Argomenti attuali nella fisica dei K

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Outline

- History
  - Fermi interaction
  - $H_{\Delta S=1}$ $H_{\Delta S=2}$
  - $K^0 - \bar{K}^0$ system
- SM-Flavour physics tests - CKM determination
- Rare Kaon decays
- Minimal Flavour violation
- CP violation in $K^+ \to 3\pi$
- Chiral Perturbation theory
- Bell Steinberger relations
- Conclusions

1963, soon after Discovery of Universality in weak interactions

Cabibbo introduces $\cos\theta_C$ to describe β-decays, $n \to pe\nu$, $\mu \to e\nu\bar{\nu}$

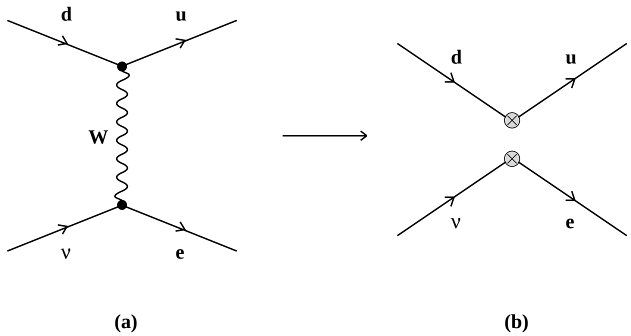
$$\mathcal{L}_F = \frac{G_F}{\sqrt{2}}\cos\theta_C J^\mu J_\mu + h.c. \qquad J_\mu = \bar{p}\gamma^\mu(1-\gamma^5)n + \bar{e}\gamma^\mu(1-\gamma^5)\nu$$

$$\frac{\sqrt{2}g^2}{8\ M_W^2} \sim 1.6637(1)\cdot 10^{-5}\mathrm{GeV}^{-2} \qquad J_\mu = \bar{u}\gamma^\mu(1-\gamma^5)d + \bar{e}\gamma^\mu(1-\gamma^5)\nu$$

decays involving the strange flavour s, the strange mesons $K^+(\bar{s}u),\ K^0(\bar{s}d)$: $K \to \mu\nu,\ K \to \pi l\nu$

$$\mathcal{L}_{\Delta S=1} = \frac{G_F}{\sqrt{2}}\sin\theta_C J^\mu J_\mu + h.c. \qquad J_\mu = \bar{u}\gamma^\mu(1-\gamma^5)s + \bar{e}\gamma^\mu(1-\gamma^5)\nu$$

Charged intermediate vector boson (W^-) propagates among currents J^μ



$$\mathcal{L}_{\Delta S=1} = \frac{G_{\rm F}}{\sqrt{2}} J^\mu J_\mu$$

J^μ charged component of a conserved current in the weak group $SU(2)_L$

$$J^i_\mu = \bar{Q}\gamma^\mu \tau^i Q \qquad Q = \begin{pmatrix} u_L \\ d_L \end{pmatrix}$$

$$Q = \begin{pmatrix} u_L \\ d_L \end{pmatrix} \longrightarrow \begin{pmatrix} u_L \\ \cos\theta_{\rm C} d_L + \sin\theta_{\rm C} s_L \end{pmatrix}$$

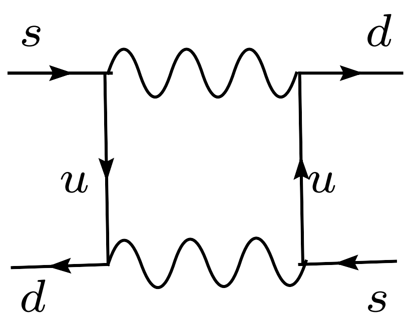
Suppression of FCNC

$K^0 \quad \bar{K}^0 \longrightarrow \begin{array}{c}\pi^+\pi^- \\ \pi^0\pi^0\end{array}$, 3π from W-exchange of hadronic currents

$$\mathcal{L}_{\Delta S=1} = \frac{G_{\rm F}}{\sqrt{2}} J^\mu J_\mu = \qquad \frac{G_{\rm F}}{\sqrt{2}} \quad \sin\theta_{\rm C}\cos\theta_{\rm C} \quad \bar{u}\gamma^\mu(1-\gamma^5)d \quad \bar{s}\gamma^\mu(1-\gamma^5)u$$

Phenomenological problems

- $\mathcal{O}(G_F)$ $A(K_L \to \mu^+\mu^-)$ from $\sin\theta_{\rm C}\cos\theta_{\rm C}\; Z_0^\mu\; \bar{s}\gamma^\mu(1-\gamma^5)d$



- $$\mathcal{L}_{\Delta S=2} = \frac{G_{\rm F}^2 M_W^2 (\sin\theta_{\rm C}\cos\theta_{\rm C})^2}{16\pi^2} \times$$

$$\bar{s}\gamma^\mu(1-\gamma^5)d \; \bar{s}\gamma_\mu(1-\gamma^5)d$$

phenomenology $\Longrightarrow \sim 10^{-4}$ suppression

CP violation in the K's

$$K^0 \quad \bar{K}^0 \stackrel{\mathcal{L}_{\Delta S=1}}{\longrightarrow} \begin{array}{c} \pi^+\pi^- \\ \pi^0\pi^0 \end{array}, 3\pi \qquad CP \mid \pi\pi\rangle = + \mid \pi\pi\rangle, \quad CP \mid 3\pi\rangle = - \mid 3\pi\rangle$$

Mismatch width versus mass eigenststates: $\langle \bar{K}^0 \mid \mathcal{L}_{\Delta S=2} \mid K^0\rangle \sim 10^{-15}\ M_K$

$$\text{Diagonalize} \qquad \begin{pmatrix} M_{11} - i\Gamma_{11}/2 & M_{12} - i\Gamma_{12}/2 \\ M_{12}^* - i\Gamma_{12}^*/2 & M_{11} - i\Gamma_{11}/2 \end{pmatrix} \qquad \begin{pmatrix} K^0 \\ \bar{K}^0 \end{pmatrix}$$

$$\Gamma_{12} = \sum_f \rho_f \langle f \mid \mathcal{L}_{\Delta S=1} \mid \bar{K}^0\rangle^* \langle f \mid \mathcal{L}_{\Delta S=1} \mid K^0\rangle$$

If $M_{ij},\ \Gamma_{ij}$ real , $K_{1,2} = \dfrac{K^0 \pm \bar{K}^0}{\sqrt{2}}$ mass-width eigenstate

$$CP \mid K_{1,2}\rangle = \pm \mid K_{1,2}\rangle$$

even if NOT predicted CP was violated in the K's

$$K_{S,L} = \frac{1}{\sqrt{2\left(1+|\epsilon|^2\right)}}\left[(1+\epsilon)\,K^0 \pm (1-\epsilon)\,\bar{K}^0\right]$$

$$\epsilon = \frac{-i\Im\left(M_{12}\right) - \frac{1}{2}\Im\left(\Gamma_{12}\right)}{m_L - m_S + i(\Gamma_S - \Gamma_L)/2}$$

TWO problems: i) large FCNC's and ii) CP violation $\Longrightarrow$ GIM + CKM mechanism

$$\begin{pmatrix} u_L \\ \cos\theta_C d_L + \sin\theta_C s_L \end{pmatrix} + \begin{pmatrix} c_L \\ -\sin\theta_C d_L + \cos\theta_C s_L \end{pmatrix}$$

Standard Model

Mismatch between the mass matrix in flavor space with 3 families

$$U = \begin{pmatrix} u \\ c \\ t \end{pmatrix}, \qquad D = \begin{pmatrix} d \\ s \\ b \end{pmatrix}, \qquad L = \begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix},$$

$$\mathcal{L}_{SM}^{Y} \quad = \quad \bar{Q} Y_D D_R H + \bar{Q} Y_U U_R H_c + \bar{L} Y_E E_R H \ + \ \text{h.c.}$$

and weak interactions generate: Q left-handed doublets, U_R, D_R,singlets

$$\frac{g}{2\sqrt{2}} \left[\bar{U} V_{CKM} \gamma^{\mu} \left(1 - \gamma_5\right) D W_{\mu}^{+} \right]$$

V_{CKM} unitarity : 3 angles +1 phase

$K \to \pi l \nu$ and CKM unitarity

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$$ V_{ub} negligible

- Superallowed transitions $\Longrightarrow |V_{ud}| = 0.9740 \pm 0.0005 \overset{\text{Unit.}}{\Longrightarrow}$

$$|V_{us}|^{\text{Unit.}} = \quad 0.2265 \pm 0.0022$$

$$|V_{us}|^{\text{PDG04}} = 0.2196 \pm 0.0026$$ Leutwyler,Roos

$$|V_{us}|^{\text{KLOE}} = \quad 0.2257 \pm 0.0021$$ (for PDG06)

Hyperons : more theoretical work needed Cabibbo *et al*

News expected from pion and nucleon beta decay

Standard Model FCNC

- SM with 3 families $\Longrightarrow$ weak int. with an unit. mat. V_{ij}: 3 angles and 1 phase (CPV)

$$\overbrace{V_{ud}, V_{cb}, V_{td}}^{\text{Wolfenstein}} \Longrightarrow \lambda,\, A\lambda^2,\, A\lambda^3(1-\rho-i\eta)$$

- FCNC only at 1-loop

$$\mathcal{H}^{SM}_{\Delta F=2} \sim \frac{G_F^2 M_W^2}{16\pi^2}\left[\frac{(V_{td}^* m_t^2 V_{tb})^2}{v^4}(\bar{d}_L\gamma^\mu b_L)^2 + \frac{(V_{td}^* m_t^2 V_{ts})^2}{v^4}(\bar{d}_L\gamma^\mu s_L)^2\right] + \text{charm}$$

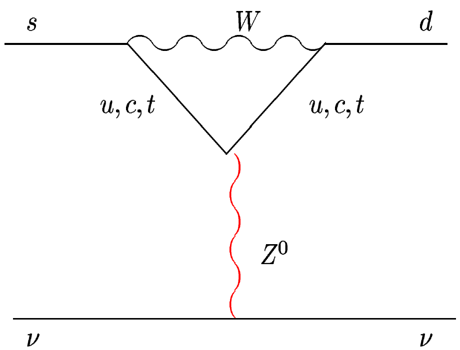
- The area of all possible CKM-unitarity triangles is an invariant:

$$|J_{CP}| \overset{Wolfenstein}{\simeq} A^2\lambda^6\eta$$

- As we shall see $B(K_L \to \pi^0\nu\overline{\nu})$ will measure this area

$K \to \pi\nu\overline{\nu}$

$$A(s \to d\nu\overline{\nu})_{\rm SM} \sim \overline{s}_L\gamma_\mu d_L \quad \overline{\nu}_L\gamma^\mu\nu_L \times \left[\sum_{q=c,t} V^*_{qs}V_{qd}\, m_q^2\right]$$



$$\sim \quad \left[A^2\lambda^5\,(1-\rho-i\eta)m_t^2 + \lambda m_c^2\right]$$

SM: $\underbrace{V - A \otimes V - A}_{\Downarrow}$ Littenberg

$$\Gamma(K_L \to \pi^0\nu\overline{\nu}) \quad \begin{cases} \text{CP violating} \\ \Rightarrow \ J = A^2\lambda^6\eta \\ \text{Only } top \end{cases}$$

Theory versus expts.

- K^+: $t >$ charm $\overset{\mathrm{NLO-QCD}}{\Longrightarrow}$ K^+ $\mathcal{O}(5\%)$, K_L $\mathcal{O}(1\%)$
- BR$(K \to \pi\nu\overline{\nu})_{\mathrm{TH}}$ K^+ $(0.8 \pm 0.1) \cdot 10^{-10}$ $K_L : (3.0 \pm 0.6) \cdot 10^{-11}$
- $B(K^+) = (1.5^{+1.3}_{-0.9}) \cdot 10^{-10}$ BNL-E787/E949 P326 at CERN

  $B(K_L) \leq 2.1 \times 10^{-7}$ at $90\% C.L.$ E391 at KEK 10% data
- K_L Model-independent bound, based on $SU(2)$ properties dim-6 operators for $\overline{s}d\overline{\nu}\nu$

  Grossman-Nir

$$B(K_L) \leq \frac{\tau_L}{\tau_+} \times B(K^{\pm})_{\mathrm{E949}} \leq 1.4 \times 10^{-9} \quad \text{at } 90\% C.L.$$

Supersymmetric Flavour Problem

- the SM Yukawa structure

$$\mathcal{L}_{SM}^{Y} \;=\; \bar{Q} Y_D D H + \bar{Q} Y_U U H_c + \bar{L} Y_E E H \;+\; \text{h.c.}$$

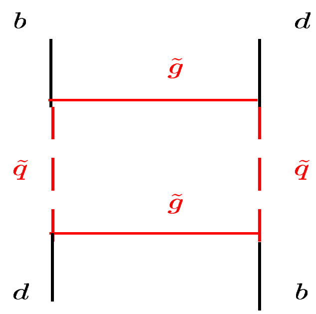
FCNC

$$\mathcal{H}_{\Delta F=2}^{SM} \sim \frac{G_F^2 M_W^2}{16\pi^2} \left[\frac{(V_{td}^* m_t^2 V_{tb})^2}{v^4} (\bar{d}_L \gamma^\mu b_L)^2 + \frac{(V_{td}^* m_t^2 V_{ts})^2}{v^4} (\bar{d}_L \gamma^\mu s_L)^2 \right] + \text{charm}$$

- Supersymmetry must be broken

$$-\mathcal{L}_{soft} = \tilde{Q}^\dagger m_Q^2 \tilde{Q} + \tilde{L}^\dagger m_L^2 \tilde{L} + \bar{\tilde{U}} a_u \tilde{Q}\; H_u + \ldots$$

- $m_Q^2, m_L^2, a_u, \ldots$ matrices in flavour space additional (to $Y_{u,d,l}$) non-trivial structures



$$\mathcal{H}^{\tilde{g}}_{\Delta F=2} \sim \frac{\alpha_s^2}{9M^2_{\tilde{Q}}} \left[(\delta^{LL}_{12})^2 \ (\bar{s}_L \gamma_\mu d_L)^2 + \cdots \right]$$

δ^{LL}_{12} departure from identity matrix m^2_Q

Gabrielli,Masiero,Silvestrini

- $\underset{\Longrightarrow}{K - \bar{K}} \quad \frac{(\delta^{LL}_{12})^2}{M^2_{\tilde{Q}}} \leq \frac{1}{(100 \mathrm{TeV})^2} \Longrightarrow$ Naturalness?

- m^2_Q obey some Flavour symmetry so that GIM is realized ($\sim I$) :

$$\mathcal{L}_{\Delta F=2} = \frac{C}{\Lambda^2_{MFV}} \left[\frac{(V^*_{td} m^2_t V_{tb})^2}{v^4} (\bar{d}_L \gamma^\mu b_L)^2 + \frac{(V^*_{td} m^2_t V_{ts})^2}{v^4} (\bar{d}_L \gamma^\mu s_L)^2 \right]$$

$C \sim 1 \Longrightarrow \Lambda_{MFV} > 5 \mathrm{TeV}$ Ali, Buras

Theory

- There is a symmetry that New Physics must obey to satisfy FCNC -constraints

$$G_F = \overbrace{\mathrm{U}(3)_Q \otimes \mathrm{U}(3)_U \otimes \mathrm{U}(3)_D \otimes \mathrm{U}(3)_L \otimes \mathrm{U}(3)_E}^{\text{global symmetry}} + \overbrace{Y_{U,D,E}}^{\text{spurions}}$$

- Technicolour — Chivukula, Georgi
- Gauge mediation — Dine,Nelson, Shirman; Giudice, Rattazzi

Motivation of a systematic study of Minimal Flavour violation (MFV)

- To write a theory of flavours (valid also beyond the SM), which is potentially consistent with Higgs stabilized $100\mathrm{TeV} \rightarrow 5\mathrm{TeV}$
- We want to know, which scales are we probing in CKM-fit or rare decays in a theory where the Flavor problem has been "solved".
- This should motivate which physics we should look
- We can study also in a systematic and general way, from an effective field theory point of view, the MFV model with two Higgs doublets
- This will allow to uncover new phenomenology (two-loop $\tan\beta$-enhanced) for $B \rightarrow X_s\gamma$, ΔM_B and $B \rightarrow \ell^+\ell^-$

MFV, one Higgs: $\Delta F = 2$

- There is only one operator for $B - \bar{B}$ and $K - \bar{K}$ to add to the SM (CKM fit)

$$\mathcal{L}_{\Delta F=2} = \frac{C}{\Lambda^2_{MFV}}[\frac{(V^*_{td} m_t^2 V_{tb})^2}{v^4}(\bar{d}_L \gamma^\mu b_L)^2 + \frac{(V^*_{td} m_t^2 V_{ts})^2}{v^4}(\bar{d}_L \gamma^\mu s_L)^2]$$

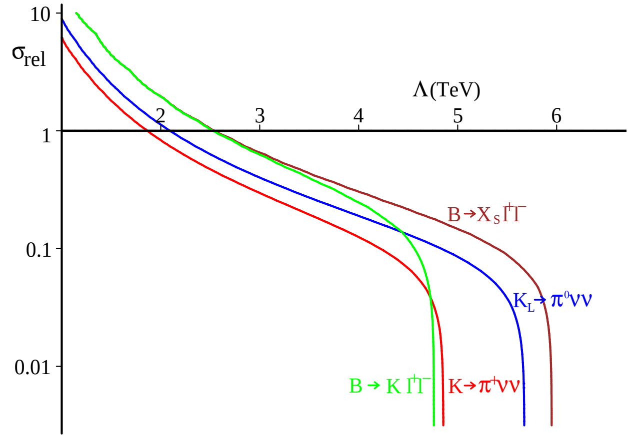
Rare FCNC decays into a lepton pair

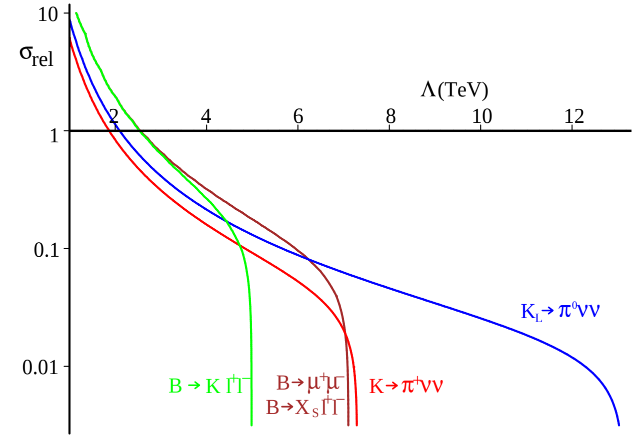
- The SM Wilson coefficients $C_{\nu\bar{\nu}}$, C_{9V} and C_{10A} receive MFV contributions. Limits by measurements $K_L \to \ell^+\ell^-$ and $\mathcal{B}(B \to K\ell^+\ell^-)$
- Actually already non-trivial MFV tests $B \leftrightarrow K$ physics
- $C_{\nu\bar{\nu}}$ limited by the measurements $K^+ \to \pi^+\nu\bar{\nu}$
- $\Rightarrow$ Limits on the MFV-scale for different operators

MFV dim-6 operator	main observables	Λ [TeV] $-$	$+$
$\mathcal{O}_0$	$\epsilon_K, \quad \Delta m_{B_d}$	6.4	5.0
$\mathcal{O}_{F1}$	$B \to X_s\gamma$	8.3	13.4
$\mathcal{O}_{G1}$	$B \to X_s\gamma$	2.3	3.8
$\mathcal{O}_{\ell 1}$	$B \to (X)\ell\bar{\ell}, \quad K \to \pi\nu\bar{\nu}, (\pi)\ell\bar{\ell}$	3.1	2.7
$\mathcal{O}_{\ell 2}$	$B \to (X)\ell\bar{\ell}, K \to \pi\nu\bar{\nu}, (\pi)\ell\bar{\ell}$	3.4	3.0
$\mathcal{O}_{H1}$	$B \to (X)\ell\bar{\ell}, \quad K \to \pi\nu\bar{\nu}, (\pi)\ell\bar{\ell}$	1.6	1.6
$\mathcal{O}_{q5}$	$B \to K\pi, \quad \epsilon'/\epsilon, \ldots$	~ 1	

Bounds on MFV operators

- Future experiments, in particular $K_L \to \pi^0 \nu \bar{\nu}$ will probe deeply MFV





- Left: 10% accuracy on the CKM , Right: 1%

Chiral Perturbation Theory

χPT effective field theory based on the two assumptions

- π's are the Goldstone boson of $SU(3)_L \otimes SU(3)_R \to SU(3)_V$
- *(chiral) power counting* i.e. the theory has a small expansion parameter: $p^2/\ \Lambda^2_{\chi SB}$: $\Lambda_{\chi SB} \sim 4\pi F_\pi \sim 1.2$ GeV

$$\mathcal{L}_{\Delta S=0} = \mathcal{L}^2_{\Delta S=0} + \mathcal{L}^4_{\Delta S=0} + \cdots = \frac{{F_\pi}^2}{4} \overbrace{\langle D_\mu U D^\mu U^\dagger + \chi U^\dagger + U \chi^\dagger \rangle}^{\pi\to l\nu,\ \pi\pi\to\pi\pi,\ K\to\pi..} + \overbrace{\sum_i L_i O_i}^{K\to\pi..} + \cdots$$

Fantastic chiral prediction $A_{\pi\pi} \sim (s - m_\pi^2)/{F_\pi}^2$ Weinberg, Colangelo *et al*

$$\mathcal{L}_{\Delta S=1} = \mathcal{L}^2_{\Delta S=1} + \mathcal{L}^4_{\Delta S=1} + \cdots = G_8 F^4 \underbrace{\langle \lambda_6 D_\mu U^\dagger D^\mu U \rangle}_{K\to 2\pi/3\pi} + \underbrace{G_8 F^2 \sum_i N_i W_i}_{K^+\to\pi^+\gamma\gamma,\, K\to\pi l^+ l^-} + \cdots$$

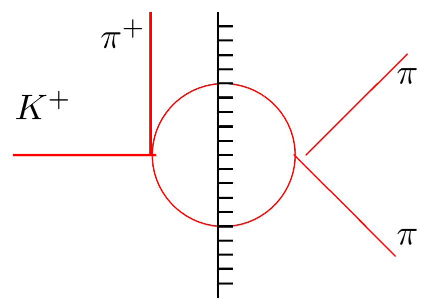
$K \to 3\pi$: **slope asymm.** $\Delta g/2g = (g_+ - g_-)/(g_+ + g_-)$

- Isospin decomposition, rescattering properties

$$\Downarrow$$

- $A(K^+ \to \pi^+\pi^+\pi^-) \quad = \quad a e^{i\alpha_0} + b e^{i\beta_0} Y + \mathcal{O}(Y^2)$

Final State Interaction



Zeldovich,Grinstein et al
Isidori,Maiani,Pugliese

Compared to $K \longrightarrow \pi\pi$

- two $\Delta I = 1/2$ transitions (a, b)
- final state small ($\alpha_0, \beta_0 \sim 0.1$)

- CPT relates

$$\begin{aligned} A(K^+ \to \pi^+\pi^+\pi^-) &= a e^{i\alpha_0} + b e^{i\beta_0} Y \\ A(K^- \to \pi^-\pi^-\pi^+) &= a^* e^{i\alpha_0} + b^* e^{i\beta_0} Y \end{aligned}$$

- The asymmetry

  can be evaluated in CHPT

$$\frac{g_+ - g_-}{g_+ + g_-} = \left[\frac{\Im b}{\Re b} - \frac{\Im a}{\Re a}\right] \sin(\alpha_0 - \beta_0) \,,$$

- Only one operator at $\mathcal{O}(p^2)$:

$$G_8 e^{i\phi} \langle \lambda_6 \partial_\mu U \partial^\mu U^\dagger \rangle$$

ϕ $\Delta I = 1/2$ electroweak phase

- Then if we stop at $\mathcal{O}(p^2)$

$$\frac{\Im b}{\Re b} = \frac{\Im a}{\Re a} \Rightarrow \Delta g / 2g = 0$$

- However $\mathcal{O}(p^4)$ is necessary in order to reproduce the phenomenological values $\frac{\Delta a}{a} \sim \frac{\Delta b}{b} \sim 30\%$

$$\Downarrow$$

- splitting $a = a^{(2)} + a^{(4)}$ and $b = b^{(2)} + b^{(4)}$

$$\frac{\Delta g}{2g} = \frac{\Im A^0}{\Re A^0} (\alpha_0 - \beta_0) \left(\frac{\Re b^{(4)}}{\Re b^{(2)}} - \frac{\Im b^{(4)}}{\Im b^{(2)}} + \frac{\Im a^{(4)}}{\Im a^{(2)}} - \frac{\Re a^{(4)}}{\Re a^{(2)}} \right)$$

G.D.,Isidori,Paver

- Since
  - $|\frac{\Im A^0}{\Re A^0}| \sim 22\epsilon' \sim 10^{-4}$ $(\alpha_0 - \beta_0) \sim 0.1$
  - to maximize Δg, we take $\mathcal{O}(p^4) \sim \mathcal{O}(p^2)$

- $\Delta g/2g \leq 10^{-5}$ NA48 $(1.5 \pm 2.9) \cdot 10^{-4}$

New Physics

- New Physics: crucial to have large $\Delta g/2g$ is to find an operator which affects $K \to 3\pi$ but not $K \to 2\pi$, limited by the experimental size of ϵ'
- In fact Masiero- Murayama explanation of a possible large size of ϵ' does the job!

- They suggest that susy (s-)particles, introduce new flavour structures affecting only the $\Delta S = 1$ and not $\Delta S = 2$ interactions

$$\mathcal{H}_{\rm mag} = C_g^+ Q_g^+ + C_g^- Q_g^- + {\rm h.c.}$$

$$Q_g^\pm = \frac{g}{16\pi^2}\left(\bar{s}_L \sigma^{\mu\nu} t^a G^a_{\mu\nu} d_R \pm \bar{s}_R \sigma^{\mu\nu} t^a G^a_{\mu\nu} d_L\right)$$

- Q_g^+ is affects only $K \to 3\pi$; Q_g^- only $K \to 2\pi$ G.D,Isidori,Martinelli

- As a result by tuning properly $C_g^\pm$ we can generate large $\Delta g/2g$ $(\leq 10^{-4})$

CPT symmetry

- Hermiticity of the Hamiltonian (probabilty conservation), QFT
- Locality
- Lorentz invariance

$\Longrightarrow$ CPT conservation

CPT violated at the Plank scale

- Quantum gravity may lead to CPT violation
- The low energy limit not known
- Interesting probe

$$|M_K - M_{\bar{K}}| < 10^{-18} M_K$$

CPT violation: Non-locality?

- Non-locality is enough? Barenboim,Lykken

Add to the Dirac lagrangian

$$\mathbf{S} = \frac{i\eta}{\pi} \int d^3x \int dt dt' \, \bar{\psi}(t, \mathbf{x}) \, \frac{1}{t - t'} \, \psi(t', \mathbf{x}).$$

Run into trouble with causality

We want still keep states that go from an initial state to a final state in a S-matrix approach

CPT violation: Break Lorentz invariance

Change the coefficient of the square of the magnetic field in the Lagrangian of quantum electrodynamics:

$$\vec{B}^2 \rightarrow (1+\epsilon)\vec{B}^2$$

This will cause the velocity of light c, given by $c^2 = 1+\epsilon$ to differ from the maximum velocity of particles, which remains equal to one.

$$\mathcal{L}^{eff}_{SU(3)\times SU(2)\times U(1)} = \tfrac{1}{2} i \overline{\psi} \Gamma^{\nu} \overleftrightarrow{\partial}_{\nu} \psi - \overline{\psi} M \psi$$

Spurions break Lorenz sym. Coleman-Glashow,Kostelecky et al.

$$\Gamma^{\nu} \equiv \gamma^{\nu} + c^{\mu\nu}\gamma_{\mu} + d^{\mu\nu}\gamma_5\gamma_{\mu} + e^{\nu} + i f^{\nu}\gamma_5 + \frac{1}{2} g^{\lambda\mu\nu}\sigma_{\lambda\mu}$$

$$M \equiv m + a_\mu \gamma^\mu + b_\mu \gamma_5 \gamma^\mu + \frac{1}{2} H^{\mu\nu} \sigma_{\mu\nu}$$

QM mechanics must be valid even if $C\not{P}T$ in the K's mass matrix

$$i\frac{d}{dt}\left[\begin{array}{c} K^0 \\ \bar{K}^0 \end{array}\right] = [M - i\Gamma/2]\left[\begin{array}{c} K^0 \\ \bar{K}^0 \end{array}\right]$$

For any superposition of K_S, K_L mass and width eigenstates

$$|\Psi\rangle = a|K_S\rangle + b|K_L\rangle$$

$$\sum_\Gamma \mid \langle\Gamma \mid T \mid \Psi\rangle \mid^2 = -\frac{d}{d\tau} \mid \Psi \mid^2$$

Bell Steinberger relations

Terms proportional to $\mid a \mid^2$ and $\mid b \mid^2$

$$\Gamma_L = \sum_\Gamma \int d\Gamma \mid \langle \Gamma \mid T \mid K_L \rangle \mid^2 \qquad \Gamma_S = \sum_\Gamma \int d\Gamma \mid \langle \Gamma \mid T \mid K_S \rangle \mid^2$$

Mixed terms, proportional to $ab^* \Longrightarrow$

$$-i(M_L^* - M_S) \overbrace{\langle K_L \mid K_S \rangle}^{\frac{2\mathrm{Re}(\tilde\epsilon)}{1+|\tilde\epsilon|^2}} = \sum_\Gamma \int d\Gamma \overbrace{(\langle \Gamma \mid T \mid K_L \rangle)^* \langle \Gamma \mid T \mid K_S \rangle}^{\alpha_f} .$$

$$\left(\frac{\Gamma_S + \Gamma_L}{2} - i\Delta m \right) \frac{2\mathrm{Re}(\tilde\epsilon)}{1+ \mid \tilde\epsilon \mid^2} = \sum_\Gamma \int d\Gamma (\langle \Gamma \mid T \mid K_L \rangle)^* \langle \Gamma \mid T \mid K_S \rangle$$

Using the Schwartz inequality

$$\left|\frac{\Gamma_S + \Gamma_L}{2} - i\Delta m\right| \frac{2\mathrm{Re}(\tilde{\epsilon})}{1+ \mid \tilde{\epsilon} \mid^2} \leq \sqrt{\Gamma_L \Gamma_S} \implies \frac{2\mathrm{Re}\tilde{\epsilon}}{1+ \mid \tilde{\epsilon} \mid^2} \leq 2.9 \times 10^{-2}$$

$\implies$ BS relation among $\Gamma_L, \Gamma_S, \Delta m, p, q$ and α_f

BS assume $\langle K_L \mid K_S \rangle = \dfrac{\mid p \mid^2 - \mid q \mid^2}{\mid p \mid^2 + \mid q \mid^2} \overset{BS}{=} \dfrac{2\mathrm{Re}(\tilde{\epsilon})}{1+ \mid \tilde{\epsilon} \mid^2}$

$C\not{P}T$ in the K's mass matrix

Diagonalize $\begin{pmatrix} M_{11} - i\Gamma_{11}/2 & M_{12} - i\Gamma_{12}/2 \\ M_{21} - i\Gamma_{21}/2 & M_{22} - i\Gamma_{22}/2 \end{pmatrix}$

$$K_{S,L} = \frac{1}{\sqrt{2\left(1 + |\epsilon_{S,L}|^2\right)}} \left[\left(1 + \epsilon_{S,L}\right) K^0 + \left(1 - \epsilon_{S,L}\right) \bar{K}^0 \right]$$

$$\begin{aligned} \epsilon_{S,L} &= \frac{-i\Im\left(M_{12}\right) - \frac{1}{2}\Im\left(\Gamma_{12}\right) \mp \frac{1}{2}\left[M_{11} - M_{22} - \frac{i}{2}\left(\Gamma_{11} - \Gamma_{22}\right)\right]}{m_L - m_S + i(\Gamma_S - \Gamma_L)/2} \\ &= \epsilon_M \mp \Delta \end{aligned}$$

$$\epsilon_M \equiv |\epsilon_M| e^{i\varphi_{SW}} \qquad \tan\varphi_{SW} = \frac{2(m_L - m_S)}{\Gamma_S - \Gamma_L}$$

$C\not{P}T$ in semileptonic decays

$$\begin{aligned}
A(K^0 \to l^+ \nu \pi^-) &= a + b = a(1 - y) \\
A(K^0 \to l^- \nu \pi^+) &= c + d = a^*(x_+ - x_-)^* \\
A(\bar{K}^0 \to l^- \nu \pi^+) &= a^* - b^* = a^*(1 + y)^* \\
A(\bar{K}^0 \to l^+ \nu \pi^-) &= c^* - d^* = a(x_+ + x_-)
\end{aligned}$$

$b, d \quad (y, x_-) \qquad C\not{P}\not{T} \qquad\qquad c, d \quad (x_+, x_-) \qquad \Delta S = -\Delta Q$

$$A_{S,L} = \frac{\Gamma^{l^+}_{S,L} - \Gamma^{l^-}_{S,L}}{\Gamma^{l^+}_{S,L} + \Gamma^{l^-}_{S,L}} = 2\Re(\epsilon_{S,L}) + 2\Re\left(\frac{b}{a}\right) \mp 2\Re\left(\frac{d^*}{a}\right)$$

- $A_S - A_L = 4(\Re(\Delta) + \Re(x_-)) \qquad\qquad A_S + A_L = 4(\Re(\epsilon_M) - \Re(y))$

$C\not{P}T$ in $K \to \pi\pi$

$$A(K^0 \to \pi\pi(I)) \equiv (A_I + B_I)e^{i\delta_I}$$
$$A(\bar{K}^0 \to \pi\pi(I)) \equiv (A_I^* - B_I^*)e^{i\delta_I}$$

- B_I is $C\not{P}T$ as $\left(\eta_{+-}=|\eta_{+-}|e^{i\phi_{+-}} \quad \eta_{00}=|\eta_{00}|e^{i\phi_{00}}\right)$

$$\phi_{+-} - \phi_{00} = 0.2 \pm 0.4^o \qquad \text{KTEV,NA48}$$

Bell Steinberger relations: CPLEAR,KTeV,NA48,KLOE

$$\left[\frac{\Gamma_S+\Gamma_L}{\Gamma_S-\Gamma_L}+i\tan\phi_{\rm SW}\right]\left[\frac{\Re(\epsilon)}{1+|\epsilon|^2}-i\Im(\Delta)\right]=\frac{1}{\Gamma_S-\Gamma_L}\sum_f \overbrace{A_L(f)A_S^*(f)}^{(\alpha_f)}$$

Also a new analysis by Gino+KLOE and me

Actual SM expectations for Bell Steinberger relations, KLOE+Gino

Channel	$B(K_S)$	$B(K_L)$	$10^5\ \alpha_f^{\mathrm{SM}}$
$\pi^+\pi^-(\gamma)$	0.69	2.1×10^{-3}	$110.8 + 105.1i$
$\pi^0\pi^0$	0.31	9.3×10^{-3}	$49.2 + 46.6i$
$\pi^\pm e^\mp \nu$	6.7×10^{-4}	0.39	$0.22 + 0.00i$
$\pi^\pm \mu^\mp \nu$	4.7×10^{-4}	0.27	$0.17 + 0.00i$
$\pi^0\pi^0\pi^0$	1.9×10^{-9}	0.21	$0.06 + 0.06i$
$\pi^+\pi^-\pi^0$	2.7×10^{-7}	0.12	$0.04 + 0.04i$
$\pi^+\pi^-\gamma_{\mathrm{DE}}$	10^{-5}	10^{-5}	< 0.01

α_f determinations for Bell Steinberger relations

$$\alpha_{\pi^+\pi^-} = \Big((1.115 \pm 0.015) + i(1.055 \pm 0.015)\Big) \times 10^{-3}$$
$$\alpha_{\pi^0\pi^0} = \Big((0.489 \pm 0.007) + i(0.468 \pm 0.007)\Big) \times 10^{-3}$$

$\alpha_{\pi\pi\pi}$ from CPLEAR, NA48, KLOE

Time dependent studies: CPLEAR

CPLEAR Study of tagged $K^0(\bar{K^0})$

$$\frac{\Gamma(K^0(t) \to f) - \Gamma(\bar{K^0}(t) \to f)}{\Gamma(K^0(t) \to f) + \Gamma(\bar{K^0}(t) \to f)}$$

$$\left[\Re\left(A_L^f A_S^{f*}\right)\cos(\Delta m t) + \Im\left(A_L^f A_S^{f*}\right)\sin(\Delta m t)\right]$$

$$\Longrightarrow \Delta m, \qquad A(K_{L,S} \to \pi^+\pi^-\pi^0)\,, \qquad A(K_{L,S} \to \pi^0\pi^0\pi^0)\,,$$

$$A(K_{L,S} \to \pi l \nu)$$

CPLEAR determination of semileptonic $\alpha_{\pi l\nu}$

$$\sum_{\pi\ell\nu}\langle\mathcal{A}_L(\pi\ell\nu)\mathcal{A}_S^*(\pi\ell\nu)\rangle = 2\Gamma(K_L\to\pi\ell\nu)\left(\Re(\epsilon)-\Re(y)-i(\Im(x_+)+\Im(\Delta))\right)$$
$$= 2\Gamma(K_L\to\pi\ell\nu)\left((A_S+A_L)/4-i(\Im(x_+)+\Im(\Delta))\right)$$

Time asymm. allow CPLEAR to obtain $\alpha_{\pi l\nu}$ from

	value	Correlation coefficients			
$\Re(\Delta)$	$(3.0\pm 3.4)\times 10^{-4}$	1			
$\Im(\Delta)$	$(-1.5\pm 2.3)\times 10^{-2}$	0.44	1		
$\Re(x_-)$	$(0.2\pm 1.3)\times 10^{-2}$	-0.56	-0.97	1	
$\Im(x_+)$	$(1.2\pm 2.2)\times 10^{-2}$	-0.60	-0.91	0.96	1

KLOE determination of semileptonic $\alpha_{\pi l \nu}$

KLOE adds the measurement of $A_S - A_L = 4[\Re(\delta) + \Re(x_-)] = (-2 \pm 10) \times 10^{-3}$. The results, referred to as the $K_{\ell 3}$ average, are given in

	value	Correlation coefficients				
$\Re(\Delta)$	$(3.4 \pm 2.8) \times 10^{-4}$	1				
$\Im(\Delta)$	$(-1.0 \pm 0.7) \times 10^{-2}$	-0.27	1			
$\Re(x_-)$	$(-0.07 \pm 0.25) \times 10^{-2}$	-0.23	-0.58	1		
$\Im(x_+)$	$(0.8 \pm 0.7) \times 10^{-2}$	-0.35	-0.12	0.57	1	
$A_S + A_L$	$(0.5 \pm 1.0) \times 10^{-2}$	-0.12	-0.62	0.99	0.54	1

Bell-Steinberger determination

Unitarity allows a determination of $\Re(\epsilon)$ not requiring CPT

$$\Re(\epsilon) = (159.6 \pm 1.3) \times 10^{-5}, \qquad \Im(\Delta) = (0.4 \pm 2.1) \times 10^{-5}$$

$$\Delta = \frac{i(m_{K^0} - m_{\overline{K}^0}) + \frac{1}{2}(\Gamma_{K^0} - \Gamma_{\overline{K}^0})}{\Gamma_S - \Gamma_L} \cos\phi_{SW} e^{i\phi_{SW}} [1 + \mathcal{O}(\epsilon)] \, .$$

$$-5.3 \times 10^{-19} \text{ GeV} < m_{K^0} - m_{\overline{K}^0} < 6.3 \times 10^{-19} \text{ GeV} \quad \text{at } 95 \ \% \text{ CL} \, .$$

improving CPLEAR $|m_{K^0} - m_{\overline{K}^0}| < 12.7 \times 10^{-19}$ GeV at 90% CL

Conclusions

- We are seeing the determination of the unitarity of the row of the CKM matrix at 0.2% fighting for small possible NP contributions; HERE we do not even know how large is NP contributions: could be very large..
- Remember CP lesson (not theoretically predicted)
- Scaling argument correct?