Introduction to hadronic collisions: theoretical concepts and practical tools for the LHC

Lecture 2

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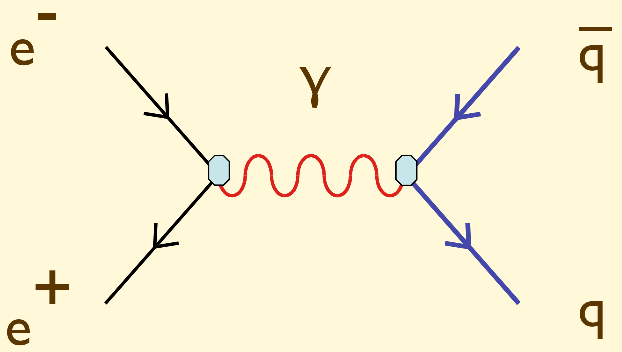
Evolution of hadronic final states

Asymptotic freedom implies that at $E_{CM} >> 1$ GeV

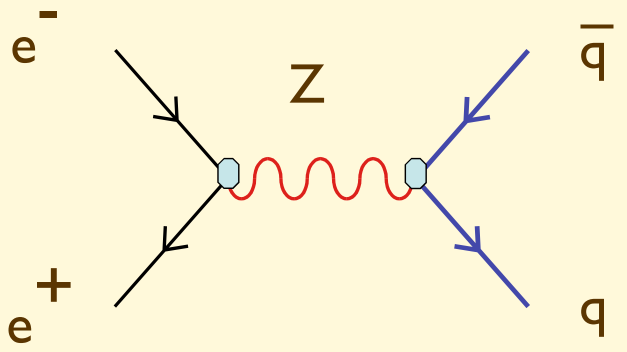
$\sigma(e^+ e^- \rightarrow \text{hadrons})$ ↔ $\sigma(e^+ e^- \rightarrow \text{quarks/gluons})$

At the Leading Order (LO) in PT:



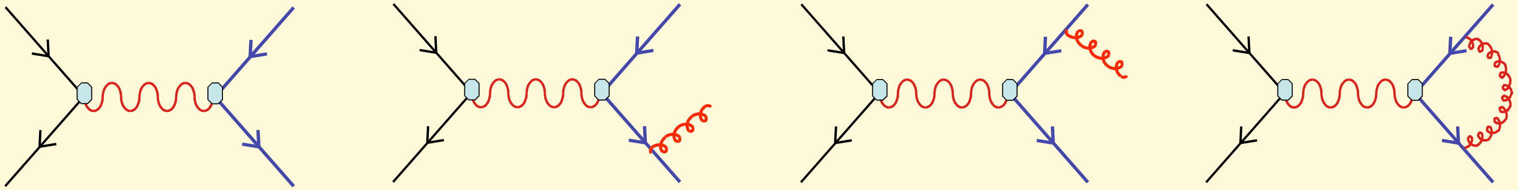
$$\sigma_0(e^+e^- \rightarrow q\bar{q}) = \frac{4\pi\alpha^2}{9s} N_c \sum_{f=u,d,\ldots} e_{q_f}^2$$

$$\frac{\sigma_0(e^+e^- \rightarrow q\bar{q})}{\sigma_0(e^+e^- \rightarrow \mu^+\mu^-)} = N_c \sum_{f=u,d,\ldots} e_{q_f}^2$$



$$\frac{\sigma_0(e^+e^- \rightarrow Z \rightarrow q\bar{q})}{\sigma_0(e^+e^- \rightarrow Z \rightarrow \mu^+\mu^-)} = N_c \frac{\sum_{f=u,d,\ldots}\left(v_{q_f}^2 + a_{q_f}^2\right)}{\left(v_\mu^2 + a_\mu^2\right)}$$

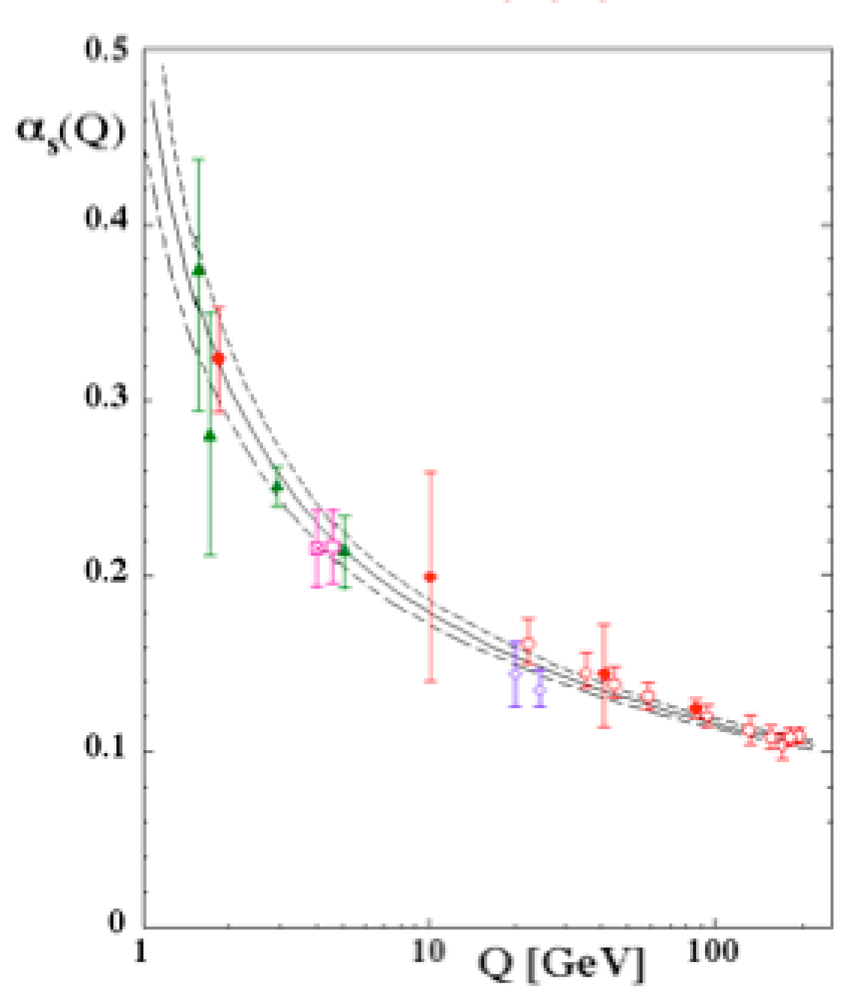
Adding higher-order perturbative terms:



+ ≥2-gluon emissions

$$\sigma_1(e^+e^- \to q\bar{q}(g)) = \sigma_0(e^+e^- \to q\bar{q}) \left(1 + \frac{\alpha_s(E_{CM})}{\pi} + O(\alpha_s^2)\right)$$

O(3%) at M_Z

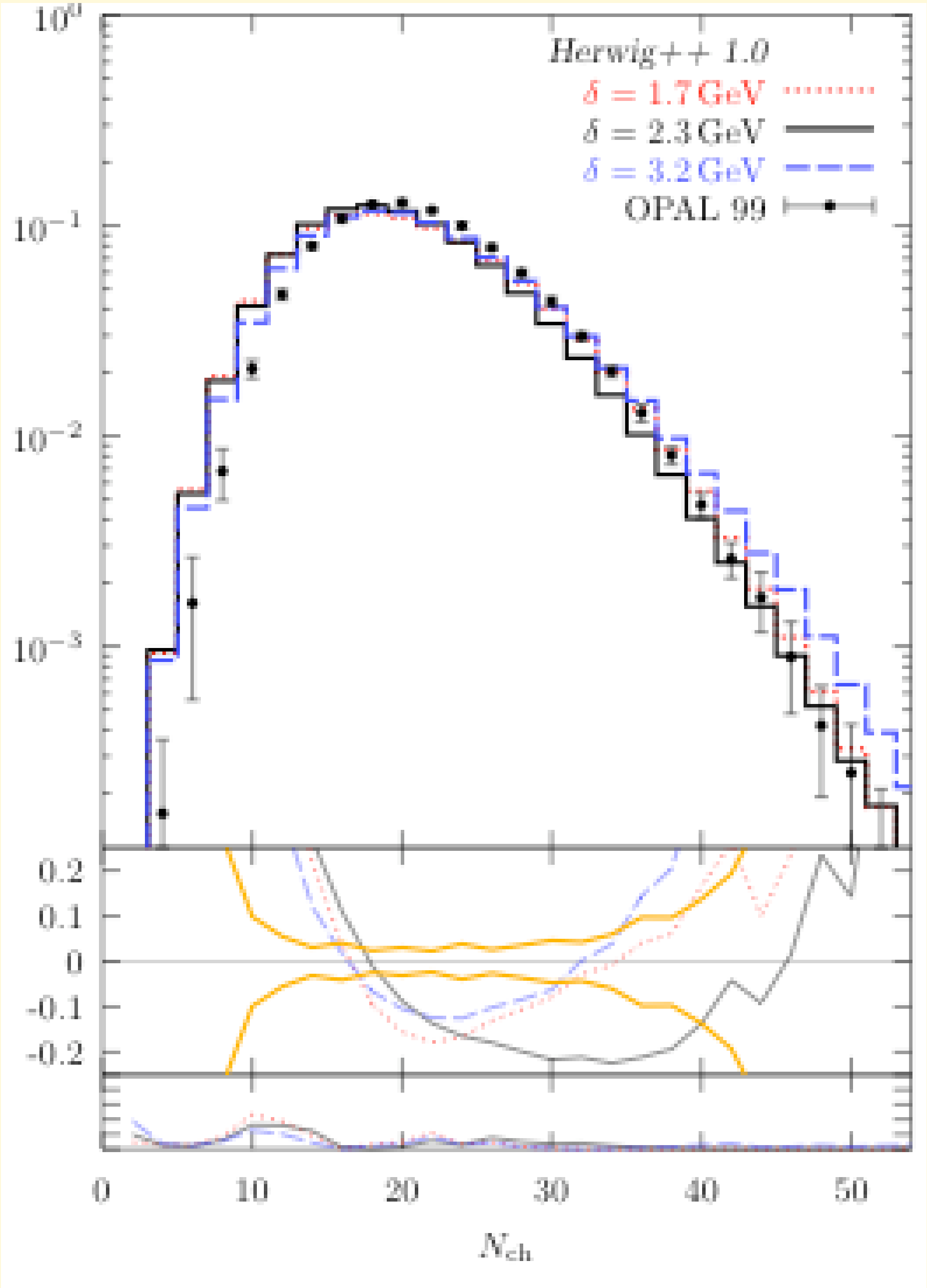


Excellent agreement with data,
provided N_c=3
Extraction of αs consistent with the Q evolution predicted by QCD

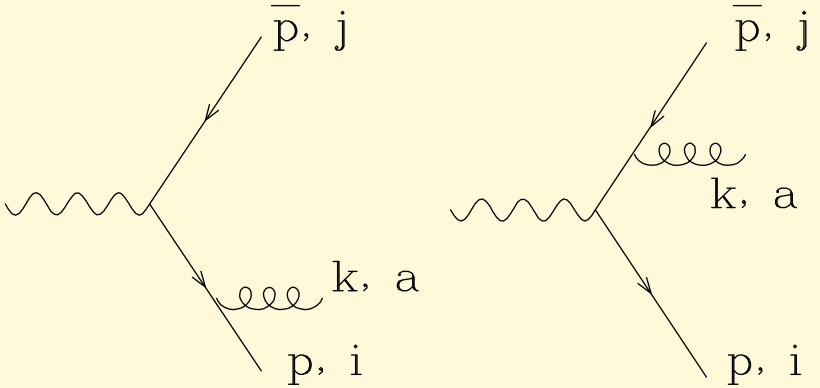
Experimentally, the final states contain a large number of particles, not the 2 or 3 which apparently saturate the perturbative cross-section.

Experimental multiplicity distribution

$\langle n_{charged} \rangle = 20.9$



Soft gluon emission



$$
\begin{aligned}
A &= \bar{u}(p)\epsilon(k)(ig)\,\frac{-i}{\not p+\not k}\,\Gamma^{\mu}\,v(\bar{p})\;\lambda^{a}_{ij} + \bar{u}(p)\,\Gamma^{\mu}\,\frac{i}{\not{\bar{p}}+\not k}\,(ig)\epsilon(k)\,v(\bar{p})\;\lambda^{a}_{ij} \\
&= \left[\frac{g}{\boxed{2p\cdot k}}\,\bar{u}(p)\epsilon(k)\,(\not p+\not k)\Gamma^{\mu}\,v(\bar{p}) - \frac{g}{\boxed{2\bar{p}\cdot k}}\,\bar{u}(p)\,\Gamma^{\mu}\,(\not{\bar{p}}+\not k)\epsilon(k)\,v(\bar{p})\right]\lambda^{a}_{ij}
\end{aligned}
$$

$p\cdot k = p_0 k_0 (1-\cos\theta) \Rightarrow$ singularities for collinear ($\cos\theta \to 1$) or soft ($k_0 \to 0$) emission

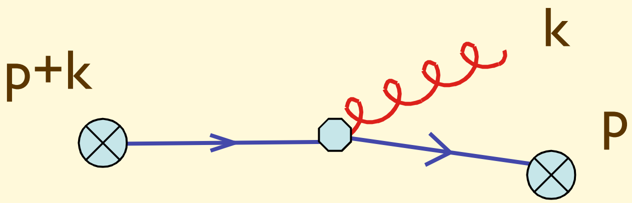
Collinear emission does not alter the global structure of the final state, since its preserves its "pencil-like-ness". **Soft emission** at large angle, however, could spoil the structure, and leads to strong interferences between emissions from different legs. So soft emission needs to be studied in more detail.

In the soft ($k_0 \to 0$) limit the amplitude simplifies and factorizes as follows:

$$
A_{soft} = g\lambda^{a}_{ij}\left(\frac{p\cdot\varepsilon}{p\cdot k} - \frac{\bar{p}\cdot\varepsilon}{\bar{p}\cdot k}\right) A_{Born}
$$

Factorization: it is the expression of the independence of long-wavelength (soft) emission on the nature of the hard (short-distance) process.

Another simple derivation of soft-gluon emission rules



charge current of a free fermion

$$\bar{\psi}(p)\,\gamma_\mu\,\psi(p+k)\,\varepsilon^\mu(k) \quad \overset{k\to 0}{\longrightarrow} \quad \bar{\psi}(p)\,\gamma_\mu\,\psi(p)\varepsilon^\mu(k) \;=\; 2p\cdot\varepsilon$$

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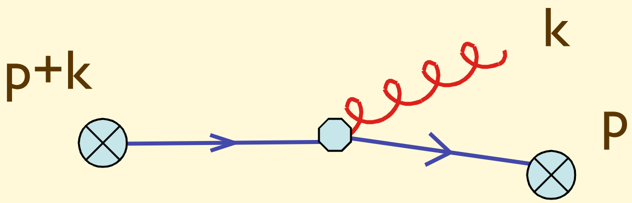

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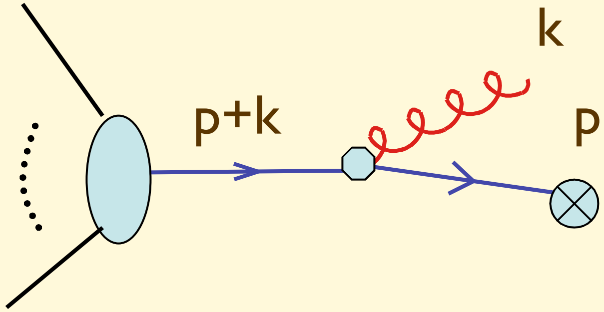

$$\frac{1}{\not{p}+\not{k}}\gamma_\mu\psi(p+k)\,\varepsilon^\mu(k) \xrightarrow{k\to 0}$$

$$\frac{1}{2p\cdot k}\not{p}\gamma_\mu\psi(p)\varepsilon^\mu(k) = \frac{p\cdot\varepsilon}{p\cdot k}$$

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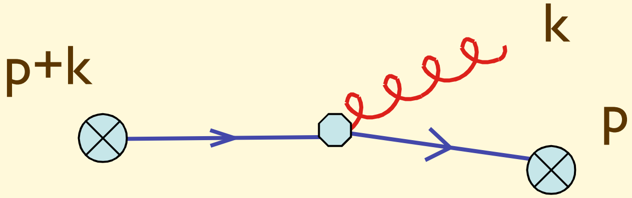

charge current of a free fermion

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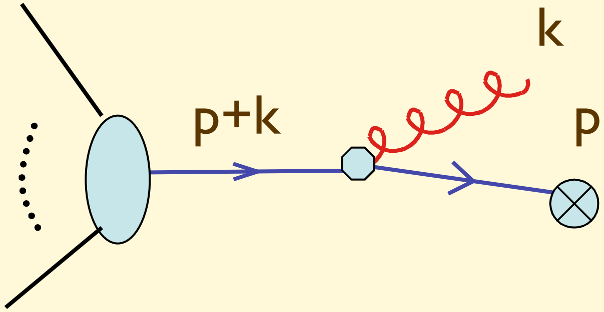

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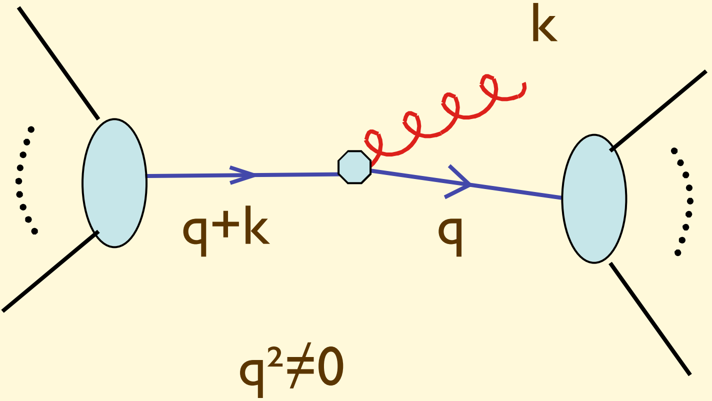
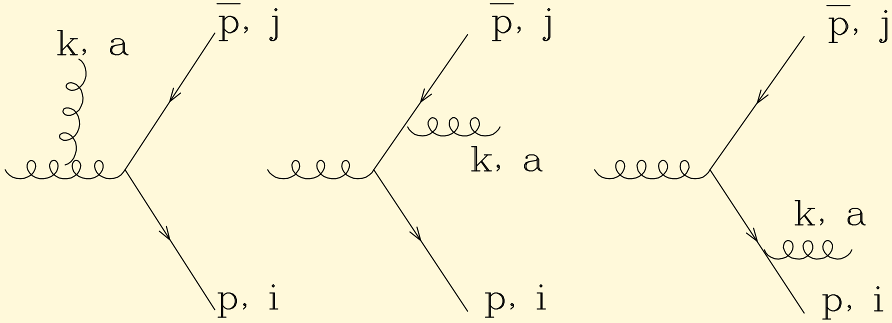


$q^2\neq0$

$$\frac{1}{\not{q}+\not{k}}\gamma_\mu\frac{1}{\not{q}}\varepsilon^\mu(k) \xrightarrow{q^2\neq0,\,k\to0} \frac{1}{q^2}\not{q}\gamma_\mu\not{q}\frac{1}{q^2}\varepsilon^\mu(k)$$

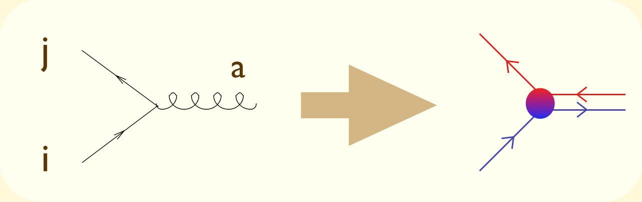
=> finite

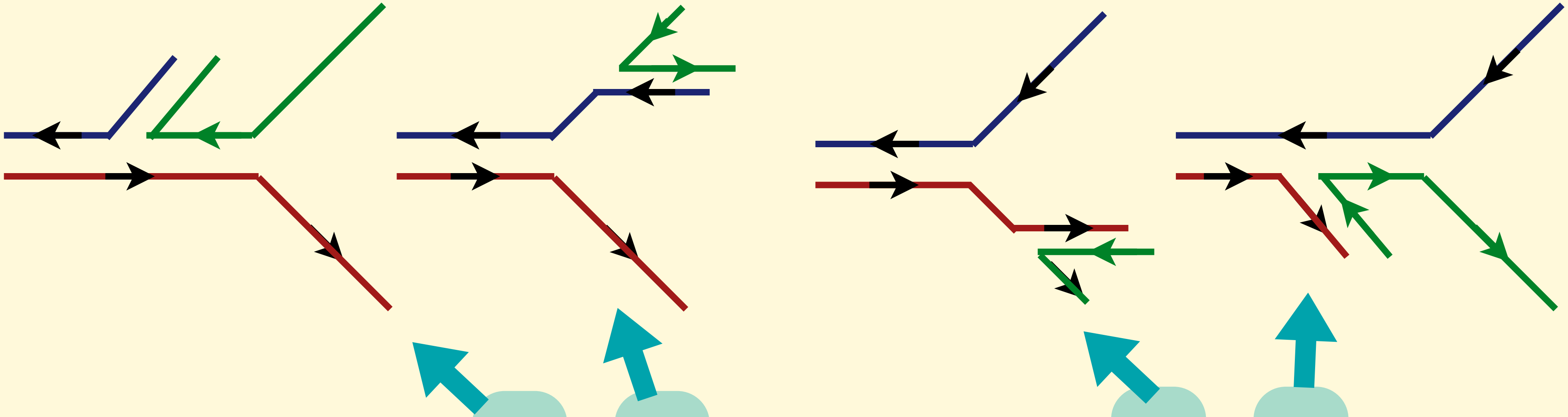
Similar, but more structured, result in the case of a fully coloured process:



$$A_{soft} = g\,(\lambda^a\lambda^b)_{ij}\left[\frac{Q\varepsilon}{Qk} - \frac{\bar{p}\varepsilon}{\bar{p}k}\right] + g\,(\lambda^b\lambda^a)_{ij}\left[\frac{p\varepsilon}{pk} - \frac{Q\varepsilon}{Qk}\right]$$

The four terms correspond to the two possible ways colour can flow, and to the two possible emissions for each colour flow:





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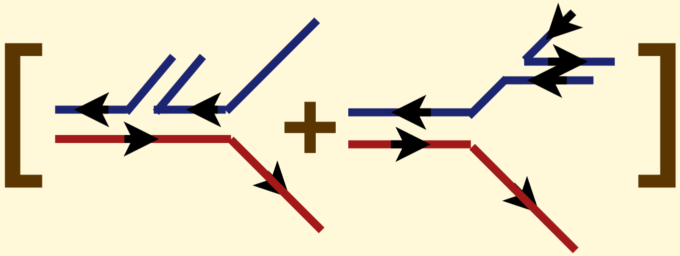
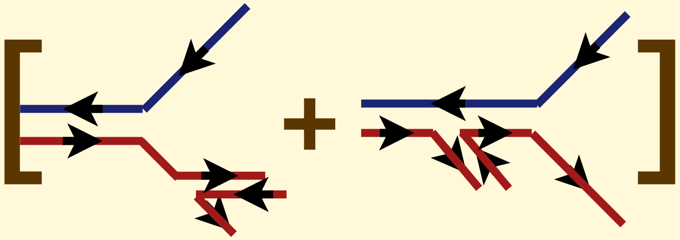
The interference between the two colour structures



$\propto (\lambda^a\lambda^b)_{ij}$



$\propto (\lambda^b\lambda^a)_{ij}$

is suppressed by $1/N_c^2$:

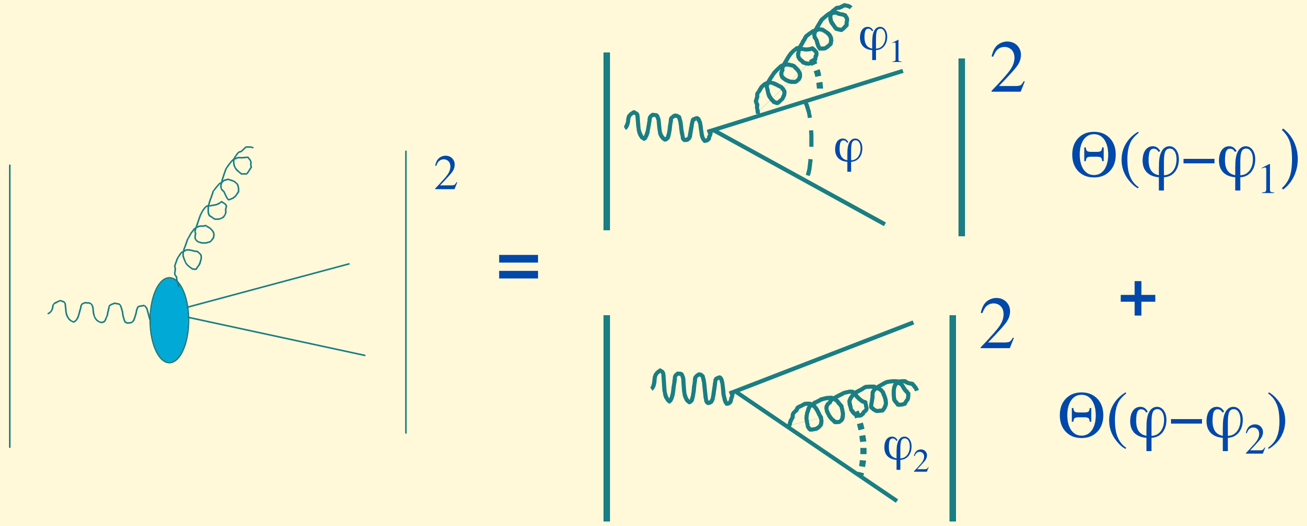
$$\sum_{a,b,i,j} |(\lambda^a\lambda^b)_{ij}|^2 = \sum_{a,b} tr\ (\lambda^a\lambda^b\lambda^b\lambda^a) = \frac{N^2-1}{2} C_F = O(N^3)$$

$$\sum_{a,b,i,j} (\lambda^a\lambda^b)_{ij}[(\lambda^b\lambda^a)_{ij}]^* = \sum_{a,b} tr(\lambda^a\lambda^b\lambda^a\lambda^b) = \frac{N^2-1}{2} \underbrace{(C_F - \frac{C_A}{2})}_{-\frac{1}{2N}} = O(N)$$

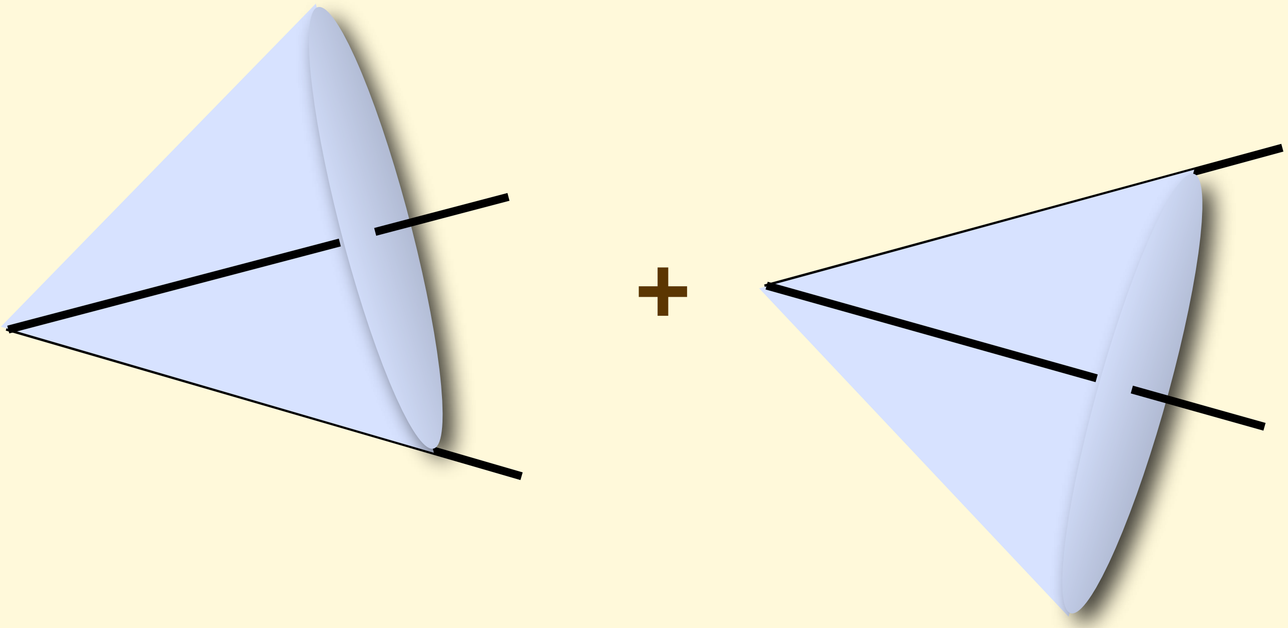
As a result, the emission of a soft gluon can be described, to the leading order in $1/N_c^2$, as the incoherent sum of the emission from the two colour currents

What about the interference between the two diagrams corresponding to the same colour flow? ➥

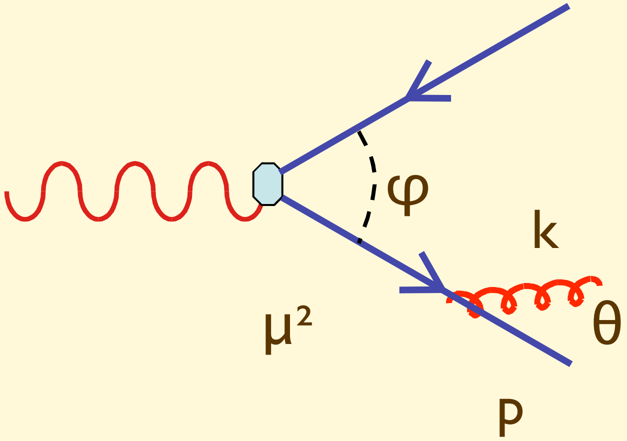
Angular ordering



Radiation inside the cones is allowed, and described by the eikonal probability, radiation outside the cones is suppressed and averages to 0 when integrated over the full azimuth



An intuitive explanation of angular ordering



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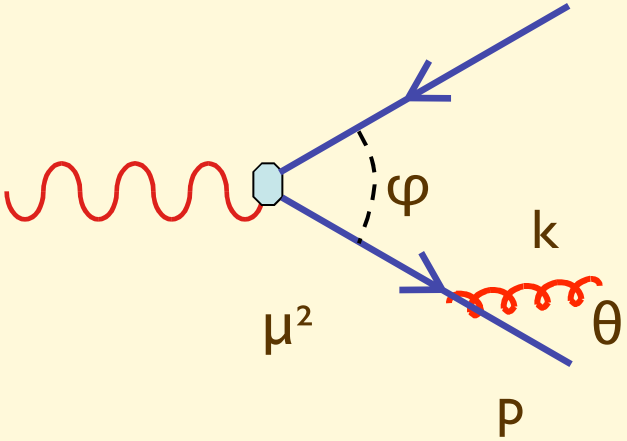

$$\mu^2 = (p+k)^2 = 2E\, k_0\, (1-\cos\theta)$$
$$\sim E\, k_0\, \theta^2 \sim E\, k_\perp\, \theta$$

Lifetime of the virtual intermediate state:

$$\tau < \gamma/\mu = E/\mu^2 = 1 / k_0\, \theta^2 = 1/k_\perp \theta$$

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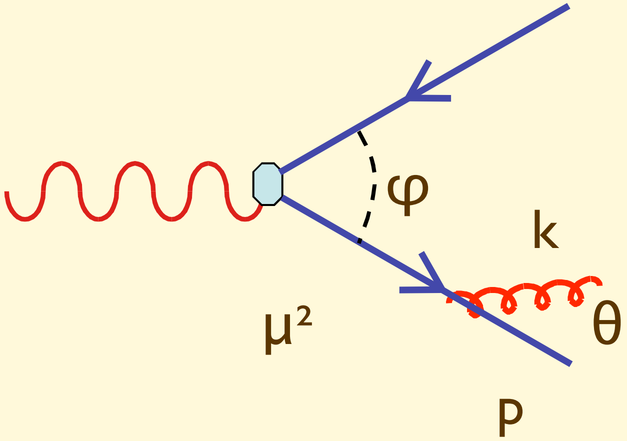

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Distance between q and qbar after τ:

$$d = \varphi\tau = (\varphi/\theta)\ 1/k_\perp$$

If the transverse wavelength of the emitted gluon is longer than the separation between q and qbar, the gluon emission is suppressed, because the q qbar system will appear as colour neutral (=> dipole-like emission, suppressed)

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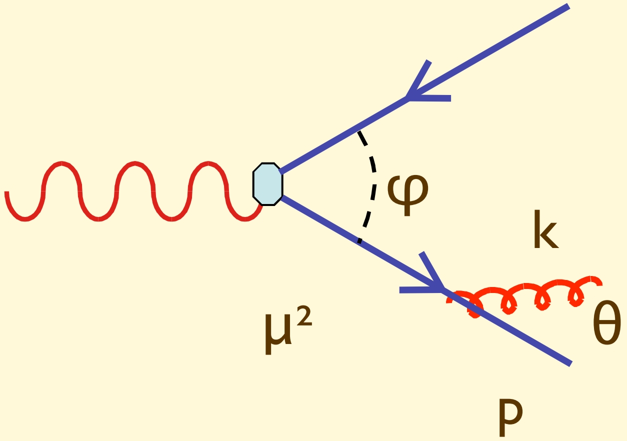

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$\mu^2 = (p+k)^2 = 2E\,k_0\,(1-\cos\theta)$
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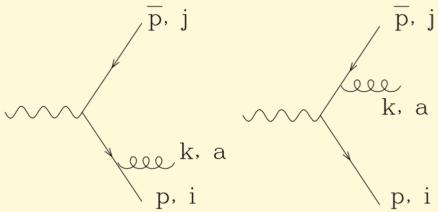
Therefore $d > 1/k_\perp$, which implies $\theta < \varphi$

The formal proof of angular ordering



$$d\sigma_g = \sum |A_{soft}|^2 \frac{d^3k}{(2\pi)^3 2k^0} \sum |A_0|^2 \frac{-2p^\mu \bar{p}^\nu}{(pk)(\bar{p}k)} g^2 \sum \varepsilon_\mu \varepsilon^*_\nu \frac{d^3k}{(2\pi)^3 2k^0}$$

$$= d\sigma_0 \frac{\alpha_s C_F}{\pi} \frac{dk^0}{k^0} \frac{d\phi}{2\pi} \frac{1-\cos\theta_{ij}}{(1-\cos\theta_{ik})(1-\cos\theta_{jk})} d\cos\theta$$

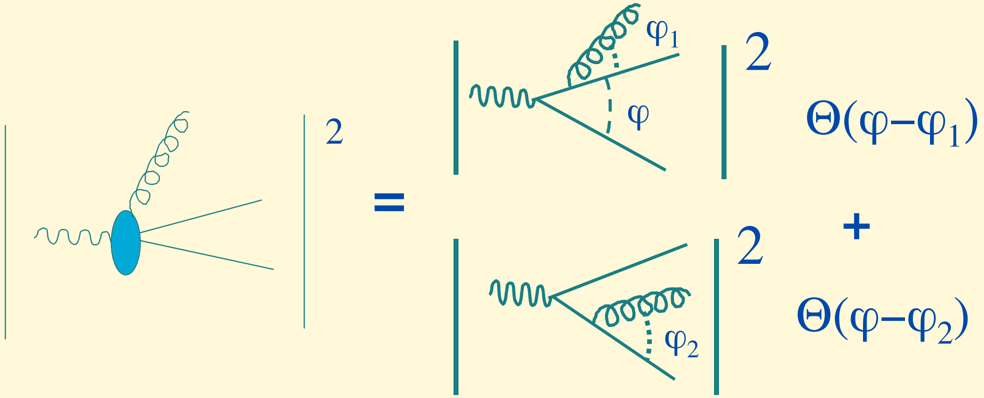
You can easily prove that:

$$\frac{1-\cos\theta_{ij}}{(1-\cos\theta_{ik})(1-\cos\theta_{jk})} = \frac{1}{2}\left[\frac{\cos\theta_{jk}-\cos\theta_{ij}}{(1-\cos\theta_{ik})(1-\cos\theta_{jk})} + \frac{1}{1-\cos\theta_{ik}}\right] + \frac{1}{2}[i \leftrightarrow j] \equiv W_{(i)} + W_{(j)}$$

where:

$$W_{(i)} \rightarrow finite\ if\ k \parallel j\ (\cos\theta_{jk} \rightarrow 1)$$
$$W_{(j)} \rightarrow finite\ if\ k \parallel i\ (\cos\theta_{ik} \rightarrow 1)$$

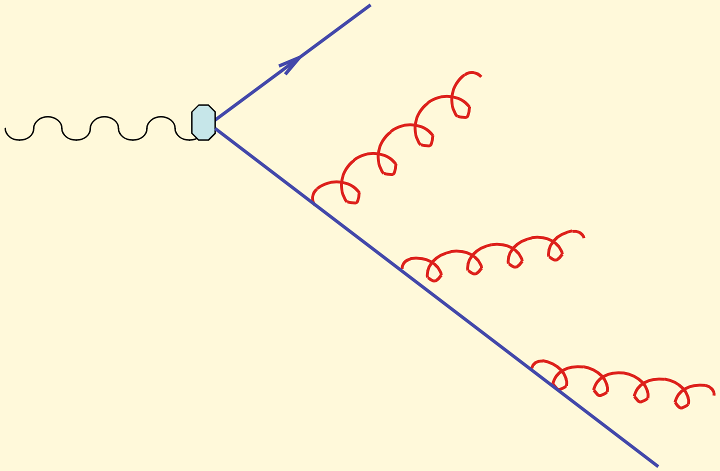
The probabilistic interpretation of W(i) and W(j) is a priori spoiled by their non-positivity. However, you can prove that after azimuthal averaging:



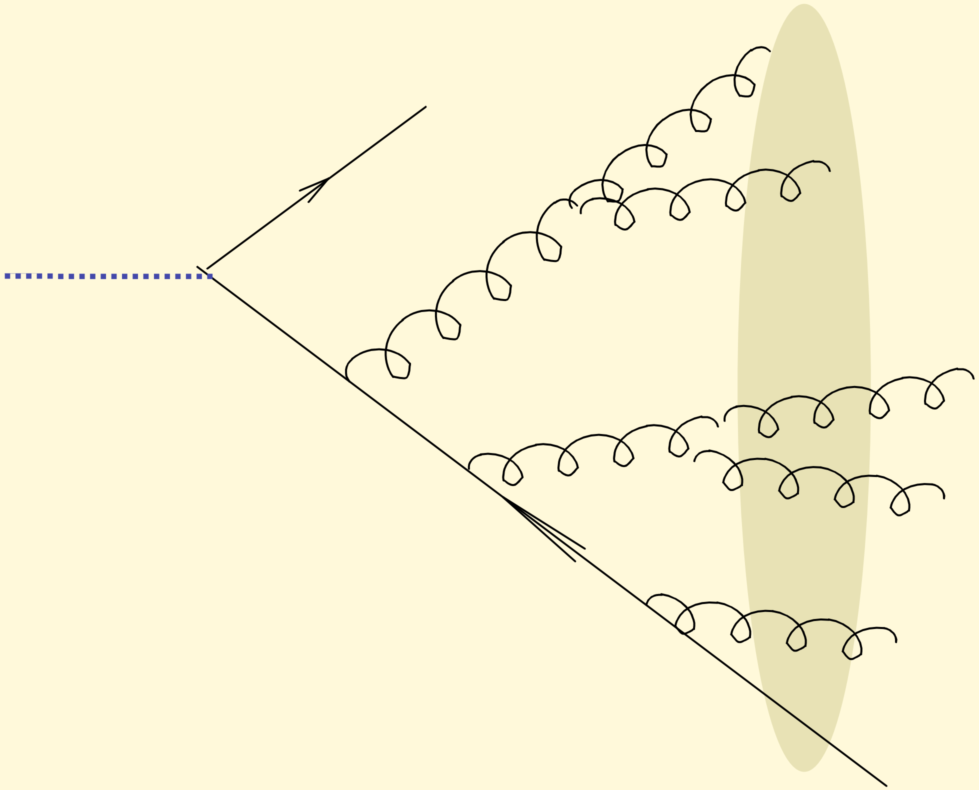
$$\int \frac{d\phi}{2\pi} W_{(i)} = \frac{1}{1-\cos\theta_{ik}} \text{ if } \theta_{ik} < \theta_{ij}\,, \quad 0 \text{ otherwise}$$

$$\int \frac{d\phi}{2\pi} W_{(j)} = \frac{1}{1-\cos\theta_{jk}} \text{ if } \theta_{jk} < \theta_{ij}\,, \quad 0 \text{ otherwise}$$

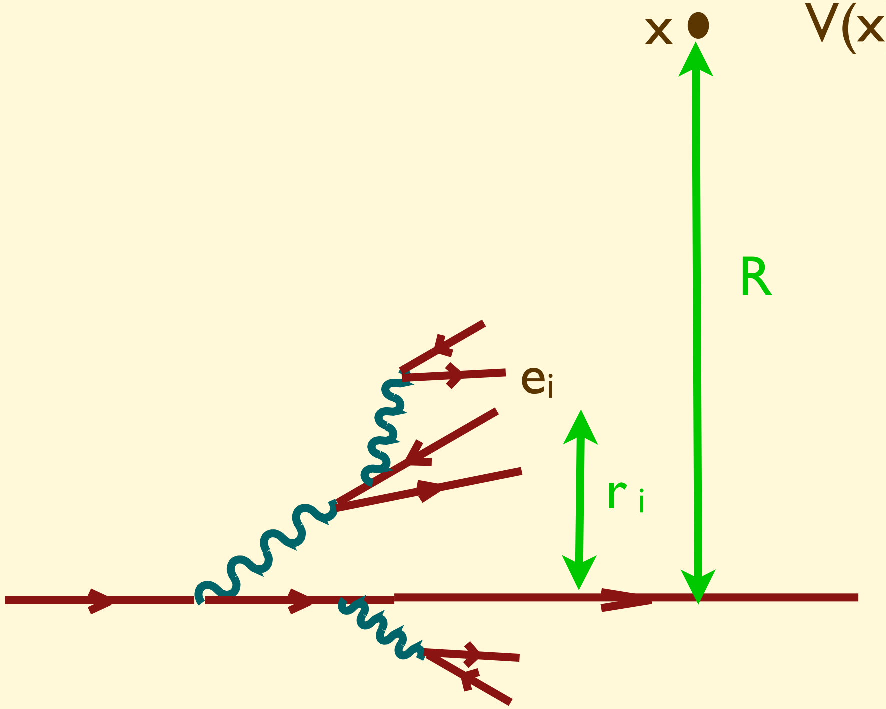
Further branchings will obey angular ordering relative to the new angles. As a result emission angles get smaller and smaller, squeezing the jet



The construction can be iterated to the next emission, with the result that emission angles keep getting smaller and smaller => **jet structure**

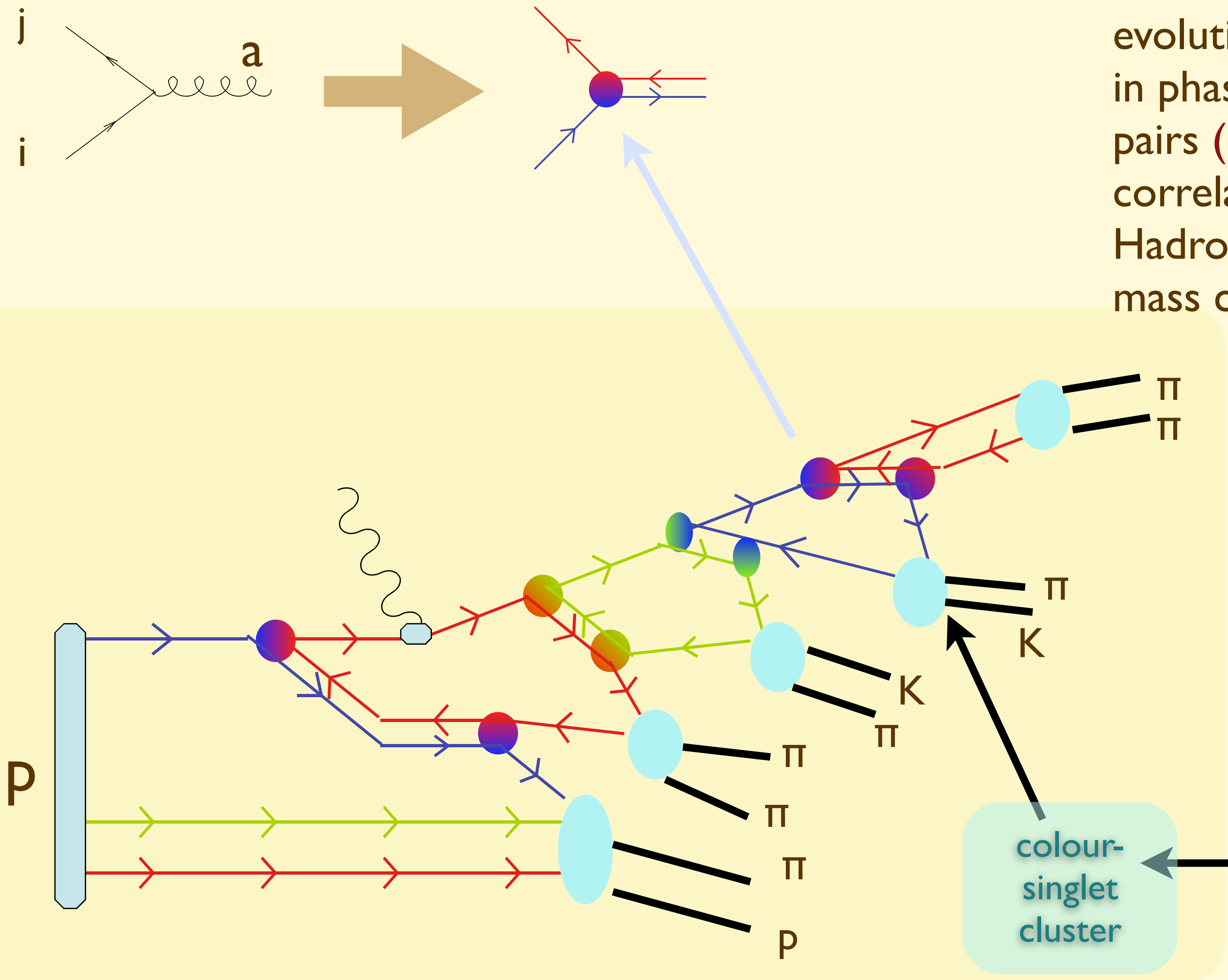


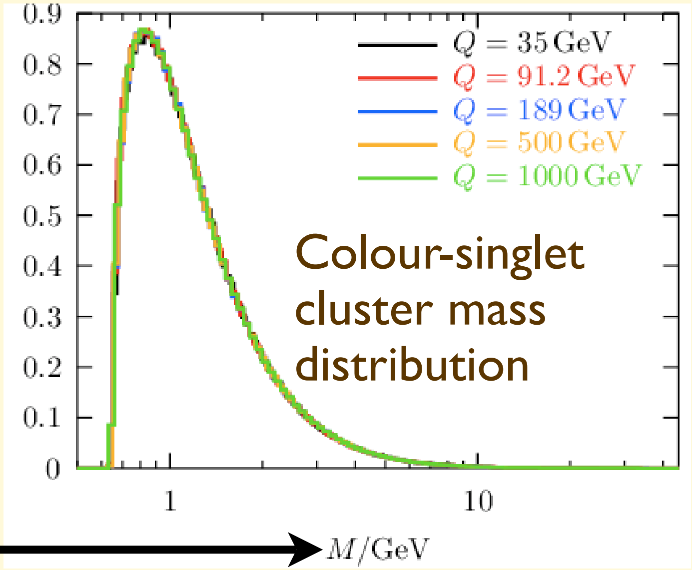
Total colour charge of the system is equal to the quark colour charge. Treating the system as the incoherent superposition of N gluons would lead to artificial growth of gluon multiplicity. Angular ordering enforces coherence, and leads to the proper evolution with energy of particle multiplicities.



$$V(x) = \frac{e}{R} + \frac{1}{R^2}\,\left|\,\Sigma\; e_i\, \boldsymbol{r}_i\,\right|$$

The structure of the perturbative evolution leads naturally to the clustering in phase-space of colour-singlet parton pairs ("preconfinement"). Long-range correlations are strongly suppressed. Hadronization will only act locally, on low-mass colour-singlet clusters.





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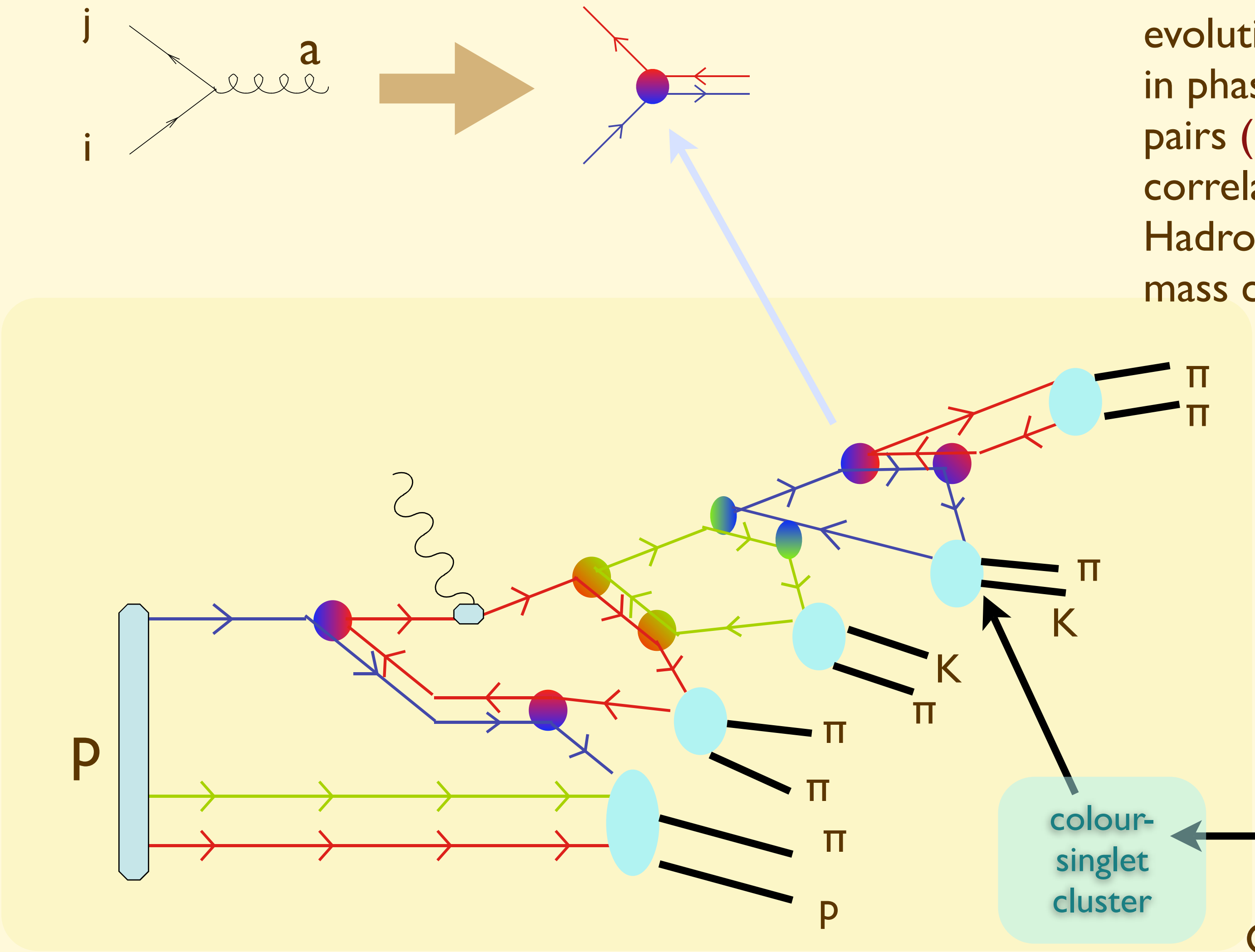

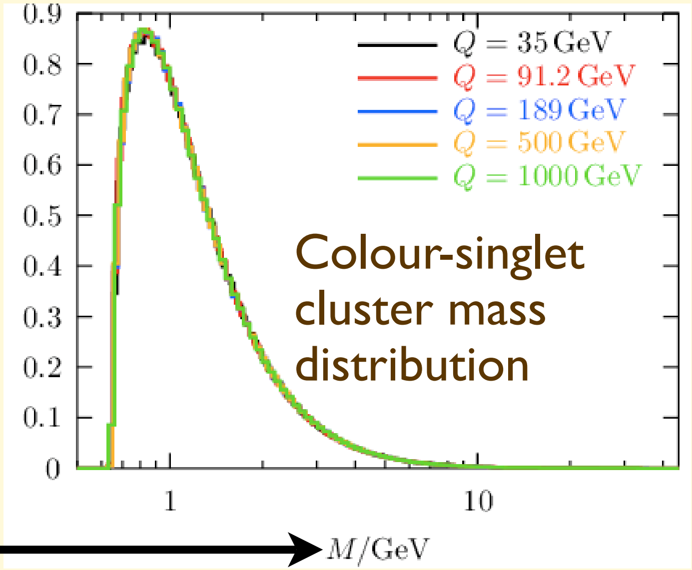

Colour-singlet cluster mass distribution

Colour is left "behind" by the struck quark. The first soft gluon emitted at large angle will connect to the beam fragments, ensuring that the beam fragments can recombine to form hadrons, and will allow the struck quark to evolve without having to worry about what happens to the proton fragments.

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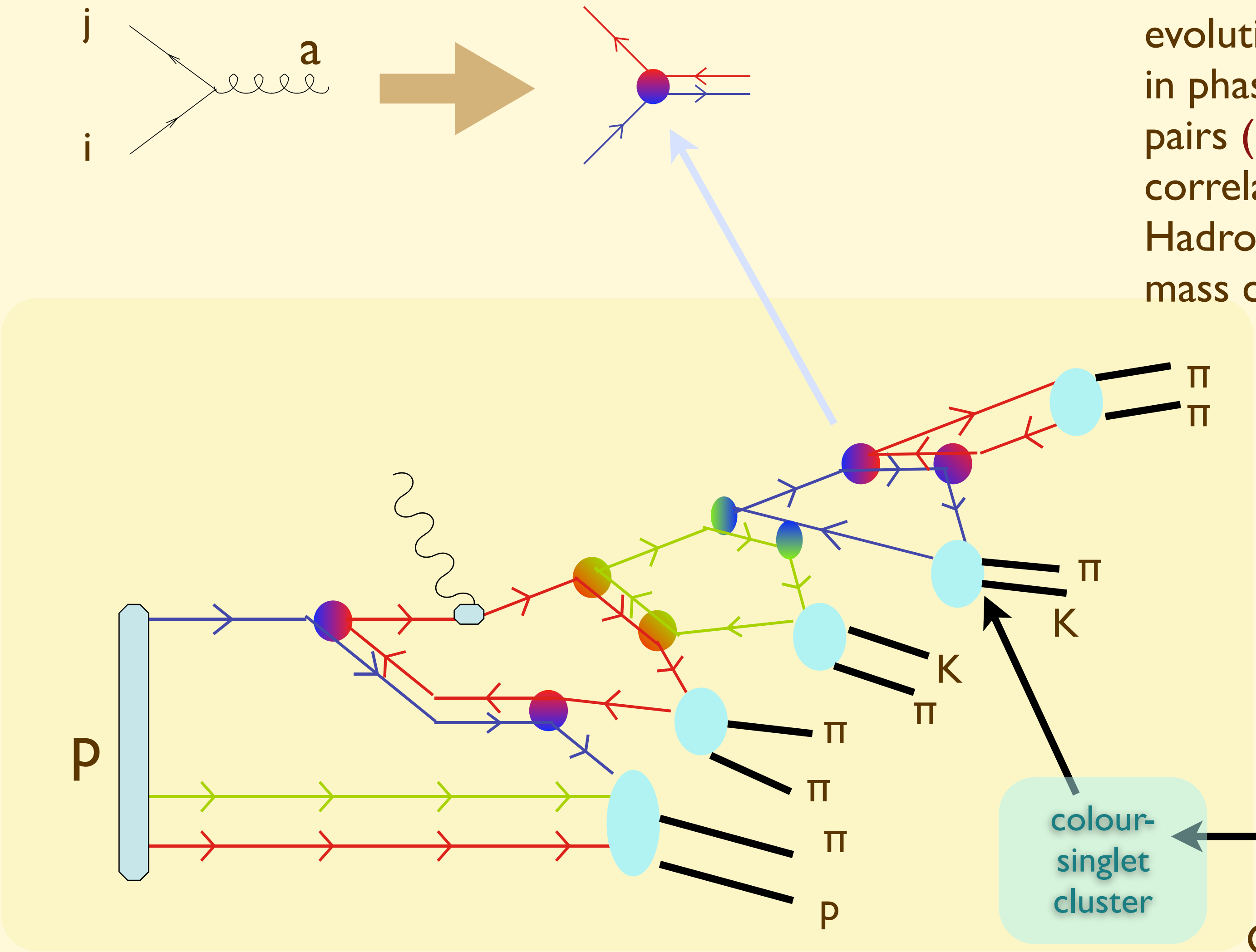

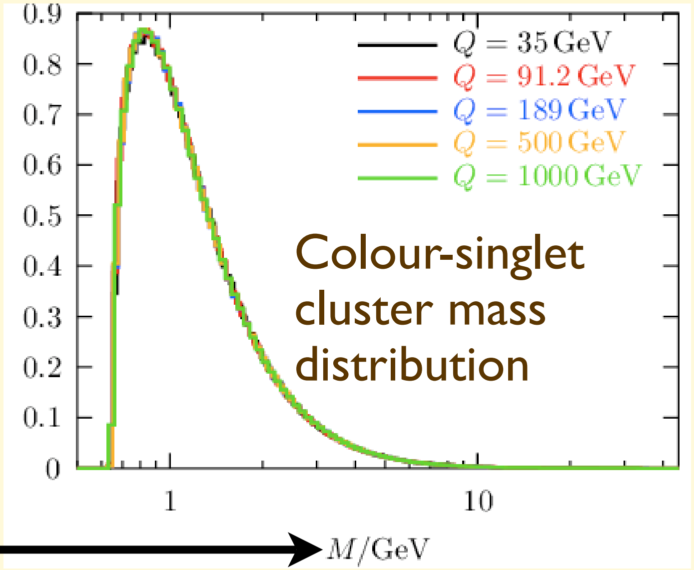

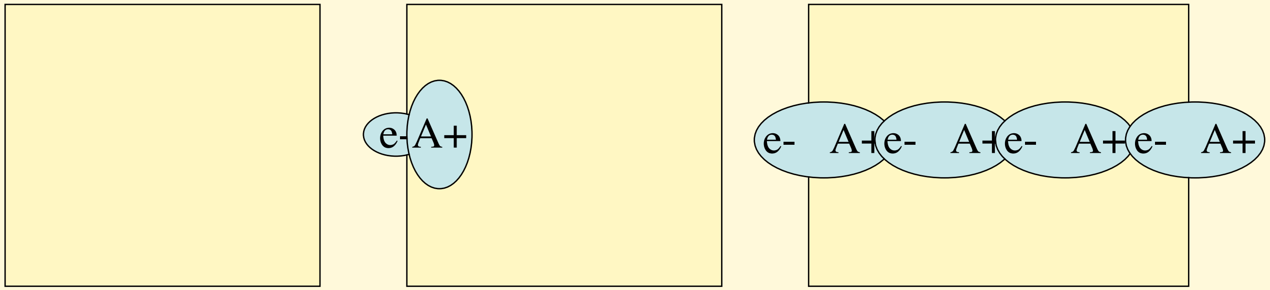

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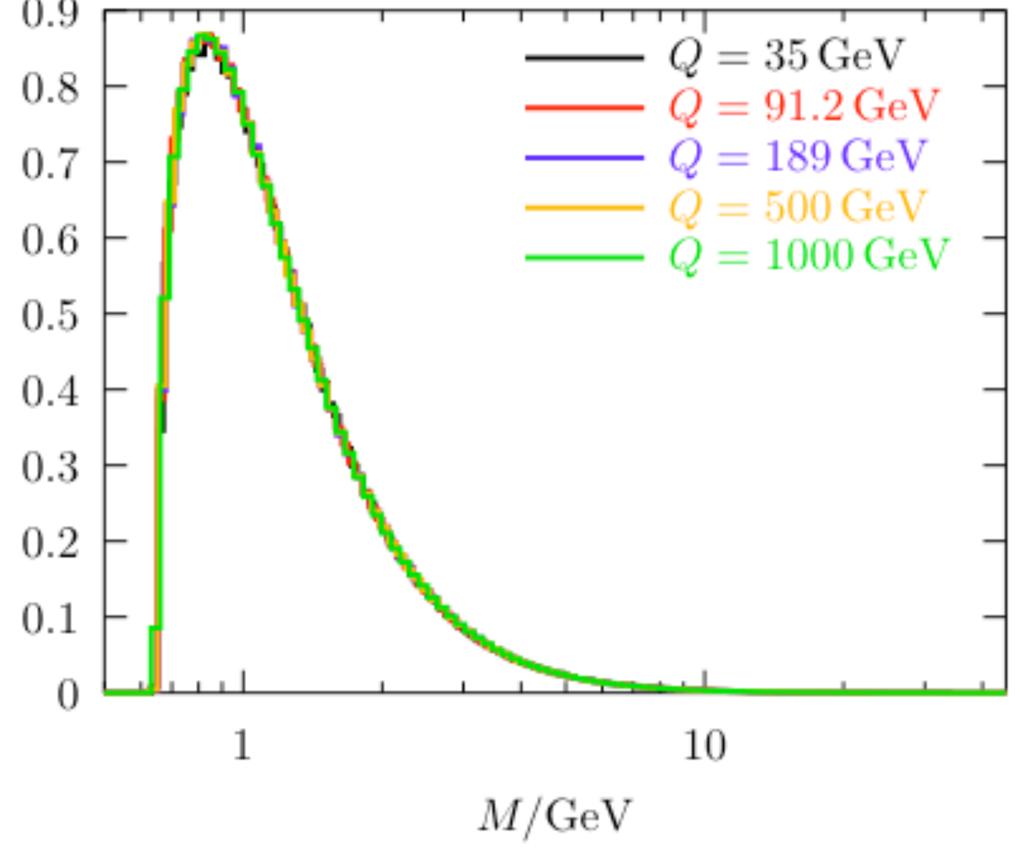
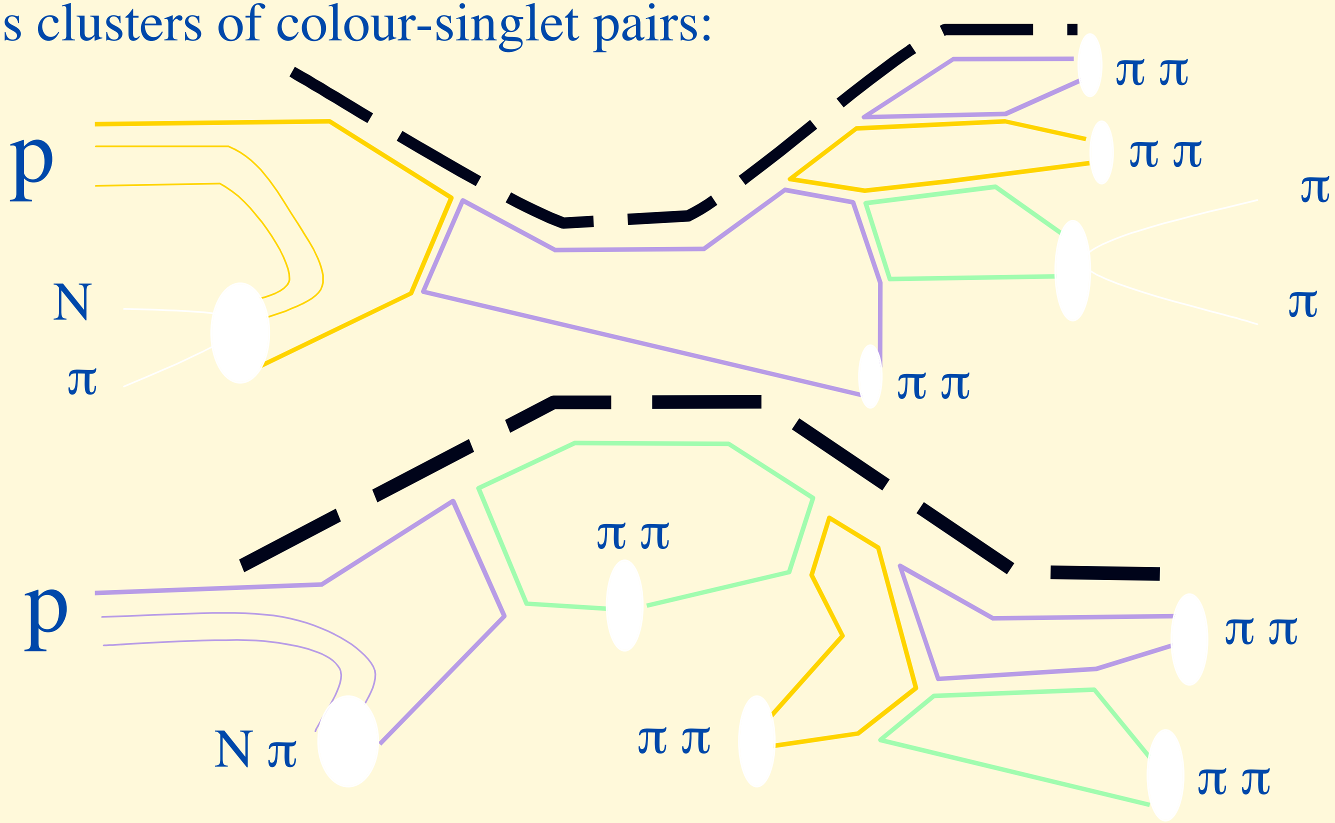


The existence of high-mass clusters, however rare, is unavoidable, due to IR cutoff which leads to a non-zero probability that no emission takes place. This is particularly true for evolution of massive quarks (as in, e.g. Z→bb or cc). Prescriptions have to be defined to deal with the "evolution" of these clusters. **This has an impact on the z→1 behaviour of fragmentation functions.**

Phenomenologically, this leads to uncertainties, for example, in the background rates for H→γγ (jet→γ).

Hadronization

At the end of the perturbative evolution, the final state consists of quarks and gluons, forming, as a result of angular-ordering, low-mass clusters of colour-singlet pairs:



Thanks to the cluster pre-confinement, hadronization is local and independent of the nature of the primary hard process, as well as of the details of how hadronization acts on different clusters. Among other things, one therefore expects:

N(pions) = C N(gluons),
C=constant~2