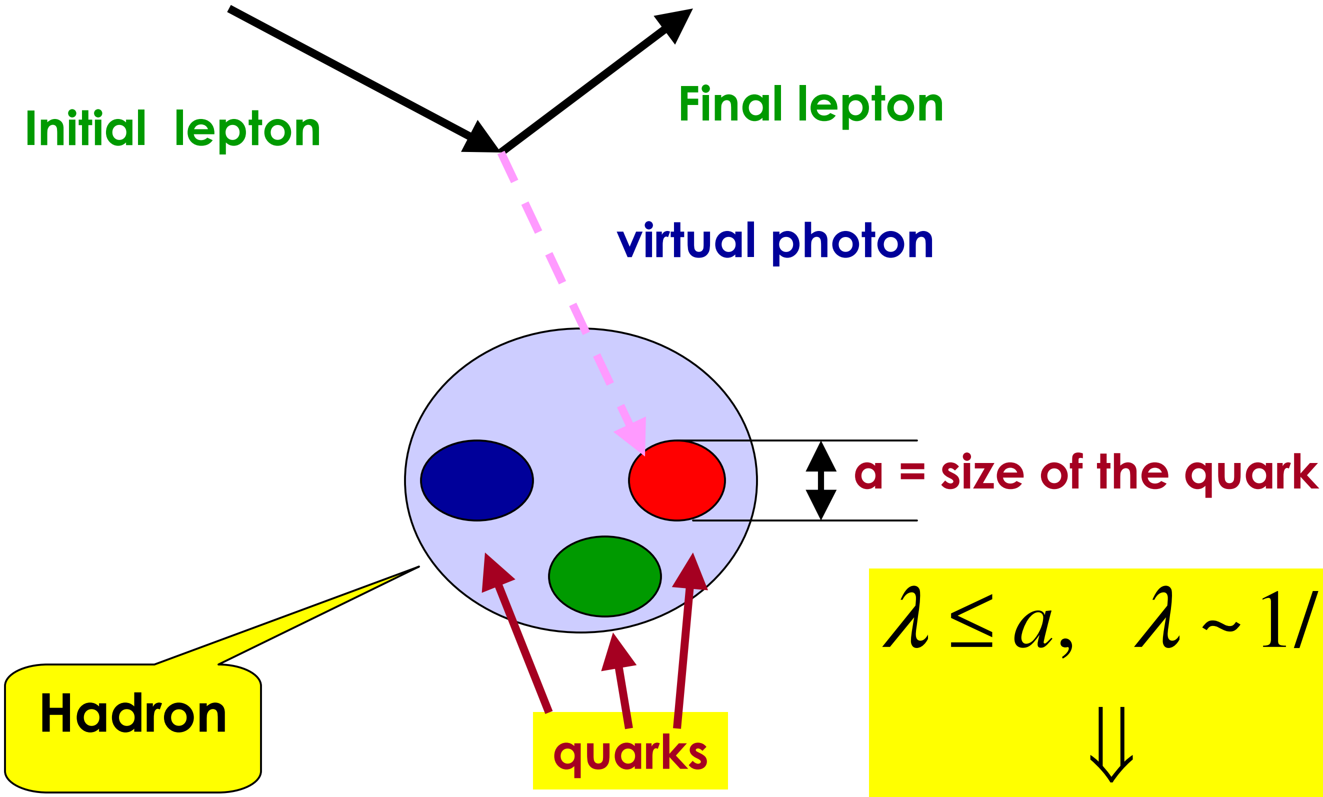
Napoli, 11 Dec 2007

Review on the theory of the Deep-Inelastic Scattering at small x

B.I. Ermolaev

talk based on results obtained in collaboration with
M. Greco and **S.I. Troyan**

Probing the hadron structure with lepton-hadron collisions

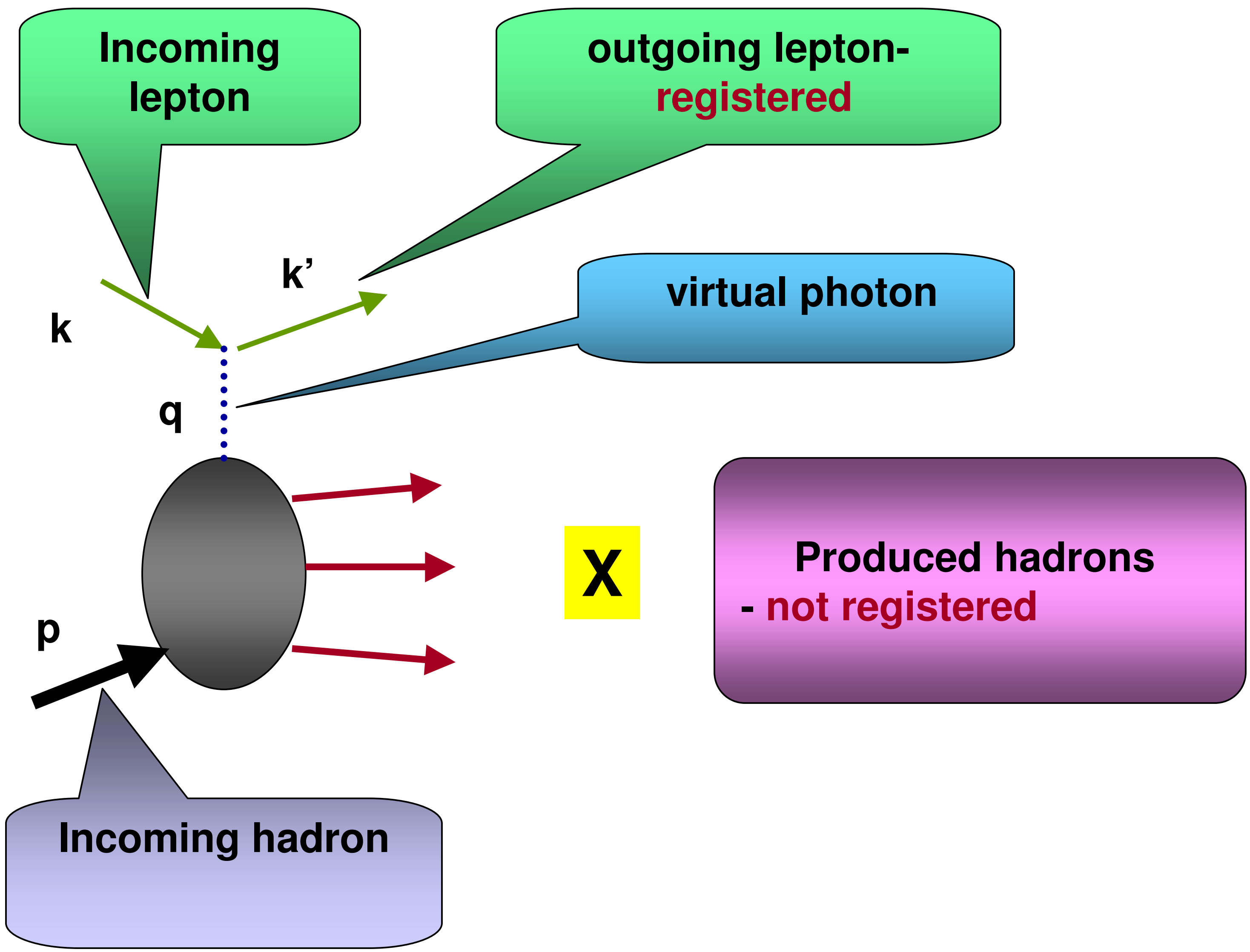


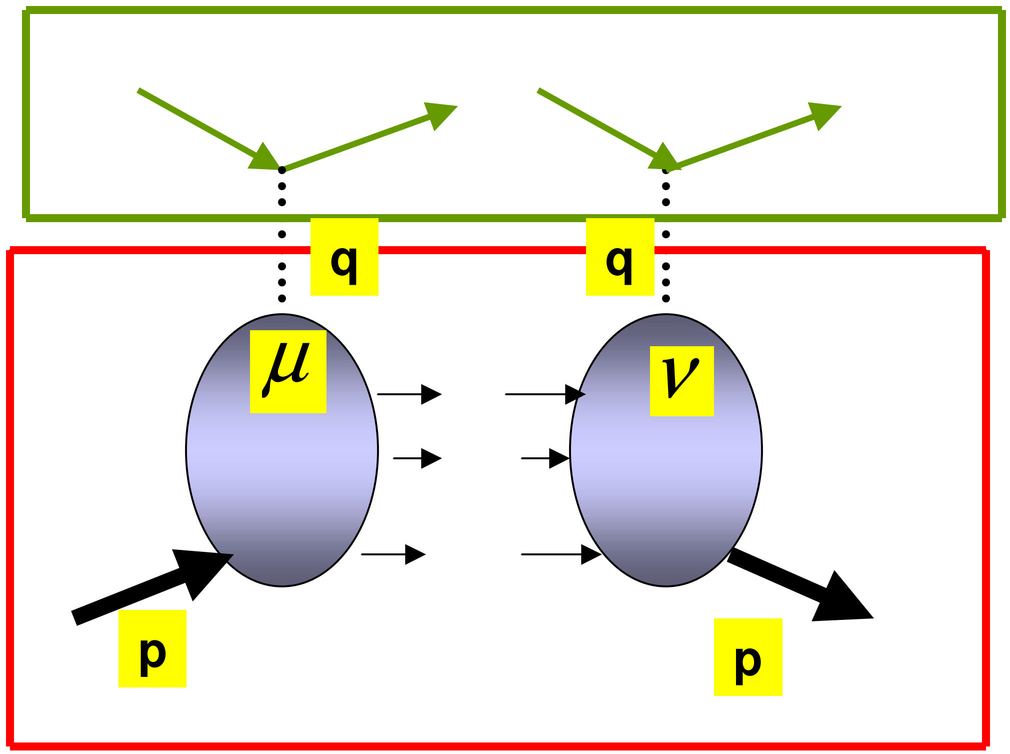
$$\lambda \leq a, \quad \lambda \sim 1/\omega$$

$$\Downarrow$$

Small a $\rightarrow$ high energies of the collisions

Deep- Inelastic Scattering





Leptonic tensor

hadronic tensor $W_{\mu\nu}$

Does not depend on spin

Spin-dependent

$$W_{\mu\nu} = W_{\mu\nu}^{unpolarized}(p,q) + W_{\mu\nu}^{spin}(p,q)$$

symmetric

antisymmetric

The spin-independent part of $W_{\mu\nu}$ is parameterized by two structure functions:

$$W_{\mu\nu}^{unpol} = \left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) F_1(x, Q^2) + \left(p_\mu - q_\mu \frac{pq}{q^2} \right) \left(p_\nu - q_\nu \frac{pq}{q^2} \right) \frac{F_2(x, Q^2)}{pq}$$

Projection operators respect Lorentz and gauge symmetries

$$q_\mu W_{\mu\nu} = q_\nu W_{\mu\nu} = 0$$

where **p** is the hadron momentum, **q** is the virtual photon momentum (**$Q^2 = -q^2 > 0$**). Both of the functions depend on **Q^2** and **$x = Q^2 / 2pq$**, **$0 < x < 1$**.

$F_2 = 2xF_1$ in the Born approximation and $F_2 \to 2xF_1$ at $x \to 0$

In the QCD framework, he spin-dependent part of $W_{\mu\nu}$ is also parameterized by two structure functions:

$$W^{spin}_{\mu\nu} = \frac{m}{pq} i\varepsilon_{\mu\nu\lambda\rho} q_\lambda \left[S_\rho g_1(x,Q^2) + \left(S_\rho - \frac{Sq}{pq} p_\rho \right) g_2(x,Q^2) \right]$$

where m, p and S are the hadron mass, momentum and spin;
q is the virtual photon momentum ($Q^2 = -q^2 > 0$). Again both functions depend on Q^2 and $x = Q^2/2pq$, $0 < x < 1$. They measure asymmetries

g_1 measures the longitudinal spin flip

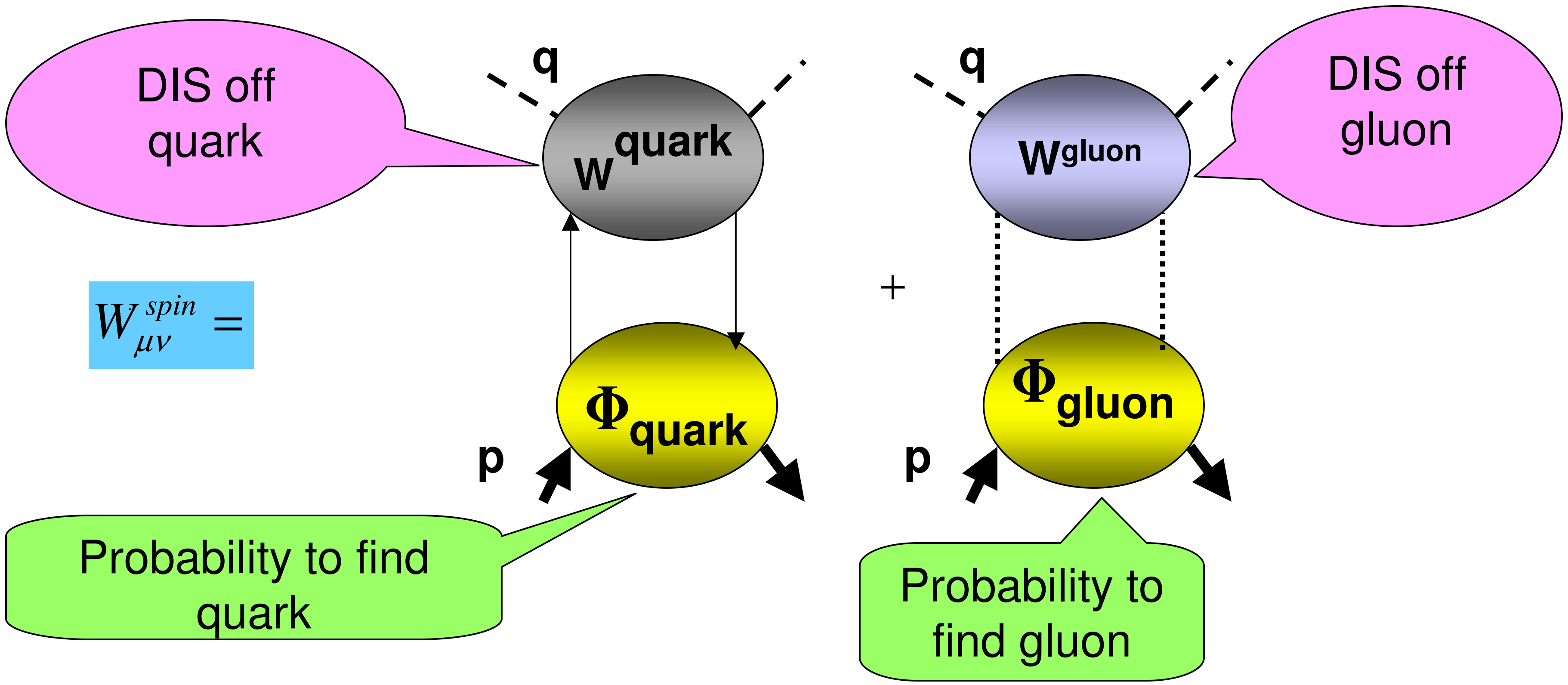
$$g_1 \propto \sigma_{L\uparrow\uparrow} - \sigma_{L\uparrow\downarrow}$$

$g_1 + g_2$ measures the transverse spin flip

$$g_1 + g_2 \propto \sigma_{T\uparrow\uparrow} - \sigma_{T\uparrow\downarrow}$$

FACTORISATON: $W_{\mu\nu}$ **is a convolution of the the partonic tensor and probabilities to find a polarized parton (quark or gluon) in the hadron :**

$$W_{\mu\nu}^{spin} =$$



DIS off quark and gluon can be studied with perturbative QCD, with calculating involved Feynman graphs.

Probabilities, Φ_{quark} and Φ_{gluon} involve non-perturbaive QCD. There is no a regular analytic way to calculate them. Usually they are defined from experimental data at large x and small Q^2, they are called the initial quark and gluon densities and are denoted δq and δg .

So, the conventional form of the hadronic tensor is:

$$W_{\mu\nu} = W_{\mu\nu}^{quark} \otimes \delta q + W_{\mu\nu}^{gluon} \otimes \delta g$$

Initial quark distribution

Initial gluon distribution

DIS off the quark,

DIS off the gluon

are calculated with methods of Pert QCD

The Standard Approach includes the Altarelli-Parisi alias DGLAP alias Q^2- Evolution Equations and the Standard Fits for initial paron densities

Evolution Equations: Altarelli-Parisi, Gribov-Lipatov, Dokshitzer

In particular, g_1:

$$g_1(x,Q^2) = C_q(x/y) \otimes \Delta q(y,Q^2) + C_g(x/y) \otimes \Delta g(y,Q^2)$$

Evolved quark distribution

Evolved gluon distribution

Coefficient function

Coefficient function

DGLAP evolution equations

$$\frac{d\Delta q}{d\ln Q^2} = \frac{\alpha_s}{2\pi} P_{qq} \otimes \Delta q + \frac{\alpha_s}{2\pi} P_{qg} \otimes \Delta g$$

$$\frac{d\Delta g}{d\ln Q^2} = \frac{\alpha_s}{2\pi} P_{gq} \otimes \Delta q + \frac{\alpha_s}{2\pi} P_{gg} \otimes \Delta g$$

$P_{qq},\ P_{qg},\ P_{gq}, P_{gg}$ are splitting functions

Mellin transform of the splitting functions = anomalous dimensions

In DGLAP, coefficient functions and anomalous dimensions are known with LO and NLO accuracy, often at integer $\omega = n$

LO

NLO

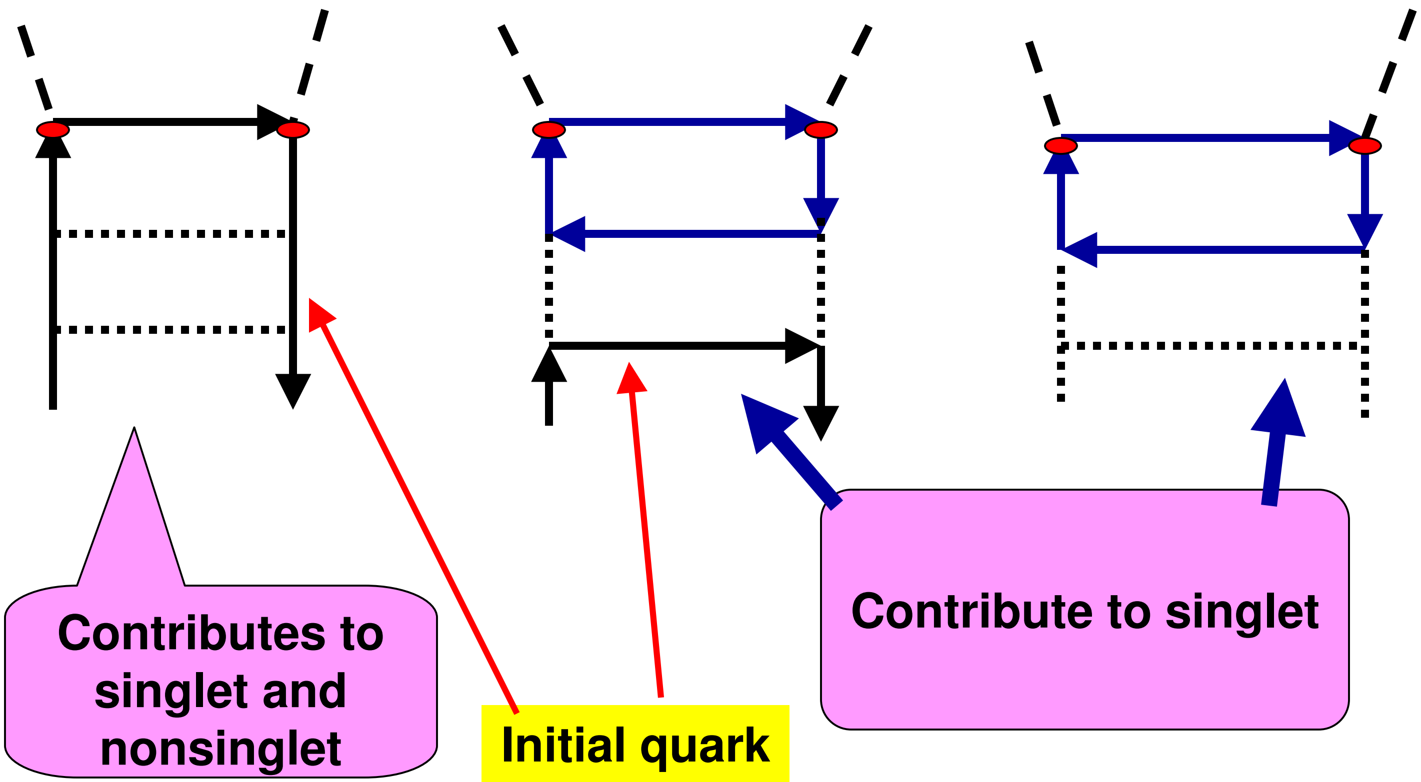
$$C(\omega) = 1 + (\alpha_s(Q^2)/2\pi)\, C^{(1)}(\omega) + \ldots$$

$$\gamma(\omega) = (\alpha_s(Q^2)/4\pi)\,\gamma^{(0)}(\omega) + (\alpha_s(Q^2)/2\pi)^2\,\gamma^{(1)}(\omega) + \ldots$$

LO

NLO

Piece of terminology



Each structure function has both the non-singlet and singlet components:

$F_1 = F_1^{NS} + F_1^{S}$ $g_1 = g_1^{NS} + g_1^{S}$

For example, for the simplest case of the non-singlets **g_1, F_1**

Initial quark density

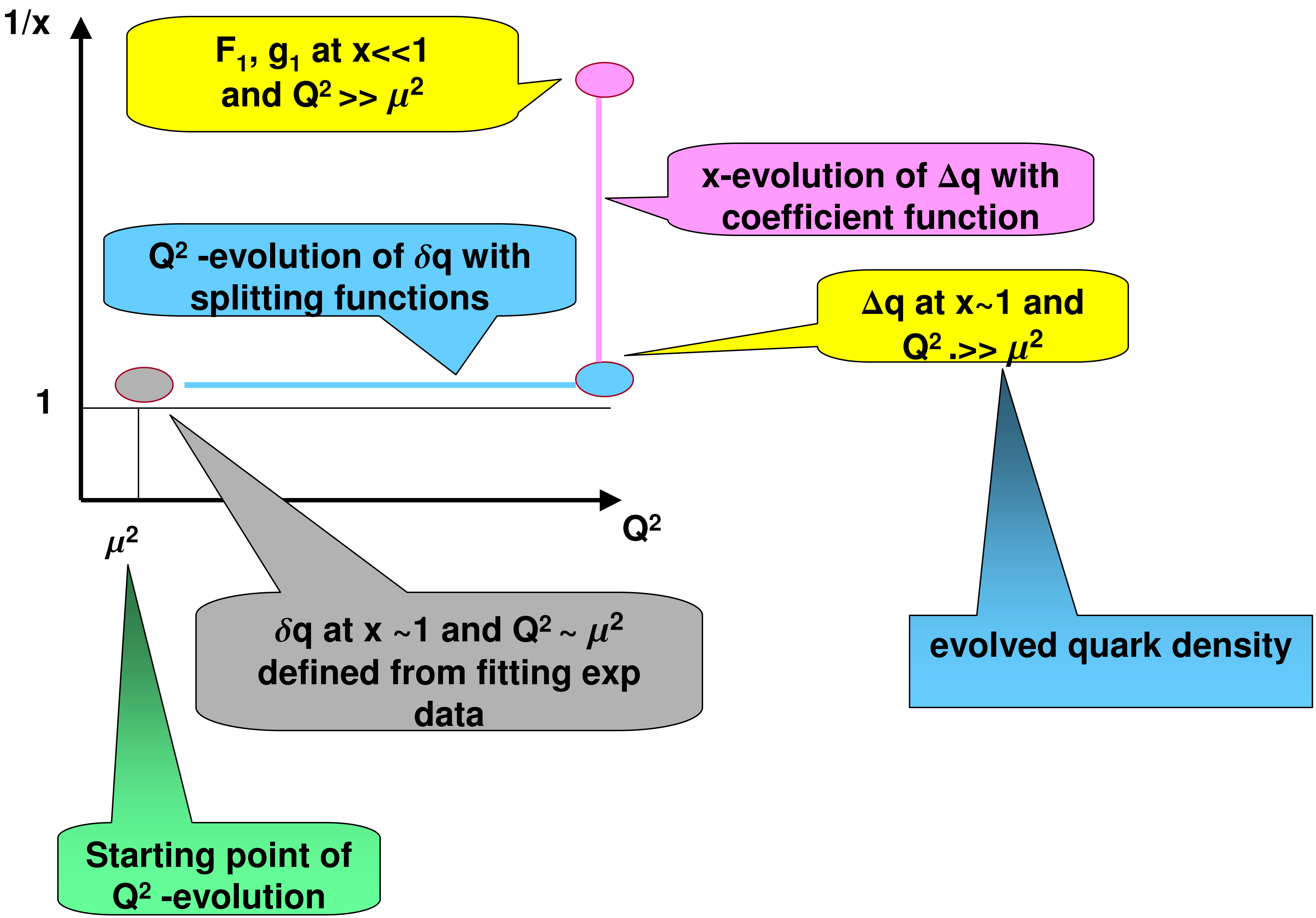
$$g_1^{NS}(x,Q^2) = (e_q^2/2) \int_{-i\infty}^{i\infty} \frac{d\omega}{2\pi i} \left(\frac{1}{x}\right)^{\omega} C(\omega)\,\delta q(\omega) \exp\left[\int_{\mu^2}^{Q^2} \frac{dk_\perp^2}{k_\perp^2} \gamma(\omega, \alpha_s(k_\perp^2))\right]$$

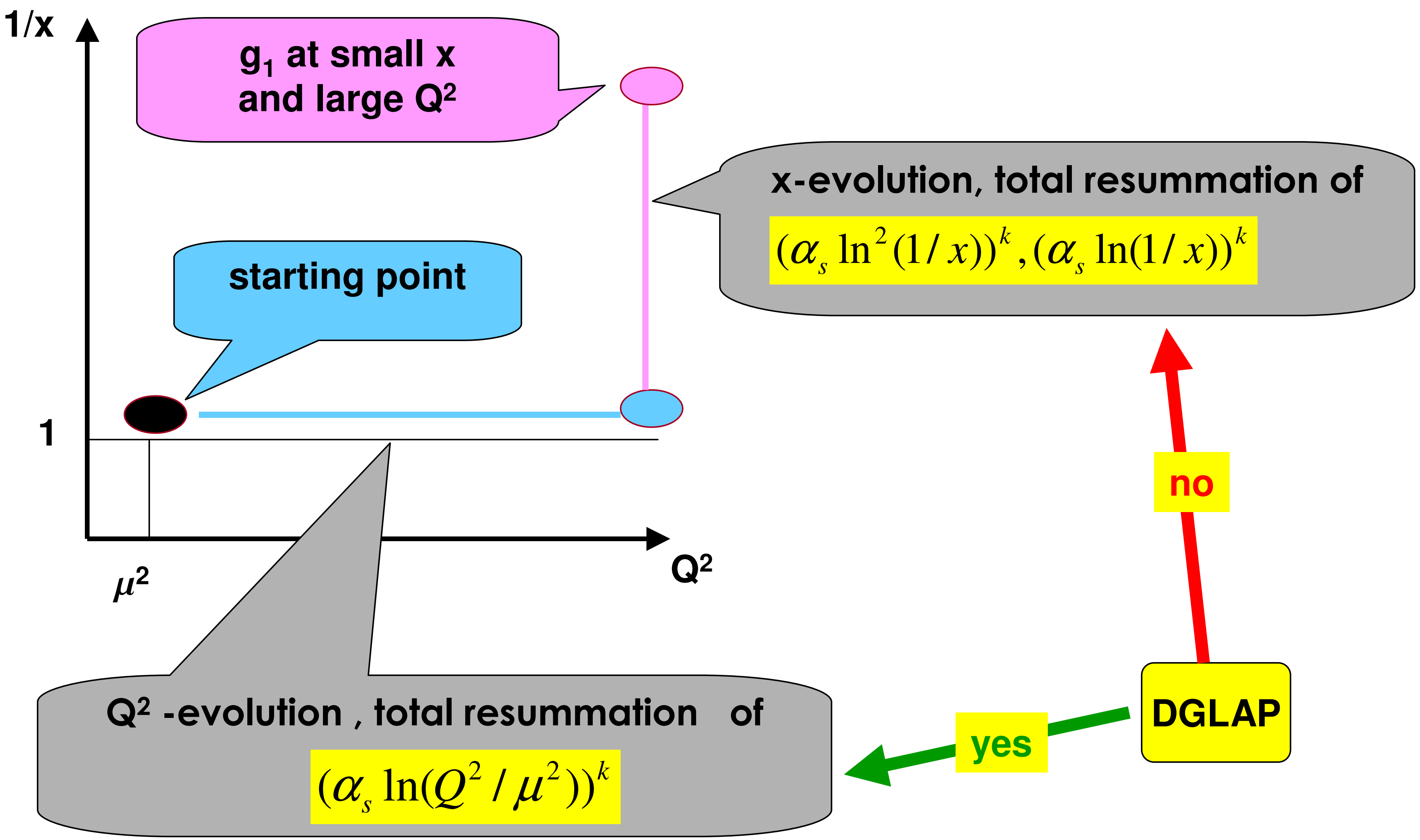
Coefficient **function**

Anomalous dimension

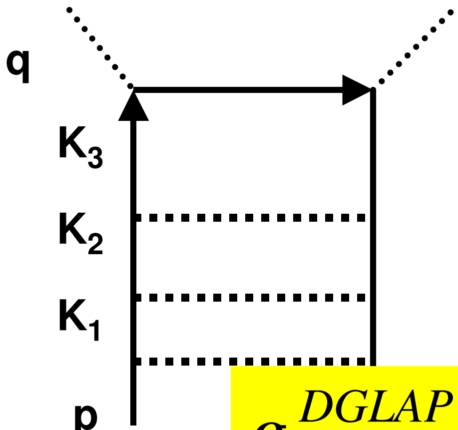
Non-Pert QCD

Pert QCD


1/x
F_1, g_1 at x<<1
and $Q^2 >> \mu^2$
x-evolution of Δq with
coefficient function
Q^2 -evolution of δq with
splitting functions
Δq at x~1 and
$Q^2 .>> \mu^2$
1
Q^2
μ^2
δq at x ~1 and $Q^2 \sim \mu^2$
defined from fitting exp
data
evolved quark density
Starting point of
Q^2 -evolution



DGLAP cannot do total resummation of logs of **x** because of the DGLAP-ordering – KEYSTONE of DGLAP



DGLAP –ordering:

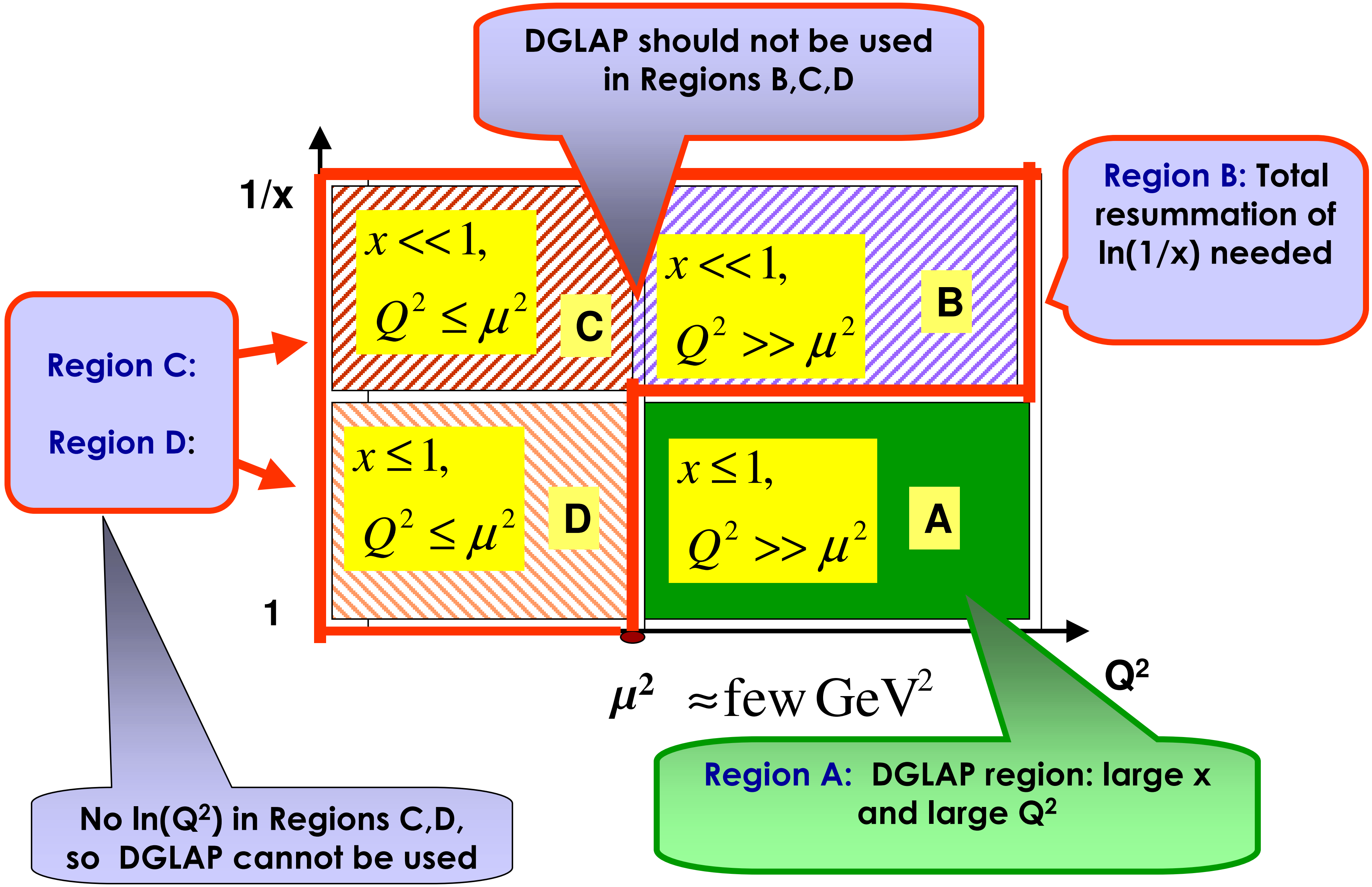
$$\mu^2 < k_{1\perp}^2 < k_{2\perp}^2 < k_{3\perp}^2 < Q^2$$

good approximation for large **x** when logs of **x** can be neglected. At **x << 1** the ordering has to be lifted

$$g_1^{DGLAP} \sim \exp\left[\ln(1/x)\ln\ln\left(Q^2/\Lambda_{QCD}^2\right)\right]^{1/2}$$

$$F_1^{DGLAP} \sim \left(\frac{1}{x}\right)\exp\left[\ln(1/x)\ln\ln\left(Q^2/\Lambda_{QCD}^2\right)\right]^{1/2}$$

providing the initial parton densities are not singular at small **x** from theoretical grounds:



DL contributions

SL contributions

$$(\alpha_s \ln^2(1/x))^k,$$

$$(\alpha_s \ln(1/x)\ln(Q^2/\mu^2))^k \qquad (\alpha_s \ln(1/x))^k,$$

$$k = 1,2..\infty$$

These logarithms are important at small x but DGLAP does include the total resummation of such logarithms, so the small-x region is beyond the reach of the Standard Approach

In practice SA solves this problem through introducing singular fits for initial parton densities, they cause a fast growth at small x and thereby mimic the resummation Week point: no theoretical grounds

Altarelli-Ball-Forte-Ridolfi, Blumlein- Botcher, Leader- Sidorov-Stamenov, Hirai et al

In the literature, there are different fits for initial parton densities. For example,

$$\delta q = N x^{-\alpha} [(1 - x)^{\beta} (1 + \gamma x^{\delta})]$$

$$\delta q = N [\ln^{\alpha}(1/x) + \gamma x \ln^{\beta}(1/x)]$$

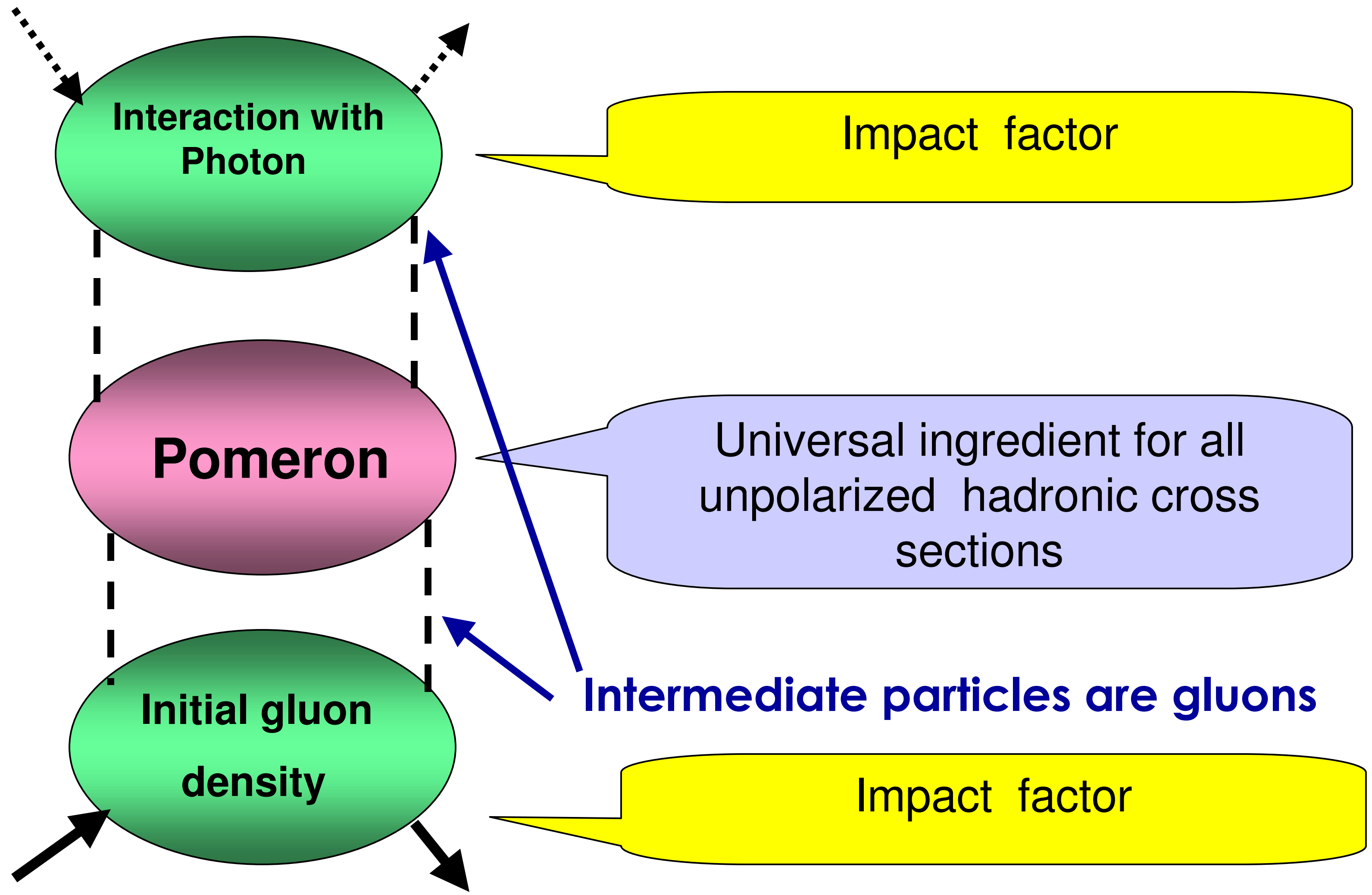
Altarelli-Ball-Forte-Ridolfi,

Parameters $N, \alpha, \beta, \gamma, \delta$ should be fixed from experiment

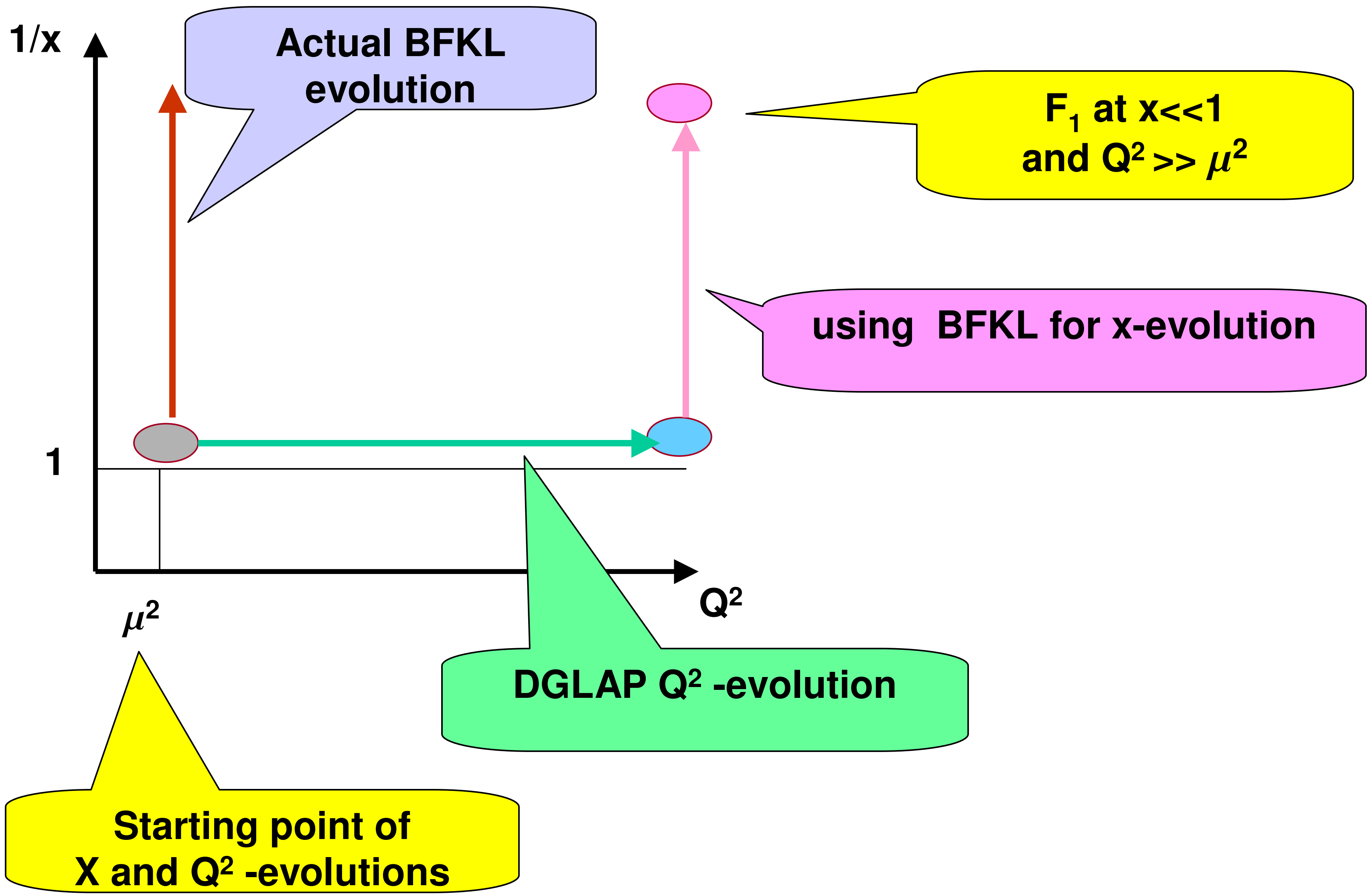
Alternative, Straightforward Way: Total resummation of leading logs of x

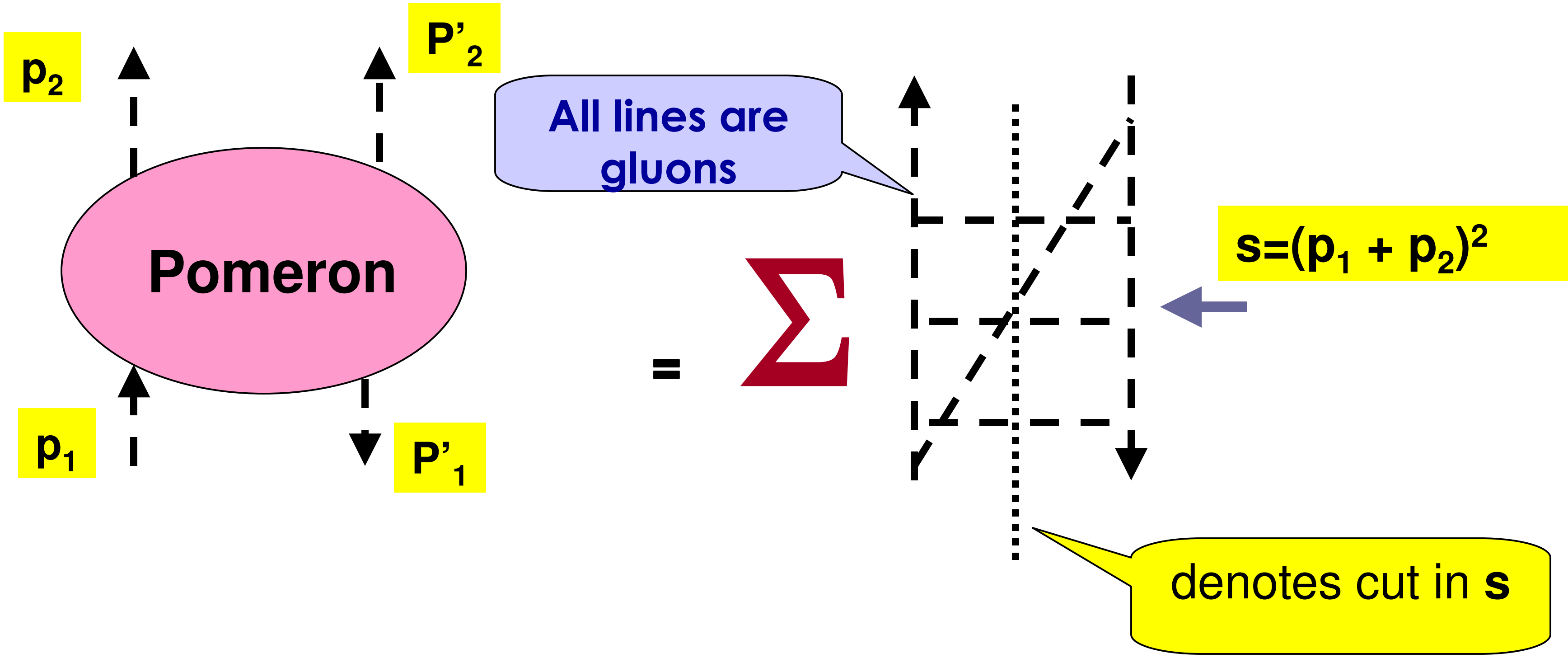
Spin-independent structure functions at small x

no model-independent results. The most popular models are using BFKL Pomeron



The way to use BFKL Pomeron





BFKL pomeron = total resummation of leading logarithms of s

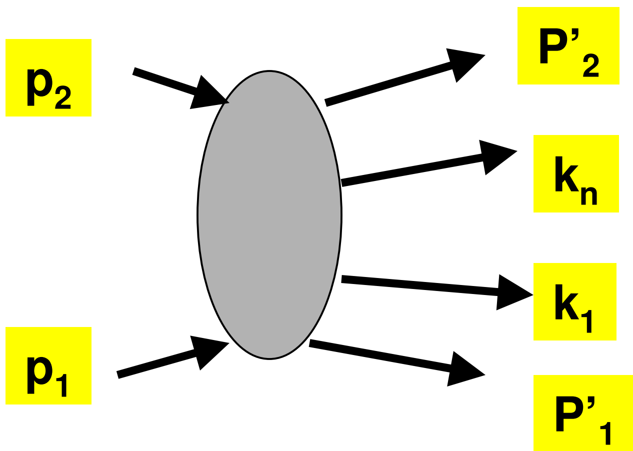
$$\left(\frac{s}{M^2}\right)\left[1 + c_1\alpha_s \ln(s/M^2) + c_2[\alpha_s \ln(s/M^2)]^2 + \ldots\right]$$

First loop=Born factor

Higher loops

Derivation of BFKL

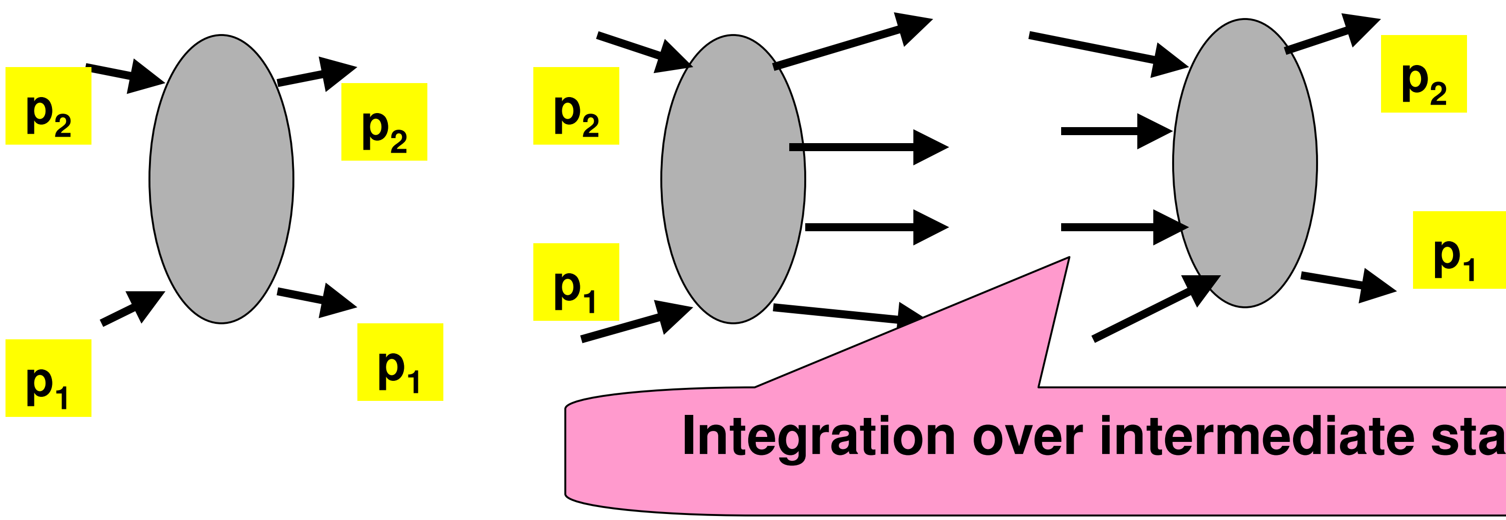
Step 1: amplitudes 2 -> 2 + in the multi-Regge kinematics



$$A_{2->2+n} = A^{Born}_{2->2+n} \prod (s_i / M^2)^{a \ln(q_i^2)}$$

Reggeons

Step 2: BFKL Pomeron = | amplitudes 2 -> 2 + n |^2



BFKL Pomeron sums up leading logarithms, although approximately

APPROXIMATIONS:

1. Intermediate particles are in the multi- Regge kinematics (LO) or in the quasi multi-Regge kinematics (NLO)

2. α_s is fixed either at unknown scale (LO) or the scale is set in a model-dependent way (NLO)

NLO corrections are too great to neglect NNLO etc $\rightarrow$ NO END

BFKL Pomeron predicts the Regge behavior for F_1 and F_2 at x <<1

$$F_1 \sim \left(\frac{1}{x}\right)^{1+\Delta_P}, \quad F_2 \sim 2xF_1$$

Pomeron intercept

Value of the BFKL intercept

$$\text{Leading order } \Delta_P^{LO} = (4N\alpha_s/\pi)\ln 2$$
$$= 3.8\alpha_s \sim 0.5,$$
$$\text{Non-leading } \Delta_P^{NLO} = 0.08$$

Fadin –Kuraev-Lipatov

Fadin-Lipatov

LO and NLO intercepts violate the Unitarity and Froissar bound

$$\sigma_{tot}^{BFKL} \sim s^{\Delta_P}$$

$$\sigma_{tot} \le c \ln^2 s$$

Identical first-loop factors

$$F_1^{BFKL} \sim \left(\frac{1}{x}\right) \exp[\Delta_P \ln(1/x)]$$

$$F_1^{DGLAP} \sim \left(\frac{1}{x}\right) \exp\,[\ln(1/x)]^{1/2}$$

Difference comes from higher-loop factors

It is difficult to discriminate between BFKL and DGLAP at the present state of experiment, all hopes for LHC

Polarized DIS

$$g_1^{LL} \sim (1/x)^{\Delta} \, (Q^2/\mu^2)^{\Delta/2}$$

Bartels- Ermolaev-
Manaenkov-Ryskin,
Ermolaev-Greco-Troyan

Resummation of Leading Logarithms

whereas DGLAP predicts

$$g_1^{DGLAP} \sim \exp\,[\ln(1/x)\ln\,\ln\,(Q^2/\Lambda^2_{QCD})]^{1/2}$$

Obviously $g_1^{LL} >> g_1^{DGLAP}$ when x→ 0

NEXT IMPORTANT STEP:

What is appropriate parameterization of α_s at small x ?

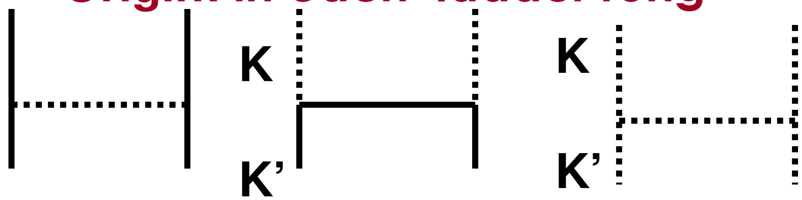
Standard parameterization $\alpha_s = \alpha_s(Q^2)$

DGLAP-parameterization

Arguments in favor of the Q^2- parameterization:

Amati-Bassetto-Ciafaloni-Marchesini - Veneziano; Dokshitzer-Shirkov

Origin: in each ladder rung



$$\alpha_s = \alpha_s(k_\perp^2)$$

DGLAP-parameterization

Ermolaev-Greco-Troyan

However, such a parameterization is good for large x only. At small **x** :

$$\alpha_s = \alpha_s((k-k')^2) \approx \alpha_s((k_\perp^2 + k_\perp'^2)/x) \approx \alpha_s(k_\perp^2)$$

When DGLAP-ordering is used and x ~1

time-like argument

Participates in the Mellin transform

DGLAP -parametrization

$$\alpha_s = \alpha_s(Q^2)$$

space-like argument,

no Mellin transform

Obviously, this parameterization and the DGLAP one converge when x is large but differ a lot at small x

So, for studying g_1 in the small-x region, it is necessary to do:

1. Total resummation of logs of **x**
2. New parameterization of the QCD coupling

The basic idea: the formula $\alpha_s(k^2) = 1/(b\ln(k^2/\Lambda^2))$

valid when $k^2 >> \Lambda^2$

$\Downarrow$

it is necessary to introduce an infrared cut-off for k^2

It is convenient to introduce the cut-off in the transverse space:

$$k_\perp^2 > \mu^2$$

Lipatov

As value of the cut-off is not fixed, one can evolve the structure functions with respect to μ → the name of the method:

Infra-Red Evolution Equations (IREE)

Highlights of the history of the method

- ☆ Analyses of two-particle cuts in Regge kinematics — Gribov
- ☆ Factorization of photons with small transverse momenta — Gribov
- ☆ Infrared cut-off in the transverse momentum space — Lipatov
- ☆ Quark-quark scattering amplitudes — Kirschner-Lipatov
- ☆
- ☆ QCD inelastic processes in Regge kinematics — Ermolaev-Lipatov
- ☆ Applications to Polarized Deep-Inelastic scattering — Bartels-Ermolaev -Manaenkov-Ryskin- Greco-Troyan

Expression for the non-singlet g_1 at large Q^2: $Q^2 >> 1\ GeV^2$

Initial quark density

$$g_1^{NS} = \frac{e_q^2}{2}\int \frac{d\omega}{2\pi i}\left(\frac{1}{x}\right)^{\omega}\left(\frac{\omega}{\omega - H(\omega)}\right)\delta q(\omega)\left(\frac{Q^2}{\mu^2}\right)^{H(\omega)}$$

Coefficient function

Anomalous dimension

New coefficient function and anomalous dimension sum up leading logarithms to all orders in α_s

Compare our non-singlet anomalous dimension to the LO DGLAP one:

expand C and H into series in $1/\omega$

$$H = \frac{A(\omega)C_F}{2\pi}\left[\frac{1}{\omega} + \frac{1}{2}\right] + \ldots$$

coincide, save the treatment of α_s

$$\gamma_{NS}^{\mathrm{LO\ DGLAP}} = \frac{\alpha_s(Q^2)C_F}{2\pi}\left[\frac{1}{n(n+1)} + \frac{3}{2} - S_2(n)\right] \approx \frac{\alpha_s(Q^2)C_F}{2\pi}\left[\frac{1}{n} + \frac{1}{2} + O(n)\right]$$

when n < 1

where

$$S_k(n) = \sum_{j=1}^{n}\frac{1}{j^k}$$

small/large x $\Longleftrightarrow$ small/large n

Compare our coefficient function and the NLO DGLAP one

$$C = \frac{\omega}{\omega - H(\omega)} = 1 + \frac{A(\omega) C_F}{2\pi} \left[\frac{1}{\omega^2} + \frac{1}{2\omega} \right] + \dots$$

coincide, save the treatment of α_s

LO

NLO

$$C_{NS}^{\mathrm{DGLAP}} = 1 + \frac{\alpha_s(Q^2) C_F}{2\pi} \left[\frac{1}{n^2} + \frac{1}{2n} + \frac{1}{2n+1} - \frac{9}{2} + \left(\frac{3}{2} - \frac{1}{n(1+n)} \right) S_1(n) + S_1^2(n) - S_2(n) \right]$$

when n < 1

$$\approx \frac{\alpha_s(Q^2) C_F}{2\pi} \left[\frac{1}{n^2} + \frac{1}{2n} + O(n) \right]$$

Expression for the singlet g_1 at large Q^2:

$$g_1^S = \frac{<e_q^2>}{2}\int \frac{d\omega}{2\pi i}\left(\frac{1}{x}\right)^{\omega}$$

$$\left[\left(C_q^{(+)}\delta q + C_q^{(+)}\delta g\right)\left(\frac{Q^2}{\mu^2}\right)^{\Omega^{(+)}} + \left(C_q^{(-)}\delta q + C_q^{(-)}\delta g\right)\left(\frac{Q^2}{\mu^2}\right)^{\Omega^{(-)}}\right]$$

$$\Omega^{(+)} > \Omega^{(-)}$$

Large Q^2 means

$$Q^2 > \mu^2; \;\; \mu \approx 5 \text{ GeV}$$

Small –x symptotics of g_1: when $x \to 0$, the saddle-point method leads to

$$g_1^{NS} \sim \frac{e_q^2}{2} (1/x)^{\Delta_{NS}} \left(Q^2/\mu^2\right)^{\Delta_{NS}/2} \delta q$$

Nonsinglet intercept $\Delta_{NS} = 0.42$

At large x, g_1^{NS} and g_1^{S} are positive

$\delta q > 0$ → $g_1^{NS} > 0$ **In the whole range of x at any Q^2**

Asymptotics of the singlet g_1 are more involved

$$g_1^S \sim \frac{\langle e_q^2 \rangle}{2} S(\Delta_S)\,(1/x)^{\Delta_S}\,\left(Q^2/\mu^2\right)^{\Delta_S/2}$$

With intercept $$\Delta_S = 0.86$$

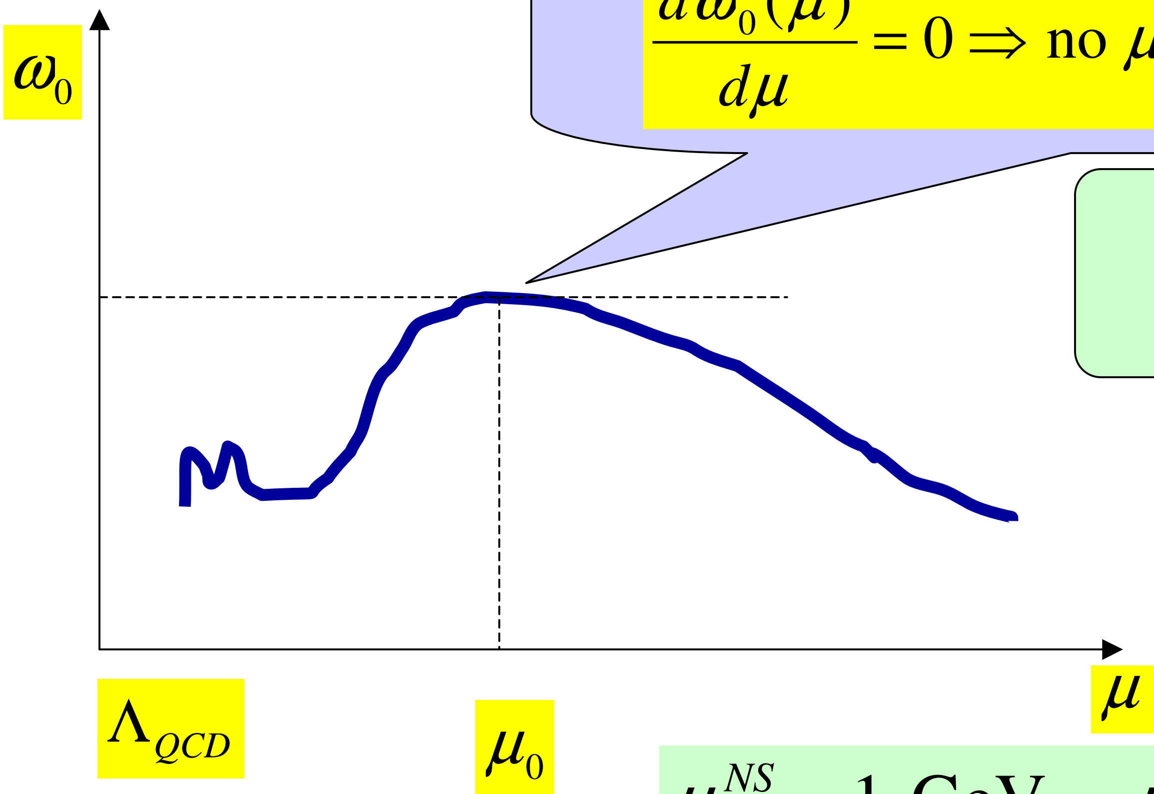
and $$S(\Delta_S) = -\,\delta q - 0.064\,\delta g$$

Interplay between the quark and gluon densities can lead to different sign of g_1 singlet at $x \ll 1$

Warning: asymptotic expressions $g_1 \sim (1/x)^{\Delta}$ are reliable at x<10-5

Obtaining intercepts: Plot $\omega_0 = \omega_0(\mu)$

$$\frac{d\omega_0(\mu)}{d\mu} = 0 \Rightarrow \text{no } \mu\text{ - depedence at } \mu = \mu_0$$



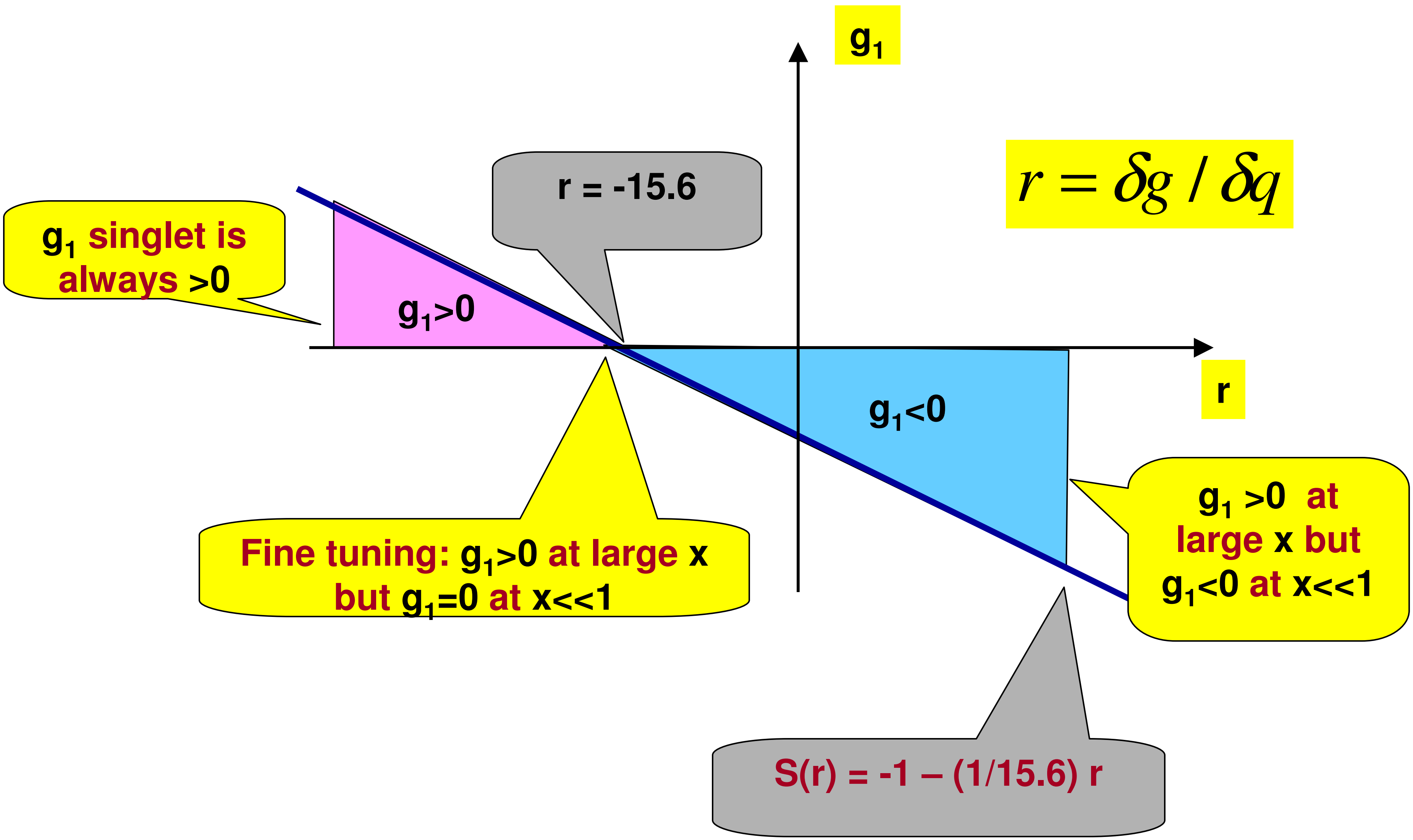
like it should be for the genuine intercept

$$\max \omega_0 \equiv \Delta$$

$$\mu_0^{NS} \approx 1 \text{ GeV}, \quad \mu_0^{S} \approx 5.5 \text{ GeV}$$

Optimal scales

At large x, g_1 singlet is positive . When x--> 0, the sign of asymtpotics of the singlet g_1 depends on the ratio between the initial parton densities



Values of the intercepts perfectly agree with results of several groups who fitted experimental data.

non-singlet intercept

Soffer-Teryaev, Kataev-Sidorov-Parente, Kotikov-Lipatov-Parente-Peshekhonov-Krivokhijine-Zotov,

singlet intercept

Kochelev-Lipka-Vento-Novak-Vinnikov

Anatomy of the singlet intercept

A. Graphs with gluons only:

$\Delta_S = 1.1$ → violates unitarity → similar to LO BFKL

B. All graphs

$\Delta_S = 0.86$ → No violation of unitarity

Comparison of our results to DGLAP at finite x –no asymptotic formulae used

Comparison depends on the assumed shape of initial parton densities.

The simplest option: use the bare quark input

$$\delta q(x) = \delta(\mathrm{x}) \qquad \Longleftrightarrow \qquad \delta q(\omega) = 1$$

in x- space

in Mellin space

Numerical comparison shows that the impact of the total resummation of logs of x becomes quite sizable at x = 0.05 approx.

PUZZLE: DGLAP should have Failed at x < 0.05.
However, it does not take place.

SOLUTION TO PUZZLE: consider in more detail standard fits for initial parton densities

$$\delta q(x) = N\, x^{-\alpha}\left[(1+\gamma x^{\delta})(1-x)^{\beta}\right]$$

normalization

singular factor

regular factors

parameters $\alpha \approx 0.58,\ \beta \approx 2.7,\ \gamma \approx 34.3,\ \delta \approx 0.75$

are fixed from fitting experimental data at large x

In the Mellin space this fit is

$$\delta q(\omega) = N[(\omega-\alpha)^{-1} + \sum_{k=1}^{\infty} c_k((\omega+k-\alpha)^{-1} + \gamma(\omega+k+1-\alpha)^{-1})]$$

Leading pole
α=0.58 >0

Non-leading poles
-k +α<0

the small-x DGLAP asymptotics of g_1 is (inessential factors dropped)

$$g_1^{DGLAP} \sim (1/\mathrm{x})^{\alpha}$$

phenomenology

Comparison it to our asymptotics

$$\mathrm{g}_1 \sim (1/x)^{\Delta_{NS}}$$

calculations

shows that the singular factor in the DGLAP fit mimics the total resummation of ln(1/x) . However, the value α = 0.58 sizably differs from our non-singlet intercept =0.4

Although our and DGLAP asymptotics lead to the x- behavior of Regge type, they predict different intercepts for the x- dependence and different Q^2 -dependence:

our calculations

$$g_1 \sim (1/x)^{\Delta} \left(Q^2/\mu^2\right)^{\Delta/2}$$

whereas DGLAP predicts the steeper x-behavior and the flatter Q^2 -behavior:

DGLAP

$$g_1^{DGLAP} \sim (1/x)^{\alpha} (\ln Q^2)^{\gamma(\alpha)}$$

x-asymptotics was checked with extrapolating available exp data to $x \rightarrow 0$.
It agrees with our values of Δ
Contradicts DGLAP

our and the DGLAP Q^2 –asymptotics have not been checked yet.

Common opinion: the total resummation is not relevant at available x
Actually: the resummation has always been accounted for through the standard fits, however without realizing it

Common opinion: fits for δq are singular but defined and large x, then convoluting them with coefficient functions weakens the singularity

$$C(x,y)\otimes\delta q(y)=\Delta q(x)$$

initial

x-evolved

Obviously, it is not true: They both are singular equally

Structure of DGLAP fit once again:

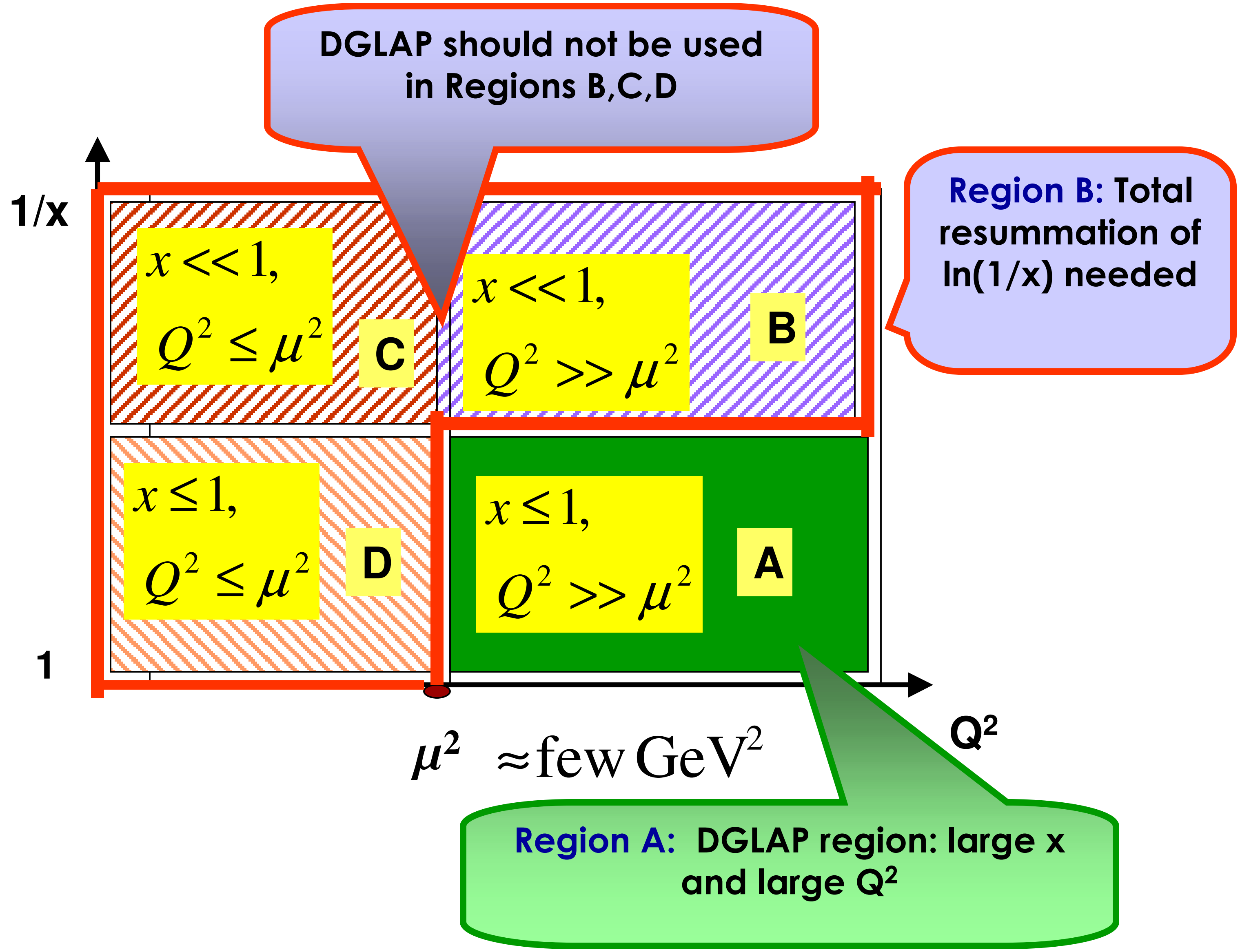
$$\delta q(x)=N\ \mathrm{x}^{-\alpha}[(1+\gamma\mathrm{x}^{\delta})(1-x)^{\beta}]$$

Can be dropped when ln(x) are resummed

x-dependence is weak at x<<1 and can be dropped

Therefore at x << 1

$$\delta q(x)\approx \mathrm{N}(1+\mathrm{ax})$$



DGLAP should not be used in Regions B,C,D
Region B: Total resummation of ln(1/x) needed
1/x
1
$x << 1,$ $Q^2 \le \mu^2$
C
$x << 1,$ $Q^2 >> \mu^2$
B
$x \le 1,$ $Q^2 \le \mu^2$
D
$x \le 1,$ $Q^2 >> \mu^2$
A
Q²
$\mu^2 \approx \text{few GeV}^2$
Region A: DGLAP region: large x and large Q²

Unified description of g_1 in Regions A&B:

large Q^2 and arbitrary x:

DGLAP

our approach

Good at large x because includes exact two-loop calculations but bad at small x as it lacks the total resummaion of ln(x)

Good at small x , includes the total resummaion of ln(x) but bad at large x because neglects some contributions essential in this region

WAY OUT – interpolation expressions combining our approach and DGLAP

1. Expand our formulae for coefficient functions and anomalous dimensions into series in the QCD coupling
2. Replace the first- and second- loop terms of the expansion by corresponding DGLAP –expressions

Non-singlet g_1: Our expressions

$$H_{LL}(\omega) = (1/2)[\omega - (\omega^2 - B(\omega))]^{1/2} \quad C_{LL}(\omega) = \omega/(\omega - H(\omega))$$

anomalous dimension

coefficient function

First tems of their expansions into the perturbation series

$$H_1 = \frac{A(\omega)C_F}{2\pi}\left[\frac{1}{\omega} + \frac{1}{2}\right] \quad C_1 = \frac{A(\omega)C_F}{2\pi}\left[\frac{1}{\omega^2} + \frac{1}{2\omega}\right]$$

New formulae combine Resummation and DGLAP:

$$H_C = H_{LL} - H_1 + H_{LO\,DGLAP} \quad C_C = C_{LL} - C_1 + C_{LO\,DGLAP}$$

formulae for the singlet anomalous dimensions and coefficient functions are written quite similarly

No singular parton densities are required

Region C: small x and small Q^2

At $Q^2 \gg \mu^2$ g_1 depends on Q^2 through logarithms:

$$g_1^{NS} = \frac{e_q^2}{2} \int \frac{d\omega}{2\pi i} \left(\frac{1}{x}\right)^{\omega} \left(\frac{\omega}{\omega - H(\omega)}\right) \delta q(\omega) \left(\frac{Q^2}{\mu^2}\right)^{H(\omega)}$$

$$= \sum_k \ln^k (Q^2/\mu^2)\, C_k(x)$$

resummation of leading ln(x)

At $Q^2 < \mu^2$ g_1 depends on Q^2 through powers:

$$g_1^{NS} = \sum_k (Q^2/\mu^2)^k\, C_k^{'}(x)$$

again resummation of leading ln(x)

Description of g_1 in Region C: small Q^2 and small x:

Generalization of our previous results through the shift

$$Q^2 \to Q^2 + \mu^2 \quad \Rightarrow \quad x \to \bar{x} = (Q^2 + \mu^2)/2pq = x + z$$

Infrared cut-off

Similar shifts have been used for DIS structure functions by many authors, however from phenomenological considerations. We do It from analysis of the involved Feynman graphs

It leads to new expressions: non-singlet g_1 at small x and arbitrary Q^2

$$z = \frac{\mu^2}{2pq} >> x = \frac{Q^2}{2pq}$$

$$g_1^{NS} = \frac{e_q^2}{2} \int \frac{d\omega}{2\pi i} \left(\frac{1}{z + x} \right)^{\omega} \left(\frac{\omega}{\omega - H(\omega)} \right) \delta q(\omega) \left(\frac{\mu^2 + Q^2}{\mu^2} \right)^{H(\omega)}$$

x -dependence

Anomalous dimension

Q^2 –dependence

Coefficient function

Initial quark density

Technology of getting universal description of g_1:

Step 1:
Resummation of leading $\ln(1/x)$ and $\ln(Q^2)$

→ g_1 in Region B: large Q^2 and small x

Step 2:
Combining above results and DGLAP

→ g_1 in Region A&B: large Q^2 and arbitrary x

Step 3:
Shift $Q^2 \to Q^2 + \mu^2$

→ g_1 in Region C&D: small Q^2 and arbitrary x

Thus, we arrive at universal and model-independent description of g_1 at arbitrary Q^2 and x without singular fits:

$$g_1^{NS} = \frac{e_q^2}{2}$$

$$\int \frac{d\omega}{2\pi i}\left(\frac{1}{z+x}\right)^{\omega} C_C(\omega)\,\delta q(\omega)\left(\frac{\mu^2+Q^2}{\mu^2}\right)^{H_C(\omega)}$$

$$z = \frac{\mu^2}{2pq}, \quad x = \frac{Q^2}{2pq}$$

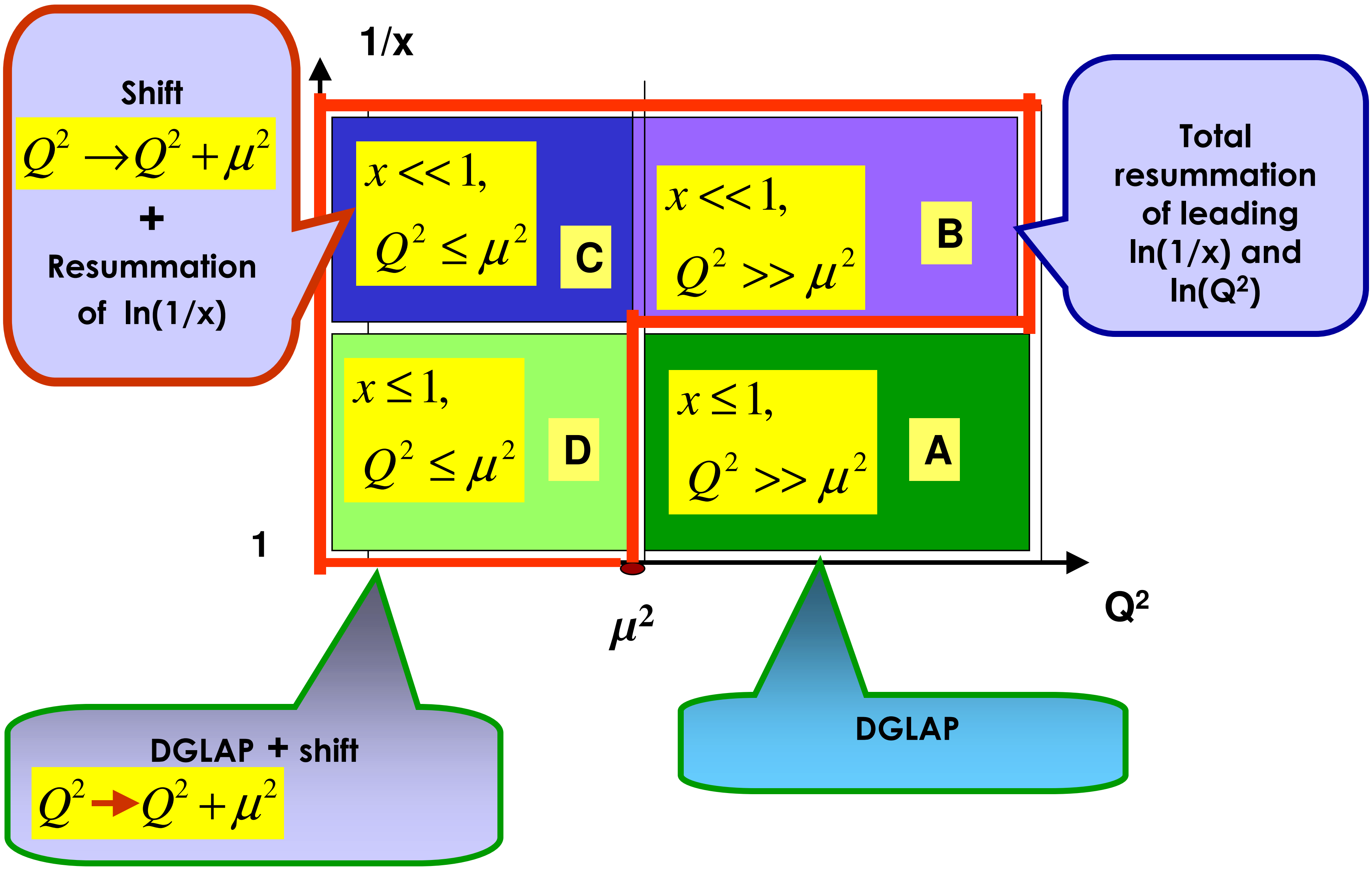
Combined coefficient function

Non-singular quark density

Combined anomalous dimension

expression for the singlet g_1 is written quite similarly

Main impact on g_1 in Regions A,B,C,D comes from:



Recent applications of our approach to:

1. COMPASS results

2. Power Q^2-corrections

Taken from wwwcompass.cern.ch

COMPASS

COMPASS is a high-energy physics experiment at the Super Proton Synchrotron (SPS) at CERN in Geneva, Switzerland. The purpose of this experiment is the study of hadron structure and hadron spectroscopy with high intensity muon and hadron beams.

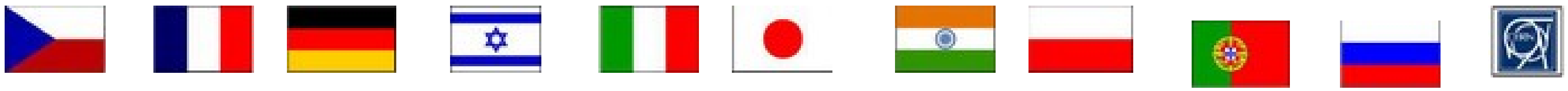
On February 1997 the experiment was approved conditionally by CERN and the final Memorandum of Understanding was signed in September 1998. The spectrometer was installed in 1999 - 2000 and was commissioned during a technical run in 2001. Data taking started in summer 2002 and continued until fall 2004. After one year shutdown in 2005, COMPASS will resume data taking in 2006.

Nearly 240 physicists from 11 countries and 28 institutions work in COMPASS

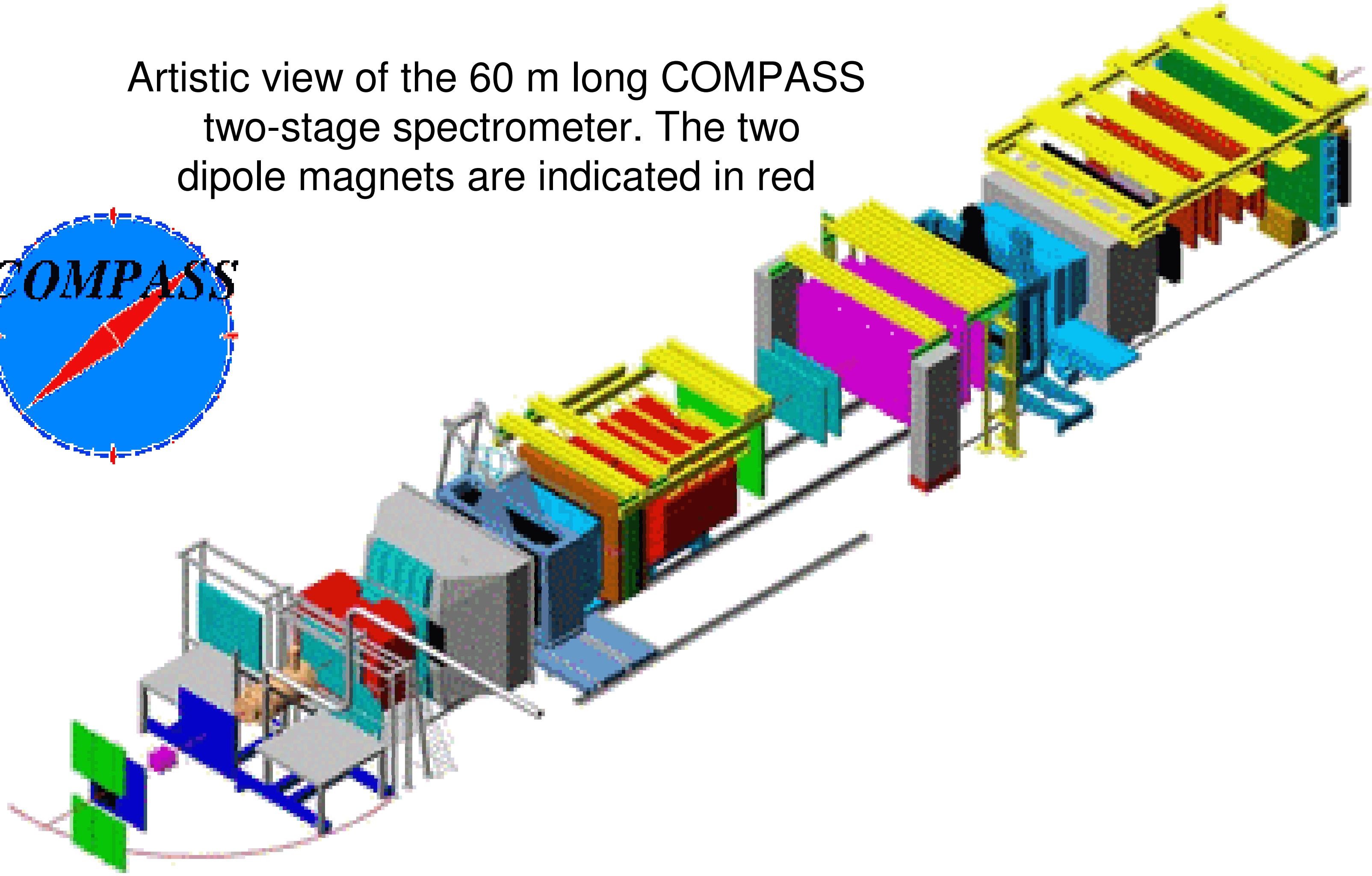
Taken from wwwcompass.cern.ch

COMPASS

COmmon Muon Proton Apparatus for Structure and Spectroscopy



Artistic view of the 60 m long COMPASS two-stage spectrometer. The two dipole magnets are indicated in red



Small Q^2
Singlet g_1

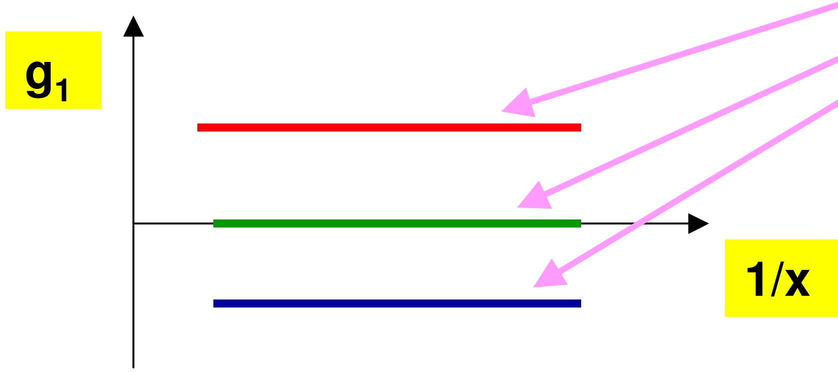
$$g_1^S = \frac{<e_q^2>}{2} \int \frac{d\omega}{2\pi i} \left(\frac{1}{z+x} \right)^{\omega} \left[C_q \delta q + C_g \delta g \right]$$

$$z = \frac{\mu^2}{2pq}, \qquad x = \frac{Q^2}{2pq}$$

$$C_g = C_g^{(+)} \left(\frac{\mu^2 + Q^2}{\mu^2} \right)^{\Omega^{(+)}} + C_g^{(-)} \left(\frac{\mu^2 + Q^2}{\mu^2} \right)^{\Omega^{(-)}}$$

$$C_q = C_q^{(+)} \left(\frac{\mu^2 + Q^2}{\mu^2} \right)^{\Omega^{(+)}} + C_q^{(-)} \left(\frac{\mu^2 + Q^2}{\mu^2} \right)^{\Omega^{(-)}}$$

when $Q^2 << \mu^2$ both x- and Q^2- dependences are flat, even for x<<1.



Location of the line is determined by the z-dependence

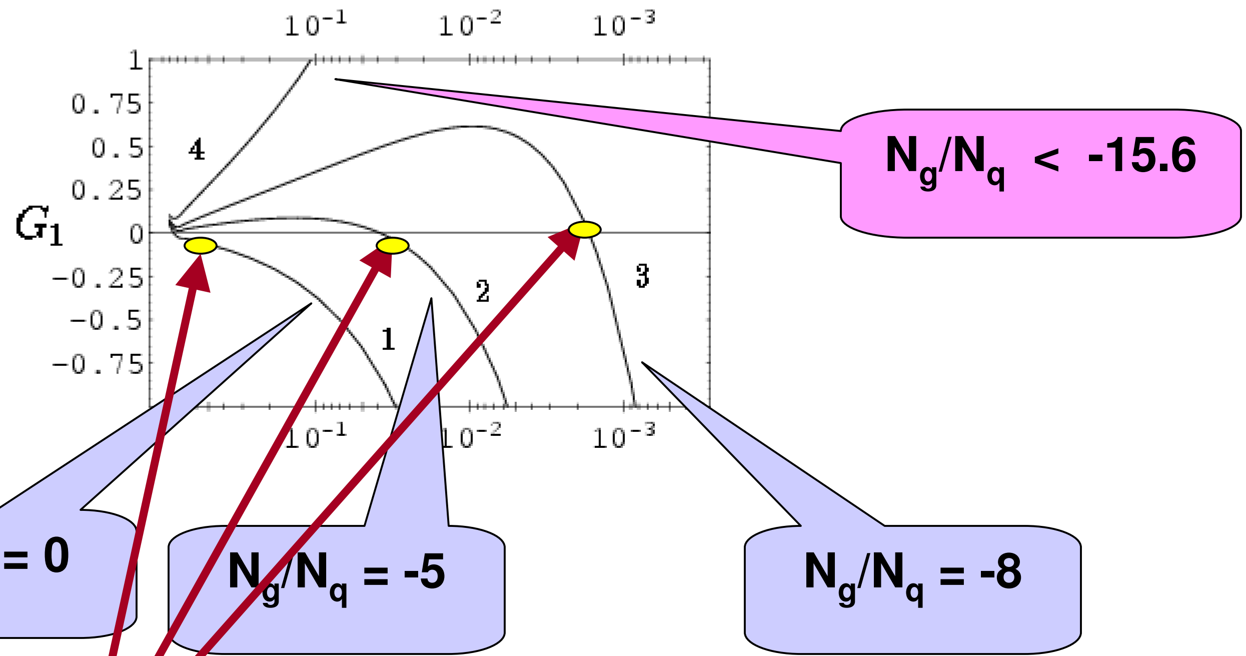
$$g_1(z) = \left(\frac{e_q^2}{2}\right)\int_{-i\infty}^{i\infty}\frac{d\omega}{2\pi i}\left(\frac{1}{z}\right)^{\omega}\left[C_q(\omega)\delta q + C_g(\omega)\delta g\right]$$

Approximating

$$\delta q \approx N_q, \delta q \approx N_q,$$

perform numerical calculations of G_1

$$g_1 = (e_q^2/2)N_q G_1,$$



Position of the turning point is sensitive to N_g/N_q , so the experimental detection of it will allow to estimate Ng/Nq

Power Corrections to non-singlet g_1

Leading twist contribution

mass scale: $Q^2 > M^2$

$$g_1(x,Q^2) = g_1^{LT}(x,Q^2)\left[1 + \sum_k C_k \left(\frac{M^2}{Q^2}\right)^k\right]$$

Power corrections

PC are supposed to come from higher twists.
No satisfactory theory
is known for the higher twists

Standard way of obtaining PC from experimental
data at small x: Leader-Stamenov- Sidorov

Compare experimental data to predictions of the Standard Approach
and assign the discrepancy to the impact of PC

$$g_1^{LT} = g_1^{DGLAP}$$

Counter-argument:

1. **DGLAP, the main ingredient of SA, is theoretically unreliable at small x, so comparing experiment to it is not so productive: it proves nothing**
2. **SA cannot explain why PC appear at $Q^2 > 1$ GeV^2 only and predict what happens at smaller Q^2**

Our approach can do it:

$$g_1^{NS} = \frac{e_q^2}{2} \int \frac{d\omega}{2\pi i} \left(\frac{w}{\mu^2 + Q^2} \right)^{\omega} C(\omega)\ \delta q(\omega) \left(\frac{\mu^2 + Q^2}{\mu^2} \right)^{H(\omega)}$$

where w = 2pq and Q^2 can be large or small, μ = 1 GeV

μ **=1 GeV**, so when Q² < 1 GeV², expansion into power series is:

$$g_1^{NS} = \frac{e_q^2}{2} \int \frac{d\omega}{2\pi i} \left(\frac{w}{\mu^2} \right)^{\omega} C(\omega)\, \delta q(\omega)$$

$$\left[1 + \sum_{k=1} T_k(\omega) \left(\frac{Q^2}{\mu^2} \right)^k \right]$$

Power corrections

Leading contribution for g_1^{NS} does not depend on Q^2

At $Q^2 > 1$ GeV^2 expansion into series is different:

$$g_1^{NS} = \frac{e_q^2}{2}\int \frac{d\omega}{2\pi i}\left(\frac{w}{Q^2}\right)^{\omega} C(\omega)\,\delta q(\omega)\left(\frac{\mu^2}{Q^2}\right)^{H(\omega)} \left[1 + \sum_{k=1} T_k(\omega)\left(\frac{\mu^2}{Q^2}\right)^{k}\right]$$

Conventionally looking Power Corrections

Leading contribution for g_1^{NS}

These Power Corrections have perturbative origin and should be accounted in the first place. Only AFTER THAT one can reliable estimate a genuine impact of higher twist contributions

Conclusion

Standard Approach

DGLAP was originally developed for operating at the region where both x and Q^2 are large. Basic ingredients of the DIS structure functions – coefficient functions and splitting functions (anomalous dimensions) are calculated in DGLAP in the first and second loops. By construction, DGLAP describes the Q^2–evolution but cannot describe the x–evolution. Accounting for the x–evolution is especially important in the small-x region.

In order to extend DGLAP to the region of small x and large Q^2, it have been complemented with rather complicated expressions for the initial parton densities δq and δg found from fitting experimental data.

DGLAP + Standard fits form Standard Approach (SA). SA describes DIS at large Q^2 and arbitrary x.

Alternatives to SA

Unpolarized DIS: No model-independent description. Models involve either phenomenological Pomerons or BFKL. Both LO and NLO BFKL intercepts are positive. They violate both the untiarity and Froissar bound. It is urgent to calculate NNLO corrections to the intercept

As the Pomeron intercept is small, it is difficult to discriminate between these approaches

Polarized DIS:

.

Model-independent description of g_1 combines total resummation of leading logarithmic contributions, DGLAP expressions, and shift of Q^2 .
It represents g_1 at arbitrary x and Q^2 .

DGLAP agrees with experimental data only when special expressions for initial parton densities are used. They include singular factors, though DGLAP offers no theoretical explanation of the origin of the factors

Actually, the singular factors mimic total resummation of leading logarithms
When the resummatiion is accounted for, the expressions for initial parton densities can be simplified down to constants