CLOCK SYNCHRONIZATION
IN
SPECIAL AND GENERAL RELATIVITY:

FROM ACES TO THE YORK MAP AND BEYOND

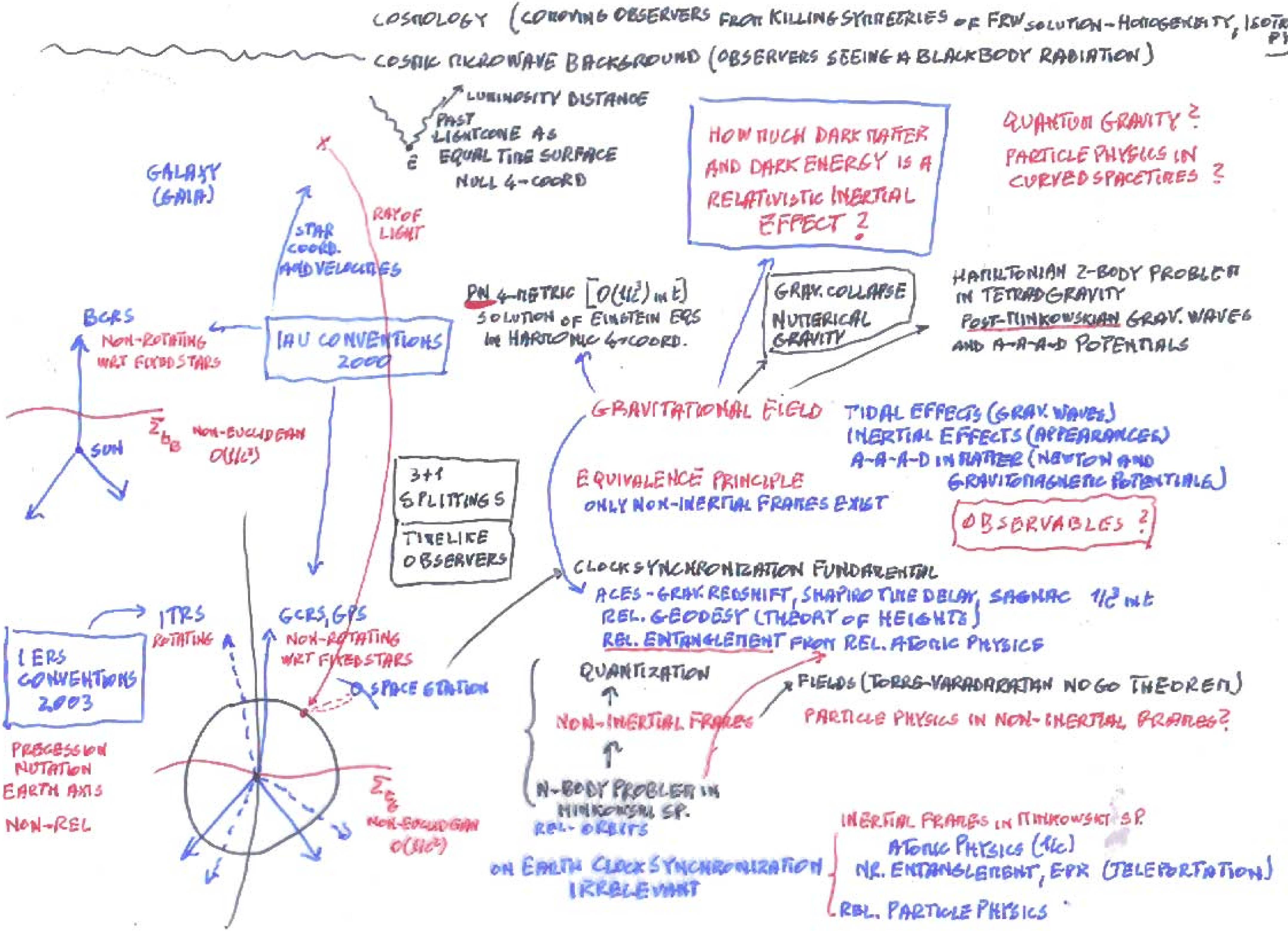
LUCA LUSANNA

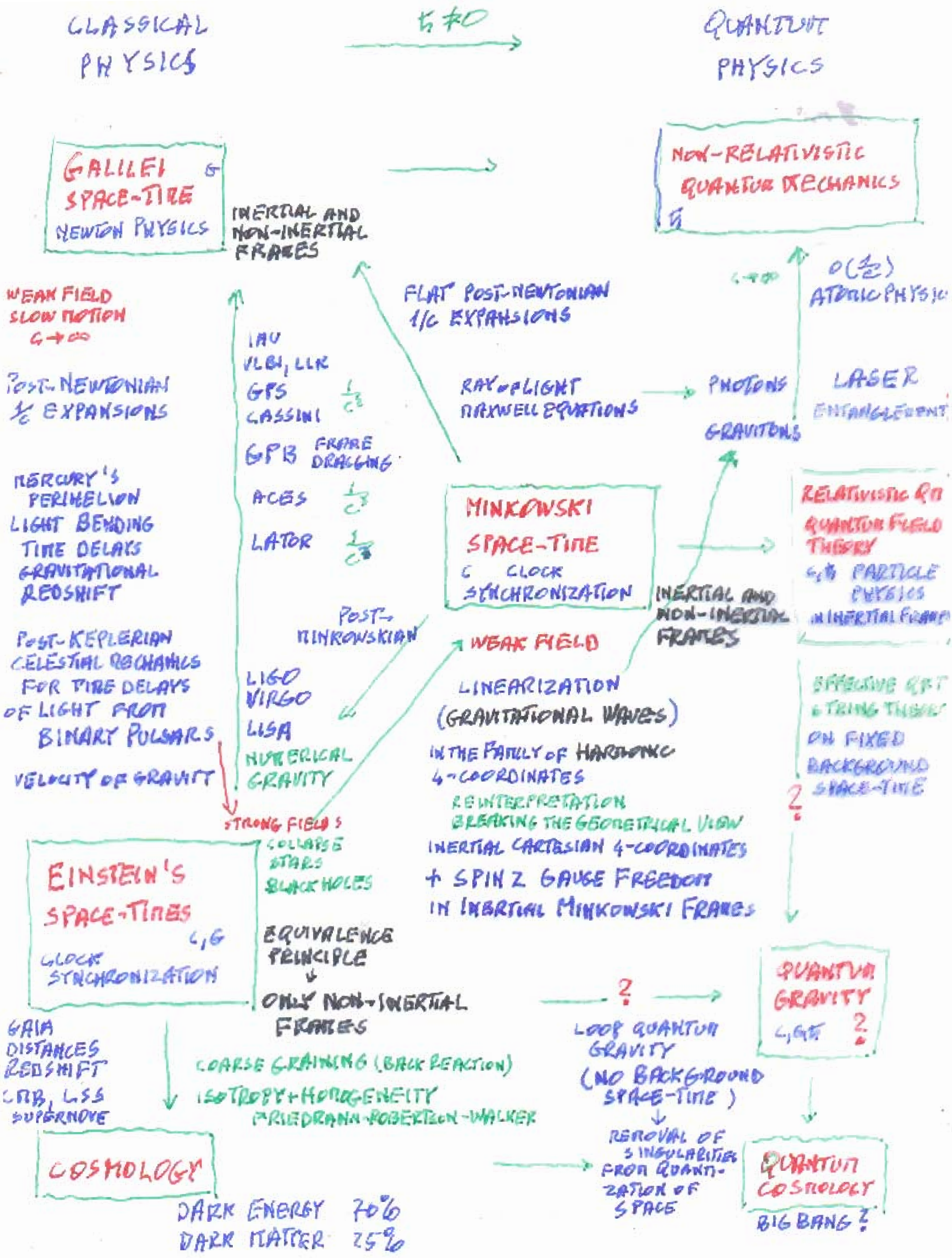
INFN FIRENZE

ROMA, PARMA, NAPOLI

DICEMBRE 2007

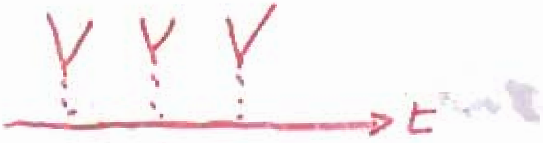
GENNAIO 2008


COSMOLOGY (COMOVING OBSERVERS FROM KILLING SYMMETRIES OF FRW SOLUTION - HOMOGENEITY, ISOTR
PY
COSMIC MICROWAVE BACKGROUND (OBSERVERS SEEING A BLACKBODY RADIATION)
LUMINOSITY DISTANCE
PAST LIGHTCONE AS EQUAL TIME SURFACE
NULL 4-COORD
E
GALAXY (GAIA)
X
RAY OF LIGHT
STAR COORD. AND VELOCITIES
BCRS
NON-ROTATING WRT FIXED STARS
IAU CONVENTIONS 2000
SUN
Σ_{b_B}
NON-EUCLIDEAN $O(1/c^3)$
PN 4-METRIC $[O(1/c^3)$ in $t]$
SOLUTION OF EINSTEIN EQS IN HARMONIC 4-COORD.
3+1 SPLITTINGS
TIMELIKE OBSERVERS
HOW MUCH DARK MATTER AND DARK ENERGY IS A RELATIVISTIC INERTIAL EFFECT ?
QUANTUM GRAVITY ?
PARTICLE PHYSICS IN CURVED SPACETIMES ?
GRAV. COLLAPSE
NUMERICAL GRAVITY
HAMILTONIAN 2-BODY PROBLEM IN TETRAD GRAVITY
POST-MINKOWSKIAN GRAV. WAVES AND A-A-A-D POTENTIALS
GRAVITATIONAL FIELD
TIDAL EFFECTS (GRAV. WAVES)
INERTIAL EFFECTS (APPEARANCES)
A-A-A-D IN MATTER (NEWTON AND GRAVITOMAGNETIC POTENTIALS)
OBSERVABLES ?
EQUIVALENCE PRINCIPLE
ONLY NON-INERTIAL FRAMES EXIST
CLOCK SYNCHRONIZATION FUNDAMENTAL
ACES - GRAV. REDSHIFT, SHAPIRO TIME DELAY, SAGNAC $1/c^3$ in t
REL. GEODESY (THEORY OF HEIGHTS)
REL. ENTANGLEMENT FROM REL. ATOMIC PHYSICS
IERS CONVENTIONS 2003
ITRS
ROTATING
GCRS, GPS
NON-ROTATING WRT FIXED STARS
SPACE STATION
PRECESSION
NUTATION
EARTH AXIS
NON-REL
Σ_{E_E}
NON-EUCLIDEAN $O(1/c^3)$
QUANTIZATION
NON-INERTIAL FRAMES
N-BODY PROBLEM IN MINKOWSKI SP.
REL. ORBITS
FIELDS (TORRE-VARADARAJAN NO GO THEOREM)
PARTICLE PHYSICS IN NON-INERTIAL FRAMES?
INERTIAL FRAMES IN MINKOWSKI SP.
ATOMIC PHYSICS (1/c)
NR. ENTANGLEMENT, EPR (TELEPORTATION)
REL. PARTICLE PHYSICS
ON EARTH CLOCK SYNCHRONIZATION IRRELEVANT


CLASSICAL PHYSICS
$\hbar \neq 0$
QUANTUM PHYSICS
GALILEI SPACE-TIME
$\hbar$
NEWTON PHYSICS
INERTIAL AND NON-INERTIAL FRAMES
NON-RELATIVISTIC QUANTUM MECHANICS
$\hbar$
WEAK FIELD SLOW MOTION
$c \to \infty$
POST-NEWTONIAN 1/c EXPANSIONS
IAU
VLBI, LLR
GPS
$\frac{1}{c^2}$
CASSINI
GPB FRAME DRAGGING
ACES
$\frac{1}{c^3}$
LATOR
$\frac{1}{c^4}$
FLAT POST-NEWTONIAN 1/c EXPANSIONS
$c \to \infty$
$O(\frac{1}{c^2})$
ATOMIC PHYSIC
RAY of LIGHT
MAXWELL EQUATIONS
PHOTONS
GRAVITONS
LASER
ENTANGLEMENT
MERCURY'S PERIHELION
LIGHT BENDING
TIME DELAYS
GRAVITATIONAL REDSHIFT
MINKOWSKI SPACE-TIME
c CLOCK SYNCHRONIZATION
INERTIAL AND NON-INERTIAL FRAMES
RELATIVISTIC QM
QUANTUM FIELD THEORY
$c, \hbar$ PARTICLE PHYSICS
IN INERTIAL FRAMES
POST-KEPLERIAN CELESTIAL MECHANICS FOR TIME DELAYS OF LIGHT FROM BINARY PULSARS
VELOCITY OF GRAVITY
POST-MINKOWSKIAN
LIGO
VIRGO
LISA
NUMERICAL GRAVITY
WEAK FIELD
LINEARIZATION
(GRAVITATIONAL WAVES)
IN THE FAMILY OF HARMONIC 4-COORDINATES
REINTERPRETATION
BREAKING THE GEOMETRICAL VIEW
INERTIAL CARTESIAN 4-COORDINATES
+ SPIN 2 GAUGE FREEDOM
IN INERTIAL MINKOWSKI FRAMES
EFFECTIVE QFT
STRING THEORY
ON FIXED BACKGROUND SPACE-TIME
?
STRONG FIELDS
COLLAPSE STARS
BLACK HOLES
EINSTEIN'S SPACE-TIMES
c, G
CLOCK SYNCHRONIZATION
EQUIVALENCE PRINCIPLE
ONLY NON-INERTIAL FRAMES
?
QUANTUM GRAVITY
$c, G, \hbar$
?
LOOP QUANTUM GRAVITY
(NO BACKGROUND SPACE-TIME)
REMOVAL OF SINGULARITIES FROM QUANTIZATION OF SPACE
GAIA
DISTANCES
REDSHIFT
CMB, LSS
SUPERNOVE
COARSE GRAINING (BACK REACTION)
ISOTROPY + HOMOGENEITY
FRIEDMANN-ROBERTSON-WALKER
COSMOLOGY
QUANTUM COSMOLOGY
BIG BANG ?
DARK ENERGY 70%
DARK MATTER 25%

NON-RELATIVISTIC PHYSICS

(NEWTON'S THEORY IN GALILEI SPACETIME)



ABSOLUTE TIME
ABSOLUTE 3-SPACE (EUCLIDEAN)

⇓

ABSOLUTE SIMULTANEITY AND SPATIAL DISTANCE

GALILEI LAW OF INERTIA → INERTIAL OBSERVERS

GALILEI RELATIVITY PRINCIPLE – INVARIANCE IN FORM OF $\vec{F} = m\vec{a}$ IN INERTIAL FRAMES (CENTERED ON INERTIAL OBSERVERS) WITH CARTESIAN 3-COORDINATES – KINEMATICAL GALILEI GROUP

NEWTON'S GRAVITY – A-A-A-D INTERACTION WITH GALILEI EQUIVALENCE PRINCIPLE

$$m\vec{a} = \vec{F}_G = m_G\,\vec{g} \qquad m = m_G$$

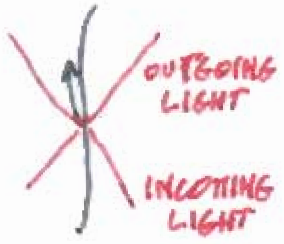
INERTIAL FRAMES – IDEAL LIMIT

RIGID NON-INERTIAL FRAMES CENTERED ON ACCELERATED OBSERVERS – INERTIAL FORCES ∝ INERTIAL MASS

$$\vec{a} = \frac{\vec{F}}{m} - \vec{W}(t) + \vec{x} \times \frac{d\vec{\omega}(t)}{dt} + 2\,\frac{d\vec{x}}{dt} \times \vec{\omega}(t) + \vec{\omega}(t) \times [\vec{x} \times \vec{\omega}(t)]$$

LINEAR JACOBI CORIOLIS CENTRIFUGAL

SPECIAL RELATIVITY (MINKOWSKI SPACETIME)



ABSOLUTE FLAT SPACETIME

$$ds^2 = \epsilon[c^2dt^2 - d\vec{x}^2] = \eta_{\mu\nu}\,dx^\mu dx^\nu$$

$$\eta_{\mu\nu} = \epsilon\,(+---), \quad \epsilon = \pm 1$$

CARTESIAN 4-COORDINATES

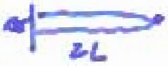
ONLY THE CONFORMAL STRUCTURE (THE LIGHT CONE) IS INTRINSICALLY GIVEN

NO NOTION OF
- INSTANTANEOUS 3-SPACE, SPATIAL DISTANCE
- SIMULTANEITY (SYNCHRONIZATION OF DISTANT CLOCKS NEEDED)
- CAUCHY 3-SURFACE FOR MAXWELL EQs
- 1-WAY VELOCITY OF LIGHT (2 SYNCHRONIZED CLOCKS NEEDED)

LIGHT POSTULATES – THE 2-WAY (OR ROUND TRIP; ONLY 1 CLOCK NEEDED) VELOCITY OF LIGHT IS A) ISOTROPIC B) CONSTANT $c = 2.997924580\ 10^8\, m s^{-1}$

METROLOGY – c and standard of time

$L \approx \frac{1}{2} c\Delta t$

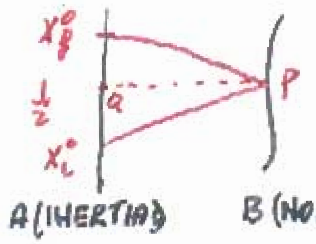


↳ but curved 3-space?

LAW OF INERTIA → INERTIAL OBSERVERS

SPECIAL RELATIVISTIC RELATIVITY PRINCIPLE - INVARIANCE IN FORM OF PHYSIC'S LAWS IN INERTIAL FRAMES (CENTERED ON INERTIAL OBSERVERS) WITH CARTESIAN 4-COORDINATES - KINEMATICAL POINCARÉ GROUP

→ IDENTIFIED BY EINSTEIN'S $\frac{1}{2}$ CONVENTION FOR CLOCK SYNCHRONIZATION



$$x^0_P \overset{def}{=} x^0_Q = x^0_i + \frac{1}{2}(x^0_f - x^0_i) = \frac{1}{2}(x^0_f + x^0_i)$$

$$\Downarrow$$

3 SPACES = EUCLIDEAN $x^0 = ct = const$ HYPERPLANES

1-WAY = 2-WAY = c


=


INERTIAL OBSERVERS = IDEAL LIMIT → ACCELERATED OBSERVES

ACCELERATION LENGTH $\begin{cases} \mathcal{L} = c^2/a & \text{TRANSLATIONAL} \\ \mathcal{L} = c/\omega & \text{ROTATIONAL} \end{cases}$ (~1 LY
(~10 AU,

LOCALITY HYPOTHESIS - an accelerated observer is a succession of instantaneously comoving inertial observers

$\lambda > \mathcal{L}$ PROBLEMS

1+3 POINT OF VIEW

ONLY THE OBSERVER'S WORLDLINE AND 4-VELOCITY ARE GIVEN

NO KNOWN METHOD TO DEFINE GLOBAL INSTANTANEOUS 3-SPACES (CAUCHY SURFACES)

FERMI COORD.

ROTATING DISK

$\omega R = c \Rightarrow g_{00}(Y^0, \vec{Y} = \omega R \hat{u} = c \hat{u})$
$= 0$

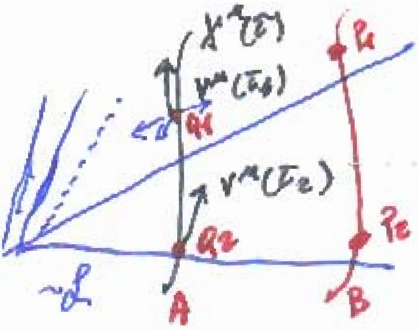
↓ $x^\mu \to y^\mu(x),\ \eta_{\mu\nu} \to g_{\mu\nu}(y)$

SAGNAC EFFECT

COORDINATE SYNGULARITIES

1+3 POINT OF VIEW

GIVEN THE WORLD-LINE OF AN ACCELERATED OBSERVER, I.E. $x^\mu(\tau)$ WITH 4-VELOCITY $v^\mu(\tau) = \dot{x}^\mu(\tau)$ [$v^2(\tau) = 1$ IF τ PROPER TIME], TRY TO BUILD AN INSTANTANEOUS 3-SPACE USING ONLY IT !



THE ONLY QUANTITY WHICH CAN BE DEFINED IS THE SPACE-LIKE VECTOR SPACE OF THE VECTORS ORTHOGONAL TO THE 4-VELOCITY AND

IDENTIFY IT WITH AN INSTANTANEOUS 3-SPACE

THEN DEFINE NON-INERTIAL 4-COORDINATES CENTERED ON THE OBSERVER

LIKE TO APPROXIMATE
LOCALLY A CURVE WITH
ITS TANGENT PLANE
IN A POINT

FERMI 4-COORDINATES (FERMI, ..., MARTZLIN, MASHHOON ----- MARCKE-WHEELER, PAURI-VALLISNERI ...)

THESE INSTANTANEOUS HYPERPLANES CROSS AT A DISTANCE OF THE ORDER OF THE ACCELERATION RADII OF THE OBSERVER

$\Rightarrow$ ONLY LOCAL 4-COORDINATE SYSTEM WITH ORIGIN IN A

$$y^\mu \mapsto g_{\mu\nu}(y)$$

$\rightarrow$ COORDINATE SINGULARITY

$\Rightarrow$ BAD (ONLY LOCAL) SIMULTANEITY NOTION IF THE OBSERVER A USES EINSTEIN'S CONVENTION TO SYNCHRONIZE THE CLOCK OF B

IF $y^\mu_A = x^\mu$ WITH PROPER TIME $\tau_A = \frac{1}{c}\int_0^A \sqrt{g_{oo}(y)}\, dy^o$

AND $y^\mu_B = y^\mu_A + \Delta y^\mu$

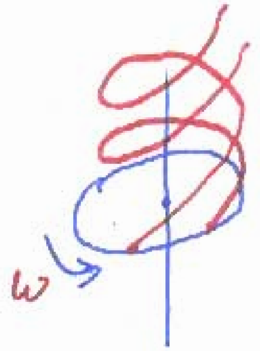
THEN EINSTEIN'S CONVENTION (WHICH IMPLIES $dx^o = 0$ or $d\tau = 0$ IF A WAS AN INERTIAL OBSERVER) APPLIED TO THE NON-INERTIAL OBSERVER A IMPLIES

1) $$c\,\Delta\tau = \sqrt{g_{oo}(y)}\,\Delta y^o + \frac{g_{oi}(y)}{\sqrt{g_{oo}(y)}}\,\Delta y^i = 0$$ LOCAL (NOT GLOBAL) SIMULTANEITY CONDITION

2) A SPATIAL DISTANCE $\Delta \ell_{AB} = \sqrt{{}^3\gamma_{ij}(y)\,\Delta y^i \Delta y^j}$ ON THE ISTANTANEOUS 3-SPACE WITH THE LANDAU-LIFSCHIZ 3-METRIC $${}^3\gamma_{ij} = g_{ij} - \frac{g_{oi}\,g_{oj}}{g_{oo}}$$

$\Rightarrow$ 1-WAY VELOCITY OF LIGHT $\neq c$ AND POINT DEPENDENT

ROTATING DISK (EINSTEIN, EHRENFEST, ...)



IT IS DESCRIBED BY MAKING A COORDINATE TRANSFORMATION FROM

INERTIAL CARTESIAN 4-COORD. x^{μ} $\longrightarrow$ ROTATING (USUALLY GALILEAN) 4-COORD. y^{μ}

$$\eta_{\mu\nu} \longrightarrow g_{\mu\nu}(y)$$

WITH THESE 4-COORD. AT A DISTANCE R FROM THE ROTATION AXIS WHERE $\omega R = c$ WE GET $g_{00}(y^{o}, \omega R\hat{n} = c\hat{n}) = 0$

$\Rightarrow$ HORIZON PROBLEM (TIMELIKE VECTORS BECOME NULL VECTORS AT R LIKE ON A BLACK-HOLE HORIZON)

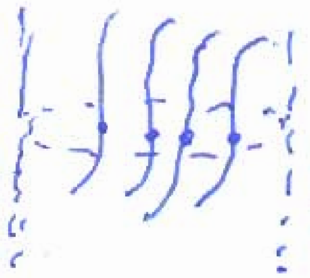
$\Rightarrow$ COORDINATE SINGULARITY

BAD NOTION OF SIMULTANEITY

CLOCKS ON THE EDGE OF THE DISK, WHICH WERE SYNCHRONIZED WITH EINSTEIN CONVENTION AT $t=0$, DEVELOP A <u>SYNCHRONIZATION GAP</u>

HOW TO DEFINE AN INSTANTNEOUS 3-SPACE? IS IT EUCLIDEAN OR RIEMANNIAN?

CONGRUENCE OF THE WORLDLINES OF THE POINTS OF THE DISK (LIKE A FLUID) – – THEIR TANGENT 4-VECTORS: $\ell^{\mu}(y)$, $\ell^{2}(y)=1$ (BUT $=0$ AT $\omega R = c$) ARE THE 4-VELOCITIES OF THE FLUID ELEMENTS



$\ell^{\mu}(y)$ ALLOWS TO DEFINE
$$\left\{\begin{array}{ll} \text{4-ACCELERATION} & a^{\mu}(y) \\ \text{EXPANSION} & \theta(y) \\ \text{SHEAR } \ 0 \to 0 & \sigma_{\mu\nu}(y) \\ \text{VORTICITY (TWIST)} & \omega_{\mu\nu}(y) \end{array}\right.$$
OF THE CONGRUENCE

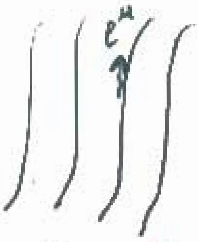
$\ell^{\mu}(y)$ IS SURFACE-FORMING (I.E. THE WORLDLINES ARE ORTHOGONAL TO A SIMULTANEITY 3-SURFACES) IF AND ONLY IF $\omega_{\mu\nu}(y) = 0$

$\omega_{\mu\nu}(y) \neq 0$ FOR THE ROTATING DISK $\Rightarrow$ SYNCHRONIZATION GAP (NO SYNCHRONIZATION IS POSSIBLE)

$\Rightarrow$ SAGNAC EFFECT

TIME AND PHASE DIFFERENCE FOR A ROUND TRIP $\left\{\begin{array}{l} \Delta t = \frac{4\omega A}{c^2} \\ \delta\phi = 8\pi\omega A/c\lambda \end{array}\right.$ CORRECTIONS IN GPS

CONGRUENZA DI OSSERVATORI TIMELIKE
O REFERENCE FRAME



CAMPO DI VELOCITÀ $l^\mu(x)\frac{\partial}{\partial x^\mu}$, 1+3 SPLITTING

$$D_\mu l_\nu = l_\mu a_\nu + \frac{1}{3}\theta\left[\eta_{\mu\nu} - \varepsilon l_\mu l_\nu\right] + \sigma_{\mu\nu} + \omega_{\mu\nu}$$

$a^\mu = l^\nu D_\nu l^\mu$ – 4-ACCELERAZIONE

$\theta = D_\mu l^\mu$ – EXPANSION – ALLARGAMENTO O CONTRAZIONE DELLE WORLDLINES INTORNO AD UNA DATA

$\sigma_{\mu\nu} = \frac{1}{2}(a_\mu l_\nu + a_\nu l_\mu) + \frac{1}{2}(D_\mu l_\nu + D_\nu l_\mu) - \frac{1}{3}\theta[\eta_{\mu\nu} - \varepsilon l_\mu l_\nu]$, $\sigma^2 = \frac{1}{2}\sigma_{\mu\nu}\sigma^{\mu\nu}$

– SHEAR – MISURA COME UNA SFERA NELLO SPAZIO TANGENTE A UNA DATA WORLDLINE γ CHE SIA LIE-TRANSPORTED LUNGO γ $[\mathcal{L}_{l^\alpha\partial_\alpha}$ [illegible] $= 0]$ E' DISTORTA A ELLISSOIDE CON ASSI PRINCIPALI GLI AUTOVETTORI DI $\sigma_{\mu\nu}$ E RATE DATA DAGLI AUTOVALORI $\boxed{\sigma_{\mu\nu} l^\nu = 0}$

$\omega_{\mu\nu} = -\omega_{\nu\mu} = \varepsilon_{\mu\nu\alpha\beta}\,\omega^\alpha l^\beta = \frac{1}{2}(a_\mu l_\nu - a_\nu l_\mu) + \frac{1}{2}(D_\mu l_\nu - D_\nu l_\mu)$, $\boxed{\omega_{\mu\nu} l^\nu = 0}$

$\omega^\mu = \frac{1}{2}\varepsilon^{\mu\alpha\beta\gamma}\omega_{\alpha\beta} l_\gamma$

– TWIST O VORTICITÀ O ROTAZIONE – misura la rotazione delle worldlines intorno ad una data

CONGRUENZA SURFACE-FORMING $\underset{\text{FROBENIUS THEOREM}}{\longleftrightarrow}$ $\omega_{\mu\nu} = 0$

DISCO ROTANTE $\omega_{\mu\nu} \neq 0$

SINCRONIZZABILITÀ DI UN REFERENCE FRAME

$\alpha = l_\mu(x)dx^\mu$ 1-FORMA

1) LOCALMENTE SINCRONIZZABILE IFF $\alpha \wedge d\alpha = 0 \rightarrow d\alpha = \beta \wedge \alpha$
2) LOCALLY PROPER TIME SYNCRONIZABLE IFF $d\alpha = 0 \rightarrow \alpha = d\beta$
3) SYNCHRONIZABLE IFF $\alpha = h\,dt$ $h(x), t(x)$ FUNZIONI
4) PROPER TIME SYNCHRONIZABLE IFF $\alpha = dt$

$\alpha \wedge d\alpha = 0 \Leftrightarrow \omega = \omega_{\mu\nu}dx^\mu \wedge dx^\nu = 0$ ROTANTI = NON SINCRONIZZABILI

MOTO RIGIDO DI BORN – $\theta = \sigma_{\mu\nu} = 0$

PIATTAFORMA – REFERENCE FRAME + TETRADI $E^\mu_{(0)} = l^\mu$

TRASPORTO DI FERMI-WALKER DI TETRADI (STANDARD DI NON-ROTAZIONE) LUNGO γ

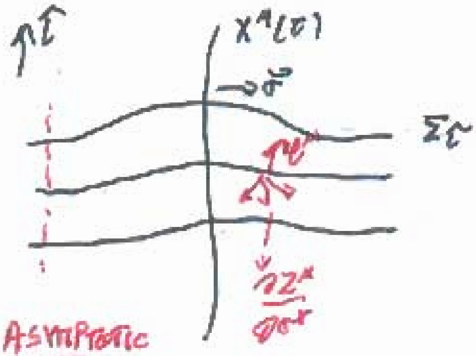
$$\frac{DE^\mu_{(a)}}{D\tau} = [a^\mu l_\nu - a_\nu l^\mu]E^\nu_{(a)}$$

3+1 POINT OF WIEW

WORLDLINE OF AN ACCELERATED OBSERVER +

+ 3+1 SPLITTING OF MINKOWKI SPACETIME = NICE FOLIATION WITH SPACELIKE LEAVES

GLOBAL NON-INERTIAL FRAME CENTERED ON THE ACCELERATED OBSERVER



ASYMPTOTIC INERTIAL OBSERVERS (FIXED STARS)

$$\begin{cases} x^\mu \longmapsto \sigma^A(x) = (\tau; \sigma^r) \\ \sigma^A \longmapsto x^\mu = z^\mu(\tau, \sigma^r) \end{cases}$$

RADAR 4-COORDINATES (BONDI)

EMBEDDING OF Σ_τ

$\downarrow$ CAN BE DEFINED IN AN OPERATIONAL WAY

$$\eta_{\mu\nu} \rightarrow g_{AB}[z(\tau,\vec\sigma)] = \frac{\partial z^\mu(\sigma)}{\partial\sigma^A}\,\eta_{\mu\nu}\,\frac{\partial z^\nu(\sigma)}{\partial\sigma^B}$$

$$\frac{\partial z^\mu(\tau,\vec\sigma)}{\partial\tau} = \underset{\text{LAPSE}}{N(\tau,\vec\sigma)}\, l^\mu(\tau,\vec\sigma) + \underset{\text{SHIFT}}{N^r(\tau,\vec\sigma)}\,\frac{\partial z^\mu(\tau,\vec\sigma)}{\partial\sigma^r}$$

MØLLER ADMISSIBILITY CONDITIONS

$$g_{\tau\tau} > 0,\quad g_{\tau r} < 0,\quad \begin{vmatrix} g_{rr} & g_{rs} \\ g_{sr} & g_{ss} \end{vmatrix} > 0,\quad \det g_{rs} < 0$$

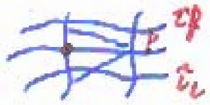
$\Downarrow$

NO RIGID ROTATIONS

$\Sigma_\tau \xrightarrow[\text{SPATIAL INFINITY}]{} $ SPACELIKE HYPERPLANE

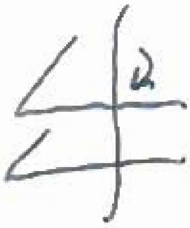
Σ_τ – INSTANTANEOUS 3-SPACE FROM A CLOCK SYNCHRONIZATION CONVENTION ≠ EINSTEIN

= CAUCHY SURFACE



$$\tau_P \overset{def}{=} \tau_i + \mathcal{E}(\sigma,\sigma^r)\,[\tau_f - \tau_i]$$

DIFFERENTIAL ROTATIONS

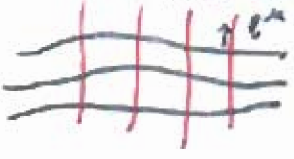


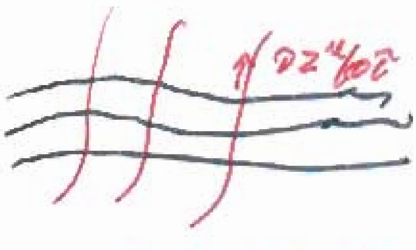
$$z^\mu(\tau,\vec\sigma) = x^\mu(\tau) + \epsilon^\mu_r\, R_{rs}(\tau,\sigma)\,\sigma^s \qquad \sigma = |\vec\sigma|$$

$$R(\tau,\sigma) = R[\alpha_i(\tau,\sigma)] \qquad \alpha_i(\tau,\sigma) = F(\sigma)\,\tilde\beta_i(\tau)$$

$$\frac{dF(\sigma)}{d\sigma} \neq 0 \qquad 0 < F(\sigma) < \frac{1}{A\sigma}$$

CONGRUENCE OF EULERIAN OBSERVERS





ROTATING NON-SURFACE FORMING CONGRUENCE (LIKE IN ROTATING DISK)

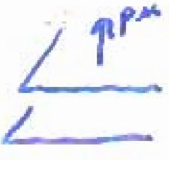
PARAMETRIZED MINKOWSKI THEORIES – $S = \int d\tau\, d^3\sigma\, L(\text{matter}, z(\tau,\vec\sigma))$

INVARIANT UNDER FRAME-PRESERVING DIFFEOMORPHISMS

$\Rightarrow z^\mu(\tau,\vec\sigma)$ GAUGE VARIABLE : CHANGE OF CLOCK SYNCHRONIZATION CONVENTION IS A GAUGE TRANSFORMATION

REST-FRAME INSTANT FORM

INERTIAL FRAME



RELATIVISTIC NOTIONS OF CENTER OF MASS

RELATIVISTIC THEORY OF ORBITS

HAMILTONIAN RELATIVISTIC 2-BODY PROBLEM:

CENTER OF MASS
AND
ORBIT RECONSTRUCTION

LUCA LUSANNA
INFN - FIRENZE

PADOVA - 27/9/2007

WITH D.ALBA, H.CRATER
J.PHYS A40 (2007) 9585
hep-th/0610200

BACKGROUND

D.ALBA, L.LUSANNA, M.PAURI
J.MATH.PHYS 43 (2002) 1677 (hep-th/0102087)
46 (2004) 062505 (hep-th/0402181)

REVIEW hep-th/0505005

PARAMETRIZED MINKOWSKI THEORIES

WORLD-LINE OF AN ACCELERATED OBSERVER (TIMELIKE)

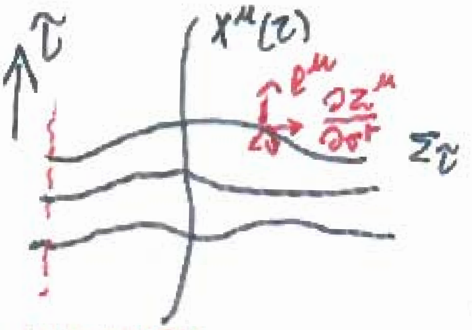
+ 3+1 SPLITTING OF MINKOWSKI SPACETIME

(= NICE FOLIATION WITH SPACELIKE 3-SURFACES =

= CLOCK SYNCHRONIZATION CONVENTION =

= DEFINITION OF THE INSTANTANEOUS 3-SPACE)

⇒ GLOBAL NON-INERTIAL FRAME CENTERED ON THE ACCELERATED OBSERV



ASYMPTOTIC INERTIAL OBSERVER

FRAME- AND OBSERVER- DEPENDENT, LORENTZ-SCALA RADAR 4-COORDINATES (BONDI) $(\tau;\sigma^r)$

[DIFFERENTLY FROM FERMI COORDINATES, THEY CAN BE OPERATIONALLY DEFINED]

$$\begin{cases} x^\mu \longmapsto \sigma^A(x) = (\tau;\sigma^r) \\ \sigma^A \longmapsto x^\mu = z^\mu(\tau,\sigma^r) \end{cases}$$

x^μ CARTESIAN 4-COORD. IN AN INERTIAL FRAME

EMBEDDING OF Σ_τ

$$\eta_{\mu\nu} \to g_{AB}[z(\tau,\vec\sigma)] = \frac{\partial z^\mu(\sigma)}{\partial\sigma^A}\,\eta_{\mu\nu}\,\frac{\partial z^\nu(\sigma)}{\partial\sigma^B}$$

$$\frac{\partial z^\mu(\tau,\vec\sigma)}{\partial\tau} = \underbrace{N(\tau,\vec\sigma)}_{\text{LAPSE}}\, l^\mu(\tau,\vec\sigma) + \underbrace{N^r(\tau,\vec\sigma)}_{\text{SHIFT}}\, \frac{\partial z^\mu(\tau,\vec\sigma)}{\partial\sigma^r}$$

↳ $= z^\mu_r(\tau,\vec\sigma)$ TANGENT TO Σ_τ

NORMAL TO Σ_τ

$$l^\mu = \frac{1}{\sqrt{g}}\,\epsilon^\mu{}_{\alpha\beta\gamma}\, z^\alpha_1 z^\beta_2 z^\gamma_3$$

MØLLER ADMISSIBILITY CONDITIONS FOR THE 3+1 SPLITTINGS

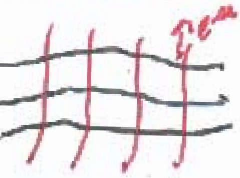
$g_{\tau\tau}(\tau,\vec\sigma) > 0$, $g_{\tau r}(\tau,\vec\sigma) < 0$, $\begin{vmatrix} g_{rr} & g_{rs} \\ g_{sr} & g_{ss} \end{vmatrix}(\tau,\vec\sigma) > 0$, det $g_{rs}(\tau,\vec\sigma) < 0$

⇓

NO RIGID REL. ROTATIONS

$\Sigma_\tau \xrightarrow{\text{SPATIAL INFINITY}}$ SPACELIKE HYPERPLANE

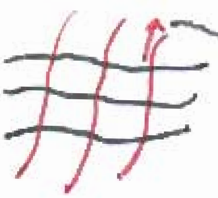
2 CONGRUENCES OF ACCELERATED OBSERVER ASSOCIATED TO EVERY 3+1 SPLITTING



NORMAL = 4-VELOCITY

SURFACE-FORMING

EULERIAN OBSERVERS



$\frac{\partial z^\mu}{\partial\tau}/\sqrt{g_{\tau\tau}}$ = 4-VELOCITY

NON-SURFACE-FORMING

(ROTATING DISK)

ISOLATED SYSTEM (PARTICLES, STRINGS, FLUIDS, FIELDS) WITH LAGRANGIAN DESCRIPTION (CONFIGURATIONS WITH THE 10 POINCARÉ GENERATORS FINITE)

COUPLING TO EXTERNAL GRAVITATIONAL FIELD $g_{\mu\nu}(x)$ $\mathcal{L}(\mathrm{matter}, g_{\mu\nu})$

$$g_{\mu\nu}(x) \mapsto g_{AB}[z^\mu(\tau,\vec{\sigma})] \Rightarrow S = \int d\tau\, d^3\sigma\, \tilde{\mathcal{L}}(\mathrm{matter}, z(\tau,\vec{\sigma}))$$

ACTION PRINCIPLE FOR MATTER AND EMBEDDING $z^\mu(\tau,\vec{\sigma})$

S INVARIANT UNDER FRAME-PRESERVING DIFFEOMORPHISMS
(SPECIAL RELATIVISTIC GENERAL COVARIANCE)

4 FIRST-CLASS CONSTRAINTS (DEPARAMETRIZED ANALOGUES OF THE SUPER-HAM. AND SUPER-MOMENTUM CONSTRAINTS OF GR)

$$\begin{cases} \mathcal{H}^\mu(\tau,\vec{\sigma}) = \rho^\mu(\tau,\vec{\sigma}) - l^\mu(\tau,\vec{\sigma})\, T^{\tau\tau}(\tau,\vec{\sigma}) - z^\mu_r(\tau,\vec{\sigma})\, T^{\tau r}(\tau,\vec{\sigma}) \approx 0 \\ \{\mathcal{H}^\mu(\tau,\vec{\sigma}_1), \mathcal{H}^\mu(\tau,\vec{\sigma}_2)\} = 0 \end{cases}$$

4-MOMENTUM OF $z^\mu(\tau,\vec{\sigma})$

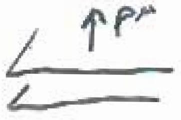
MATTER ENERGY DENSITY

MATTER 3-MOMENTUM DENSITY

$\Rightarrow$ $z^\mu(\tau,\vec{\sigma})$ GAUGE VARIABLES $\Rightarrow$ GAUGE EQUIVALENCE OF ADMISSIBLE NON-INERTIAL FRAMES (I.E. OF CLOCK SYNCHRONIZATION CONVENTIONS AND OF CHOICE OF 3-COORDINATES ON THE INSTANTANEOUS 3-SPACES)

REST-FRAME INSTANT FORM OF DYNAMICS

INERTIAL 3+1 SPLITTING CENTERED ON AN INERTIAL OBSERVER (DESCRIBING THE COLLECTIVE VARIABLES OF THE ISOLATED SYSTEM) WHOSE INSTANTENEOUS 3-SPACES ARE ORTHOGONAL TO THE 4-MOMENTUM OF THE ISOLATED SYSTEM (INTRINSICALLY DEFINED BY ITS CONFIGURATIONS)



$$z^\mu(\tau,\vec{\sigma}) = x^\mu(\tau) + \epsilon^\mu_r(P)\,\sigma^r$$

NON-INERTIAL REST FRAMES

ASYMPTOTIC INERTIAL OBSERVERS = FIXED STAR

REST FRAME INSTANT FORM OF ISOLATED SYSTEMS

$$z^\mu(\tau, \sigma^r) = x^\mu(\tau) + \epsilon^\mu_r(u(P))\,\sigma^r \qquad \begin{cases} \epsilon^\mu_o(u(P)) = u^\mu(P) = \dfrac{P^\mu}{\sqrt{P^2}} \\ \epsilon^\mu_r(u(P)) = \left(-u_r(P);\ \delta^i_r - \dfrac{u^i(P)\,u_r(P)}{1+u^o(P)}\right) \end{cases}$$

$$x^\mu(\tau) = u^\mu(P)\,\tau$$

STANDARD WIGNER BOOST FOR TIMELIKE POINCARÉ ORBITS

$$P^\mu = L^\mu{}_\nu(P, \overset{\circ}{P})\,\overset{\circ}{P}{}^\nu, \quad \overset{\circ}{P}{}^\mu = \sqrt{P^2}\,(1; \vec{0}), \quad \epsilon^\mu_A(u(P)) = L^\mu{}_A(P, \overset{\circ}{P})$$

$\Rightarrow$ 3-INDICES ON Σ_τ = WIGNER SPIN 1 3-INDICES

ON Σ_τ $\begin{cases} \Rightarrow \vec{A} \cdot \vec{B} = \text{LORENTZ SCALAR} \\ \Rightarrow A_r \text{ TRANSFORMS UNDER WIGNER ROTATIONS} \\ \qquad A_r \to R_{rs}(P, \Lambda) A_s \text{ UNDER EXTERNAL LORENTZ BOOSTS} \end{cases}$

$\Rightarrow \Sigma_\tau$ WIGNER HYPERPLANE WITH WIGNER COVARIANCE

ISOLATED SYSTEM – FROM THE LAGRANGIAN BUILD THE ENERGY-MOMENTUM TENSOR AND THE CONSERVED 10 POINCARÉ GENERATORS

$$\begin{cases} P^\mu = \int_{\Sigma_\tau} d^3\sigma \text{ -----} \\ J^{\mu\nu} = \int_{\Sigma_\tau} d^3\sigma \text{ -----} \end{cases}$$

THEY ARE NON-LOCAL QUANTITIES WHICH KNOW THE WHOLE INSTANTANEOUS 3-SPACE

LORENTZ BOOSTS $K^r = J^{or}$ – DIFFERENTLY FROM GALILEI BOOSTS THEY DEPEND ON THE INTERACTIONS $\boxed{[P^i, K^j] = \delta^{ij} P^o}$

COLLECTIVE VARIABLES FOR THE ISOLATED SYSTEM

– PROBLEM OF THE RELATIVISTIC CENTER OF MASS

THERE ARE ONLY 3 (NON-LOCAL) COLLECTIVE VARIABLES WHICH CAN BE INTRINSICALLY DEFINED BY USING ONLY THE 10 POINCARÉ GENERATORS

LET X^μ_S BE THE 4-POSITION CANONICALLY CONJUGATED TO P^μ

$$\{X^\mu_S, P^\nu\} = -\eta^{\mu\nu}$$

$$J^{\mu\nu} = X^\mu_S P^\nu - X^\nu_S P^\mu + S^{\mu\nu}$$

DEFINITION OF THE SPIN TENSOR

PSEUDO-4-VECTOR (OR CENTER OF SPIN)

1) EXTERNAL CANONICAL NON-COVARIANT 4-CENTER OF MASS

q. $$\tilde{x}^\mu = z^\mu(\tau, \tilde{\sigma}^r) = x^\mu_S - \frac{1}{\pi(p^0+\pi)}\Big[p_\nu S^{\nu\mu} + \pi\Big(S^{0\mu} - S^{0\nu}\frac{p_\nu p^\mu}{\pi^2}\Big)\Big]$$

$$\pi = \sqrt{p^2}$$

$$\tilde{\sigma}^r = q^r = \tilde{x}^r - \frac{p^r}{p^0}\tilde{x}^0 = \frac{p^0 R^r + \pi Y^r}{p^0+\pi}$$

EXTERNAL 3-CENTER OF MASS (THE CLASSICAL ANALOGUE OF THE NEWTON-WIGNER POSITION OPERATOR)

$$J^{\mu\nu} = \tilde{x}^\mu p^\nu - \tilde{x}^\nu p^\mu + \tilde{S}^{\mu\nu}$$ NEW SPIN TENSOR

UNIVERSAL BREAKING OF LORENTZ COVARIANCE (SOURCE OF THE NO-INTERACTION THEOREM) CONFINED TO A DECOUPLED FREE PSEUDO-PARTICLE (POINT PARTICLE CLOCK ?: WHO WILL OBSERVE IT, DUE TO THE NON-LOCALITY OF ITS CONSTRUCTION ?)

2) EXTERNAL NON-CANONICAL NON-COVARIANT MØLLER 4-CENTER OF ENERGY

↳ PSEUDO-4-VECTOR

$$R^\mu = z^\mu(\tau, \sigma^r_R)$$

LORENTZ BOOST

$$\sigma^r_R = R^r = -\frac{K^r}{p^0} = \tilde{x}^r - \frac{p^r}{p^0}\tilde{x}^0 - \frac{(\vec{S}\times\vec{P})^r}{p^0(p^0+\pi)}$$

MØLLER 3-CENTER OF ENERG

$$\frac{\Sigma_i m_i \vec{x}_i}{\Sigma_i m_i} \rightarrow \frac{\Sigma_i E_i \vec{x}_i}{\Sigma_i E_i}$$

3) EXTERNAL NON-CANONICAL COVARIANT FOKKER-PRYCE 4-CENTER OF INERTIA

$$Y^\mu = z^\mu(\tau, \sigma^r_Y)$$

$$\sigma^r_Y = q^r + \frac{(\vec{S}\times\vec{P})^r}{\pi(p^0+\pi)} = R^r + \frac{(\vec{S}\times\vec{P})^r}{\pi p^0}$$

FOKKER-PRYCE 3-CENTER OF INERTIA

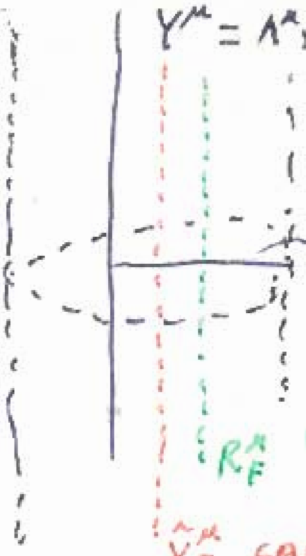
IT IS THE ONLY 4-VECTOR

THESE 3 COLLECTIVE VARIABLES COINCIDE ONLY IN THE REST FRAME AND HAVE THE SAME 4-VELOCITY $p^\mu/\pi = u^\mu(p)$

IT IS THE INERTIAL OBSERVER (TIME AXIS OF THE REST-FRAME)
$x^\mu(\tau)$

$\vec{x}, \vec{R}, \vec{Y}$ are the only c.o.m. 3-coordinates, which can be built entirely only in terms of the Poincaré generators of the system $P^\mu, J^{\mu\nu}$

PAULI-PROSPERI



$Y^\mu = \Lambda^\mu{}_\nu \tilde{x}^\nu_{RF}$ $\{Y^\mu, Y^\nu\} \ne 0$ 4-VECTOR

FOKKER-PRYCE CENTER OF INERTIA

INVARIANT RADIUS ρ (MØLLER)

R^μ_F MØLLER CENTER OF ENERGY

$\{R^\mu_F, R^\nu_F\} \ne 0$, NOT 4-VECTOR

$\tilde{x}^\mu_F$ CANONICAL CENTER OF MASS (NEWTON-WIGNER

$\{\tilde{x}^\mu_F, \tilde{x}^\nu_F\} = 0$, NOT 4-VECTOR

MØLLER

INTRINSIC RADIUS OF NONCOVARIANCE (CLASSICAL INTRINSIC UNIT OF LENGTH)

$$\rho = \frac{\sqrt{-W^2}}{P^2 c} = \frac{|\vec{S}|}{\sqrt{P^2}\, c} \qquad \begin{cases} P^2 > 0 \\ W^2 = -P^2 \vec{S}^2 \ne 0 \end{cases}$$

It is impossible to localize in a frame-independent way the classical canonical c.o.m. (the only one which can be quantized in the standard way)

1) WAVE QUANTIZATION $\rho \mapsto \lambda_c \cdot |s|$ (COMPTON WAVELENGTH x TOTAL SPIN)

α) PAIR PRODUCTION happens when one tries to localize inside the worldtube

$\Delta x = \frac{\hbar}{\Delta P} + \frac{\hbar \Delta P}{S^2}$

β) $\Delta x \geq \frac{\hbar}{\Delta P}$ and $\boxed{\Delta x > \rho}$ $\Rightarrow \frac{\hbar}{\Delta P} > \rho \Rightarrow \Delta P < \frac{\hbar}{\rho} = \frac{1}{|s| \lambda_c} = \frac{\hbar c}{|s|} \sqrt{P^2}$

γ) HEGERFELDT THEOREMS $\begin{cases} \text{IF } \hat{\vec{x}} = \hat{\vec{x}}^+ \Rightarrow \text{VIOLATION OF EINSTEIN CAUSALITY} \\ \text{IF } \hat{\vec{x}} \text{ NOT SELF-ADJOINT} \Rightarrow \text{BAD LOCALIZATION} \end{cases}$

$\Rightarrow \hat{\vec{x}}^\mu$ WITH THE SAME ONTOLOGY ... PROBLEMS OF THE WAVE FUNCTION OF THE UNIVERSE - MAYBE NOT TO BE QUANTIZED (← CLASSICAL LIMIT OF THE COPENHAGEN INTERPRETATION) ; MACH'S PRINCIPLE ? Q-GRAVITY

2) MØLLER $R_{MATTER} < \rho$

α) peripheral rotation velocity > c

β) classical T^{oo} NOT DEFINITE POSITIVE everywhere in every frame

$\Rightarrow \rho$ REMNANT of the ENERGY CONDITIONS of GENERAL RELATIVITY;

$l_{PLANCK} \leq \rho \leq$ (cosmological scale) UNIVERSE EFFECTIVE RADIUS

INFRARED CUTOFF

EXTERNAL POINCARÉ GROUP

$$\begin{cases} P^o = \sqrt{\underline{M}^2+\vec{P}^2} \\ \vec{P} \\ J^{ij} = \tilde{x}^i P^j - \tilde{x}^j P^i + \varepsilon^{ijk}\,\underline{S}^k \\ K^i = J^{oi} = \tilde{x}^o P^i - \tilde{x}^i \sqrt{\underline{M}^2+\vec{P}^2} - \dfrac{\varepsilon^{ijk} P^j \underline{S}^k}{\underline{M}+\sqrt{\underline{M}^2+P^2}} \end{cases}$$

ONLY THE INVARIANT MASS M AND THE SPIN $\vec{S}$ DEPEND ON THE NATURE OF THE ISOLATED SYSTEM

[U(2) ALGEBRA]

FROM THE EXTERNAL POINT OF VIEW EVERY ISOLATED SYSTEM IS REDUCED TO A DECOUPLED PSEUDO-PARTICLE ENDOWED OF MASS M AND SPIN $\vec{S}$

[EXTERNAL POLE-DIPOLE STRUCTURE ≠ FROM MULTIPOLAR EXPANSIONS OF PAPAPETROU...]

INTERNAL POINT OF VIEW – INSIDE EACH WIGNER HYPERPLANE (WITH WIGNER COVARIANCE) M AND $\vec{S}$ HAVE TO BE EXPRESSED IN TERMS OF INTERNAL RELATIVE VARIABLES

INTERNAL POINCARÉ GENERATORS (UNFAITHFUL REALIZATION)

$$\begin{cases} M = \int_{\Sigma_\tau} d^3\sigma\, T^{\tau\tau}(\tau,\vec{\sigma}) \\ \vec{P} \approx 0 \\ \vec{J} = \vec{S} \\ \vec{K} \approx 0 \end{cases}$$

$\vec{P} \approx 0$ REST-FRAME CONDITIONS] INTERACTION-INDEPENDENT (INSTANT FORM)

$\vec{K} \approx 0$ GAUGE FIXING TO THE REST-FRAME CONDITIONS – THEY IDENTIFY THE INERTIAL OBSERVER $X^\mu(\tau)$ WITH THE FOKKER-PRYCE CENTER OF INERTIA

$\to = \int_{\Sigma_\tau} d^3\sigma\, \vec{\sigma}\, T^{\tau\tau}(\tau,\vec{\sigma})$

$\vec{P} \approx 0,\ \vec{K} \approx 0$ ELIMINATE THE INTERNAL CENTER OF MASS $(\vec{q}_{int} \approx \vec{R}_{int} \approx \vec{Y}_{int})$ TO AVOID A DOUBLE COUNTING OF THE COLLECTIVE VARIABLE

$\vec{P} \approx 0$

⇒ INTERNAL 3-CENTER OF MASS ≈ MØLLER 3-CENTER OF ENERGY ≈ FOKKER-PRYCE 3-CENTER OF INERTIA

$$\vec{q} \approx \vec{Y} \approx \vec{R} = -\frac{\vec{K}}{M} \approx 0$$

CANONICAL INTERNAL C.O.M $\vec{q}$ KNOWN FUNCTION OF $M, \vec{P}, \vec{S}, \vec{K}$

N-BODY SYSTEM WITH POSITIVE ENERGY IN THE REST-FRAME INSTANT FORM

$$\begin{cases} x^\mu_i(\tau) = X^\mu + \underset{\sim}{\varepsilon^\mu_r(u(P))}\, \eta^r_i(\tau) \\ p^\mu_i(\tau) = +\sqrt{m_i^2 + \vec{\kappa}_i^2(\tau)}\, u^\mu(P) + \varepsilon^\mu_r(u(P))\, \kappa_{ir}(\tau) \end{cases}$$

X^μ = FOKKER-PRYCE CENTER OF INERTIA

$i = 1, .., N$ DERIVED QUANTITIES

CANONICAL VARIABLES

$$\downarrow$$

$$\boxed{\begin{array}{c|c} \tilde{x}^\mu & \vec{\eta}_i \\ P^\mu & \vec{\kappa}_i \end{array}}$$

NON-LOCAL

$\begin{cases} \tilde{x}^\mu(\tau) \\ P^\mu \end{cases}$ EXTERNAL NON-COVARIANT C.O.M (NEWTON-WIGNER)

$\begin{cases} \eta_i(\tau) \\ \kappa_i(\tau) \end{cases}$ RESTRICTED BY $\vec{P} \approx 0$, $\vec{K} \approx 0$ ELIMINATION OF INTERNAL C.O.M. $\vec{q}$

FREE CASE

$$\boxed{\begin{array}{c} \vec{\eta}_i \\ \vec{\kappa}_i \end{array}} \longrightarrow \boxed{\begin{array}{c|c} \vec{q}_0 \approx 0 & \vec{\rho}_{qa} \\ \vec{P} \approx 0 & \vec{\pi}_{qa} \end{array}}$$

$a = 1, .., N-1$

CANONICAL TRANSF. POINT ONLY IN THE MOMENTA

HAM. EQS FOR RELATIVE VARIABLES WITH HAMILTONIAN M

INTERACTING CASE – INTERACTION-DEPENDENT CAN. TRANSF (NON-POINT)

$\vec{q} = \vec{q}_0 + \vec{q}_{int}$ USE CANONICAL BASIS OF FREE CASE

AND USE $\vec{K} \approx 0$ TO FIND $\vec{q} \approx \vec{f}(\vec{\rho}_{qa}, \vec{\pi}_{qa})$

$$\Rightarrow \quad \vec{\eta}_i \simeq \vec{q} + \cancel{\text{cos}} \approx \vec{F}_i(\vec{\rho}_{qa}, \vec{\pi}_{qa}) \approx \vec{f}(\vec{\rho}_{qa}, \vec{\pi}_{qa}) + \vec{F}_i(\vec{\rho}_{qa}, \vec{\pi}_{qa})$$

NON-RELATIVISTIC CASE

$$\boxed{\begin{array}{c} \vec{x}_i \\ \vec{p}_i \end{array}} \longrightarrow \boxed{\begin{array}{c|c} \vec{x}_{cm} & \vec{\rho}_a \\ \vec{P} & \vec{\pi}_a \end{array}} \xrightarrow{\text{REST-FRAME}} \boxed{\begin{array}{c|c} \vec{x}_{cm} \approx 0 & \vec{\rho}_a \\ \vec{P} \approx 0 & \vec{\pi}_a \end{array}}$$

CAN. TRANSF. POINT BOTH IN COORDINATES AND MOMENTA

LOCAL SEPARATION OF UNIQUE CENTER OF MASS

2-BODY SYSTEM

$$\begin{cases} x_i^\mu(\tau) = z^\mu(\tau, \vec{\eta}_i(\tau)) = x^\mu + \epsilon^\mu_r(u(P))\, \eta_i^r(\tau) \\ p_i^\mu(\tau) = \sqrt{m_i^2 + \vec{\kappa}_i^2(\tau)}\, u^\mu(P) + \epsilon^\mu_r(u(P))\, \kappa_{ir}(\tau) \end{cases} \Rightarrow p_i^2 = m_i^2$$

↓ POSITIVE ENERGY SOLUTION OF →

$\vec{\eta}_i(\tau)$, $\vec{\kappa}_i(\tau)$, $\{\eta_i^r(\tau), \kappa_{js}(\tau)\} = \delta_{ij}\,\delta^r_s$

CANONICAL VARIABLES (WIGNER SPIN-1 3-VECTORS) FOR THE PARTICLES INSIDE EVERY WIGNER HYPERPLANE

1) FREE PARTICLES

$$\begin{cases} M_0 = \sqrt{m_1^2 + \vec{\kappa}_1^2} + \sqrt{m_2^2 + \vec{\kappa}_2^2} \\ \vec{P} = \vec{\kappa}_1 + \vec{\kappa}_2 \approx 0 \\ \vec{J} = \vec{S} = \vec{\eta}_1 \times \vec{\kappa}_1 + \vec{\eta}_2 \times \vec{\kappa}_2 \\ \vec{K}_0 = -\sqrt{m_1^2 + \vec{\kappa}_1^2}\, \vec{\eta}_1 - \sqrt{m_2^2 + \vec{\kappa}_2^2}\, \vec{\eta}_2 \approx 0 \end{cases}$$

INTERNAL 3-CENTER OF MASS

$$\vec{q} \approx \vec{R} = -\frac{\vec{K}}{M} = \frac{\sum_i \sqrt{m_i^2 + \vec{\kappa}_i^2}\, \vec{\eta}_i}{\sum_i \sqrt{m_i^2 + \vec{\kappa}_i^2}} \approx 0 \qquad \vec{q} \xrightarrow[c \to \infty]{} \frac{\sum_i m_i \vec{\eta}_i}{\sum_i m_i} = \vec{\eta}_{com}$$

CANONICAL TRANSFORMATION

$$\boxed{\begin{matrix} \vec{\eta}_i \\ \vec{\kappa}_i \end{matrix}} \longrightarrow \boxed{\begin{array}{c|c} \vec{q} \approx 0 & \vec{\rho}_q \\ \vec{P} \approx 0 & \vec{\pi}_q \end{array}}$$

RELATIVE VARIABLES

POINT ONLY IN THE MOMENTA

INTERACTION-DEPENDENT

NON-RELATIVISTIC

$$\boxed{\begin{array}{c|c} \vec{\eta}_{com} \approx 0 & \vec{\rho} \\ \vec{P} \approx 0 & \vec{\pi} \end{array}} \qquad \vec{\rho} = \vec{\eta}_1 - \vec{\eta}_2 \qquad \vec{\pi} = \frac{m_2 \vec{\kappa}_1 - m_1 \vec{\kappa}_2}{m_1 + m_2}$$

POINT BOTH IN POSITION AND MOMENTA

INTERACTION-INDEPENDENT

$$\begin{array}{l|l} \vec{\eta}_1 = \vec{\eta}_{com} + \frac{m_2}{m}\vec{\rho} \approx \frac{m_2}{m}\vec{\rho} & \vec{\kappa}_1 = \frac{m_1}{m}\vec{P} + \vec{\pi} \approx \vec{\pi} \\ \vec{\eta}_2 = \vec{\eta}_{com} - \frac{m_1}{m}\vec{\rho} \approx \frac{m_1}{m}\vec{\rho} & \vec{\kappa}_2 = \frac{m_2}{m}\vec{P} - \vec{\pi} \approx -\vec{\pi} \end{array} \qquad \begin{matrix} \vec{\eta}_{com} \\ \vec{P} \approx \end{matrix}$$

$m = m_1 + m_2$

CANONICAL TRANSFORMATION [illegible]

$$\begin{cases} M_0 = \sqrt{m_1^2+\vec{k}_1^2} + \sqrt{m_2^2+\vec{k}_2^2} = \sqrt{m_0^2+\vec{P}^2} \approx m_0 \\ m_0 = \sqrt{\Pi_0^2-\vec{P}^2} = \sqrt{m_1^2+\vec{\pi}_q^2} + \sqrt{m_2^2+\vec{\pi}_q^2} \end{cases}$$

$$\begin{cases} \vec{\rho}_q = \vec{\rho} + \left(\frac{\sqrt{m_1^2+\vec{k}_1^2}}{\sqrt{m_2^2+\vec{\pi}_q^2}} + \frac{\sqrt{m_2^2+\vec{k}_2^2}}{\sqrt{m_1^2+\vec{\pi}_q^2}} \right) \frac{\vec{P}\cdot\vec{\rho}\ \ \vec{\pi}_q}{\Pi_0\sqrt{\Pi_0^2-\vec{P}^2}} \approx \vec{\rho} = \vec{\eta}_1 - \vec{\eta}_2 \\ \vec{\pi}_q = \vec{\pi} + \frac{m_1-m_2}{2(m_1+m_2)}\vec{P} - \\ \quad - \frac{\vec{P}}{\sqrt{\Pi_0^2-\vec{P}^2}} \left[\frac{1}{2}\left(\sqrt{m_1^2+\vec{k}_1^2} - \sqrt{m_2^2+\vec{k}_2^2} \right) - \frac{\Pi_0 - \sqrt{\Pi_0^2-\vec{P}^2}}{\vec{P}^2}\ \vec{P}\cdot\left(\vec{\pi} + \frac{m_1-m_2}{2(m_1+m_2)}\vec{P} \right) \right] \end{cases}$$

$$\approx \vec{\pi} = \frac{m_2\vec{k}_1 - m_1\vec{k}_2}{m_1+m_2}$$

$$\vec{q} = \frac{\sqrt{m_1^2+\vec{k}_1^2}\ \vec{\eta}_1 + \sqrt{m_2^2+\vec{k}_2^2}\ \vec{\eta}_2}{\sqrt{\Pi_0^2-\vec{P}^2}} + \frac{\vec{\rho}\times\vec{P}}{\sqrt{\Pi_0^2-\vec{P}^2}\left(\Pi_0+\sqrt{\Pi_0^2-\vec{P}^2}\right)} - \frac{\left(\sqrt{m_1^2+\vec{k}_1^2}\ \vec{\eta}_1 + \sqrt{m_2^2+\vec{k}_2^2}\ \vec{\eta}_2\right)\cdot\vec{P}\ \vec{P}}{\Pi_0\sqrt{\Pi_0^2-\vec{P}^2}\left(\Pi_0+\sqrt{\Pi_0^2-\vec{P}^2}\right)}$$

$$\approx \frac{\Sigma_i\sqrt{m_i^2+\vec{k}_i^2}\ \vec{\eta}_i}{\Sigma_i\sqrt{m_i^2+\vec{k}_i^2}}$$

SOLUTION OF $\vec{K} \approx 0$

$\vec{q} \approx 0$

$$\begin{cases} \vec{\eta}_i = \vec{q} - \frac{(\vec{\rho}_q\times\vec{\pi}_q)\times\vec{P}}{\sqrt{m_0^2+\vec{P}^2}\left(m_0+\sqrt{m_0^2+\vec{P}^2}\right)} + \frac{1}{2}\left[(-)^{i+1} - \frac{2m_0\,\vec{\pi}_q\cdot\vec{P} + (m_1^2-m_2^2)\sqrt{m_0^2+\vec{P}^2}}{m_0^2\sqrt{m_0^2+\vec{P}^2}} \right] \cdot \\ \quad \cdot \left[\vec{\rho}_q - \frac{\vec{\rho}_q\cdot\vec{P}\quad \vec{\pi}_q}{m_0\sqrt{m_0^2+\vec{P}^2}\left(\frac{\sqrt{m_1^2+\vec{k}_1^2}}{\sqrt{m_2^2+\vec{\pi}_q^2}} + \frac{\sqrt{m_2^2+\vec{k}_2^2}}{\sqrt{m_1^2+\vec{\pi}_q^2}} \right)^{-1} + \vec{\pi}_q\cdot\vec{P}} \right] \\ \vec{k}_i = \left\{ (-)^{i+1}\vec{\pi}_q + \left[\frac{1}{2} + \frac{(-)^{i+1}}{m_0\sqrt{m_0^2+\vec{P}^2}} \left(\vec{\pi}_q\cdot\vec{P}\left[1 - \frac{m_0\left(\sqrt{m_0^2+\vec{P}^2}-m_0\right)}{\vec{P}^2} \right] + \right. \right. \right. \\ \quad \left. + (m_1^2-m_2^2)\sqrt{m_0^2+\vec{P}^2} \right. \approx (-)^{i+1}\vec{\pi} \end{cases}$$

$$\sqrt{m_i^2+\vec{k}_i^2} = \frac{1}{2}\sqrt{m_0^2+\vec{P}^2}\left[1 + (-)^{i+1}\frac{m_1^2-m_2^2}{m_0^2} \right] + (-)^{i+1}\frac{\vec{\pi}_q\cdot\vec{P}}{m_0}$$

$$\vec{\eta}_i \approx \vec{q} + \frac{1}{2}\left[(-)^{i+1} - \frac{m_1^2-m_2^2}{m_0^2} \right]\vec{\rho}_q \underset{\vec{q}=0}{\approx} \frac{1}{2}\left[(-)^{i+1} - \frac{m_1^2-m_2^2}{m_0^2} \right]\vec{\rho}$$

ORBIT RECONSTRUCTION

FREE CASE $\vec{P} \approx 0, \; \vec{q} \approx 0$

$$x^\mu_i(\tau) \approx u^\mu(P)\,\tau + \epsilon^\mu_r(u(P))\,\rho^r_q(\tau)\,\frac{1}{2}\Big[(-)^{i+1} - \frac{m_1^2 - m_2^2}{\left(\sqrt{m_1^2+\vec{\pi}_q^2} + \sqrt{m_2^2+\vec{\pi}_q^2}\right)^2}\Big]$$

IT DEPENDS ONLY ON THE RELATIVE VARIABLES

$\vec{\rho}_q \approx \vec{\rho}, \; \vec{\pi}_q \approx \vec{\pi}$ WHOSE DYNAMICS IS DETERMINED BY THE INTERNAL INVARIANT MASS

$$M_0 \approx \mathcal{M}_0 = \sqrt{m_1^2+\vec{\pi}_q^2} + \sqrt{m_2^2+\vec{\pi}_q^2}$$ AS HAMILTONIAN

$$p^\mu_i(\tau) \approx \sqrt{m_i^2+\vec{\pi}_q^2}\; u^\mu(P) + (-)^{i+1}\,\epsilon^\mu_r(u(P))\,\pi_{q\,r}$$

INTERACTING CASE - SINCE THE CANONICAL TRANSFORMATION LEADING TO THE INTERNAL CENTER OF MASS AND RELATIVE VARIABLES $(\vec{q}^{\,(int)}, \vec{P} \approx 0, \vec{\rho}_q^{\,(int)}, \vec{\pi}_q^{\,(int)})$ IS NOT KNOWN,

WE MUST USE THE VARIABLES $(\vec{q}, \vec{P} \approx 0, \vec{\rho}_q, \vec{\pi}_q)$

OF THE FREE CASE.

NOW $\vec{K} \approx 0$ GIVES $\vec{q} \approx \underset{\substack{\downarrow \\ \text{INTERACTION-DEPENDENT}}}{\vec{q}(\vec{\rho}_q, \vec{\pi}_q)}$ INSTEAD OF $\vec{q} \approx 0$.

$$x^\mu_i(\tau) \approx u^\mu(P)\,\tau + \epsilon^\mu_r(u(P))\Big[q^r(\vec{\rho}_q, \vec{\pi}_q) + \frac{1}{2}\rho^r_q\Big((-)^{i+1} - \frac{m_1^2 - m_2^2}{\mathcal{M}^2}\Big)\Big]$$

WITH $\vec{\rho}_q(\tau), \vec{\pi}_q(\tau)$ SOLUTION OF THE HAMILTON EQUATIONS WITH HAMILTONIAN $M \approx \mathcal{M}$

THE SIMPLEST INTERACTING 2-BODY SYSTEM

$$\begin{cases} M = \sqrt{m_1^2+\vec{\kappa}_1^2+\underline{\Phi(\vec{\rho}^2)}} + \sqrt{m_2^2+\vec{\kappa}_2^2+\underline{\Phi(\vec{\rho}^2)}} \\ \vec{P} = \vec{\kappa}_1+\vec{\kappa}_2 \approx 0 \\ \vec{J} = \vec{S} = \vec{\eta}_1 \times \vec{\kappa}_1 + \vec{\eta}_2 \times \vec{\kappa}_2 \\ \vec{K} = -\vec{\eta}_1 \sqrt{m_1^2+\vec{\kappa}_1^2+\underline{\Phi(\vec{\rho}^2)}} - \vec{\eta}_2 \sqrt{m_2^2+\vec{\kappa}_2^2+\underline{\Phi(\vec{\rho}^2)}} \approx 0 \end{cases} \qquad \vec{\rho} = \vec{\eta}_1 - \vec{\eta}_2$$

THE INTERNAL POINCARÉ ALGEBRA IS SATISFIED

$$\Pi \approx \mathcal{M} = \sqrt{m_1^2+\vec{\pi}^2+\Phi(\vec{\rho}^2)} + \sqrt{m_2^2+\vec{\pi}^2+\Phi(\vec{\rho}^2)}$$

HAMILTONIAN FOR THE RELATIVE MOTION

$$m_0 = \sqrt{m_1^2+\vec{\pi}^2} + \sqrt{m_2^2+\vec{\pi}^2}$$

$$\vec{K} \approx 0$$

$$\Rightarrow \quad \vec{q} \approx \vec{q}(\vec{\rho}, \vec{\pi}) = \frac{m_1^2-m_2^2}{2}\left(\frac{1}{m_0^2} - \frac{1}{\mathcal{M}^2}\right)\vec{\rho}$$

$$\Rightarrow \quad \vec{\eta}_i \approx \frac{(-)^{i+1}}{2}\left(1 - \frac{m_1^2-m_2^2}{\mathcal{M}^2}\right)\vec{\rho} \xrightarrow[c\to\infty]{} (-)^{i+1}\frac{m_i}{m_1+m_2}\vec{\rho}$$

IN THE PAPER THERE ARE THE ORBITS FOR COULOMB-LIKE POTENTIALS

$$\Phi(\vec{\rho}^2) = -2\mu \frac{e^2}{\sqrt{\vec{\rho}^2}}$$

N.B. FOR $\Pi = \sqrt{m_1^2+\vec{\kappa}_1^2} + \sqrt{m_2^2+\vec{\kappa}_2^2} + \dfrac{e^2}{4\pi|\vec{\eta}_1-\vec{\eta}_2|}$ (REAL COULOMB POTENTIAL)

IT IS NOT KNOWN THE FORM OF THE INTERNAL BOOST $\vec{K}$

(IT IS NOT KNOWN THE LAGRANGIAN, FROM WHICH TO GET THE ENERGY-MOMENTUM TENSOR)

$$\begin{cases} Mc = \sum_i \sqrt{m_i^2c^2 + \left(\vec{\kappa}_i(\tau) - \frac{Q_i}{c}\vec{A}_\perp(\tau,\vec{\eta}_i(\tau))\right)^2} + \sum_{i\neq j} \frac{Q_iQ_j}{4\pi|\vec{\eta}_i(\tau)-\vec{\eta}_j(\tau)|} \\ \qquad + \frac{1}{2}\int d^3\sigma\, [\vec{\pi}_\perp^2 + \vec{B}^2](\tau,\vec{\sigma}) \\ \vec{P} = \sum_i \vec{\kappa}_i(\tau) + \frac{1}{c}\int d^3\sigma\, (\vec{\pi}_\perp \times \vec{B})(\tau,\vec{\sigma}) \approx 0 \\ \vec{J} = \vec{S} = \sum_i \vec{\eta}_i(\tau) \times \vec{\kappa}_i(\tau) + \frac{1}{c}\int d^3\sigma\, [\vec{\sigma} \times (\vec{\pi}_\perp \times \vec{B})](\tau,\vec{\sigma}) \\ \vec{K} = -\sum_i \vec{\eta}_i(\tau)\sqrt{m_i^2c^2 + \left(\vec{\kappa}_i(\tau) - \frac{Q_i}{c}\vec{A}_\perp(\tau,\vec{\eta}_i(\tau))\right)^2} + \\ \quad + \frac{1}{c}\sum_{i\neq j} Q_iQ_j \left(\frac{1}{\Delta_{\eta_j}} \frac{\partial c(\vec{\eta}_i(\tau)-\vec{\eta}_j(\tau))}{\partial \vec{\eta}_j} - \vec{\eta}_j(\tau)\, c(\vec{\eta}_i(\tau)-\vec{\eta}_j(\tau)) \right) \\ \quad + \sum_i \frac{Q_i}{c}\int d^3\sigma\, \vec{\pi}_\perp(\tau,\vec{\sigma})\, c(\vec{\sigma}-\vec{\eta}_i(\tau)) - \\ \quad - \frac{1}{2c}\int d^3\sigma\, \vec{\sigma}\, (\vec{\pi}_\perp^2 + \vec{B}^2)(\tau,\vec{\sigma}) \approx 0 \end{cases}$$

$Q_i^2 = 0$

$Q_iQ_j \neq 0$

$\vec{\pi}_\perp = \frac{1}{c}\vec{E}_\perp = \frac{\partial \vec{A}_\perp}{\partial \tau}$

$c(\vec{\sigma}) = \frac{1}{4\pi|\vec{\sigma}|}$

CRATER - LL - ANN.PHYS. 289(2001)87
hep-th/0001046
ALBA, CRATER, LL
INT.J.MOD.PHYS A16(2001) 3365
hep-th/0103109

2 PARTICLES WITH DARWIN POTENTIAL

$$\begin{cases} Mc = \sqrt{m_1^2c^2 + \vec{\kappa}_1^2(\tau)} + \sqrt{m_2^2c^2 + \vec{\kappa}_2^2(\tau)} + \frac{Q_1Q_2}{4\pi|\underbrace{\vec{\eta}_1(\tau)-\vec{\eta}_2(\tau)}_{\vec{\rho}}|} + V_{DARWIN} \\ \vec{P} = \vec{\kappa}_1(\tau) + \vec{\kappa}_2(\tau) \approx 0 \\ \vec{J} = \vec{S} = \vec{\eta}_1(\tau) \times \vec{\kappa}_1(\tau) + \vec{\eta}_2(\tau) \times \vec{\kappa}_2(\tau) \\ \vec{K} = -\sum_i^2 \vec{\eta}_i(\tau)\sqrt{m_i^2c^2 + \vec{\kappa}_i^2(\tau)} - \\ \quad - \frac{Q_1Q_2}{2c}\left[\vec{\eta}_1 \frac{\vec{\kappa}_1}{\sqrt{m_1^2c^2+\vec{\kappa}_1^2}} \cdot \left(\frac{1}{2}\frac{\partial K_{12}}{\partial \vec{\rho}} - 2\vec{A}_{\perp S2}(\vec{\kappa}_2,\vec{\rho})\right) + \vec{\eta}_2 \frac{\vec{\kappa}_2}{\sqrt{\vec{\kappa}_2^2+m_2^2c^2}} \cdot \left(\frac{1}{2}\frac{\partial K_{12}}{\partial \vec{\rho}} - 2\vec{A}_{\perp S1}(\vec{\kappa}_1,\vec{\rho})\right)\right] \end{cases}$$

$$- \frac{Q_1Q_2}{2c}\left(\sqrt{m_1^2c^2+\vec{\kappa}_1^2}\frac{\partial}{\partial \vec{\kappa}_1} + \sqrt{m_2^2c^2+\vec{\kappa}_2^2}\frac{\partial}{\partial \vec{\kappa}_2}\right) K_{12}(\vec{\kappa}_1,\vec{\kappa}_2,\vec{\rho}) -$$

$$- \frac{Q_1Q_2}{4\pi c}\int d^3\sigma \left(\frac{\vec{\pi}_{\perp S1}(\vec{\sigma}-\vec{\eta}_1,\vec{\kappa}_1)}{|\vec{\sigma}-\vec{\eta}_1|} + \frac{\vec{\pi}_{\perp S2}(\vec{\sigma}-\vec{\eta}_2,\vec{\kappa}_2)}{|\vec{\sigma}-\vec{\eta}_2|} \right) - \qquad \approx 0$$

$$- \frac{Q_1Q_2}{c}\int d^3\sigma \left[\vec{\pi}_{\perp S1}(\vec{\sigma}-\vec{\eta}_1,\vec{\kappa}_1) \cdot \vec{\pi}_{\perp S2}(\vec{\sigma}-\vec{\eta}_2,\vec{\kappa}_2) + \vec{B}_{S1}(\vec{\sigma}-\vec{\eta}_1,\vec{\kappa}_1) \cdot \vec{B}_{S2}(\vec{\sigma}-\vec{\eta}_2,\vec{\kappa}_2) \right]$$

QUANTUM MECHANICS OF SCALAR AND SPINNING PARTICLES IN NON-INERTIAL FRAMES

D. ALBA and L.L.
INT. J. MOD. PHYS
21 (2006) 2781 relativistic
21 (2006) 5917 non-relativistic
hep-th/0502060
0504060

MULTITEMPORAL QUANTIZATION

QUANTIZE ONLY THE PARTICLES BUT NOT THE INERTIAL APPEARANCES [$z^\mu(\tau,\vec{\sigma})$ C-NUMBER]

TORRE-VARADARAJAN - CLAS. QUAN. GRAV 16 (1999) 2651
ARAGEORGIS, EARMAN, RUETSCHE
STUDIES HIST. PHIL. MODERN PHYS
33 (2002) 151

OPEN PROBLEM

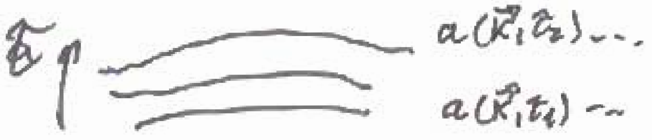
QUANTIZATION OF KG FIELD IN NON-INERTIAL FRAMES WITH THE PREVIOUS MULTITEMPORAL METHOD → STANDARD PARTICLE MODEL IN NON-INERTIAL FRAMES

AVOID THE TORRE-VARADARAJAN NO-GO THEOREM (NO UNITARY EVOLUTION IN THE TOMONOGA-SCHWINGER FORMALISM)

EVOLUTION (HAMILTONIAN) ONLY IN A 3+1 SPLITTING → FOURIER TRANSFORM ON INSTANTANEOUS 3-SPACES → FOCK SPACE

INSTEAD OF DETECTORS REPLACING PARTICLES (QFT IN CURVED SPACETIMES)

1) 3+1 SPLITTINGS WITH LEAVES ADMITTING FOURIER TRANSFORM

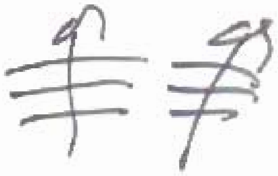


↕ BOGOLIUBOV TRANSFORMATION

$a'_k = \sum_l [\alpha_{kl} a_l + \beta_{kl} a^*_l]$

2) FIND 3+1 SPLITTINGS WITH HILBERT-SCHMIDT $\sum_{kl} |\beta_{kl}|^2 < \infty$

3) UNITARY EQUIVALENCE OF HS 3+1 SPLITTINGS



TRY TO REFORMULATE THE STANDARD MODEL OF ELEMENTARY PARTICLES IN NON-INERTIAL FRAMES (THE ONLY ONES EXISTING IN EINSTEIN GLOBALLY HYPERBOLIC SPACETIME DUE TO THE EQUIVALENCE PRINCIPLE)

QFT - HAAG THEOREM

NO UNITARY TRANSFORMATION FROM ASYMPTOTIC IN-STATES TO INTERPOLATING STATES WITH INTERACTION

(NON EXISTENCE OF INTERACTION PICTURE)

ATOMIC PHYSICS WITH FIXED NUMBER OF ATOMS

$O(\frac{1}{c})$ INTERACTION WITH THE ELECTROMAGNETIC FIELD

NEITHER GALILEI NOR POINCARE GROUP

LASER LIGHT DESCRIBED BY INTERPOLATING EM FIELD

(LIMIT OF QED IN COULOMB GAUGE WITH FIXED NUMBER OF PARTICLES
COHEN-TANNOUDJI)

ALBA, CRATER, LL - REST-FRAME INSTANT FORM OF SCALAR (OR SPINNING) CHARGED PARTICLES + EM FIELD

(GRASSMANN-VALUED ELECTRIC CHARGES TO REGULARIZE SELF-ENERGIES)

DARWIN (OR SALPETER) POTENTIALS AS THE EFFECTIVE POTENTIALS FOR 1-PHOTON EXCHANGE I

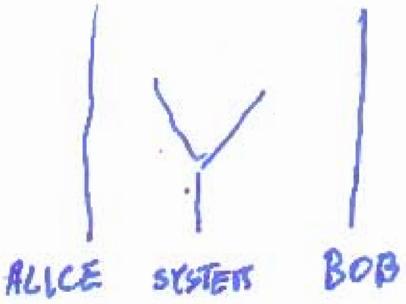
⇒ RELATIVISTIC ATOMIC PHYSICS WITH POINCARÉ GROUP

AT THE CLASSICAL LEVEL THERE IS A CANONICAL TRANSFORMATION

FROM
$$\boxed{\begin{array}{c}\text{PARTICLES}\\ +\\ \text{EM FIELD}\end{array}} \longrightarrow \boxed{\begin{array}{c}\text{DRESSED PARTICLES INTERACTING}\\ \text{WITH DARWIN POTENTIAL}\\ +\\ \text{FREE RADIATION EM FIELD}\end{array}}$$

IF UNITARILY IMPLEMENTABLE → WAY OUT FROM HAAG THEOREM FOR ATOMIC PHYSICS

NON-RELATIVISTIC QUANTUM MECHANICS IN INERTIAL FRAMES



$$\boxed{\begin{array}{lll} \vec{x}_1 & \vec{x}_2 & \vec{x}_3 \dots \\ \vec{p}_1 & \vec{p}_2 & \vec{p}_3 \dots \end{array}} \longmapsto \boxed{\begin{array}{l|lll} \vec{x} & \vec{r}_1 & \vec{r}_2 & \dots \\ \vec{p} & \vec{q}_1 & \vec{q}_2 & \dots \end{array}}$$

IS A POINT CANONICAL TRANSFORMATION

SEPARABILITY OF SUBSYSTEMS $H(\vec{x}_1) \otimes H(\vec{x}_2) \otimes H(\vec{x}_3) \dots$

UNITARILY EQUIVALENT TO $H(\vec{x}) \otimes H(\vec{r}_1) \otimes H(\vec{r}_2) \dots$

THEN

LOCALITY

AND THEORY OF MEASUREMENT

$\downarrow$

EPR, BELL'S INEQUALITIES

COLLAPSE OF WAVE FUNCTION

OR ....

$\downarrow$

ENTANGLEMENT

TELEPORTATION

NO-SIGNALLING THEOREM

RADAR COMMUNICATION

BUT MAXWELL EQUATIONS ARE ABSENT

PHOTONS ?

$\downarrow$

GO TO MINKOWSKI SPACETIME

RAY OF LIGHTS (EIKONAL APPROXIMATION OF MAXWELL EQS.)

MOVING ON NULL GEODESICS

AND CARRYING TWO POLARIZATION STATES

PERES-TERNO

QUANTUM INFORMATION AND RELATIVITY THEORY

RMP 76 (2004) 93

$$\Psi = \Psi(P) \otimes \Psi_{\underset{\text{HELICITY}}{\text{SPIN}}}$$

POINCARÉ TRANSFORMATION $\begin{cases} P \to P' = \Lambda P \\ s^i \to R(\Lambda, P)^i{}_j s^j \end{cases}$

$$\Psi' = \Psi(P') \otimes \Psi'_{\underset{\text{HELICITY}}{\text{SPIN}}}(P)$$

$\llcorner$ WIGNER ROTATION

SELF-ENTANGLEMENT

FOR 2 PARTICLES SYSTEM MORE COMPLICATED

RELATIVISTIC QUANTUM MECHANICS (AND QFT)

TWO ISOLATED SYSTEMS – IF WE USE THE HILBERT SPACE $H(1) \otimes H(2)$

$\Rightarrow$ THERE ARE STATES IN WHICH 1 IS IN THE ABSOLUTE FUTURE (OR PAST) OF 2

(S-MATRIX – IN- AND OUT- STATES OF THIS TYPE <u>ONLY</u> AT $t \to \pm\infty$)

TO ELIMINATE THESE CONFIGURATIONS WE NEED A CLOCK SYNCHRONIZATION CONVENTION IDENTIFYING AN INSTANTANEOUS 3-SPACE WHERE 1 AND 2 ARE SIMULTANEOUS (I.E. WITH A SPACELIKE SEPARATION)

$\Downarrow$

RELATIVISTIC 2-BODY PROBLEM $H\left(\begin{smallmatrix}\text{EXTERNAL}\\ \text{C.O.M}\end{smallmatrix}\right) \otimes H\left(\begin{smallmatrix}\text{RELATIVE}\\ \text{VARIABLES}\end{smallmatrix}\right)$

NOT UNITARILY EQUIVALENT TO $H(1) \otimes H(2)$

$\Downarrow$

1) $\boxed{\text{NON-SEPARABILITY OF SUBSYSTEMS (EVEN IF NON-INTERACTING)}}$

THE BEST APPROXIMATION TO SEPARABILITY IS TO LOOK FOR CANONICAL BASES OF RELATIVE VARIABLES ADAPTED TO ALICE AND BOB

2) RELATIVISTIC C.O.M. – IT IS A NON-LOCAL NOTION, BECAUSE DETERMINED BY THE POINCARÉ GENERATORS (ALL 3-SPACE Σ_t INVOLVED

NEWTON-WIGNER (CANONICAL NON-COVARIANT – WAY OUT FROM NO-INTERACTION THEOREM)

a) IF SELF-ADJOINT, THEN WAVE PACKETS SPREAD WITH INFINITE VELOCITY (HEGERFELDT) $\Rightarrow$ BAD LOCALIZATION

b) ALGEBRAIC QFT (REEH-SCHLIEDER THEOREM ...) $\Rightarrow$ PARTICLES DO NOT EXIST!

N.W LOCALIZATION $\neq$ LOCAL OR QUASI-LOCAL LOCALIZATION

↳ NON-LOCAL LOCALIZATION WITH POWER TAILS (HEGERFELDT)

MØLLER CENTER OF ENERGY (NEITHER CANONICAL NOR COVARIANT)

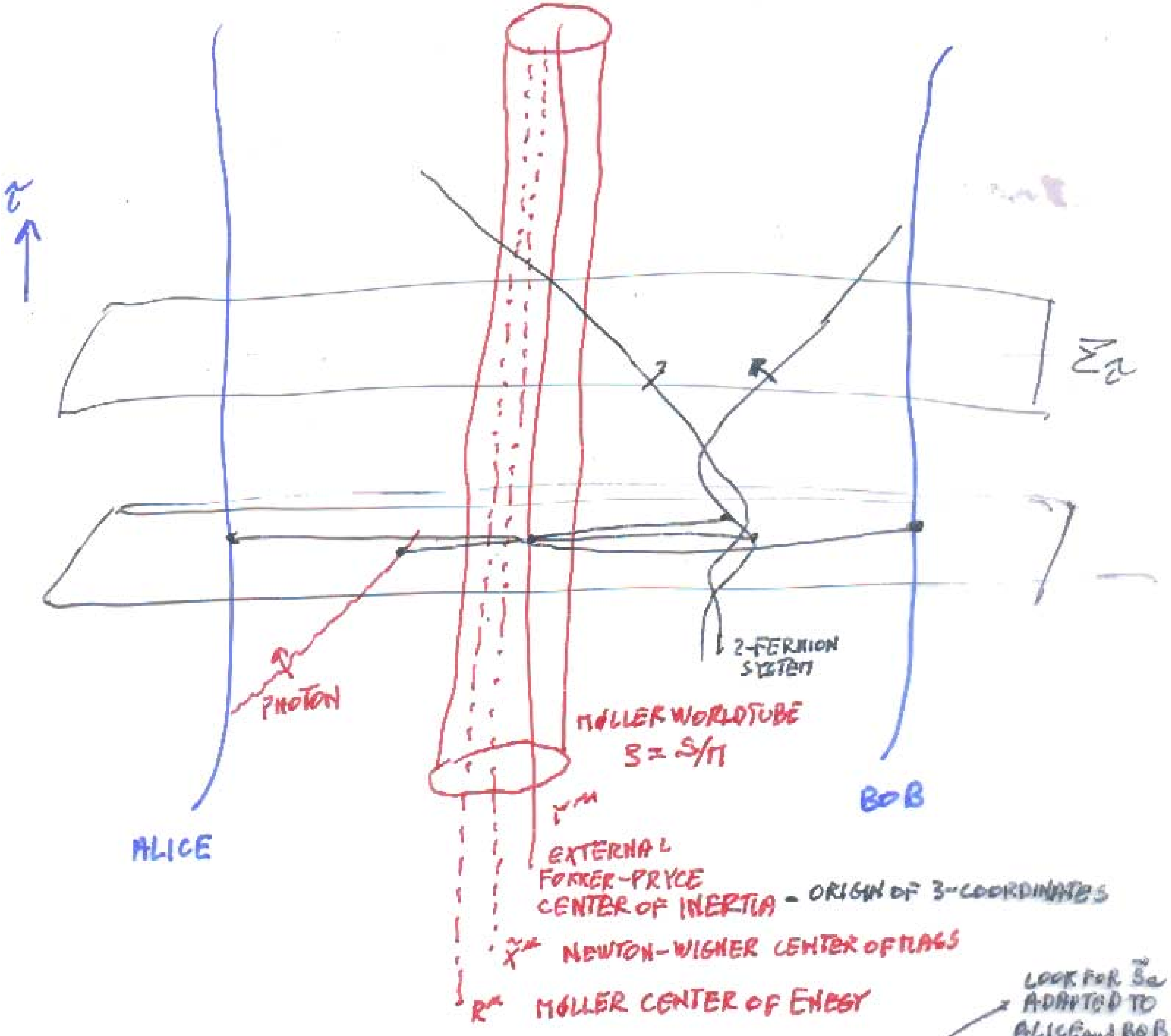
FOKKER-PRYCE CENTER OF INERTIA (COVARIANT, NON-CANONICAL)

3) RELATIVISTIC CAUSALITY BUT NON-FACTUAL CAUCHY PROBLEM –

TO HAVE PREDICTABILITY WE MUST GIVE THE CAUCHY DATA ON THE WHOLE 3-SPACE Σ_t (A-A-AD REAPPERS FROM THE CANONICAL REDUCTION OF GAUGE THEORIES LIKE THE EM FIELD)

4) LEARN HOW TO DESCRIBE PHYSICS <u>ONLY</u> IN TERMS OF RELATIVE VARIABLES

DIFFERENCE BETWEEN FREE AND INTERACTING CASES (WEAK RELATIONALISM)



$$\boxed{\begin{array}{cc} x_1^\mu & x_2^\mu \dots \\ p_1^\mu & p_2^\mu \dots \end{array}} \mapsto \boxed{\begin{array}{c|cc} \tilde{x}^\mu & \vec{\eta}_1 & \vec{\eta}_2 \dots \\ p^\mu & \vec{\kappa}_1 & \vec{\kappa}_2 \dots \end{array}} \; \Big| \begin{array}{l} \vec{\mathcal{P}} \approx 0 \\ \vec{q}_+^{\,int} \approx 0 \end{array} \longmapsto \boxed{\begin{array}{c|c|cc} \tilde{x}^\mu & \vec{q}_+^{\,int} \approx 0 & \vec{\rho}_1 & \vec{\rho}_2 \dots \\ p^\mu & \vec{\mathcal{P}} \approx 0 & \vec{\pi}_1 & \vec{\pi}_2 \dots \end{array}}$$

LOOK FOR $\vec{\rho}_a$ ADAPTED TO ALICE and BOB

EXTERNAL C.O.M.

INTERNAL C.O.M.

NON-POINT CANONICAL TRANSF.
(NOT UNITARILY IMPLEMENTABLE)

HILBERT SPACE $H(\tilde{x}^\mu) \otimes H(\vec{\rho}_1, \vec{\rho}_2 \dots, \text{EM FIELD})$

↳ ATOMIC SPECTRA

HOW TO IMPOSE THE SEMI-CLASSICAL NATURE OF ALICE AND BOB (MACROSCOPIC OBJECTS)?

- WAVE FUNCTIONS PEAKED ON CLASSICAL TRAJECTORIES (EHRENFEST THEOR.), BUT STARTING FROM A WAVEFUNCTION DEPENDING ON RELATIVE VARIABLES.
(LEARN FROM ORBIT RECONSTRUCTION AT CLASSICAL LEVEL)

GENERAL RELATIVITY (EINSTEIN SPACETIMES)

GEOMETRICAL VIEW OF THE GRAVITATIONAL FIELD

EVERYTHING IS DYNAMICAL

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu}$$

SPACETIME ≈ GRAVITATIONAL FIELD

⇒ CAUCHY SURFACE NEEDED

DOUBLE ROLE OF THE METRIC TENSOR $g_{\mu\nu}(x)$ [x^μ 4-COORDINATES IN A CHART OF THE ATLAS OF M^4]

A) POTENTIAL OF THE GRAVITATIONAL FIELD

B) DYNAMICAL CHRONOGEOMETRICAL STRUCTURE OF SPACETIME $ds^2 = g_{\mu\nu}(x)\, dx^\mu dx^\nu$

$g_{\mu\nu}(x)$ TEACHES RELATIVISTIC CAUSALITY TO THE OTHER FIELDS!

NULL GEODESICS ($ds^2=0$) VARYING WITH THE POINT (TRAJECTORIES OF PHOTONS GLUONS..

RELATIVITY PRINCIPLE → PRINCIPLE OF GENERAL COVARIANCE – $G_{\mu\nu}$ 4-TENSOR

POINCARÉ GROUP → DIFFEOMORPHISM GROUP → INVARIANCE IN FORM

EQUIVALENCE PRINCIPLE → GLOBAL INERTIAL FRAMES DO NOT EXIST!

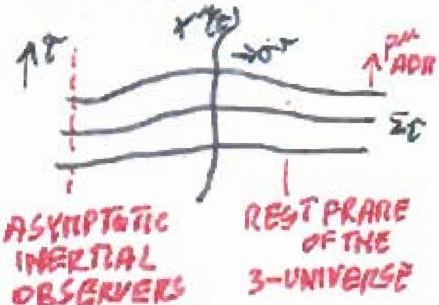
ONLY FREE FA

SPACETIMES

$M^4 \sim R^3 \times R$

A) GLOBALLY HYPERBOLIC (TIME), SPATIALLY NON-COMPACT, TOPOLOGICALLY TRIVIAL
↳ AND HAMILTONIAN FORMULATION

B) ASYMPTOTICALLY FLAT AT SPATIAL INFINITY AND WITHOUT TRANSLATION
↳ ASYMPTOTIC ADM POINCARÉ GROUP



GLOBAL NON-INERTIAL FRAMES CENTERED ON ACCELERATED OBSERVERS

$g_{\mu\nu}(x) \xrightarrow[\text{INFINITY}]{\text{SPATIAL}} \eta_{\mu\nu}$ ASYMPTOTIC BACKGROUND

ASYMPTOTICALLY MINKOWSKIAN SPACETIME

↳ DEPARAMETRIZATION TO THE REST-FRAME INSTANT FORM $P^\mu_{ADM} \to P^\mu$, $J^{\mu\nu}_{ADM} \to J^{\mu\nu}$

(IN ABSENCE OF MATTER CHRISTODOULOU-KLAINERMANN SPACETIMES)

HAMILTONIAN FORMULATION TO DISENTANGLE THE GAUGE VARIABLE FROM THE PHYSICAL DEGREES OF FREEDOM OF THE GRAVITATIONAL FIELD

⇒ FIXED GAUGE WITH DETERMINISTIC HAMILTON EQS AND A WELL POSED CAUCHY PROBLEM

TETRAD GRAVITY (FERMIONS) $g_{AB}(\tau,\vec\sigma) = E^{(\alpha)}_A(\tau,\vec\sigma)\; \eta_{(\alpha)(\beta)}\; E^{(\beta)}_B(\tau,\vec\sigma)$

USE ADM ACTION

$$E^{(\alpha)}_{A} = L^{(\alpha)}{}_{(o)}(\varphi_{(a)}) \begin{pmatrix} N \\ \vec{0} \end{pmatrix} + L^{(\alpha)}{}_{(a)}(\varphi_{(a)}) \begin{pmatrix} n_{(a)} \\ {}^3e_{(a)r} \end{pmatrix}$$

$${}^3e_{(a)r} = R_{(a)(b)}(\alpha_{(c)})\, V_{ra}(\theta^n)\, \phi^2\, e^{\sum_{\bar{a}} \gamma_{\bar{a}a} R_{\bar{a}}}$$

$$g_{\tau\tau} = N^2 - n_{(a)} n_{(a)} \qquad g_{\tau r} = {}^3e_{(a)r}\, n_{(a)}$$

$$g_{rs} = \sum_a {}^3e_{(a)r}\, {}^3e_{(a)s} = \phi^4 \sum_a V_{ra}(\theta^n) V_{sa}(\theta^n)\, e^{2\sum_{\bar{a}} \gamma_{\bar{a}a} R_{\bar{a}}}$$

$\alpha_{(a)}, \varphi_{(a)}$ O(3,1) GAUGE FREEDOM OF TETRADS – generators $\pi^{\varphi}_{(a)} \approx 0$, $\pi_{(a)} \approx 0$

GYROSCOPES AND THEIR TRANSPORT

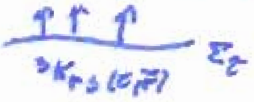
θ^n 3-COORDINATE GAUGE FREEDOM ON Σ_τ

(π^{θ}_{n} from $\mathcal{H}_{(a)} \approx 0$)

$n_{(a)}$ GRAVITOMAGNETISM GAUGE FREEDOM $\pi_{n_{(a)}} \approx 0$

$\phi^6 = {}^3e$ VOLUME ELEMENT FROM $\mathcal{H} \approx 0$ $\Rightarrow \boxed{\phi \approx \phi(R_{\bar{a}}, \Pi_{\bar{a}}, \theta^n, \Pi_\phi)}$

$\pi_\phi \propto {}^3K$ GAUGE FREEDOM IN CLOCK SYNCHRONIZATION



n GAUGE FREEDOM IN LOCAL UNIT OF PROPER TIME $\pi_n \approx 0$

$$H_D = E_{ADM} + \int d^3\sigma \left[n\,\mathcal{H} + n_{(a)}\,\mathcal{H}_{(a)} \right]$$

$R_{\bar{a}}$, $\Pi_{\bar{a}}$ TIDAL EFFECTS

INERTIAL POTENTIALS FROM GAUGE VARIABLES

θ^n → CORIOLIS, ... (Γ-Γ in E_{ADM})

$n_{(a)}$ → GRAVITOMAGNETIC POTENTIAL

π_ϕ → INSTANTANEOUS 3-SPACE INERTIAL POTENTIAL NOT EXISTING IN NEWTON THEORY

n → PROPER TIME

FIX THE GAUGE ⟷ KINEMATICAL NON-INERTIAL FRAME

⇓

DETERMINISTIC HAMILTON EQS FOR THE TIDAL $R_{\bar{a}}, \Pi_{\bar{a}}$

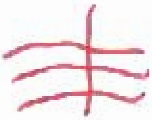
+

CAUCHY DATA ON A Σ_τ of γ

⇓

EINSTEIN'S SPACETIME WITH $g_{AB}(\tau,\vec{\sigma})$ IN COORDINATES $(\tau,\vec{\sigma})$

AND A DYNAMICAL NON-INERTIAL FRAME



ONE OF WHOSE EQUAL-TIME SURFACES
IS THE CAUCHY SURFACE OF THE SOLUTION

⇓

DYNAMICAL INSTANTANEOUS 3-SPACE
DYNAMICAL CLOCK SYNCHRONIZATION CONVENTION

L.L. and PAURI

GEN. REL. GRAV 38 (2006) 187 and 229
gr-qc/0403081 and 0407007

STUDIES HIST. AND PHIL. MODERN PHYS.
37 (2006) 692 gr-qc/0604087

Book RELATIVITY AND THE DIMENSIONALITY OF GR
SPRINGER 2007, ed. V. Petkov
gr-qc/0611045

NEITHER SUBSTANTIVALISM
NOR RELATIONISM

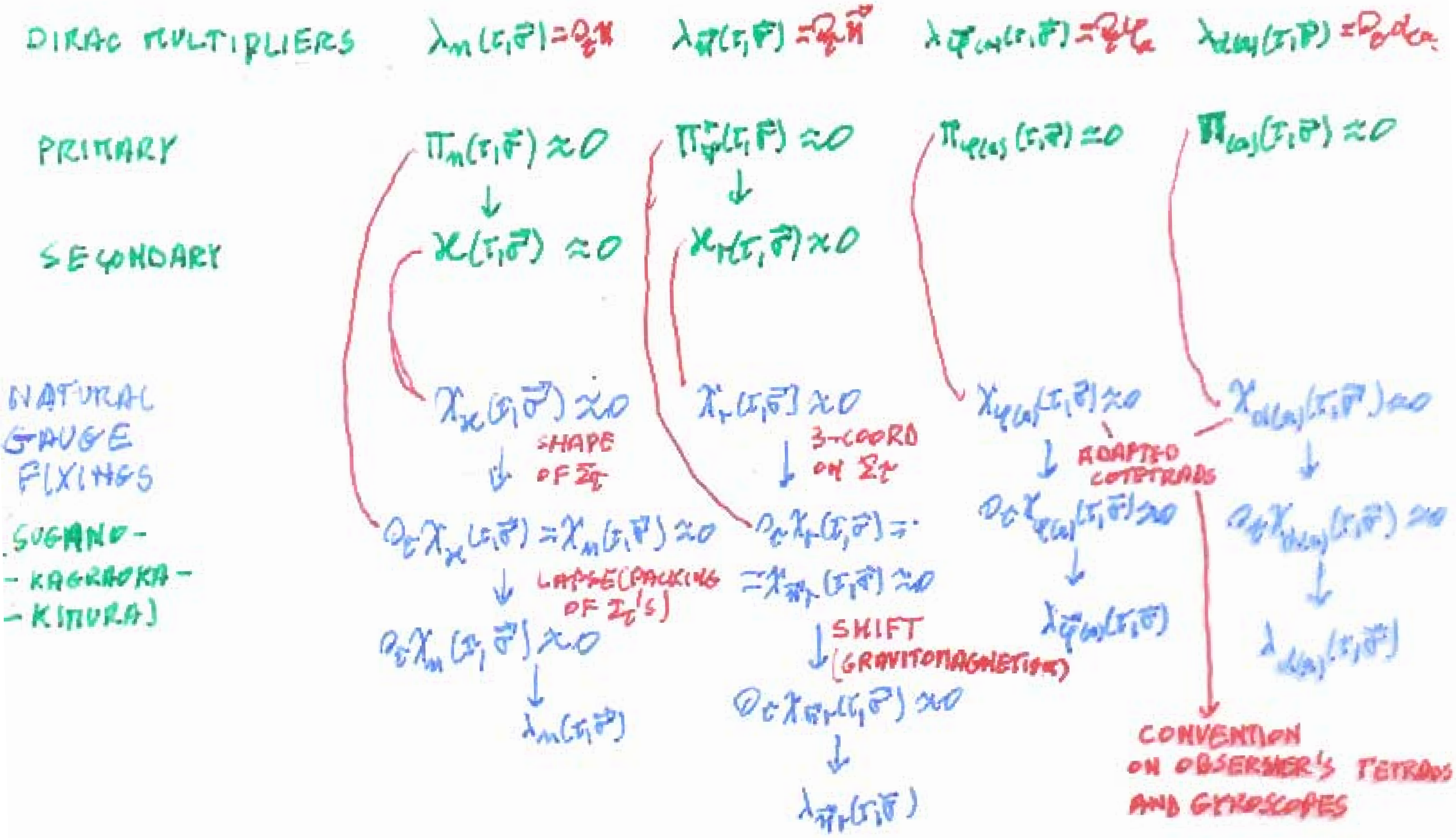
BUT SOME TYPE OF STUCTURALISM

SCHEME OF GAUGE-FIXINGS

SECONDARY CONSTRAINTS

$$H_D = \check{E}_{ADM} + \int d^3\sigma\, \Sigma_\alpha \lambda_\alpha(\tau,\vec\sigma)\, \Phi_\alpha(\tau,\vec\sigma) + \int d^3\sigma\, [N\mathcal{H} + N_r\mathcal{H}^r](\tau,\vec\sigma) \approx \check{E}_{ADM}$$

PRIMARY CONSTRAINTS



COMPLETELY FIXED HAM. GAUGE $\equiv$ 4-COORDINATE SYSTEM +

ON SOLUTIONS OF HAM. EQS (I.E. EINSTEIN EQS)

\+ 3+1 SPLITTING WITH LEAVES

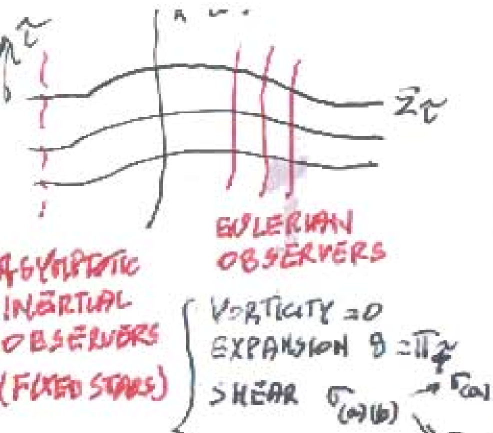
Σ_τ (IN GENERAL NOT HYPERPLANES)

DEFINED BY AN EMBEDDING

$z^\mu_\sigma(\tau,\vec\sigma)$

||

CONVENTION FOR THE SYNCHRONIZATION OF CLOCKS (≠ EINSTEIN'S CONVENTION)



VORTICITY $= 0$
EXPANSION $\theta = \pi_{\tilde{\phi}}$
SHEAR $\sigma_{(a)(b)}$ $\to \sigma_{(a)(a)} \Leftrightarrow \pi_{\bar{a}}$
$\searrow \sigma_{(a)(b)}|_{a\neq b} \Leftrightarrow \pi_i^{(\theta)}$
ACCELERATION $\frac{\vec{\partial} n}{n}$

0604086 TO APPEAR ON GEN.REL.GRAV

SCHWINGER TIME GAUGE → YORK CANONICAL BASIS

$\to$ from $\mathcal{H} \approx 0$

$\alpha_{(a)} \approx 0$	$\varphi_{(a)} \approx 0$	n	$\bar{n}_{(a)}$	θ^i	$\tilde{\phi} = \phi^6$	$R_{\bar{a}}$
≈ 0	$\approx$	≈ 0	≈ 0	$\pi_i^{(\theta)}$	$\pi_{\tilde{\phi}}$	$\pi_{\bar{a}}$

$\hookrightarrow$ from $\mathcal{H}_{(a)} \approx 0$

3-SCALARS

$$\begin{cases} {}^4g_{\tau\tau} = \epsilon\left[(1+n)^2 - \vec{n}^2_{(a)}\right] \\ {}^4g_{\tau r} = -\epsilon\, \vec{n}_{(a)}\, {}^3\vec{e}_{(a)r} \\ {}^4g_{rs} = -\epsilon\, {}^3g_{rs} = -\epsilon\, {}^3\vec{e}_{(a)r}\, {}^3\vec{e}_{(a)s}, \quad {}^3\vec{e}_{(a)r} = \phi^2 Q_a V_{ra}(\theta^i), \quad Q_a = e^{\sum_{\bar{a}} \gamma_{\bar{a}a} R_{\bar{a}}} \end{cases}$$

$$\begin{cases} {}^4\overset{o}{E}{}^A_{(o)} = l^A = \frac{1}{1+n}\left(1;\, -\bar{n}_{(a)}\, {}^3\vec{e}^{\,r}_{(a)}\right) \\ {}^4\overset{o}{E}{}^A_{(a)} = \left(0;\, {}^3e^r_{(a)}\right) \end{cases}$$

$$\begin{cases} H_D = E_{ADM} + \int d^3\sigma \Big[n\,\mathcal{H} + \vec{n}_{(a)}\mathcal{H}_{(a)} + \lambda_n \pi_n + \lambda_{\bar{n}_{(a)}} \pi_{\bar{n}_{(a)}} + \underset{\approx 0}{\mu_{(a)}}\, \pi_{(a)} + \underset{\approx 0}{\hat{\mu}_{(a)}}\, \pi_{\varphi_{(a)}} \Big] \\ \vec{P}_{ADM} \approx 0 \quad \text{REST-FRAME CONDITIONS} \\ \vec{K}_{ADM} \approx 0 \quad \text{GAUGE-FIXING} \end{cases}$$

EXTERNAL CENTER OF MASS OF THE 3-UNIVERSE DECOUPLED

$$\begin{cases} \mathcal{H} = \frac{c^3}{16\pi G}\left[\phi\left(-8\hat{\Delta}\phi + {}^3\hat{R}\phi\right) - \phi^6 \sum_{a\neq b} \sigma^2_{(a)(b)}\right] - \underline{\mathcal{M}} - \frac{4\pi G}{c^3}\phi^{-6}\sum_{\bar{a}} \pi^2_{\bar{a}} + \frac{6\pi G}{c^3}\phi^6 \underline{\pi^2_{\tilde{\phi}}} \approx 0 \\ \mathcal{H}_{(a)} = \sum_{rb} D_{r(a)(b)}\left[\frac{c^3}{8\pi G}\phi^4 \sum_{c\neq b} Q_c^{-1} V_{rc}\, \sigma_{(b)(c)} + \phi^4 Q_b^{-1} V_{rb} \underline{\pi_{\tilde{\phi}}} - \phi^2 Q_b^{-1} V_{rb} \sum_{\bar{a}} \gamma_{\bar{a}b} \pi_{\bar{a}}\right] - \phi^2 Q_a^{-1} \sum_v V_{ra} \underline{\mathcal{M}_v} \approx 0 \\ E_{ADM} = c\int d^3\sigma \left\{ \underline{\mathcal{M}} - \frac{c^3}{16\pi G}\left(S - \phi^6 \sum_{a\neq b}\sigma^2_{(a)(b)}\right) - \frac{6\pi G}{c^3}\underline{\pi^2_{\tilde{\phi}}} + \frac{4\pi G}{c^3}\phi^{-6}\sum_{\bar{a}}\pi^2_{\bar{a}} \right\}(\tau,\vec{\sigma}) = c\, P^o_{ADM} \end{cases}$$

→ SIGN OF ${}^3K$ RELEVANT

$\hookrightarrow$ $\Gamma\Gamma - \Gamma\Gamma$ 3-COORDINATE-DEPENDENT (INERTIAL POTENTIALS LIKE IN NEWTON THEORY)

$$H_D = \int d^3\sigma \Big\{ (1+n)\mathcal{M} + \frac{4\pi G}{c^3}(1+n)\phi^{-6}\sum_{\bar{a}}\pi^2_{\bar{a}} - \frac{c^3}{16\pi G}\Big[S + n\phi\left(-8\hat{\Delta}\phi + {}^3\hat{R}\phi\right) - (1+n)\phi^6 \sum_{a\neq b}\sigma^2_{(a)(b)}\Big] - \frac{6\pi G}{c^3}(1+n)\phi^6 \underline{\pi^2_{\tilde{\phi}}}$$
$$+ \sum_a \bar{n}_{(a)}\mathcal{H}_{(a)} + \lambda_n \pi_n + \lambda_{\bar{n}_{(a)}} \pi_{\bar{n}_{(a)}} \Big\}(\tau,\vec{\sigma})$$

IT DEPENDS ON THE GAUGE VARIABLES $\theta^i, \pi_{\tilde{\phi}}, n, \bar{n}_{(a)}$
AND DIRAC MULTIPLIERS $\lambda_n \overset{\circ}{=} \partial_\tau n$, $\lambda_{\bar{n}_{(a)}} \overset{\circ}{=} \partial_\tau \bar{n}_{(a)}$ $\Rightarrow$ INERTIAL POTENTIALS IN NON-INERTIAL FRAMES

ADM

$n, n_r, {}^3g_{rs}, {}^3\pi^{rs} \propto \sqrt{{}^3g}\,({}^3K^{rs} - {}^3K\,{}^3g^{rs})$

$\mathcal{H} \approx 0,\ \mathcal{H}_r \approx 0,\ \pi_n \approx 0,\ \pi_{n_r} \approx 0$

$$\begin{cases} \partial_\tau\, {}^3g_{rs} = n_{r|s} + n_{s|r} - 2(1+n)\,{}^3K_{rs} \\ \partial_\tau\, {}^3K_{rs} = (1+n)\left({}^3R_{rs} + {}^3K\,{}^3K_{rs} - 2\,{}^3K_{ru}\,{}^3K^u{}_s\right) - \\ \quad - n_{|s|r} + n^u{}_{|s}\,{}^3K_{ur} + n^u{}_{|r}\,{}^3K_{us} + n^u\,{}^3K_{rs|u} \end{cases}$$

12 EQUATIONS

$${}^3K_{rs} = \frac{1}{3}\underset{\overset{\parallel}{{}^3K = \pi_{\tilde\phi}}}{\theta}\,{}^3g_{rs} + \sum_{ab}\sigma_{(a)(b)}\,{}^3\tilde e_{(a)r}\,{}^3\tilde e_{(b)s}$$

$\partial_\tau\, \theta \doteq \cdots$ RAYCHAUDHURI EQUATION (CAUSTICS OF WORLDLINES OF EULERIAN OBSERVERS)

YORK CANONICAL BASIS

$$\begin{cases} \partial_\tau R_{\bar a} \overset{\circ}{=} \{R_{\bar a}, H_D\} \\ \partial_\tau \Pi_{\bar a} \overset{\circ}{=} \{\Pi_{\bar a}, H_D\} \\ + \text{matter equations} \end{cases}$$

4 HYPERBOLIC EQS FOR TIDAL EFFECTS WITH INERTIAL FORCES DEPENDING ON GAUGE VARIABLES $\theta^i, \pi_{\tilde\phi}, n, \bar n_{(a)}$ AND DIRAC MULTIPLIERS $\lambda_n, \lambda_{\bar n_{(a)}}$

$\tilde\phi = \phi^6$ SOLUTION OF $\mathcal{H} \approx 0$, $\pi_i^{(\theta)}\left(\sigma_{(a)(b)}|_{a\neq b}\right)$ SOLUTION OF $\mathcal{H}_{(a)} \approx 0$

$$\begin{cases} \partial_\tau \tilde\phi \overset{\circ}{=} \{\tilde\phi, H_D\} \\ \partial_\tau \pi_i^{(\theta)} \overset{\circ}{=} \{\pi_i^{(\theta)}, H_D\} \end{cases}$$

4 CONTRACTED BIANCHI IDENTITIES (POSSIBLE RESTRICTIONS ON THE GROUP MANIFOLD OF 4-DIFFEOMORPHISMS)

$$\begin{cases} \partial_\tau \pi_{\tilde\phi} \overset{\circ}{=} \{\pi_{\tilde\phi}, H_D\} \quad \text{RAYCHAUDHURI EQ} \\ \partial_\tau \theta^i \overset{\circ}{=} \{\theta^i, H_D\} \end{cases}$$

4 EQS FOR LAPSE n AND SHIFT $\bar n_{(a)}$ TO BE USED AFTER HAVING GIVEN THE 4 GAUGE FIXINGS

$$\begin{cases} \pi_{\tilde\phi} \approx \cdots \\ \theta^i \approx \cdots \end{cases}$$

IDENTIFICATION OF THE INSTANTANEOUS 3-SPACE Σ_τ (CAUCHY SURFACE AND CLOCK SYNCHRONIZATION)

FIXATION OF 3-COORDINATES ON Σ_τ

$n, \bar n_{(a)}$

$\Rightarrow \lambda_n \approx \partial_\tau n,\ \lambda_{\bar n_{(a)}} \approx \partial_\tau \bar n_{(a)}$

THE INERTIAL EFFECTS CONNECTED TO $\pi_{\tilde\phi}$ (CLOCK SYNCHRONIZATION) DO NOT EXIST IN NEWTON THEORY (ABSOLUTE TIME)

→ DARK MATTER AS A RELATIVISTIC INERTIAL EFFECT? COOPERSTOCK-TIEU (GRAVITOMAGNETISM) ALTERNATIVE TO MOND

→ BASIS OF DARK ENERGY BY USING COARSE GRAINED COSMOLOGY (BUCHERT, ELLIS)

→ HOMOGENEITY AS MEAN OVER VOLUMES AT LARGE SCALES

1) YORK-LICHNEROWICZ CONFORMAL APPROACH

→ YORK CANONICAL BASIS

2) 1+3 POINT OF VIEW OF G. ELLIS IN EISTEIN'S EQS FOR COSMOLOGY

→ EXPANSION, SHEAR, ACCELERATION OF EULERIAN OBSERVER

1)+2) ADM HAMILTONIAN EQS REWRITTEN IN YORK CANONICAL BASIS

3) ONLY ASYMPTOTIC BACKGROUND

4) NATURAL GAUGE - 3-ORTHOGONAL $^3g_{rs}$ DIAGONAL

ONLY 1 Σ_τ INVOLVED

A) EXPLORE THE INERTIAL EFFECT CONNECTED TO THE GAUGE FREEDOM IN THE CHOICE OF THE INSTANTANEOUS 3-SPACE (CHOICE OF CLOCK SYNCHRONIZATION

B) HARMONIC GAUGE - VERY COMPLICATED IN YORK CANONICAL BASIS

↓

POST-NEWTONIAN GRAVITY (IAU 2000 CONVENTIONS)

GRAVITATIONAL WAVES

DAMOUR ....., BLUMBERG .....

$g_{\mu\nu} \approx \eta_{\mu\nu} + h_{\mu\nu}$ FIXED MINKOWSKI BACKGROUND, 1/c EXPANSIONS

WAVE EQUATIONS RETARDED GREEN FUNCTIONS ALL Σ_τ's FROM

NO IN-RADIATION $\tau \to -\infty$ INVOLVED

⇒ 3.5 PN, BINARIES, N-BODY PROBLEM, MULTIPOLES

UNDER STUDY - 2-BODY PROBLEM IN 3-ORTHOGONAL GAUGE

WITH GRASSMANN REGULARIZATION OF SELF-ENERGIES

ONLY ASYMPTOTIC BACKGROUND

POST-MINKOWSKIAN (RESUMMATION OF 1/c EXPANSION)

SEE THE INERTIAL POTENTIAL FROM $\Pi_\phi \approx {}^3K$ ON EACH Σ_τ

(SIMULATION OF DARK MATTER)

DARK MATTER AS A BYPRODUCT OF GRAVITY ?

A) MOND – MODIFY NEWTON LAW $\frac{\vec{F}}{m} = \vec{a} \rightarrow \frac{\vec{F}}{m} = \vec{f}(\vec{a}, \vec{a}_o) \xrightarrow[\text{BIG } \vec{a}]{} \vec{a}$

FIT ROTATION CURVES OF GALAXIES ($|\vec{a}_o|$ ~ HUBBLE CONSTANT)

B) COOPERSTOCK – TIEU

INT.J.MOD.PHYS A22 (2007) 2293
astro-ph/0610370
MOD.PHYS.LETT. 21 (2006) 2133

GALAXY AS A UNIFORMLY ROTATING FLUID WITHOUT PRESSURE AND SYMMETRIC ABOUT ITS ROTATION AXIS + STATIONARY AXIALLY SYMMETRIC 4-METRIC

GRAVITOMAGNETIC EFFECT (g_{oi}, SHIFT FUNCTION)

C) RELATIVISTIC INERTIAL EFFECT FROM THE INERTIAL POTENTIAL CONNECTED TO THE GAUGE VARIABLE $\pi_\phi = {}^3K$ (CHOICE OF THE INSTANTANEOUS 3-SPACE, I.E. CLOCK SYNCHRONIZATION CONVENTION): IT INFLUENCES ALSO THE SHIFT FUNCTIONS. $m\vec{a} = \vec{F} + \vec{F}_{INERTIAL}$

?

DARK ENERGY AS A BYPRODUCT OF GRAVITY ?

T. BUCHERT – DARK ENERGY FROM STRUCTURE: A STATUS REPORT
0707.2153 TO APPEAR IN SPECIAL ISSUE OF GEN. REL. GRAV. ON DARK ENERGY

G.F.R. ELLIS AND T. BUCHERT – THE UNIVERSE SEEN AT DIFFERENT SCALES
PHYS. LETT. A347 (2005) 38

α) the 4-VELOCITY FIELD OF A FLUID IDENTIFIES A PREFERRED CONGRUENCE OF TIMELIKE OBSERVERS. IF ZERO VORTICITY ALSO A PREFERRED 3+1 SPLITTING (CLOCK SYNCHRONIZATION CONVENTION) IS SELECTED

CMB OBSERVERS, COSMOLOGICAL FRW OBSERVERS

β) GIVEN PREFERRED 3+1 SPLITTING MAKE AVERAGES OF SCALARS ON VOLUMES V ON Σ_τ

⇒ COARSE GRAINED COSMOLOGY – HOMOGEINITY EMERGING AT BIG SCALES

$\theta = \pi_\phi = {}^3K \quad \langle \partial_\tau \theta \rangle \neq \partial_\tau \langle \theta \rangle$, FROM ADM EQS. $\partial_\tau \theta = ----$ (RAYCHAUDHURI EQ)

↳ EXTRINSIC (KINEMATICAL) AND INTRINSIC BACK-REACTION

⇒ MODIFIED FRW EQS WITH BACKREACTION TERMS SIMULATING DARK ENERGY

ITS QUANTITATIVE RELEVANCE IS AN OPEN PROBLEM

STUDY IN THE YORK CANONICAL BASIS WITH CLEAR SEPARATION OF INERTIAL EFFECTS FROM TIDAL EFFECTS.

REPLACE TIDAL VARIABLES $R_{\bar a}, \Pi_{\bar a}$, $\bar a = 1,2$, (3-SCALARS)

WITH 2 PAIRS OF CANONICALLY CONJUGATED 4-SCALARS

NEWMAN-PENROSE WEYL SCALARS (NULL TETRAD DEPENDENT)

↓

THE 4 ALGEBRAICALLY INDEPENDENT 4-SCALARS
WEYL EIGENVALUES
$Tr(gCgC)$, $Tr(gC\epsilon C)$, $Tr(gCgCgC)$, $Tr(gCgC\epsilon C)$

QUANTUM GRAVITY – BACKGROUND INDEPENDENT QUANTIZATION
OF THE 4 WEYL EIGENVALUES
WITHOUT QUANTIZATION OF THE GAUGE VARIABLES
(INERTIAL EFFECTS AS APPEARANCES OF PHENOMENA)

LIKE IN RELATIVISTIC QUANTUM MECHANICS
IN NON-INERTIAL FRAMES

INVERSE PROBLEM

FIX COMPLETELY THE GAUGE BUT WITH $^3K(\tau,\vec{\sigma})$ AN ARBITRARY FUNCTION (ISTANT. 3-SPACE AND CLOCK SYNCHR. CONVENTION ARBITRARY)

$\Rightarrow$ FAMILY OF GAUGES WITH LAPSE AND SHIFT FUNCTIONS DEPENDING ON $^3K(\tau,\vec{\sigma})$

1) LET THE MATTER BE A BALL OF DUST (A GALAXY)

STUDY THE HAMILTON EQS FOR $R_{\bar{a}}, \Pi_{\bar{a}}$, DUST

LOOK FOR A $^3K(\tau,\vec{\sigma})$, NAMELY FOR A GLOBAL **NON-INERTIAL** FRAME, CENTERED ON THE EARTH, IN WHICH WE GET THE OBSERVED ROTATION CURVE OF THE GALAXY LIKE IT HAPPENS WITH MOND OR IN THE SIMPLE COOPERSTOCK-TIEU MODEL

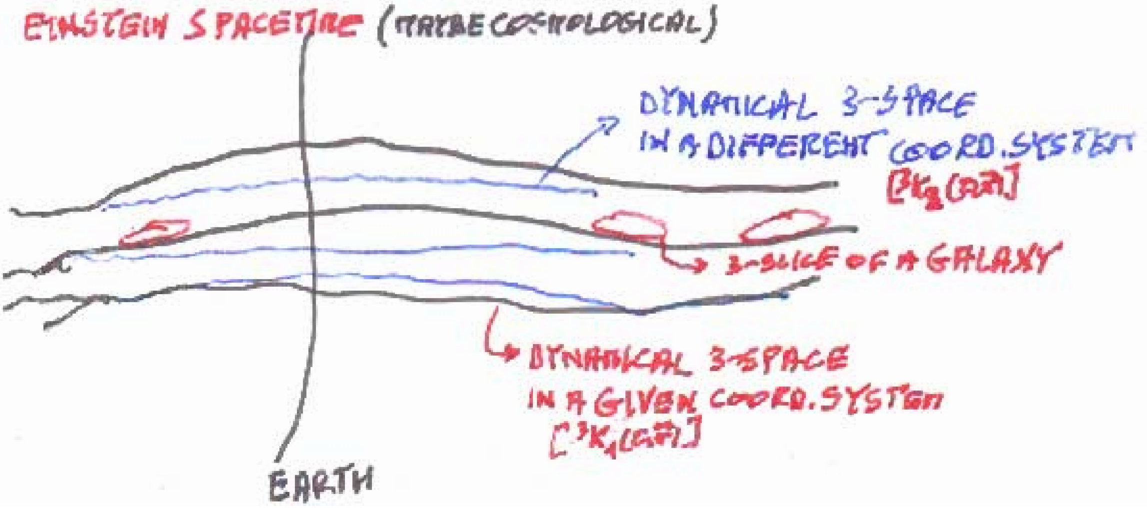
DARK MATTER AS AN INERTIAL EFFECT?

2) LET THE MATTER BE THE ELECTROMAGNETIC FIELD

HOW THE DISTRIBUTION OF THE MAGNETIC FIELD VARIES WITH $^3K(\tau,\vec{\sigma})$?

(COSMIC MAGNETIC FIELD PROBLEM)

EINSTEIN SPACETIME (MAYBE COSMOLOGICAL)



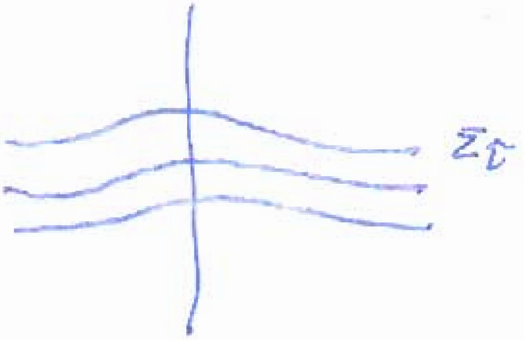
WHICH NON-INERTIAL FRAME GIVES A BETTER DESCRIPTION OF THE 3-UNIVERSE

$$\begin{cases} E_{ADM} = \int d^3\sigma\, \mathcal{E}_{ADM}(\tau,\vec\sigma) \approx \hat{E}_{ADM} = \int d^3\sigma\, \partial_r \mathcal{T}^{\tau r}_{ADM}(\tau,\vec\sigma) \\ P^r_{ADM} = \int d^3\sigma\, \mathcal{P}^r_{ADM}(\tau,\vec\sigma) \approx \hat{P}^r_{ADM} = \int d^3\sigma\, \partial_s \mathcal{T}^{rs}_{ADM}(\tau,\vec\sigma) \end{cases}$$

$\Rightarrow$ EXTRACT ADM PSEUDO ENERGY-MOMENTUM TENSOR

$$\mathcal{T}^{AB}_{ADM}(\tau,\vec\sigma)$$

IT DEPENDS UPON $\begin{cases} \text{INERTIAL EFFECTS (GAUGE VARIABLES), IN PARTICULAR ON } {}^3K(\tau,\vec\sigma) \\ \text{TIDAL EFFECTS} \end{cases}$



$\sum_r \mathcal{T}^{rr}_{ADM}(\tau,\vec\sigma)$ MAY HAVE SIGN CHANGING FROM A REGION OF Σ_τ TO ANOTHER ONE

$$\Updownarrow$$

$${}^3P$$

COMPARE WITH

$$\mathcal{T}^{AB}(\tau,\vec\sigma) = \left[\rho\, u^A u^B + (P - \zeta\vartheta)\, h^{AB} - 2\eta\, \sigma^{AB} + q^A u^B - q^B u^A \right](\tau,\vec\sigma)$$

EXTRACT $\rho(\tau,\vec\sigma)$, $P(\tau,\vec\sigma)$, $u^A(\tau,\vec\sigma)$ ......

$\Rightarrow$ PSEUDO-VISCOUS-FLUID PERVADING INSTANTANEOUS 3-SPACE Σ_τ, WITH PROPERTIES CHANGING FROM POINT TO POINT

IS DARK ENERGY A COSMOLOGICAL SIMULATION OF THIS PSEUDO-FLUID ?

IAU 2000 RESOLUTIONS

ASTRO-PH/0303376

$2 \times \pm 1$

1) BARYCENTRIC CELESTIAL REFERENCE SYSTEM (BCRS)

↳ SOLAR SYSTEM SPATIAL AXES → FIXED STARS (ICRS, HYPPARCOS)

~ INERTIAL (IGNORE EXTERNAL GALACTIC AND EXTRAGALACTIC MATTER)

$$\begin{cases} t = TCB & \text{BARYCENTRIC COORD.-TIME} \\ x^i & \\ g_{\mu\nu} \xrightarrow[r\to\infty]{} \varepsilon(\eta\text{---}) & \text{ASYMPTOTICALLY FLAT} \\ w, w^i & \text{GRAV. POTENTIALS} \end{cases}$$

$$\begin{cases} g_{00} = \varepsilon\left[1 - \frac{2w}{c^2} + \frac{2w^2}{c^4}\right] + O(c^{-5}) \\ g_{0i} = -\varepsilon\left[-\frac{4}{c^3} w^i\right] + O(c^{-5}) \\ g_{ij} = -\varepsilon\left[\left(1 + \frac{2w}{c^2}\right)\delta_{ij}\right] + O(c^{-4}) \end{cases}$$

HARMONIC GAUGE

$$\begin{cases} \Box w = -4\pi G\, \sigma + O(c^{-4}) \\ \Delta w^i = -4\pi G\, \sigma^i + O(c^{-2}) \end{cases}$$

$$\begin{cases} w(t,\vec{x}) = G \int d^3x' \frac{\sigma(t,\vec{x}')}{|\vec{x}-\vec{x}'|} + \frac{G}{2c^2}\frac{\partial^2}{\partial t^2}\int d^3x' |\vec{x}-\vec{x}'|\, \sigma(t,\vec{x}' \\ w^i(t,\vec{x}) = G \int d^3x' \frac{\sigma^i(t,\vec{x}')}{|\vec{x}-\vec{x}'|} \end{cases}$$

$\sigma = \frac{1}{c^2}(T^{00} + T^{ss})$, $\sigma^i = \frac{1}{c} T^{0i}$ SOLAR SYSTEM $T^{\mu\nu}$

RETARDED SOLUTION

2) GEOCENTRIC CELESTIAL REFERENCE SYSTEM (GCRS)

QUASI-INERTIAL · { SPATIAL AXES NON-ROTATING W.R.T BCRS
ACCELERATED GEOCENTER OF THE GEOID

$$\begin{cases} T = TCG & \text{GEOCENTRIC COORD.-TIME} \\ X^a & \\ G_{\alpha\beta} & \\ W, W^a & \end{cases}$$

$$\begin{cases} G_{00} = \varepsilon\left[1 - \frac{2W}{c^2} + \frac{2W^2}{c^4}\right] + O(c^{-5}) \\ G_{0a} = -\varepsilon\left[-\frac{4}{c^3} W^a\right] + O(c^{-5}) \\ G_{ab} = -\varepsilon\left[\left(1 + \frac{2W}{c^2}\right)\delta_{ab}\right] + O(c^{-4}) \end{cases}$$

$$\begin{cases} W(T,\vec{X}) = W_{EARTH}(T,\vec{X}) + W_{TIDAL}(T,\vec{X}) + W_{INERTIAL}(T,\vec{X}) \\ W^a(T,\vec{X}) = W^a_{EARTH}(T,\vec{X}) + \underbrace{W^a_{TIDAL}(T,\vec{X})}_{\text{QUADRATIC IN } \vec{X}} + \underbrace{W^a_{INERTIAL}(T,\vec{X})}_{\text{LINEAR IN } \vec{X}} \end{cases}$$

→ GEODESIC, THOMAS, LENSE-THIRRING PRECESSIONS

↓ ← ACCELERATION GEOID (MULTIPOLE

A) COORD. TRANSF. $(t, x^i) \leftrightarrow (T, X^a)$ TIME-DEP. LORENTZ TRANS. IN PRESENCE OF PN GRAVIT

B) POT. TRANSF $(w, w^i) \leftrightarrow (W, W^a)$

C) $TCB - TCG = \frac{1}{c^2}\left[\int_{t_0}^{t}\left(\frac{\vec{v}_E^2}{2} + U_{ext}(\vec{x}_E)\right)dt + \vec{v}_E \cdot \vec{r}_E\right] + O(c^{-4})$ $r_E^i = x^i - x_E^i$

GEOCENTER

$TCG - TT = L_G \cdot (JD - 2443144.5) \cdot 86400$ $L_G \simeq 6.9\text{---}\ 10^{-10}$

TERRESTRIAL TIME ~ TAI STANDARD OF TIME = COORD. TIME NOT PROPER TIME

(EQS. OF MOTION NOT KNOWN! TO THE EXPERIMENTALIST)

$$ds^2 = g_{\mu\nu}\, dx^\mu dx^\nu \qquad x^0 = ct$$

IAU RESOLUTIONS
ASTRO-PH/0602086
0303376
B. GUINOT – APPLICATION OF GR TO METROLOGY
METROLOGIA 34 (1997) 261

$$d\tau = \frac{1}{c}\sqrt{|ds^2|}$$

IDEAL CLOCK – POINTLIKE (FREE FALL)
– ITS READINGS ARE STRICT MEASURES OF THE PROPER TIME τ

REAL (ATOMIC) CLOCK – FINITE EXTENSION (EQUIVALENCE PRINCIPLE)

UNIVERSAL COUPLING POSTULATE OF GR –

– TO A GOOD APPROXIMATION THE VALUES READ ON A CLOCK AT ANY INSTANT (CLOCK READINGS), WHATEVER THE NATURE OF THE CLOCK AS LONG AS IT IS INSENSITIVE TO ACCELERATIONS, ARE PROPORTIONAL TO $\int_A^B d\tau$ ALONG THE OBSERVER WORLDLINE (IN FREE FALL)

NB THE STANDARDS WHICH PROVIDE THE UNIT OF TIME MUST BE INFINITELY SMALL DUE TO THE DIFFERENTIAL FORM ds^2

ATOMIC SECOND → DURATION OF 9.192.631.770 PERIODS OF THE RADIATION CORRESPONDING TO THE TRANSITION BETWEEN THE TWO HYPERFINE LEVELS OF THE GROUND STATE OF THE CAESIUM-133 ATOM – THE PROPER TIME INTERVAL $\Delta\tau = \int \frac{1}{c}\sqrt{|ds^2|}$ IS THE INTERVAL BETWEEN THE EMISSION OF TWO SUCCESSIVE CRESTS OF THE RADIATION ON THE ATOM WORLDLINE

SI (SYSTEME INTERNATIONAL)

TCB OF BCRS
TCG GCRS

TAI (INTERNATIONAL ATOMIC TIME) – TIME SCALE BASED ON THE SI SECOND ON THE EARTH'S SURFACE AT SEA LEVEL (ROTATING GEOID) – BIPM SÈVRES FRANCE – COORDINATE TIME APPROXIMATING PROPER TIME

→ THE LABORATORIES ON THE GROUND ARE EXTENDED AND ACCELERATED (NO STRICT FREE FALL) → MEAN LOCAL PROPER TIME

UTC (COORDINATE UNIVERSAL TIME)
TT (TERRESTRIAL TIME)

PERLICK [GEN. REL. GRAV. 19, 1059 (1987)]

1ST CLOCK EFFECT – TWIN PARADOX
2nd CLOCK EFFECT – IF STANDARD CLOCK SYNCHRONIZATION IN A CONSISTENT WAY CANNOT BE DONE [TILL NOW ABSEN IN ATOMIC CLOCK]

ESA – ACES AND EGE (COSMIC VISION)

SINCE WITH THE PROPOSED MISSIONS

- GRAVITOMAGNETISM
- LIGHT DEFLECTION FROM STRONG GRAVITATIONAL FIELDS (SUN, JUPITER)

ARE OUT OF REACH, THE MISSION HAS TO CONCENTRATE ON

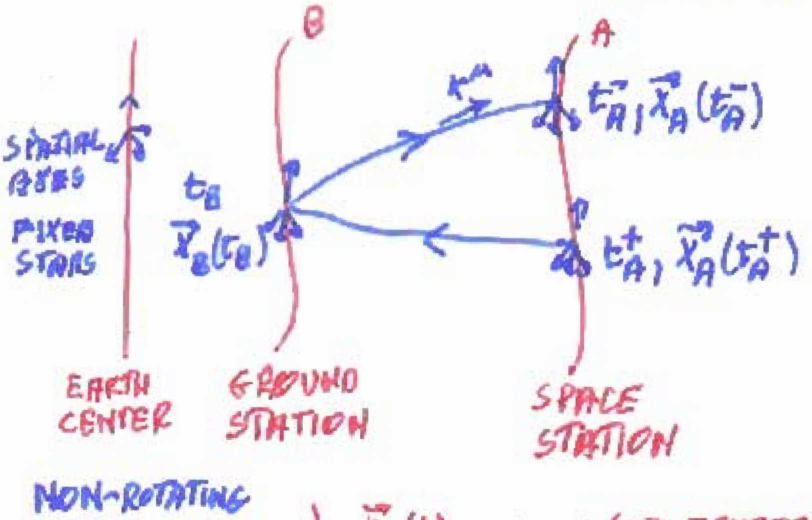
- GRAVITATIONAL REDSHIFT → RELATIVISTIC GEODESY
- SHAPIRO TIME DELAY
- DEVIATION FROM EINSTEIN'S $\frac{1}{2}$ CLOCK SYNCHRONIZATION CONVENTION

IN THE GEOCENTRIC CELESTIAL REFERENCE SYSTEM (IAU 2000)

AND ON THEIR IMPLICATIONS FOR METROLOGY AND GEODESY

THE THEORETICAL BACKGROUND IS CONTAINED IN

L. BLANCHET, C. SALOMON, P. TEYSSANDIER AND P. WOLF
RELATIVISTIC THEORY FOR TIME AND FREQUENCY TRANSFER TO ORDER C^3
ASTRONOMY and ASTROPHYSICS <u>370</u>, 320 (2000)



WORK DONE FOR ACES

$t = t_{GCRS}$

$$\left\{\begin{array}{l} d_{AB} \sim 4\cdot 10^5\,m \\ v_A \sim 7.7\cdot 10^3\,m/s \\ v_B \sim 465\,m/s \end{array}\right.$$

$\frac{d_{AB}}{c} \sim 10^{-3}\,s$

$\frac{v_A}{c} \sim 10^{-5}$

$\frac{v_B}{c} \sim 10^{-6}$

$GM_E \sim 4\cdot 10^{4}\,m^3/s^2$

$U_A/c^2, U_B/c^2 \sim 10^{-10}$

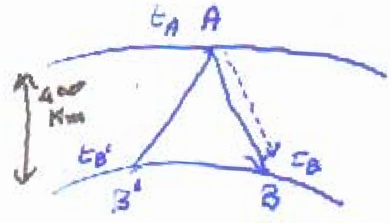
$$\left\{\begin{array}{l} \vec{x}_A(t) \text{ KNOWN (JPL EPHEMERIDES, ...)} \\ \vec{x}_B(t) = \mathbb{T}\,R(\theta)\,\vec{x}_{ITRS\sim GPS\sim WGS84} \end{array}\right.$$

KNOWN (GEOCENTRIC RECTANGULAR EARTH-FIXED 3-COORDINATES. (IERS 2003)

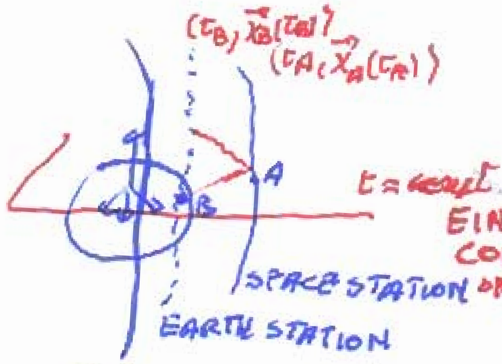
$1/c^3$ REL. EFFECTS

ACES

BLANCHET, SALOMON, TEYSSANDIER, WOLF, A.A. 370 (2000) 320



INERTIAL BARYCENTRIC CELESTIAL REF. SYS.



EINSTEIN'S CONVENTION OF SIMULTANEITY ↑

GEOCENTER

QUASI-INERTIAL GEOCENTRIC CELESTIAL REF. SYS.

FOR SHORT TIME INTERVALS APPROXIMATED AS AN INERTIAL FRAME

PHARAO CESIUM CLOCK $10^{-15}-10^{-16}$

SPACE HYDROGEN MASER

ONLY

TWO-WAY FREQUENCY TRANSFER MEASURED

FUTURE

ONE-WAY FREQUENCY TRANSFER

ONE-WAY AND TWO-WAY TIME TRANSFER $\boxed{5 ps}$

⟷ NON-ROTATING LORENTZ BOOST + PN ACCELERATION TERMS

EARTH ROTATION APPROXIMATED WITH A TERM $\omega_E^2 R$ IN B'S ACCELERATION $\vec{a}_B(t_B)$

THEORY

ONE-WAY TIME TRANSFER $T_{AB} = t_B - t_A$

FLAT NULL GEODESIC $c^2(t_A-t_B)^2 - \vec{\Delta}^2_{AB}(t_A,t_B) = 0$ $\quad \vec{\Delta}_{AB} = R_{AB}\hat{N}_{AB} = \vec{x}_A(t_A) - \vec{x}_B(t_B)$

↓

$T_{AB} = \frac{R_{AB}(t_A,t_B)}{c}$ RETARDED DISTANCE

POST-NEWT. NULL GEODESIC (time part)

$$T_{AB} = \frac{R_{AB}(t_A,t_B)}{c} + \frac{2GM}{c^3} \ln \frac{|\vec{x}_A(t_A)| + |\vec{x}_B(t_B)| + R_{AB}(t_A,t_B)}{|\vec{x}_A(t_A)| + |\vec{x}_B(t_B)| - R_{AB}(t_A,t_B)}$$

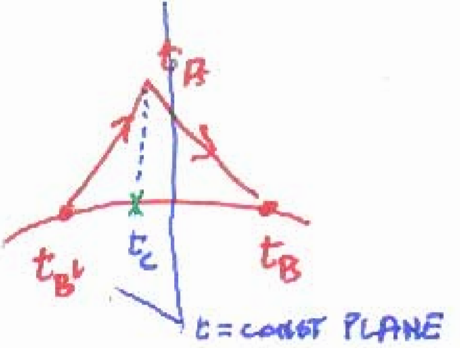
SHAPIRO TIME DELAY OF THE GEOID

EXPRESS $R_{AB}(t_A,t_B)$ IN TERMS OF THE INSTANTANEOUS DISTANCE

$$\vec{D}_{AB}(t_A) = \vec{x}_A(t_A) - \vec{x}_B(t_A)$$

$$T_{AB}^{(t_A)} = \underbrace{\frac{|\vec{D}_{AB}|}{c} + \frac{\vec{D}_{AB}\cdot\vec{v}_B(t_A)}{c^2}}_{\text{1st ORDER SAGNAC (200 ns)}} + \frac{1}{c^3}\Big\{ \underbrace{|\vec{D}_{AB}|\Big[\vec{v}_B^{\,2}(t_A) + \frac{(\vec{D}_{AB}\cdot\vec{v}_B(t_A))^2}{|\vec{D}_{AB}|^2} + \overset{\text{EARTH ROTATION}}{\vec{D}_{AB}\cdot\vec{a}_B(t_A)}\Big]}_{\text{2nd ORDER SAGNAC (5 ps)}} +$$

$$+ \underbrace{2GM \ln \frac{|\vec{x}_A(t_A)| + |\vec{x}_B(t_A)| + |\vec{D}_{AB}|}{|\vec{x}_A(t_A)| + |\vec{x}_B(t_B)| - |\vec{D}_{AB}|} \Big\}}_{\text{SHAPIRO TIME DELAY (11 ps)}} + O\left(\frac{1}{c^4}\right)$$



1-WAY TIME TRANSFERS

$$T_{AB} = t_B - t_A \qquad T_{B'A} = t_A - t_{B'}$$

IF MEASURED

$$\Rightarrow \boxed{t_A \equiv t_C} = t_{B'} + E(t_B - t_{B'})$$

SYNCHRONIZATION CONVENTION

$$E = \frac{1}{2}\left(1 + \frac{T_{B'A} - T_{AB}}{\underbrace{T_{B'A} + T_{AB}}_{\text{2-WAY TIME TRANSFER}}}\right)$$

$$= \frac{1}{2}\left(1 + \alpha + \frac{\beta}{c} + \frac{\gamma}{c^2} + \frac{\delta}{c^3} + \dots\right)$$

$E = \frac{1}{2}$

ONLY IF

$T_{AB} = T_{B'A}$

PROPER TIME OF CLOCK A

$$\frac{dt_A}{dt_A} = 1 - \frac{1}{c^2}\left[\frac{1}{2}\vec{V}_A^2(t_A) + U_{EARTH}(\vec{x}_A(t_A))\right] + O\left(\frac{1}{c^4}\right)$$

↳ WOLF, PETIT (1995)

1-WAY FREQUENCY TRANSFER

$$\frac{\nu_A}{\nu_B} = \frac{1 - \frac{1}{c^2}\left[U_E(\vec{x}_B(t_A)) + \frac{\vec{v}_B^2(t_A)}{2}\right]}{1 - \frac{1}{c^2}\left[U_E(\vec{x}_A(t_A)) + \frac{\vec{v}_A^2(t_A)}{2}\right]} \frac{q_A}{q_B}(t_A)$$

POSTNEWTONIAN

$$\begin{cases} q_A = 1 - \frac{\vec{N}_{AB}\cdot\vec{V}_A}{c} - \frac{4GM}{c^3} \cdots\cdots + O\left(\frac{1}{c^4}\right) \\ q_B = 1 - \frac{\vec{N}_{AB}\cdot\vec{V}_B}{c} - \frac{4GM}{c^3} \cdots\cdots + O\left(\frac{1}{c^4}\right) \end{cases}$$

2-WAY FREQUENCY TRANSFER $(B' \to B)$ — TO BE MEASURED

FIRST ORDER DOPPLER SHIFT REMOVED (SIMULTANEOUS TRANSMI-SSION OF 3 MICROWAVE SIGNALS

$$\frac{\nu_B}{\nu_A} = \frac{1}{2}\underbrace{\frac{\nu_B}{\nu_{B'}}}_{\text{MEASURED IN B}} + \Delta_{AB}(t_B) + \frac{1}{2}$$

LIKE A FIRST ORDER DOPPLER SHIFT (2.7·10^{-5})

$$\frac{\delta\nu}{\nu} = \Delta_{AB} = \frac{1}{c^2}\left[\underbrace{U_B - U_A}_{} - \frac{1}{2}\vec{V}_{AB}^2 - \vec{R}_{AB}\cdot\vec{a}_B\right]\left(1 + \frac{\vec{N}_{AB}\cdot\vec{V}_{AB}}{c}\right)$$

↓ ACES $5\cdot10^{-17}$

GRAV REDSHIFT OF GEOID ($\sim 5\cdot10^{-11}$)

2nd ORDER DOPPLERSHIFT ($\sim 3\cdot10^{-10}$)

SAGNAC ($\sim 2\cdot10^{-11}$)

$\vec{V}_{AB} = \vec{V}_A(t_A) - \vec{V}_B(t_B)$

$$+ \frac{R_{AB}}{c^3}\left(-\vec{V}_A\cdot\vec{a}_B + \vec{R}_{AB}\cdot\frac{d\vec{a}_B}{dt_B} + 2\vec{V}_B\cdot\vec{a}_B - \vec{V}_B\cdot\nabla U_B\right) + O\left(\frac{1}{c^4}\right)$$

NEGLIGIBLE $(< 10^{-17})$

IT IS POSSIBLE TO EXPRESS t_B IN TERMS OF

$$\begin{cases} t_A^+ & \text{EMISSION TIME} \\ t_A^- & \text{ARRIVAL TIME} \\ \hat{n}^+\big|_{t_A^+} & \text{TANGENT AT THE NULL GEODESIC AT THE EMISSION TIME} \\ & = \text{EMISSION ANGLES } \vartheta_A^+, \varphi_A^+ \end{cases}$$

$\vec{v}_A, \vec{a}_A$ - A VELOCITY AND ACCELERATION AT THE EMISSION TIME

WITH A STRAIGHTFORWARD CALCULATION

$$t_B = t_A^+ + \underbrace{\frac{1}{2}\left[1 - \{\overbrace{\frac{1}{c}\left(\vec{v}_A \cdot \hat{n}_+ + \vec{a}_A \cdot \hat{n}_+ (t_A^- - t_A^+)\right)}^{\approx 10\,nsec} - \overbrace{\frac{2}{c^2}\left(\vec{v}_A^2 - (\vec{v}_A \cdot \hat{n}_+)^2\right)}^{\lesssim 0.1\,psec}\right]}_{\mathcal{E}(t_A^-, t_A^+, \hat{n}_+, \vec{v}_B, \vec{a}_A) \neq \frac{1}{2}} (t_A^- - t_A^+]$$

SPECIAL RELATIVISTIC CORRECTIONS AT $\mathcal{E} = \frac{1}{2}$

$$\hat{n}_+ = \frac{\vec{x}_B - \vec{x}_A(t_A^+)}{R_A^+} + \frac{GM}{c^2}\hat{\alpha}_+$$

$\hookrightarrow$ GENERAL RELATIVISTIC CORRECTION TO $\mathcal{E}$ AT ORDER $\frac{1}{c^3}$

$\Rightarrow$ POSSIBILITY OF ABSOLUTE SYNCHRONIZATION OF THE CLOCKS IN THE EARTH STATION WITH THE CLOCK IN THE SPACE STATION AT THE LEVEL OF 1 nsec OR LESS IN THE GEOCENTRIC NON-INERTIAL FRAME (ACES 100ps)

TILL NOW $\mathcal{E} = \frac{1}{2}$ IS USED IN THE MICROWAVE LINK!

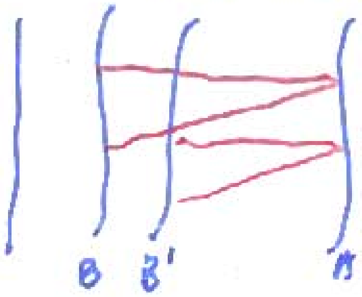
(2-WAY TIME TRANSFER, T2L2)

NO DATA!
TILL NOW.
ONLY ESTIMATES

$\Downarrow$

COMPARISON OF TWO EARTH CLOCKS

RELATIVISTIC GEODESY!
(SVEHLA)



BOTH THE TRAJECTORIES AND THE ACES CALCULATION USE

$$\begin{cases} g_{00} = \varepsilon\left(1 - \frac{2U}{c^2}\right) \\ g_{0a} = 0 \\ g_{ab} = -\varepsilon\left(1 + \frac{2U}{c^2}\right)\delta_{ab} \end{cases}$$

THE ACES CALCULATION USES $U = U_E = \frac{GM}{r}$ (MONOPOLE APPROX)

INSTEAD OF

SEE LINET-TEYSSANDIER, PR D66 (2002) 024045 FOR HIGHER MULTIPOLES IN STATIONARY AXISYMMETRIC PN SPACETIMES

$$\frac{GM}{r}\left[1 + \sum_{2}^{\infty}{}_{\ell} \sum_{0}^{\ell}{}_{m} \left(\frac{R_E}{r}\right)^{\ell} P_{\ell m}(\cos\theta)\left(C^E_{\ell m}(t,\bar{x})\cos m\varphi + S^E_{\ell m}(t,\bar{x})\sin m\varphi\right)\right]$$

⇒ PN NULL GEODESIC FROM A TO B TO ORDER $1/c^2$

NB – IT IS POSSIBLE TO FIND THE PN NULL GEODESIC WITH ARBITRARY U

AND WITH $g_{0a} = \varepsilon \frac{4}{c^3} W_a$ (GRAVITOMAGNETISM)

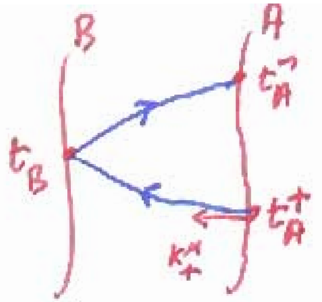
NB – THE 3-SPACE $t = const$ IS NOT EUCLIDEAN

⇒ THE SPATIAL 3-DISTANCE ALONG A 3-GEODESIC =

= EUCLIDEAN DISTANCE + $\frac{1}{c^2}$ ------

$$l_{12}(t) = \sqrt{(\vec{x}_1 - \vec{x}_2)^2} + \frac{1}{c^2}\int_0^1 ds\left[\frac{(\vec{x} - \vec{x}_1)}{(\vec{x}_2 - \vec{x}_1)^2} \cdot \frac{d\vec{\xi}(t, \vec{x}_0(s))}{ds} + U(t, \vec{x}_0(s))\right]$$

$$\begin{cases} \vec{x}_0(s) = \vec{x}_1 + (\vec{x}_2 - \vec{x}_1)\, s \\ \vec{x}(s) = \vec{x}_0(s) + \frac{1}{c^2}\left(\vec{\xi}(s) - \vec{\xi}(1)\right) \\ \xi^a(s) = -\frac{(x_2^b - x_1^b)(x_2^c - x_1^c)}{\text{[illegible]}}\left(\int_0^s - \int_0^1\right) ds_1 \int_0^{s_1} ds_2 \left[\delta_{ab}\partial_c U + \delta_{ac}\partial_b U - \delta_{bc}\partial_a U\right](t, \vec{x}_0(s)) \end{cases}$$



$$r_A^\pm = |\vec{x}_A(t_A^\pm)|\ ,\quad r_B = |\vec{x}_B(t_B)|$$

$$R_{AB}^\pm = |\vec{x}_B(t_B) - \vec{x}_A(t_A^\pm)|$$

$$\Delta \simeq t_A^- - t_A^+ \sim 10^{-3}s$$

$$t_B = t_A^+ + \varepsilon\ (t_A^- - t_A^+) = t_A^+ + \varepsilon\ \Delta$$

USUALLY IT IS PUT $= \frac{1}{2}$ LIKE IN INERTIAL FRAMES IN MINKOWSKI (EINSTEIN'S SYNCHRONIZATION)

$$\varepsilon = \frac{1}{2} + \text{-----}$$

↳ WE WANT ε AS A FUNCTION OF THE EMISSION DATA:

$$t_A^+,\ \vec{x}_A(t_A^+),\ \vec{v}_A^+,\ \vec{a}_A^+$$

AND OF THE REABSORPTION TIME t_A^-

$\dfrac{\vec{x}_A(t_A^+)}{r_A^+}$ UNIT VECTOR

PN NULL GEODESICS

$$\begin{cases} t_B - t_A^+ = \dfrac{R_{AB}^+}{c} + \dfrac{\overset{1+\gamma}{2}GM}{c^3} \ln \dfrac{r_A^+ + r_B + R_{AB}^+}{r_A^+ + r_B - R_{AB}^+} = \dfrac{1}{c} V(\vec{x}_A(t_A^+), \vec{x}_B(t_B)) \\ \underbrace{t_A^- - t_B = \dfrac{R_{AB}^-}{c}}_{\text{MINKOWSKI LIGHTCONE}} + \dfrac{2GM}{c^3} \ln \dfrac{r_A^- + r_B + R_{AB}^-}{r_A^- + r_B - R_{AB}^-} \end{cases}$$

$$\Downarrow$$

$$\begin{cases} \Delta = t_A^- - t_A^+ = \dfrac{R_{AB}^+ + R_{AB}^-}{c} + \dfrac{2GM}{c^3}\left(\ln \dfrac{r_A^+ + r_B + R_{AB}^+}{r_A^+ + r_B - R_{AB}^+} + \ln \dfrac{r_A^- + r_B + R_{AB}^-}{r_A^- + r_B - R_{AB}^-} \right) \\ t_B = \dfrac{1}{2}(t_A^+ + t_A^-) + \dfrac{R_{AB}^+ - R_{AB}^-}{2c} + \dfrac{GM}{c^3}\left(\ln \dfrac{r_A^+ + r_B + R_{AB}^+}{r_A^+ + r_B - R_{AB}^+} - \ln \dfrac{r_A^- + r_B + R_{AB}^-}{r_A^- + r_B - R_{AB}^-} \right) \end{cases}$$

A ORBIT

$$\vec{x}_A(t_A^-) = \vec{x}_A(t_A^+ + \Delta) = \vec{x}_A(t_A^+) + \vec{v}_A^+ \Delta + \frac{1}{2} \vec{a}_A^+ \Delta^2 + O(\Delta^3)$$

↳ ~ 1 ms

$$r_A^- = |\vec{x}_A(t_A^-)| = r_A^+ + \vec{v}_A^+ \cdot \frac{\vec{x}_A(t_A^+)}{r_A^+} \Delta + \frac{1}{2}\Delta^2 \left[\frac{(\vec{v}_A^+)^2}{r_A^+} + \vec{a}_A^+ \cdot \frac{\vec{x}_A(t_A^+)}{r_A^+} - \frac{1}{r_A^+}\left(\vec{v}_A^+ \cdot \frac{\vec{x}_A(t_A^+)}{r_A^+}\right)^2 \right] + O(\Delta^3)$$

TANGENT TO THE NULL GEODESIC IN THE EMISSION POINT t_A^+

$$k_+^\mu = \text{const}\,(1\,;\,\hat{n}^+)$$

$\hat{n}^+(\theta_+, \varphi_+)$ EMISSION ANGLES

$$\hat{n}^+ = \left.\frac{\partial V(\vec{x}_B(t_B), \vec{x})}{\partial \vec{x}}\right|_{\vec{x} = \vec{x}_A(t_A^+)}$$

WE BEGIN TO NEED A COHERENT PN FORMULATION OF

1) NULL GEODESICS BEYOND THE STATIONARY APPROXIMATION

2) TIME LIKE GEODESICS VERSUS {GPS / NASA COORDINATES (BASED ON EINSTEIN-INFELD-HOFFMANN EQUATIONS) FOR THE ORBIT OF THE SPACE STATION AND OF SATELLITES TILL 10cm.

3) RELATIVISTIC FORMULATION OF THE ITRS EARTH-FIXED COORDINATES FOR THE TRAJECTORY OF EARTH STATIONS

4) GRAVITATIONAL REDSHIFT AND SHAPIRO TIME-DELAY FOR NON-STATIONARY PN SOLUTIONS

5) NON-EUCLIDEAN DISTANCE IN PN INSTANTANEOUS 3-SPACE

⋮

OPEN PROBLEMS WITH THE NEW ACCURACIES OF OPTICAL CLOCKS (10^{-18})

METROLOGY { STANDARD OF TIME
UNIT OF LENGTH

STANDARD CLOCK IN SPACE PETIT WOLFF

LIGHT PROPAGATION, LOCAL POSITION INVARIANCE, PPN .....

RELATIVISTIC GEODESY – MEASURE OF AU AND THEORY OF HEIGHTS

⋮

ALSO NEEDED FOR LATOR, GAIA, BEPI-COLOMBO, LISA .....

ESA THEORETICAL TOPICAL TEAM FOR ACES

SPECIAL RELATIVITY (RELATIVISTIC ORBITS) + QUANTUM MECHANICS

⇒ SPATIAL NON-SEPARABILITY FROM CLOCK SYNCHRONIZATION

⇒ NEW TYPE OF NON-LOCALITY IN RELATIVISTIC ENTANGLEMENT

HOW TO INTRODUCE (AT LEAST SEMICLASSICAL) GRAVITY FOR TELEPORTATION FROM EARTH TO SPACE STATION? (SEE QUEST, ZEILINGER)

LL, M.PAURI - DYNAMICAL EMERGENCE OF INSTANTANEOUS 3-SPACES IN A CLASS OF MODELS OF GR

to appear in the book "RELATIVITY AND THE DIMENSIONALITY OF THE WORLD"

GR-QC/0611045

D. ALBA, LL - THE YORK MAP AS A SHANMUGADHASAN CANONICAL TRANSFORMATION IN TETRAD GRAVITY AND THE ROLE OF NON-INERTIAL FRAMES IN THE GEOMETRICAL VIEW OF THE GRAVITATIONAL FIELD

GR-QC/0604086 TO APPEAR IN GRG

- ~~[illegible]~~ GENERALIZED RADAR 4-COORDINATES AND EQUAL-TIME CAUCHY SURFACES FOR ARBITRARY ACCELERATED OBSERVERS

REVIEWS

TO APPEAR IN INT. J. MOD. PHYS D (GR-QC/0501090)

L.LUSANNA, THE CHRONO-GEOMETRICAL STRUCTURE OF SPECIAL AND GENERAL RELATIVITY: A RE-VISITATION OF CANONICAL GEOMETRODYNAMICS

Lectures at the 42 Karpacz Winter School (2006)

"Current Mathematical Topics in Gravitation and Cosmology"

GR-QC/0604120 INT. J. GEOM. METHODS MOD. PHYS. 4 (2007) 79

L.LUSANNA, Lectures at SIGRAV SCHOOL 2006

(see GGI)

WWW.FI.INFN.IT/GGI-GRAV-SPACE/EGS$_s$.html

SPECIAL RELATIVITY

D.ALB, H.W.CRATER, LL - HAMILTONIAN RELATIVISTIC 2-BODY PROBLEM: CENTER OF MASS AND ORBIT RECONSTRUCTION

HEP-TH/0610200 J.PHYS A40 (2007) 9585