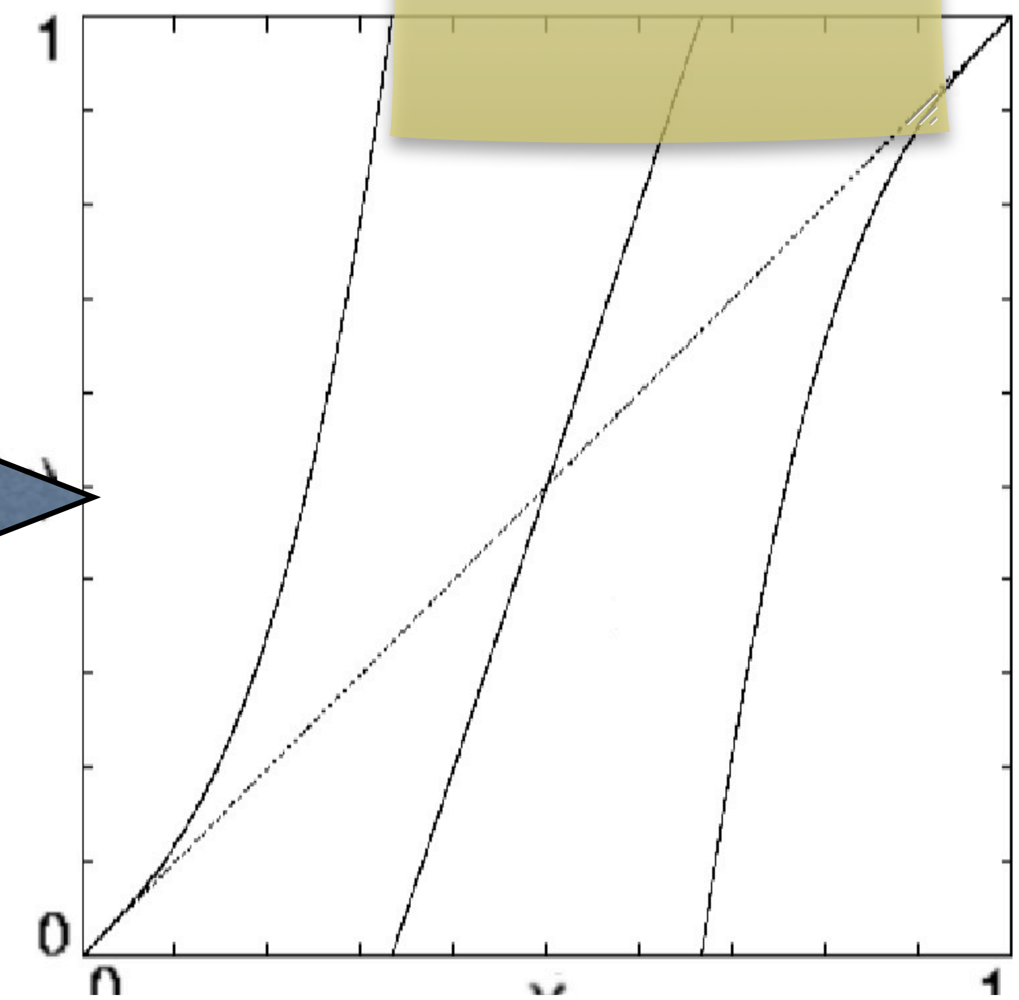
# Unbounded vs torus dynamics

forse qualcosa di meglio



# Unbounded vs torus dynamics

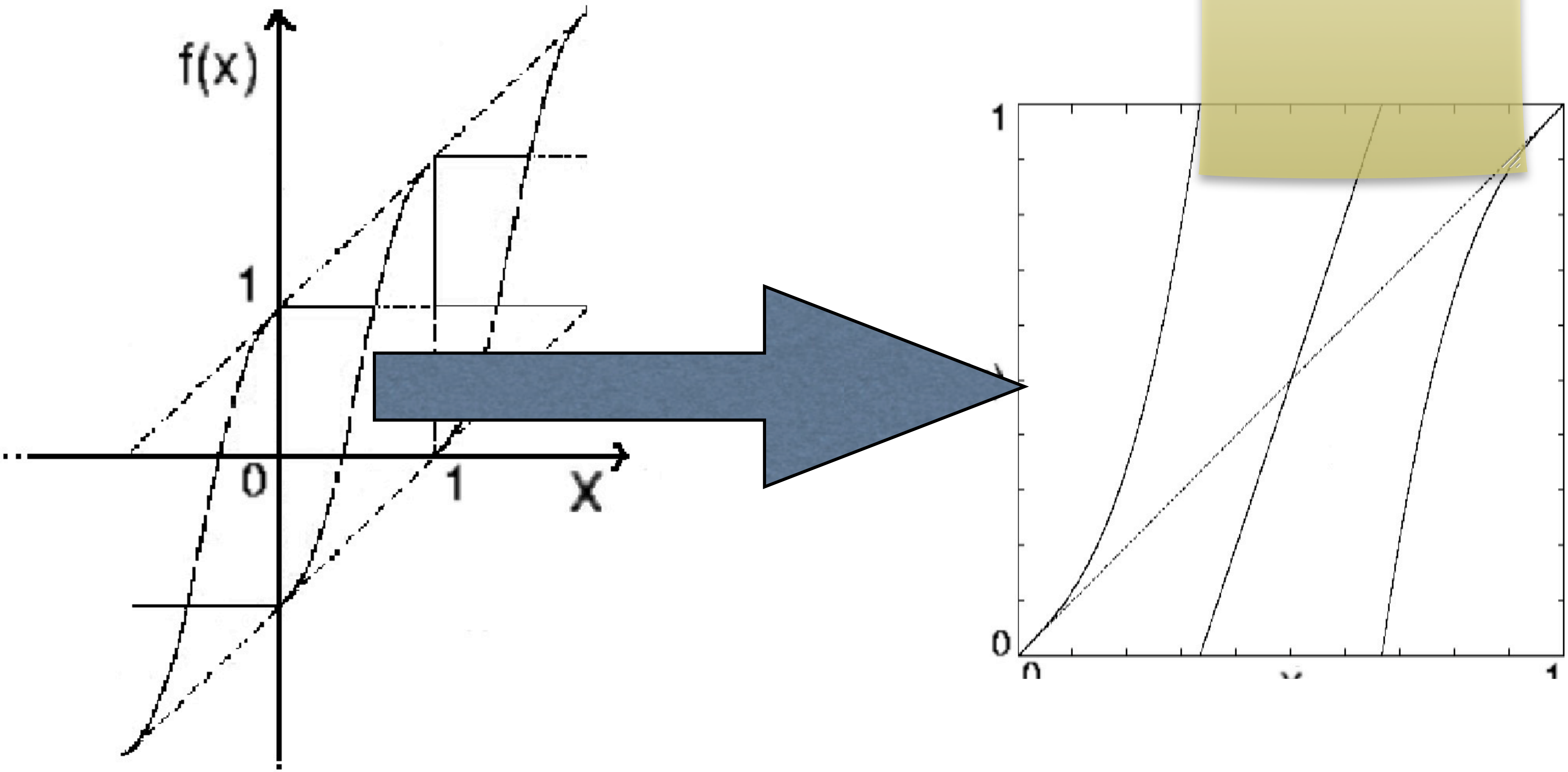



forse qualcosa di meglio

# Unbounded vs torus dynamics

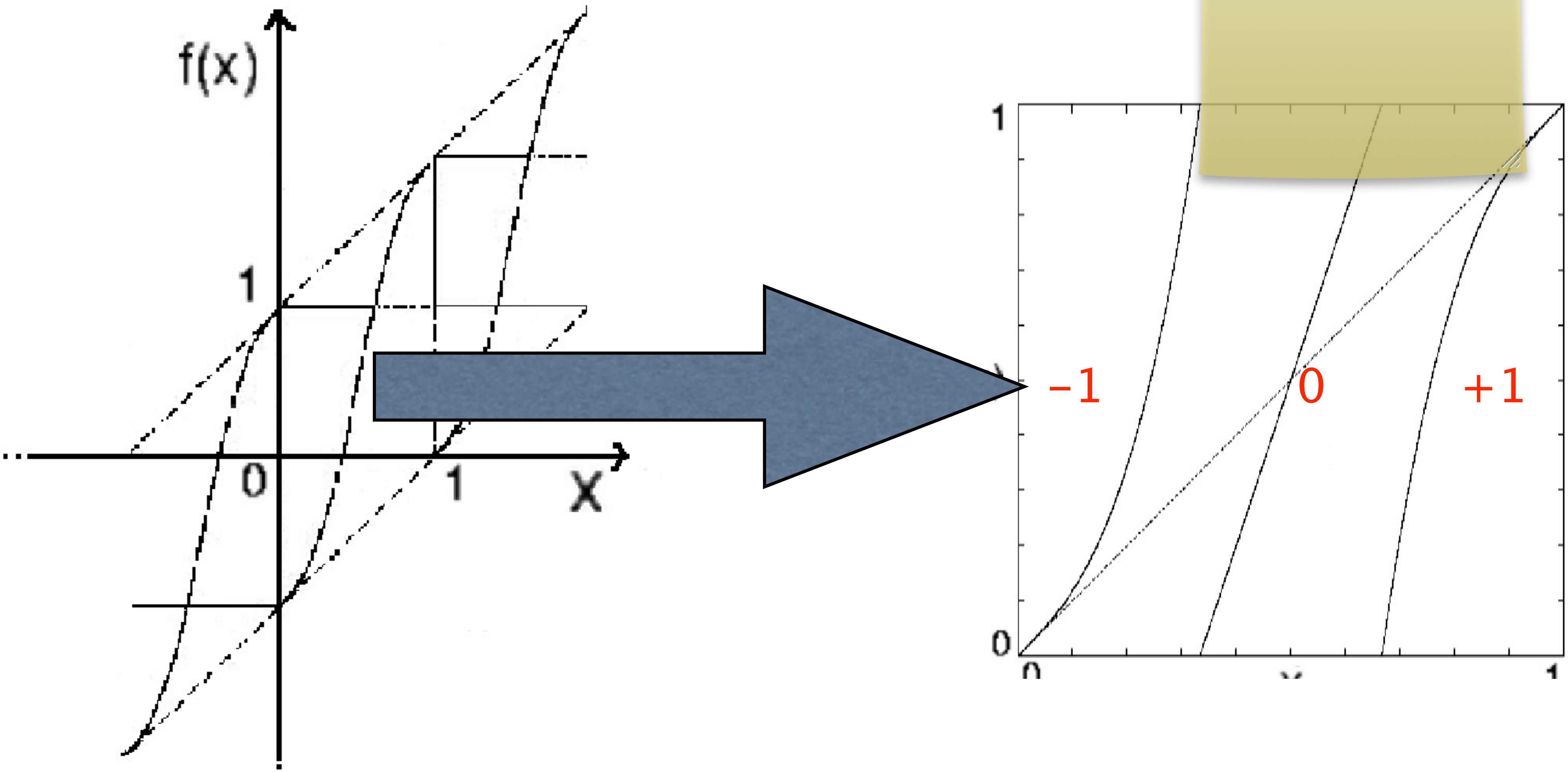
forse qualcosa di meglio



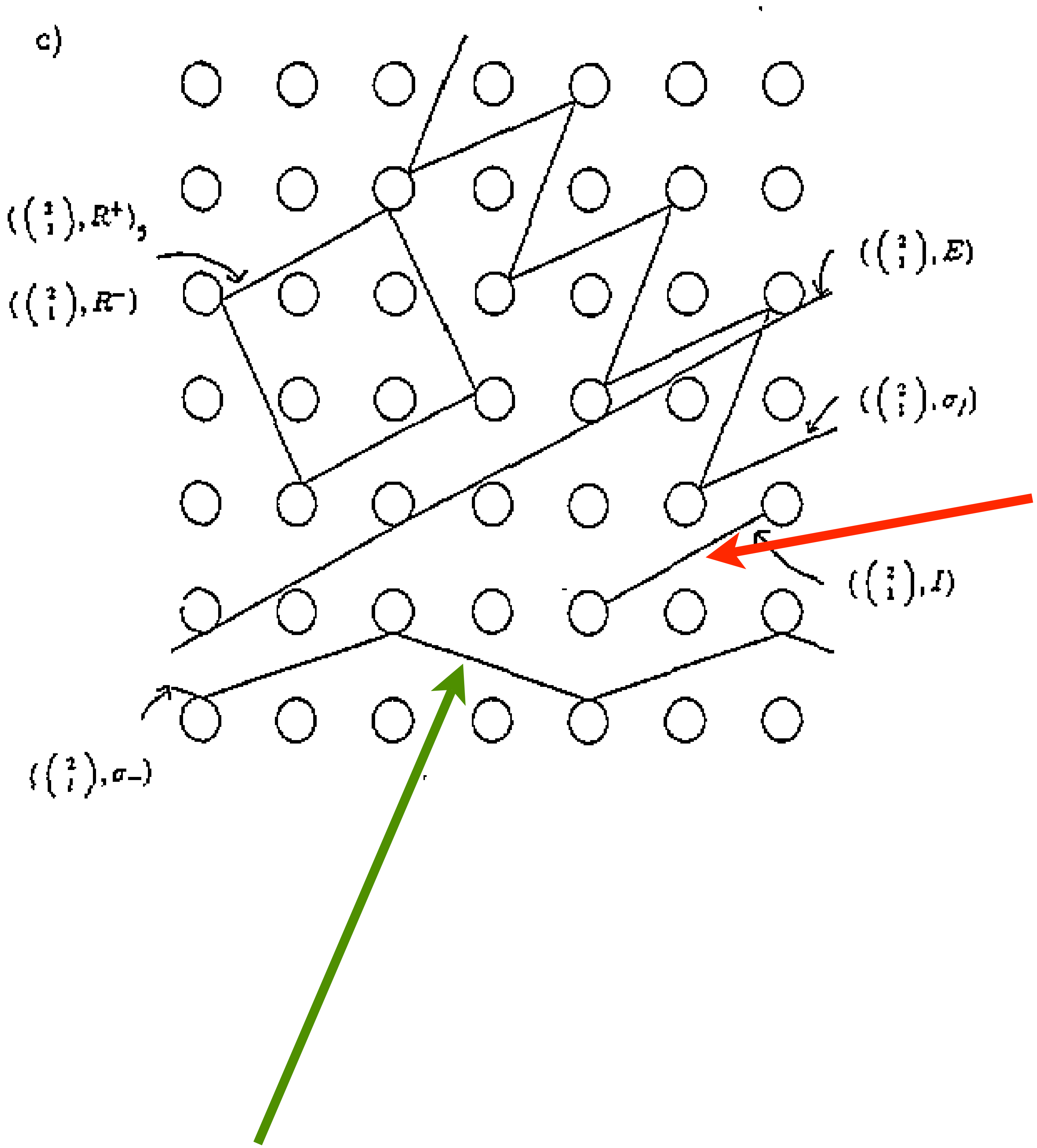

Correspondence is complete once we assign "jumping numbers" σ

# Unbounded vs torus dynamics

forse qualcosa di meglio




Correspondence is complete once we assign "jumping numbers" σ




standing mode

running mode, periodic orbit for billiard

# Statistical mechanics approach

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| $i =$ | 1 | 2 | 3 | 4 | 5 | $\cdots$ | $N-1$ | $N$ |
| $s_i =$ | $+1$ | $-1$ | $-1$ | $+1$ | $+1$ | | $-1$ | $+1$ |

## 1d Ising partition function

$$\mathcal{Z}_N(\beta, H) = \sum_{\{s_i\}} e^{-\beta E_I(\{s_i\})} = Tr\, T^N$$

leading eigenvalue yields Gibbs free energy

spectral gap rules spatial correlation decay

# dynamical systems transfer operator

$$\int_\Omega dx\,\varrho(x)(F\circ T)(x) = \int_\Omega dx(\mathcal{L}\varrho)(x)F(x)$$

evolution on observables (Koopman)

evolution on probability densities (Perron-Frobenius)

spectral analysis of $\mathcal{L}$

1 is the leading eigenvalue (invariant measure)

the spectral gap rules temporal correlation decay

# transfer operators and transport

$$(\mathcal{L}\varrho)(x) = \sum_{y:Ty=x} \frac{1}{T'(y)}\varrho(y) = \int_\Omega dz\, \varrho(z)\delta(x - T(z))$$

the spectral problem is in general highly non trivial: even the choice of a function space is delicate (this is not a mathematical detail: ugly observables generally decay at a slower rate).

we introduce a "generalized" transfer operator accounting for transport properties

$$\left(\mathcal{L}_\beta h\right)(x) = \int_\Omega dz\, h(z)\, e^{\beta(T(z)-z)}\delta(x - T(z))$$

*what's the use?*

$$\mathcal{G}_n(\beta) = \langle e^{\beta(T^n(x_0)-x_0)}\rangle_0 \sim \lambda(\beta)^n$$

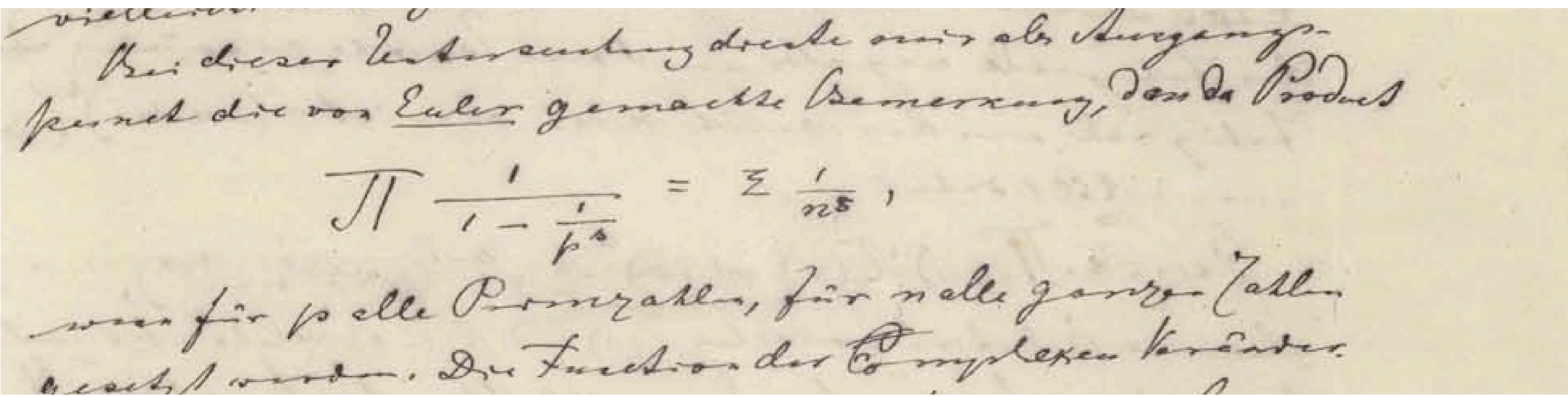
*the generating function grows asymptotically as the leading eigenvalue*

*moments are obtained by Taylor expansion of G*

# how to compute $\lambda(\beta)$

smallest zeroes of generalized zeta functions

product over the set of unstable periodic orbits of the dynamical systems


Bei dieser Untersuchung diente mir als Ausgangspunkt die von Euler gemachte Bemerkung, dass das Product

$$\prod \frac{1}{1-\frac{1}{p^s}} = \Sigma \frac{1}{n^s},$$

wenn für p alle Primzahlen, für n alle ganzen Zahlen gesetzt werden. Die Function der complexen Veränder-

$$\zeta_\beta^{-1}(z) = \prod_{\{p\}} \left(1 - z^{n_p} \frac{e^{\beta \cdot \sigma_p}}{|\Lambda_p|}\right)$$

$$\zeta_\beta^{-1}(z) = \prod_{\{p\}} \left(1 - z^{n_p} \frac{e^{\beta\cdot\sigma_p}}{|\Lambda_p|}\right)$$

*set of unstable periodic orbits*

$$\zeta_\beta^{-1}(z) = \prod_{\{p\}} \left(1 - z^{n_p} \frac{e^{\beta \cdot \sigma_p}}{|\Lambda_p|}\right)$$

$$\zeta_\beta^{-1}(z) = \prod_{\{p\}} \left(1 - z^{n_p} \frac{e^{\beta\cdot\sigma_p}}{|\Lambda_p|}\right)$$

*their period*

$$\zeta_\beta^{-1}(z) = \prod_{\{p\}} \left(1 - z^{n_p} \frac{e^{\beta \cdot \sigma_p}}{|\Lambda_p|}\right)$$

$$\zeta_\beta^{-1}(z) = \prod_{\{p\}} \left(1 - z^{n_p} \frac{e^{\beta \cdot \sigma_p}}{|\Lambda_p|}\right)$$

*their instability*

$$\zeta_\beta^{-1}(z) = \prod_{\{p\}} \left(1 - z^{n_p} \frac{e^{\beta \cdot \sigma_p}}{|\Lambda_p|}\right)$$

$$\zeta_\beta^{-1}(z) = \prod_{\{p\}} \left(1 - z^{n_p} \frac{e^{\beta \cdot \sigma_p}}{|\Lambda_p|}\right)$$

their "space shift"

# *transport and analytic properties*

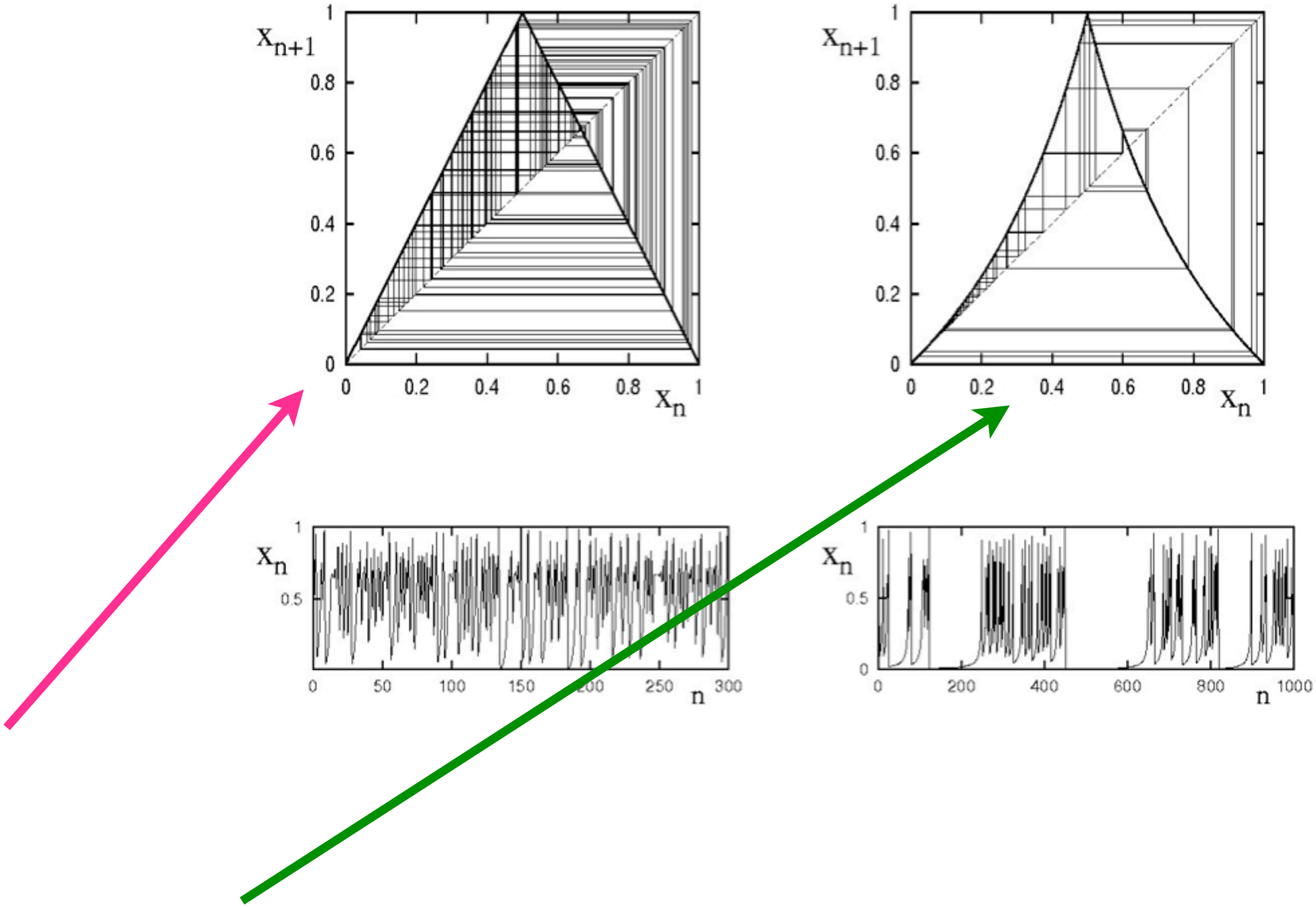
$$\langle (x_n - x_0)^k \rangle_0 \sim \frac{\partial^k}{\partial \beta^k} \frac{1}{2\pi\imath} \int_{a-\imath\infty}^{a+\imath\infty} ds\, \mathrm{e}^{sn} \frac{d}{ds} \ln \left[ \zeta_{\beta,(0)}^{-1}(\mathrm{e}^{-s}) \right] \Big|_{\beta=0}$$

$$D = \lim_{n\to\infty} \frac{1}{2n} \frac{d^2}{d\beta^2} \left( \frac{1}{2\pi\imath} \int_{a-\imath\infty}^{a+\imath\infty} ds\, e^{sn} \frac{\partial_s \zeta_{\beta,(0)}^{-1}(\mathrm{e}^{-s})}{\zeta_{\beta,(0)}^{-1}(\mathrm{e}^{-s})} \right)_{\beta=0}$$

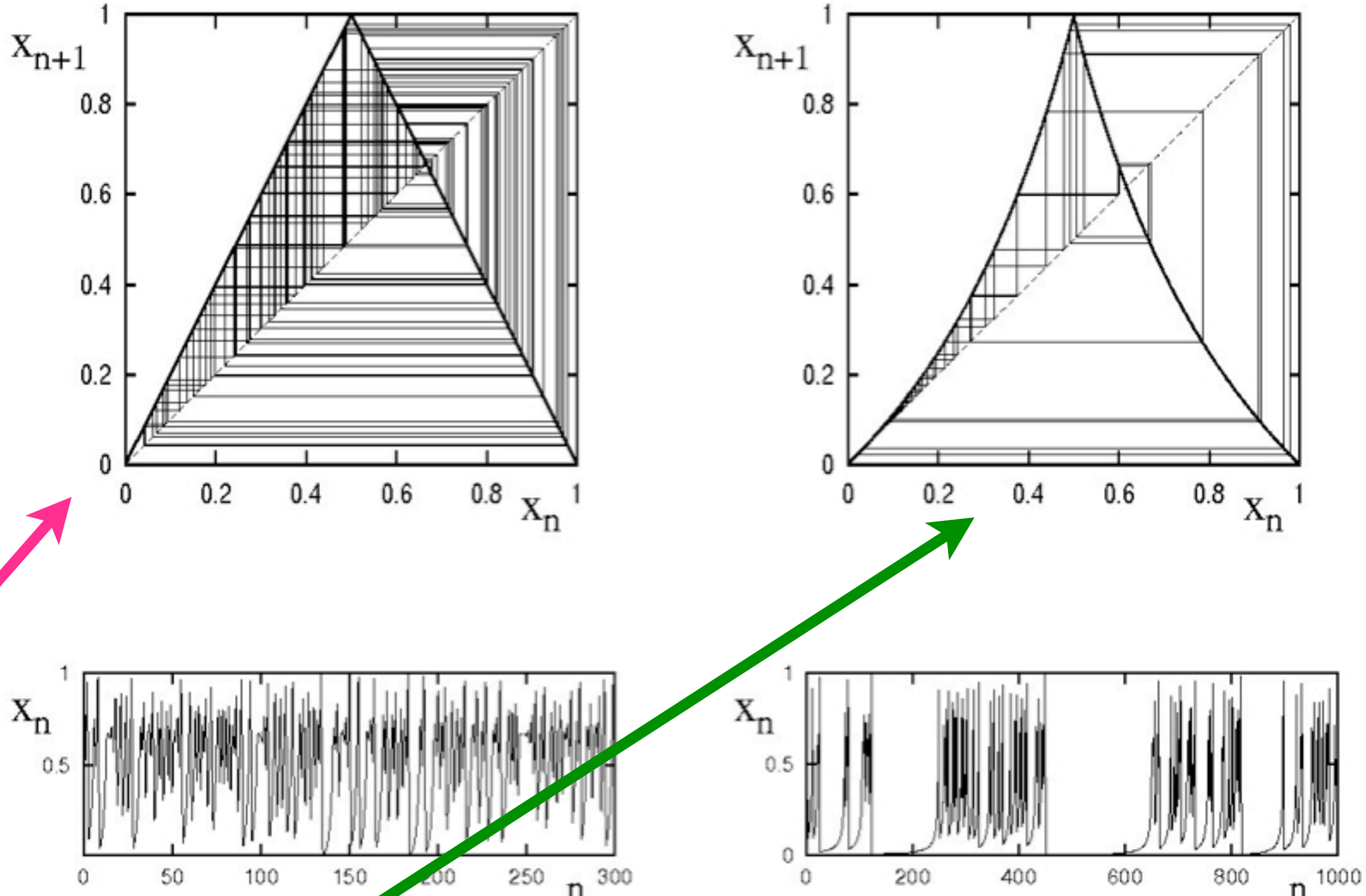
*analytic properties of zeta functions near their first zero give the asympotics of moments (via Tauberian theorems for Laplace transforms)*

*RA, Cristadoro, Dahlqvist*

# *Qualitative 1-d intermittency*



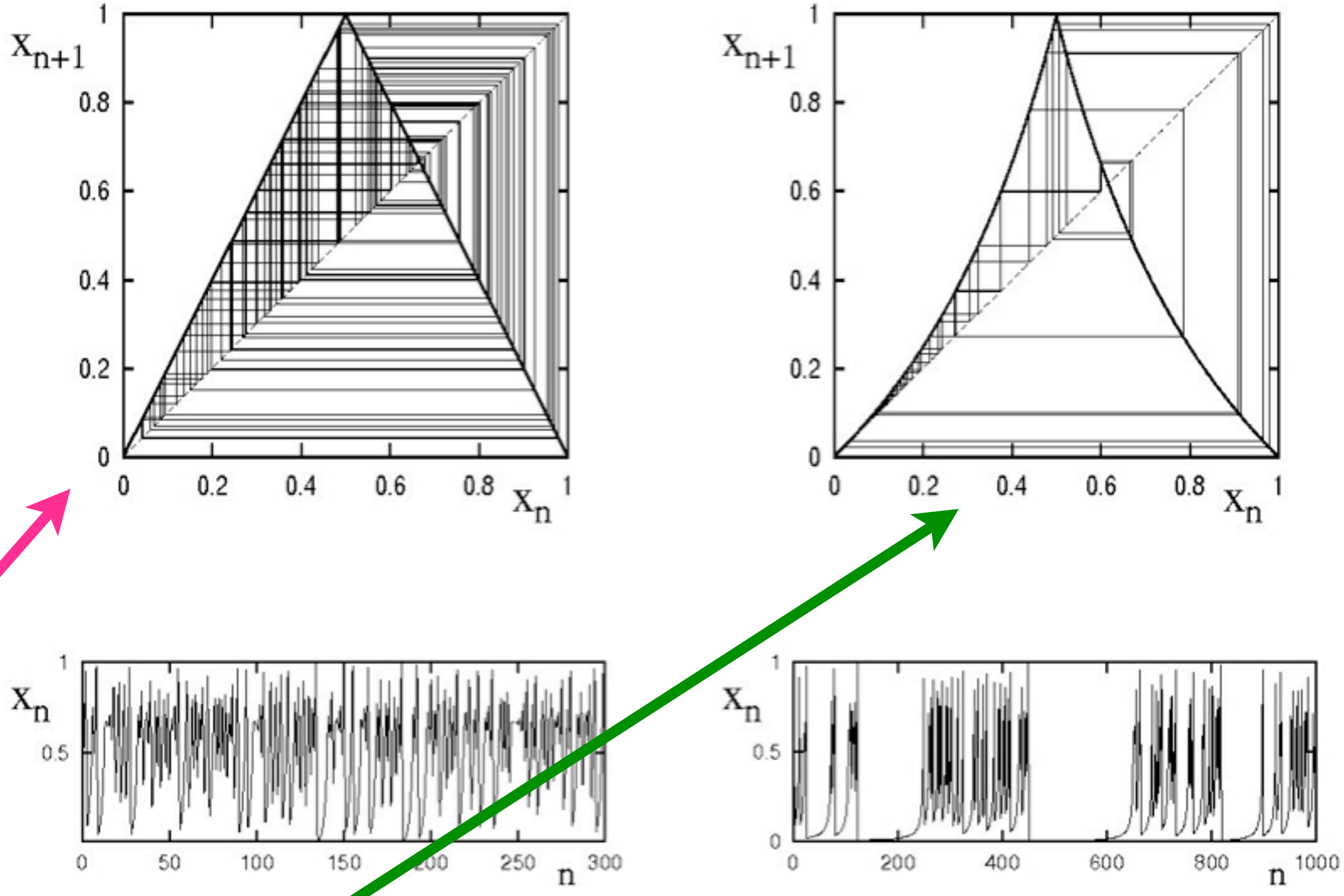
# Qualitative 1-d intermittency



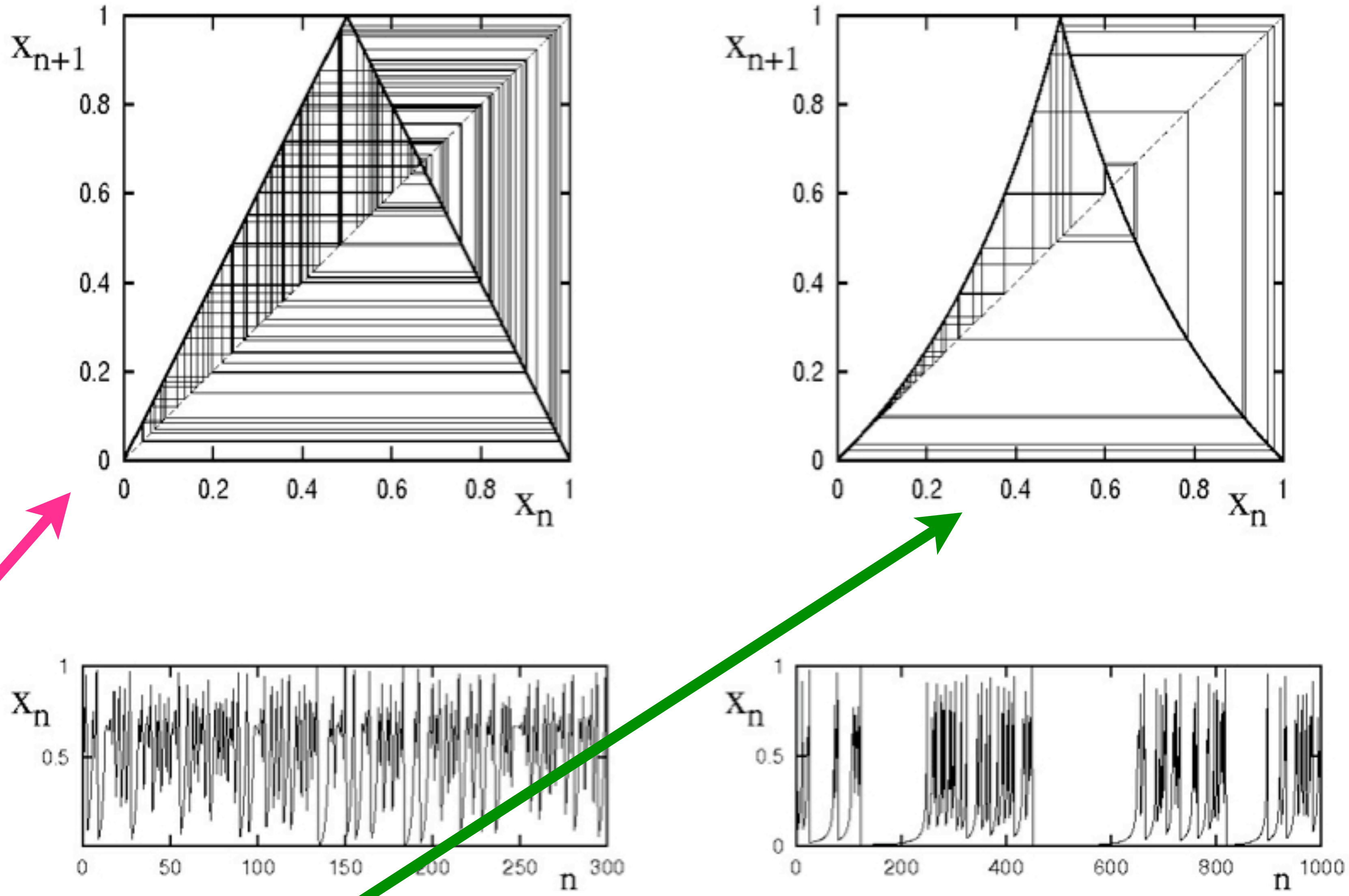

"strong chaos" exponential instability $\Lambda_n \approx a^n$

"weak chaos" power-law instability $\Lambda_n \approx n^a$

# Qualitative 1-d intermittency



# *Qualitative 1-d intermittency*




*"strong chaos" simple zero (polynomial)*

*"weak chaos" non analytic behavior*

# take-home message

even low-dimensional dynamical systems can provide a rich variety of transport properties (diffusion, anomalous scaling, ratchet behavior)

analysis in terms of periodic orbits (zeta functions) yields exact results for some models, in the realm of a purely deterministic approach

# RA, G. Cristadoro: Deterministic (anomalous) transport