Non perturbative Physics from Open Strings and D-branes

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Foreword

- This talk is mainly based on:
  - M. Billo, M. F., I. Pesando, F. Fucito, A. Lerda, A. Liccardo, JHEP **0302**, 045 (2003), [hep-th/0211250]
  - M. Billo, M. F., I. Pesando and A. Lerda, JHEP **0405**, 023 (2004), [hep-th/0402160]
  - M. Billo, M. F., S. Sciuto, G. Vallone and A. Lerda, JHEP **0605**, 069 (2006), [hep-th/0511036]
  - M. Billo, M. F., F. Fucito and A. Lerda, JHEP **0611**, 012 (2006), [hep-th/0606013]
  - M. Billo, M. F., I. Pesando, P. Di Vecchia, A. Lerda, R. Marotta, JHEP **0710**, 091 (2007), [arXiv:0708.3806]
  - M. Billo, M. F., I. Pesando, P. Di Vecchia, A. Lerda, R. Marotta, JHEP **0709**, 051 (2007), [arXiv:0709:0245]

- Another seminal paper
  - M. B. Green and M. Gutperle, JHEP **0002** (2000) 014 [hep-th/0002011]
- Recent literature on the subject:
  - R. Blumenhagen, M. Cvetic and T. Weigand, Nucl. Phys. B **771** (2007) 113 [hep-th/0609191].
  - L. E. Ibanez and A. M. Uranga, JHEP **0703** (2007) 052 [hep-th/0609213].
  - N. Akerblom, R. Blumenhagen, D. Lust, E. Plauschinn and M. Schmidt-Sommerfeld, JHEP **0704** (2007) 076.
  - R. Argurio, M. Bertolini, G. Ferretti, A. Lerda and C. Petersson, JHEP **0706** (2007) 067.
  - N. Akerblom, R. Blumenhagen, D. Lust and M. Schmidt-Sommerfeld, JHEP **0708** (2007) 044.
  - R. Blumenhagen, M. Cvetic, R. Richter and T. Weigand, JHEP **0710** (2007) 098.
  - L. E. Ibanez and A. M. Uranga, JHEP **0802** (2008) 103.
  - I. Garcia-Etxebarria, F. Marchesano and A. Uranga, arXiv:0805.0713.
  - ...

Non perturbative effects in gauge theories

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- In $\mathcal{N} = 2$ theories they contribute to the SW Prepotential
- In SQCD they induce the ADS superpotential for $N_f = N_c - 1$
- More generally they are responsible for new couplings in the field theory effective action

Gauge Instantons

A gauge instanton in four dimensions is a non trivial solution $A_\mu^a(x)$ of the field equations such that $*F^a_{\mu\nu} = F^a_{\mu\nu}$ and

$$S = \frac{1}{g^2} \int d^4x \, Tr\left(F_{\mu\nu}F^{\mu\nu}\right) = \frac{8\pi^2}{g^2}\, k$$

The $k = 1,\ SU(2)$ instanton in the regular gauge is ['t Hooft]

$$A_\mu^a = 2\,\frac{\eta^a_{\mu\nu}(x-x_0)^\nu}{(x-x_0)^2+\rho^2}$$

It explicitly depends on 5 moduli: $x_0^\mu \rightarrow$ center and $\rho \rightarrow$ size, but there are other 3 moduli: the orientation θ that may be introduced via "large" gauge transformation $A_\mu \rightarrow U(\theta)A_\mu U^\dagger(\theta)$.

The $SU(N)$ instanton can be obtained by embedding $A_\mu^{SU(2)}$ in $SU(N)$:

$$A_\mu = U \begin{pmatrix} 0_{N-2,N-2} & 0 \\ 0 & A_\mu^{SU(2)} \end{pmatrix} U^\dagger$$

and it has $5 + (4N-5)$ moduli.

ADHM Construction

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- In this way $A_\mu(x)$ depends on a number of parameters which are more then the the real instanton moduli, but are subject to a number of bosonic and fermioni constraints: ADHM constraint

The semiclassical quantization of gauge theories in instantonic backgrounds has been carefully studied in the past

$$\Downarrow$$

Instanton Calculus: to compute the instantonic contribution to the gauge degrees of freedom effective action we have to integrate over the instanton moduli space with an appropriate measure

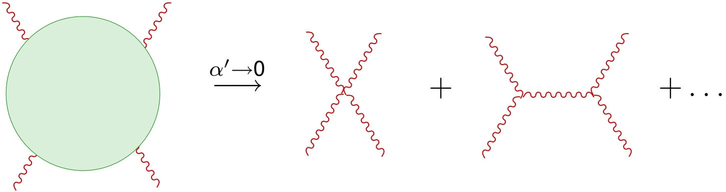
$$\Downarrow$$

In Supersymmetric theories there is a contribution only to amplitudes that saturate the fermionic zero modes!

Gauge Theory from Strings

String theory is a very powerful tool to analyze perturbative field theories, and in particular gauge theories.

Behind this, there is a rather simple and well-known fact: in the field theory limit $\alpha' \to 0$, a single string scattering amplitude reproduces a sum of different Feynman diagrams



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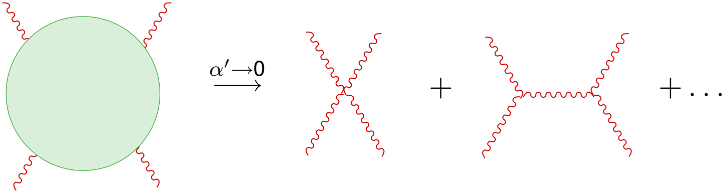

String theory S-matrix elements $\Longrightarrow$ vertices and effective actions in field theory

In general, a N-point string amplitude $\mathcal{A}_N$ is given schematically by

$$\mathcal{A}_N = \int_\Sigma \langle V_{\phi_1} \cdots V_{\phi_N} \rangle_\Sigma$$

where

- V_{ϕ_i} is the vertex for the emission of the field ϕ_i: $V_{\phi_i} \equiv \phi_i \, \mathcal{V}_{\phi_i}$
- Σ is a Riemann surface of a given topology
- $\langle \ldots \rangle_\Sigma$ is the v.e.v. with respect to the vacuum defined by Σ.

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The simplest world-sheets Σ are:

spheres for **closed strings** and **disks** for **open strings**

- For any **closed string** field ϕ_{closed}, one has

$$\langle\, \mathcal{V}_{\phi_{\text{closed}}} \,\rangle_{\text{sphere}} = 0 \quad \Rightarrow \quad \langle\, \phi_{\text{closed}} \,\rangle_{\text{sphere}} = 0$$

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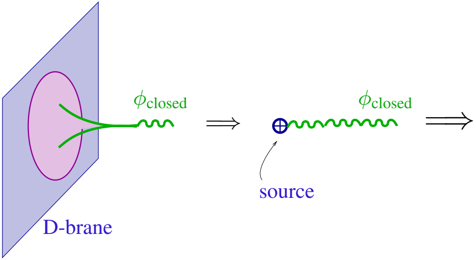
- spheres and disks describe the trivial vacuum around which ordinary perturbation theory is performed
- spheres and disks are inadequate to describe non-perturbative backgrounds where fields have non trivial profile!

However, after the discovery of D-branes, the perspective has drastically changed, and nowadays we know that also some non-perturbative properties of field theories can be analyzed using string theory!

The solitonic brane solutions of SUGRA with RR charge, D branes, have a perturbative description in terms of closed strings emitted from a disks with Dirichlet boundary conditions



$$\langle \phi_{\rm closed} \rangle_{\rm disk} \neq 0$$

[Di Vecchia et al. 1997,...]

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Is it possible that also instanton effects in (Super) Yang Mills theories have a perturbative description in string theory?

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- Gauge instanton effects can be described using open string perturbation theory on these multiple brane configurations!
- In the brane construction new effects may be present: stringy instantons

Plan of the talk

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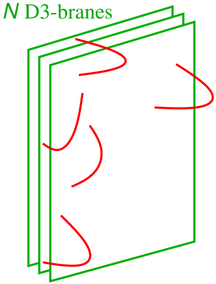
   - Instantons in closed string backgrounds
   - Stringy instanton effects
6. Conclusions and perspectives

String description of SYM theories and their instantons

The effective action of a SYM theory can be given a simple and calculable string theory realization:

- The gauge degrees of freedom are realized by open strings attached to N D3 branes.



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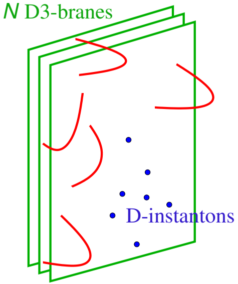

- The instanton sector of charge k is realized by adding k D(–1) branes (D-instantons).

Instantons and D-instantons

- Consider the effective action for a stack of N D3 branes

$$\text{D. B. I.} + \int_{\text{D3}} \left[C_4 + \frac{1}{2} C_0 \, \text{Tr}(F \wedge F) \right]$$

The topological density of an instanton configuration corresponds to a localized source for the R-R scalar C_0, i.e., to a distribution of D-instantons inside the D3's.

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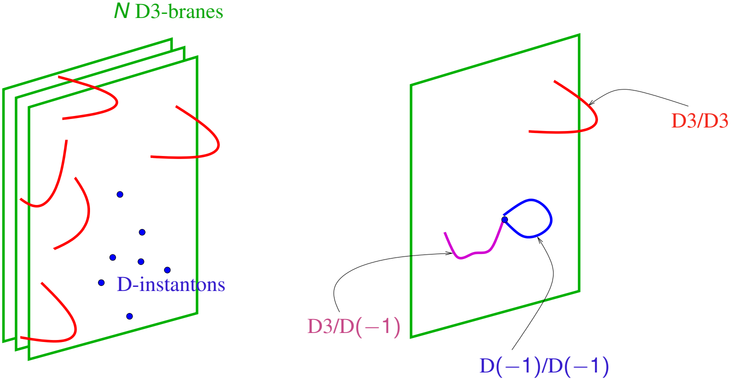
- Instanton-charge k solutions of $\mathrm{SU}(N)$ gauge theories correspond to k D-instantons inside N D3-branes.

[Witten 1995, Douglas 1995, Dorey 1999, ...]

	0	1	2	3	4	5	6	7	8	9
D3	—	—	—	—	*	*	*	*	*	*
D(−1)	*	*	*	*	*	*	*	*	*	*

Open String degrees of freedom

In this brane system there are different open string sectors



- D3/D3 strings: gauge theory fields
- D3/D(-1) and D(-1)/D(-1) strings: instanton moduli!

- D3/D(-1) system on $\mathbb{R}^4 \times \mathbb{C}^3$

  $$\Downarrow$$

  $\mathcal{N} = 4$ SYM + instantons

- D3/D(-1) system on $\mathbb{R}^4 \times \mathbb{C} \times \mathbb{C}^2/\mathbb{Z}_2$

  $$\Downarrow$$

  $\mathcal{N} = 2$ SYM + instantons

- D3/D(-1) system on $\mathbb{R}^4 \times \mathbb{C}^3/(\mathbb{Z}_2 \times \mathbb{Z}_2)$

  $$\Downarrow$$

  $\mathcal{N} = 1$ SYM + instantons

Let us discuss this construction for $\mathcal{N} = 2$ theories

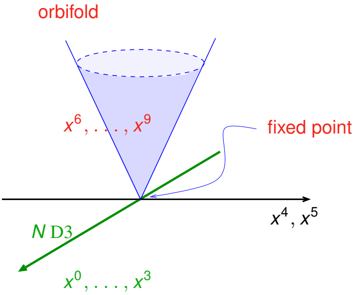
$\mathcal{N} = 2$ SU(N) SYM theory from fractional branes

- It is realized by the massless d.o.f. of open strings attached to N fractional D3-branes in the orbifold background

$$\mathbb{R}^4 \times \mathbb{R}^2 \times \mathbb{R}^4/\mathbb{Z}_2$$

where

$$\mathbb{Z}_2 \;:\; \{x^6, ..., x^9\} \quad \longrightarrow \quad \{-x^6, ..., -x^9\}$$



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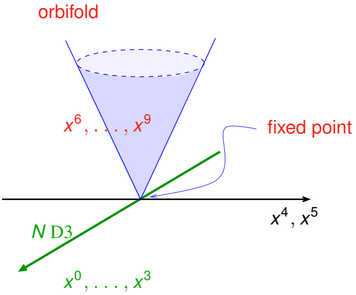

- The orbifold breaks 1/2 SUSY in the bulk, the D3 branes a further 1/2:

$$32 \times \frac{1}{2} \times \frac{1}{2} = 8 \ \text{ real supercharges} \quad \Longrightarrow \quad \mathcal{N} = 2 \text{ SUSY}$$

	0	1	2	3	4	5	6	7	8	9
D3	−	−	−	−	*	*	*	*	*	*

Since

$$SO(10) \rightarrow SO(4) \times U(1) \times SO(4)$$

the ten dimensional string coordinates X^M, ψ^M and spin fields $S^{\mathcal{A}}$ split as follows

$$X^M \longrightarrow X^\mu, X, \bar{X}, X^i \quad , \quad \psi^M \longrightarrow \psi^\mu, \Psi, \bar{\Psi}, \psi^i$$

$$S^{\mathcal{A}} \longrightarrow S_\alpha S_- S_A, S^{\dot\alpha} S^+ S^A, S_\alpha S_+ S_{\dot A}, S^{\dot\alpha} S^- S^{\dot A}$$

For example

$$\psi^\mu \in ((\mathbf{2},\mathbf{2}),0,(\mathbf{1},\mathbf{1})) \quad , \quad \bar{\Psi} \in ((\mathbf{1},\mathbf{1}),-1,(\mathbf{1},\mathbf{1}))$$

$$S_\alpha S_- S_A \in ((\mathbf{2},\mathbf{1}),-1/2,(\mathbf{2},\mathbf{1})) \quad , \quad S_\alpha S_+ S_{\dot A} \in ((\mathbf{2},\mathbf{1}),+1/2,(\mathbf{1},\mathbf{2}))$$

String vertex operators and fields

- String vertex operators:

$$V_A \simeq A_\mu \, \psi^\mu \, \mathrm{e}^{ip\cdot X} \, \mathrm{e}^{-\varphi}$$

$$V_\Lambda \simeq \Lambda^{\alpha A} \, S_\alpha S^- S_A \, \mathrm{e}^{ip\cdot X} \, \mathrm{e}^{-\frac{1}{2}\varphi}$$

$$V_\phi \simeq \phi \, \overline{\Psi} \, \mathrm{e}^{ip\cdot X} \, \mathrm{e}^{-\varphi}$$

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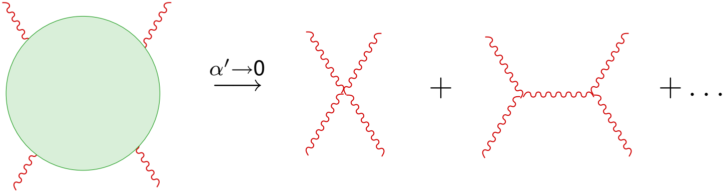
- Field content: $\mathcal{N} = 2$ vector superfield

$$\Phi(x, \theta) = \phi(x) + \theta\Lambda(x) + \frac{1}{2}\,\theta\sigma^{\mu\nu}\theta\, F^+_{\mu\nu}(x) + \cdots$$

Gauge action from disks on D3's



- String amplitudes on disks attached to the D3 branes in the limit

$$\alpha' \to 0 \text{ with gauge quantities fixed}$$

give rise to the $\mathcal{N} = 2$ SYM action

$$S_{\rm SYM} = \int d^4x \, \mathrm{Tr} \Big\{ \frac{1}{2} F_{\mu\nu}^2 + 2\, D_\mu \bar{\phi}\, D^\mu \phi - 2\, \bar{\Lambda}_A \bar{\not{D}}\, \Lambda^A$$

$$+ i\sqrt{2}\, g\, \bar{\Lambda}_A \epsilon^{AB} [\, \phi, \bar{\Lambda}_B] + i\sqrt{2}\, g\, \Lambda^A \epsilon_{AB} [\, \bar{\phi}, \Lambda^B] + g^2 [\, \phi, \bar{\phi}\,]^2 \Big\}$$

Effective action

- We are interested in the low-energy effective action on the Coulomb branch parametrized by the v.e.v.'s of the adjoint chiral superfields:

$$\langle \Phi_{uv} \rangle \equiv \langle \phi_{uv} \rangle = a_{uv} = a_u \, \delta_{uv} \quad , \quad u, v = 1, ..., N \quad , \quad \sum_u a_u = 0$$

  breaking $\mathrm{SU}(N) \to \mathrm{U}(1)^{N-1}$

- Up to two-derivatives, $\mathcal{N} = 2$ SUSY constrains the effective action for Φ to be of the form

$$S_{\text{eff}}[\Phi] = \int d^4x \, d^4\theta \, \mathcal{F}(\Phi) + \text{c.c}$$

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$$\mathcal{F}(\Phi) = \frac{\mathrm{i}}{2\pi}\Phi^2 \log \frac{\Phi^2}{\Lambda^2} + \sum_{k=1}^{\infty} \mathcal{F}^{(k)}(\Phi)$$

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- We will now discuss how to obtain the instanton corrections $\mathcal{F}^{(k)}$ to the prepotential $\mathcal{F}$ in our string set-up.

Instanton calculus in string theory

Moduli vertices and instanton parameters

Open strings with at least one end on a D(–1) carry no momentum: they are moduli, rather than dynamical fields.

D(–1)/D(–1) open strings have DD boundary conditions in all directions and the spectrum is:

	ADHM	Meaning	Vertex	Chan-Paton
NS	a'_μ	*centers*	$\psi^\mu \, e^{-\varphi}$	adj. U(k)
	χ	*aux.*	$\overline{\Psi} \, e^{-\varphi(z)}$	⋮
	D_c	*Lagrange mult.*	$\bar{\eta}^c_{\mu\nu} \psi^\nu \psi^\mu$	⋮
R	$M^{\alpha A}$	*partners*	$S_\alpha S_- S_A \, e^{-\frac{1}{2}\varphi}$	⋮
	$\lambda_{\dot\alpha A}$	*Lagrange mult.*	$S^{\dot\alpha} S^+ S^A \, e^{-\frac{1}{2}\varphi}$	⋮

In the direction 0, 1, 2, 3 the string coordinate X_μ and ψ_μ satisfy mixed ND or DN boundary conditions $\Rightarrow$ the moding is shifted by 1/2 and hence we have an unusual spectrum:

- the lowest level of the NS sector is a bosonic spinor of $\mathrm{SO}(4)$
- the lowest level of the R sector is a fermionic scalar of $\mathrm{SO}(4)$

	ADHM	Meaning	Vertex	Chan-Paton
NS	$w_{\dot\alpha}$	*sizes*	$\Delta S^{\dot\alpha}\, \mathrm{e}^{-\varphi}$	$k \times N$
	$\bar{w}_{\dot\alpha}$	*sizes*	$\overline{\Delta} S^{\dot\alpha}\, \mathrm{e}^{-\varphi}$	$N \times k$
R	μ^A	*partners*	$\Delta S_- S_A \mathrm{e}^{-\frac{1}{2}\varphi}$	$k \times N$
	$\bar{\mu}^A$	$\vdots$	$\overline{\Delta} S_- S_A\, \mathrm{e}^{-\frac{1}{2}\varphi}$	$N \times k$

Δ and $\overline{\Delta}$ are the Twist Fields whose insertion modify the boundary conditions from D3 to D(–1) type and viceversa.

Super-coordinates and centered moduli

- Among the D(–1)/D(–1) moduli we can single out the instanton center x_0^μ and its super-partners $\theta^{\alpha A}$:

$$
\begin{aligned}
a'^{\mu} &= x_0^\mu \, \mathbb{1}_{k\times k} + y_c^\mu \, T^c \quad (T^c = \text{generators of } \mathrm{SU}(k)) \\
M^{\alpha A} &= \theta^{\alpha A} \, \mathbb{1}_{k\times k} + \zeta_c^{\alpha A} \, T^c
\end{aligned}
$$

  The moduli x_0^μ and $\theta^{\alpha A}$ decouple from many interactions and play the rôle of $\mathcal{N} = 2$ superspace coordinates.

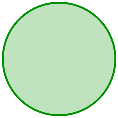
- We will distinguish the moduli $\mathcal{M}_{(k)}$ into

$$
\mathcal{M}_{(k)} \to \left\{ x_0, \theta \, ; \, \widehat{\mathcal{M}}_{(k)} \right\}
$$

  where $\widehat{\mathcal{M}}_{(k)}$ are the so-called centered moduli.

Disk amplitudes and effective actions

D3 disks



Disk amplitudes and effective actions

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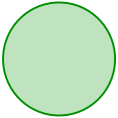
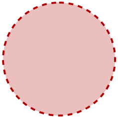


D(–1) disks



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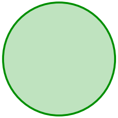

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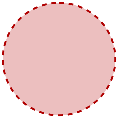
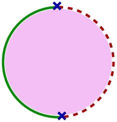


D3/D(-1)
mixed disks



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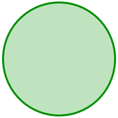

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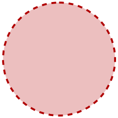

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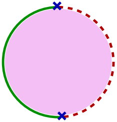


Disk amplitudes

↓ field theory limit $\alpha' \to 0$

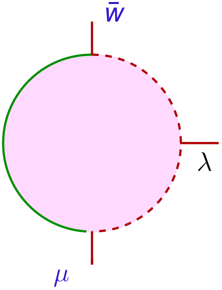
Effective actions

D3 disks → $\mathcal{N} = 2$ SYM action

D(–1) and mixed disks → ADHM measure

An example of a mixed disk amplitude

Consider the following mixed disk diagram



which corresponds to the following amplitude

$$\left\langle\!\left\langle V_\lambda V_{\bar{w}} V_\mu \right\rangle\!\right\rangle \equiv C_0 \int \frac{\prod_i dz_i}{dV_{\text{CKG}}} \times \langle V_\lambda(z_1) V_{\bar{w}}(z_2) V_\mu(z_3)\rangle = \ldots = \mathrm{tr}_k \left\{ \mathrm{i}\, \lambda^{\dot\alpha}_A\ \bar{w}_{\dot\alpha}\ \mu^A \right\}$$

where $C_0 = 8\pi^2/g^2$ is the disk normalization.

The action for the instanton moduli

From all D(–1) and mixed disk diagrams with insertion of all moduli vertices, we can extract the ADHM moduli action (at fixed k)

$$\mathcal{S}_{\text{mod}} = \mathcal{S}^{(k)}_{\text{bos}} + \mathcal{S}^{(k)}_{\text{fer}} + \mathcal{S}^{(k)}_{\text{c}}$$

with

$$\begin{aligned}
\mathcal{S}^{(k)}_{\text{bos}} &= \mathrm{tr}_k\Big\{ -2\,[\chi^\dagger, a'_\mu][\chi, a'^\mu] + \chi^\dagger \bar{w}_{\dot\alpha} w^{\dot\alpha}\chi + \chi \bar{w}_{\dot\alpha} w^{\dot\alpha}\chi^\dagger \Big\} \\
\mathcal{S}^{(k)}_{\text{fer}} &= \mathrm{tr}_k\Big\{ \mathrm{i}\,\frac{\sqrt{2}}{2}\,\bar\mu^A \epsilon_{AB}\mu^B \chi^\dagger - \mathrm{i}\,\frac{\sqrt{2}}{4}\,M^{\alpha A}\epsilon_{AB}[\chi^\dagger, M^B_\alpha]\Big\} \\
\mathcal{S}^{(k)}_{\text{c}} &= \mathrm{tr}_k\Big\{ -\mathrm{i}D_c\big(\bar{w}_{\dot\alpha}(\tau^c)^{\dot\alpha\dot\beta} w_{\dot\beta} + \mathrm{i}\bar\eta^c_{\mu\nu}\,[a'^\mu, a'^\nu]\big) \\
&\qquad -\mathrm{i}\lambda^{\dot\alpha}_A\big(\bar\mu^A w_{\dot\alpha} + \bar{w}_{\dot\alpha}\mu^A + [a'_{\alpha\dot\alpha}, M'^{\alpha A}]\big)\Big\}
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\end{aligned}$$

- In $\mathcal{S}^{(k)}_{\text{c}}$ the bosonic and fermionic ADHM constraints appear

Take for simplicity $k = 1$ ($\longrightarrow [\ ,\ ] = 0$). The classical vacuum is such that

$$w_{u\dot{\alpha}}\,\chi = 0 \quad , \qquad \bar{w}_{\dot{\alpha}u}(\tau^c)^{\dot{\alpha}\dot{\beta}} w_{u\dot{\beta}} = 0 \quad \text{ADHM constraints}$$

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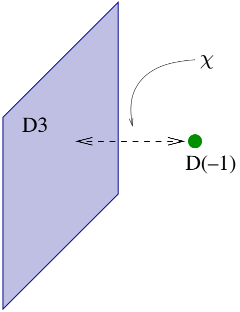
There are two types of solutions:

$$\chi \neq 0 \quad , \quad w_{u\dot\alpha} = 0$$



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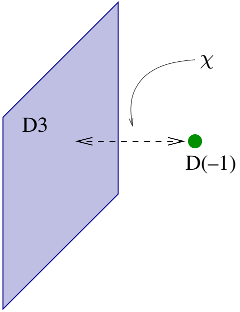
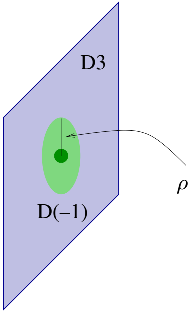


$$\chi = 0 \ , \quad w_{u\dot\alpha} = \rho \begin{pmatrix} 1_{2\times 2} \\ 0_{(N-2)\times 2} \end{pmatrix}$$



Properties of the moduli action $\mathcal{S}_{\text{mod}}$

- $\mathcal{S}_{\text{mod}}$ depends only on the centered moduli $\widehat{\mathcal{M}}_{(k)}$ but does not depend on the center x_0^μ and its super-partners $\theta^{\alpha A}$ $\Longrightarrow$ the Goldstone modes of the broken (super)translations

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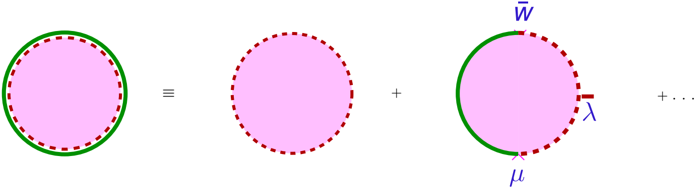
- $\mathcal{S}_{\text{mod}}$ depends only on the centered moduli $\widehat{\mathcal{M}}_{(k)}$ but does not depend on the center x_0^μ and its super-partners $\theta^{\alpha A} \Longrightarrow$ the Goldstone modes of the broken (super)translations
- The other neutral (anti-chiral) fermionic zero-modes $\lambda_{\dot{\alpha}A}$ appear in $\mathcal{S}_{\text{mod}}$ only as Lagrangian multipliers for the fermionic ADHM constraints.

Integration over all moduli leads to the instanton partition function

[Polchinski 1994, ..., Dorey et al. 1999, ...]

$$Z^{(k)} = \int d^4x_0 \, d^4\theta \, d\widehat{\mathcal{M}}_{(k)} \; e^{-\frac{8\pi^2 k}{g^2} - \mathcal{S}_{\rm mod}(\widehat{\mathcal{M}}_{(k)})}$$

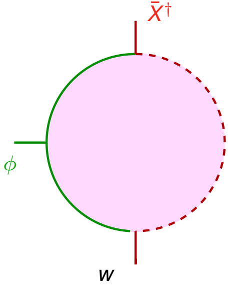
where the exponent is



$$\overset{\alpha'\to 0}{\simeq} \quad -\frac{8\pi^2 k}{g^2} \quad - \quad \mathcal{S}_{\rm mod}(\widehat{\mathcal{M}}_{(k)})$$

Field-dependent moduli action

- The interactions between the moduli and the scalar fields ϕ of the gauge theory are described by mixed disk diagrams with insertions of V_ϕ and $V_{\bar{\phi}}$



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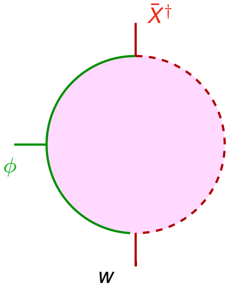

$\Longrightarrow \mathcal{S}_{\text{mod}}(\phi, \bar{\phi}; \widehat{\mathcal{M}}_{(k)})$

- Exploiting the broken SUSY, we can promote $\phi(x)$ to the full chiral superfield $\Phi(x, \theta)$ $\Longrightarrow$
  Superfield-dependent action $\mathcal{S}_{\text{mod}}(\Phi, \bar{\Phi}; \widehat{\mathcal{M}}_{(k)})$

Instanton contributions to the prepotential

Integrating over the moduli one gets the instanton induced effective action for Φ:

$$S_{\text{eff}}^{(k)}[\Phi] = \int d^4x\, d^4\theta\, d\widehat{\mathcal{M}}_{(k)}\, e^{-\frac{8\pi k}{g^2} - \mathcal{S}_{\text{mod}}(\Phi,\bar{\Phi};\widehat{\mathcal{M}}_{(k)})}$$
$$= \int d^4x\, d^4\theta\, \mathcal{F}^{(k)}(\Phi)$$

Correspondingly, the prepotential $\mathcal{F}^{(k)}$ for the low energy $\mathcal{N} = 2$ theory is given by the centered instanton partition function

$$\mathcal{F}^{(k)}(\Phi) = \int d\widehat{\mathcal{M}}_{(k)}\, e^{-\frac{8\pi k}{g^2} - \mathcal{S}_{\text{mod}}(\Phi,\bar{\Phi};\widehat{\mathcal{M}}_{(k)})}$$

- For example, for the $\mathcal{N} = 2$ SYM theory with $\mathrm{SU}(2)$ (broken to $\mathrm{U}(1)$), one finds

$$\mathcal{F}^{(k)}(\Phi) = c_k \, \Phi^2 \, \left(\frac{\Lambda}{\Phi}\right)^{4k}$$

where Λ is the dynamically generated scale, and the coefficients c_k can be obtained by evaluating the integral over the instanton moduli:

$$c_1 = \frac{1}{2} \quad , \quad c_2 = \frac{5}{16} \quad , \quad \ldots$$

(in perfect agreement with the Seiberg-Witten exact solution of the theory).

- ...

Instanton classical solution

- The mixed disks are the sources for a non-trivial gauge field whose profile is exactly that of the classical instanton!!

[Billó et al. 2002,...]

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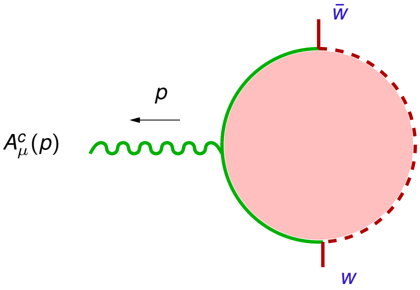
Let us consider the following mixed-disk amplitude:



$$\equiv \left\langle \mathcal{V}_{A^c_\mu(p)} \right\rangle_{\text{mixed disk}}$$

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[Billó et al. 2002,...]

Using the explicit expressions of the vertex operators, for $SU(2)$ with $k = 1$ one finds

$$\left\langle \mathcal{V}_{A^c_\mu(p)} \right\rangle_{\text{mixed disk}} \equiv \left\langle V_{\bar{w}}\, \mathcal{V}_{A^c_\mu}(p)\, V_w \right\rangle$$

$$= -i\, p^\nu\, \bar{\eta}^c_{\mu\nu} \left(\bar{w}^{\dot{\alpha}}\, w_{\dot{\alpha}} \right) \mathrm{e}^{-i\, p \cdot x_0} \equiv A^c_\mu(p; w, x_0)$$

- On this mixed disk the gauge vector field has a non-vanishing tadpole!

- Taking the Fourier transform of $A^c_\mu(p; w, x_0)$, after inserting the free propagator $1/p^2$, we obtain

$$A^c_\mu(x) \equiv \int \frac{d^4p}{(2\pi)^2} \; A^c_\mu(p; w, x_0)\, \frac{1}{p^2}\, \mathrm{e}^{i\,p\cdot x} = 2\,\rho^2\,\bar{\eta}^c_{\mu\nu}\,\frac{(x-x_0)^\nu}{(x-x_0)^4}$$

where we have used the solution of the ADHM constraints and defined $\bar{w}^{\dot\alpha}\,w_{\dot\alpha} = 2\,\rho^2$.

- This is the leading term in the large distance expansion of an $\mathrm{SU}(2)$ instanton with size ρ and center x_0 in the singular gauge!!

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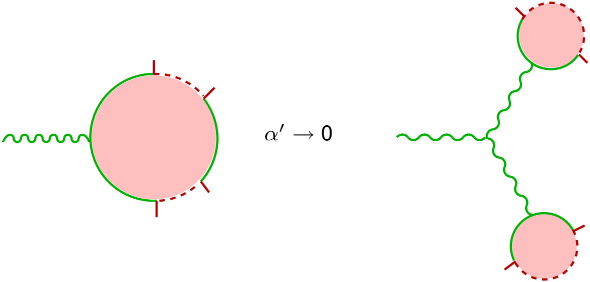
- This is the leading term in the large distance expansion of an $\mathrm{SU}(2)$ instanton with size ρ and center x_0 in the singular gauge!!
- In fact

$$\begin{aligned} A^c_\mu(x)\Big|_{\text{instanton}} &= 2\,\rho^2\,\bar\eta^c_{\mu\nu}\,\frac{(x-x_0)^\nu}{(x-x_0)^2\,\left[(x-x_0)^2+\rho^2\right]} \\ &= 2\,\rho^2\,\bar\eta^c_{\mu\nu}\,\frac{(x-x_0)^\nu}{(x-x_0)^4}\left(1-\frac{\rho^2}{(x-x_0)^2}+\ldots\right) \end{aligned}$$

- The subleading terms in the large distance expansion can be obtained from mixed disks with more insertions of moduli.

- For example, at the next-to-leading order we have to consider the following mixed disk which can be easily evaluated for $\alpha' \to 0$

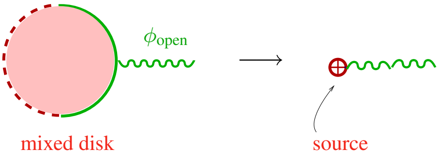


- Its Fourier transform gives precisely the 2nd order in the large distance expansion of the instanton profile

$$A^c_\mu(x)^{(2)} = -2\,\rho^4\,\bar{\eta}^c_{\mu\nu}\,\frac{(x-x_0)^\nu}{(x-x_0)^6}$$

Summary

- Mixed disks are sources for open strings



$$\longrightarrow \left\langle \mathcal{V}_{\phi\ \text{open}} \right\rangle_{\text{mixed disk}} \neq 0$$

- The gauge field emitted from mixed disks is precisely that of the classical instanton

$$\left\langle \mathcal{V}_{A_\mu} \right\rangle_{\text{mixed disk}} \quad \Leftrightarrow \quad A_\mu \Big|_{\text{instanton}}$$

- This procedure can be easily generalized to the SUSY partners of the gauge boson.

New applications

Gauge theories in gravitational background

Gauge theories in non trivial gravitational (*i.e.* closed string) backgrounds are very interesting because, in general, they are characterized by

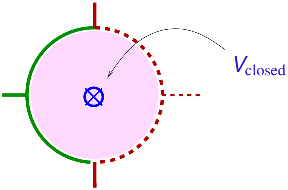
- new geometry in (super)space-time
- new types of interactions and couplings

Closed string backgrounds produce deformations in the gauge theories. For instance:

- non-commutative theories arise from NSNS background $B_{\mu\nu}$
- non-anticommutative theories from specific RR backgrounds
- non-supersymmetric theories may arise from supersymmetric models in presence of non trivial closed string fluxes in the internal space

- The instanton calculus through mixed disks can be easily generalized in the presence of a non-trivial closed string background

- One simply computes mixed disks with one or more insertions of closed string vertex operators



- These new disks produce new terms in the ADHM moduli action and suitably "deform" the instanton calculus.

A very interesting example:

Instanton calculus in a graviphoton background of $\mathcal{N} = 2$ SUGRA

[Billò et al. 2004]

This RR background allows:

- to find the gravitational corrections to the prepotential of the $\mathcal{N} = 2$ SYM theory
- to deform the ADHM measure in such a way that the instanton contributions can be computed via localization techniques
- to clarify a recent conjecture by N. Nekrasov on the so-called Ω-background
- to establish a nice correspondence with the topological string

Another interesting application:

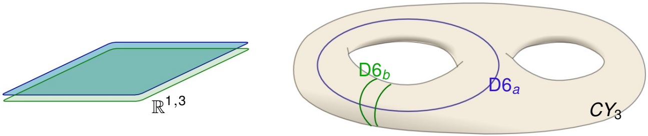
Stringy instantons

[Blumenhagen et al. 2006, Ibanez-Uranga 2006, ...]

To engineer realistic 4-d SYM theories one has to use systems of wrapped D-branes in $\mathbf{R}^{1,3} \times CY_3$.

Standard model like models, with chiral matter and interesting phenomenology:

- Type IIB : magnetized D9 branes, wrapping the CY_3
- Type IIA : intersecting D6, wrapping a 3-cycle in CY_3 (easier to visualize)



- the different number of families with different coupling constants arises from multiple intersections

- the low energy theory can be naturally described by SUGRA with vector and matter multiplets

- the interactions in the various sectors can be derived directly from string amplitudes

- the are novel non-perturbative stringy effects in the effective action (!!)

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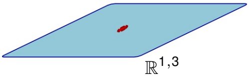
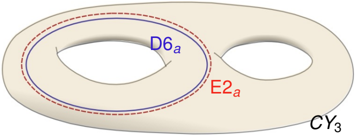
- In type IIA point-like configurations in $\mathbb{R}^{1,3}$ that correspond to gauge instantons are Euclidean D2 branes (E2) that wrap the same 3-cycle as the D6 branes
- The D6/E2 system is analogous to the D3/D(–1) system in flat space:
  - there are 4 mixed ND directions
  - the ADHM construction is obtained from open strings





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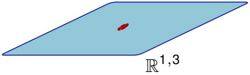

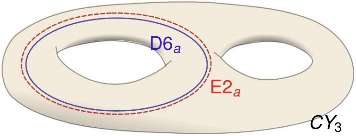

- In type IIB we should consider instead $D9_a$/$E5_a$ branes (with the same magnetization) in the internal space, wrapped on the entire CY space.

Other possibility: exotic instantons

- Type IIA E2 branes wrapped differently from the color D6 branes are still point-like in $\mathbb{R}^{1,3}$ but they do not correspond to ordinary gauge instanton configurations.

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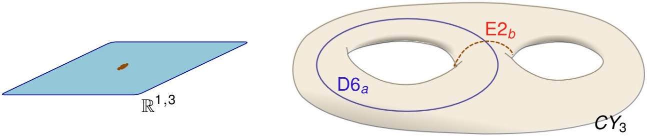
- Type IIA E2 branes wrapped differently from the color D6 branes are still point-like in $\mathbb{R}^{1,3}$ but they do not correspond to ordinary gauge instanton configurations.
  - there are more than 4 mixed ND directions
  - there are no bosonic moduli from the mixed sectors ($\rightarrow$ no w's and $\bar{w}$'s)
  - there are no ADHM-like constraints



- Their “field theory” interpretation is not yet clear but still they may give important non-perturbative contributions to the effective action, e.g. Majorana masses for neutrinos, moduli stabilizing terms, . . .

[Blumenhagen et al, 2006; Ibanez and Uranga, 2006;...]

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[Blumenhagen et al, 2006; Ibanez and Uranga, 2006;...]

- In type IIB we consider instead systems with $D9_a$ and $E5_b$ branes wrapped on the entire CY space but with different magnetizations (*i.e.* $a \neq b$).

[Billó, Di Vecchia, M.F, Lerda, Marotta, Pesando]

Conclusions and perspectives

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- Non-trivial closed string backgrounds can be easily incorporated
- Generalizations of the gauge instantons to truly stringy configurations are possible and lead to very interesting effects