STRINGHE, BUCHI NERI ED ENTROPIA

LA RICERCA DEI MICROSTATI PERDUTI

The explanation of black hole entropy in terms of microscopic states is one of the major successes of string theory

Plan of the talk

- Some properties of black holes
- B-H thermodynamics: where are the microstates?
- Black branes in SUGRA
- Black holes in SUGRA
- String theory to the rescue
  the recovery of lost microstates
- Beyond extremality

Napoli, 19/4/07

GENERAL RELATIVITY

Black Hole: region of space rendered by gravity causally irrelevant to all its environment.

Einstein gravity

1915-16 Einstein-Hilbert action

$$S = \frac{1}{16\pi G_4} \int d^4x \sqrt{-g} \left\{ R(g) + \mathcal{L}_{mat} \right\}$$

Eq. of motion

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G_4 T_{\mu\nu}$$

G_4: 4-d Newton constant

1916 Schwarzschild solution

$$ds^2 = -\left(1-\frac{r_s}{r}\right)dt^2 + \left(1-\frac{r_s}{r}\right)^{-1}dr^2 + r^2 d\Omega_2^2$$

Not even light rays can leave the region $r < r_s$

$\Rightarrow$ BLACK HOLE

r_s: Schw. radius

In weak field for the Newton grav. potential:

$$\frac{2V}{c^2} = 1 + g_{00}$$

$r_s = \frac{2MG_4}{c^2}$ $\Rightarrow$ M: mass of B-H

Event horizon — boundary in spacetime between region inaccessible to distant observers and outside world $g_{rr} = \infty$

Schw. metric: 2-sphere at $r = r_s$

Some properties:

- Event horizon: coordinate singularity
- $T_{\mu\nu} = 0 \Rightarrow R_{\mu\nu} = R = 0 \qquad \forall r \neq 0$
- $R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} = 12 \, r_s^2 / r^6$

$\Rightarrow \quad r = 0$ curvature singularity

Birkhoff th.:

Schw. sol. is the unique spherically symmetric sol. of <u>vacuum</u> Ein. eq's.

1916 1918 Reissner-Nordström B-H

Einstein-Maxwell th.

- $$ds^2 = -\left(1 - \frac{2GM}{rc^2} + \frac{GQ^2}{r^2c^4}\right) dt^2 + \left(1 - \frac{2GM}{rc^2} + \frac{GQ^2}{r^2c^4}\right)^{-1} dr^2 + r^2 d\Omega_2^2$$

- $$A_0 = \frac{Q}{r}$$

$\Rightarrow$ 2 horizons $\quad r_{\pm} = M \pm \sqrt{M^2 - Q^2}$

r_+ : event horizon

To avoid a naked singularity

$$GM^2 \geq Q^2$$

$(r_+ = r_-)$ EXTREMAL CASE $GM^2 = Q^2$

$$ds^2_{ext} = -\left(1 - \frac{\sqrt{G}\,Q}{rc^2}\right)^2 dt^2 + \left(1 - \frac{\sqrt{G}\,Q}{rc^2}\right)^{-2} dr^2 + r^2 d\Omega_2^2$$

NO FORCE:
$$F_{tot} = -\frac{GM_1M_2}{r^2} + \frac{Q_1Q_2}{r^2} = 0$$

Isotropic coordinates:

$$\rho = r - \sqrt{G}\,Q/c^2 = r - r_H$$

(covers only the "external" region)

$$ds^2 = -H(\rho)^{-2} dt^2 + H(\rho)^2 \left(d\rho^2 + \rho^2 d\Omega_2^2\right)$$

$$A_0 = \frac{Q}{\rho + r_H} = \frac{c^2}{\sqrt{G}}\left(1 - H(\rho)^{-1}\right)$$

where $H(\rho)$ is harmonic : $\partial_i \partial^i H = 0$

$$H(\rho) = 1 + \frac{r_H}{r}$$

Multi B-H solution

Only requirement on $H(\vec{x})$: be harmonic

$$H(\vec{x}) = 1 + \sum_{i=1}^{N} \frac{r_H^i}{|\vec{x} - \vec{x}_i|}$$

$$ds^2 = -H(\vec{x})^{-2} dt^2 + H(\vec{x})^2 d\vec{x}\cdot d\vec{x}$$

metric of N extreme B-H of radii r_H^i in static equilibrium

- The "points" $\vec{x}_i$ are really 2-spheres

- Distance of a point at r_0 from the horizon:

$$\Delta = \int_0^{r_0} \sqrt{g_{rr}}\, dr = \infty$$

B-H THERMODYNAMICS

"No hair" th.

A B-H is completely specified by

M, J and Q

$\Rightarrow$ single state object

In particular: (Gauss law)

$$Q = \frac{1}{4\pi} \int_{S^2} {*F}$$

Charge cons.: the charge of a B-H is that of the objects that collapsed in it.

Wheeler paradox ('60)

if a B-H had no entropy, the fall of a body into it would lead to a decrease of total entropy

4-laws of B-H mechanics

Schw. metric

$$ds^2 = -\left(1-\frac{r_H}{r}\right)dt^2 + \left(1-\frac{r_H}{r}\right)^{-1}dr^2 + r^2 d\Omega_2^2$$

or any metric

$$ds^2 = -g_{rr}^{-1}\,dt^2 + g_{rr}\,dr^2 + r^2 d\Omega_2^2$$

such that at horizon g_{rr} has a single pole

$$g_{rr} \sim \frac{\chi}{r-r_+}$$

Introducing the proper distance

$$d\sigma^2 = g_{rr}\,dr^2$$

near horizon geometry:

$$ds^2 \simeq -\sigma^2 d\tau^2 + d\sigma^2 + r_+^2 d\Omega_2^2$$

$$\tau \equiv t/2\chi$$

QUANTUM MECHANICS

QUANTUM DYNAMIC of MATTER FIELDS

in a *fixed* geometrical bckgd

(i.e. no dynamical gravit. field)

Path integral $\Rightarrow$ euclidean time $\omega = i\tau$

Schwarschild bckgd: near horizon metric:

$$ds_E^2 \sim \sigma^2 d\omega^2 + d\sigma^2 + r_H^2 d\Omega_2^2$$

To avoid a conical singularity

$$\omega \equiv \omega + 2\pi$$

Periodic euclidean time $\Rightarrow$ Finite temperature

$$t_E \equiv t_E + \beta \Rightarrow \kappa T = \hbar / \beta$$

Black Hole temperature : $\kappa T_H = \dfrac{\hbar c}{4\pi \chi}$

$\hbar \Rightarrow$ quantum origin

vacuum fluct.'s of matter fields

Hawking radiation

Schwarz. B-H

$$k_B T_H = \frac{\hbar c^3}{8\pi G_4 M} \qquad \left(T_H \sim 2\cdot 10^{-2} \frac{M_T}{M} \ {}^\circ K\right)$$

Reiss.-Nord. B-H $\left(r_\pm = M \pm \sqrt{M^2 - Q^2}\right)$

$$k T_{RN} = \frac{\hbar c}{4\pi} \frac{r_+ - r_-}{r_+^2}$$

Extremal case $r_+ = r_- \Leftrightarrow GM^2 = Q^2$

$$T_{ext} = 0$$

At finite T SUSY is broken

SUSY $\Rightarrow$ EXTREMALITY

Matter fields beyond gravity

BLACK HOLE ENTROPY

$Q=0$

B-H mass = energy

$$dU = T dS \Rightarrow c^2 dM = T_H dS$$

$$\Rightarrow \quad S_{BH} = \frac{4\pi k G M^2}{\hbar c} = k \frac{A}{4 \ell_p^2}$$

$A = 4\pi r_s^2$ = horizon area

$\ell_p^2 = \hbar G / c^3$ = Planck area

In D space-time dim. $S_{BH} = \frac{A_{D-2}}{4 \ell_p^{D-2}}$

Wheeler paradox solved, but:

- microscopic origin of S_{BH}:
  where are the microstates?
- $S = \eta A \quad \eta = \frac{1}{4}$ universal
- loop quantum gravity $\Rightarrow$ Immirzi parameter
- 1996: Strom. & Vafa solution in string theory

BLACK HOLES in STRING THEORY

String th. might be useful to study B-H since matter & gravity are put togheter in a coherent fashion:

- 1^{st} success of string theory

GRAVITY EXISTS!

in 10-D spacetime $M_n = n/\ell_s$

SOLITONIC (MACROSCOPIC) VIEW

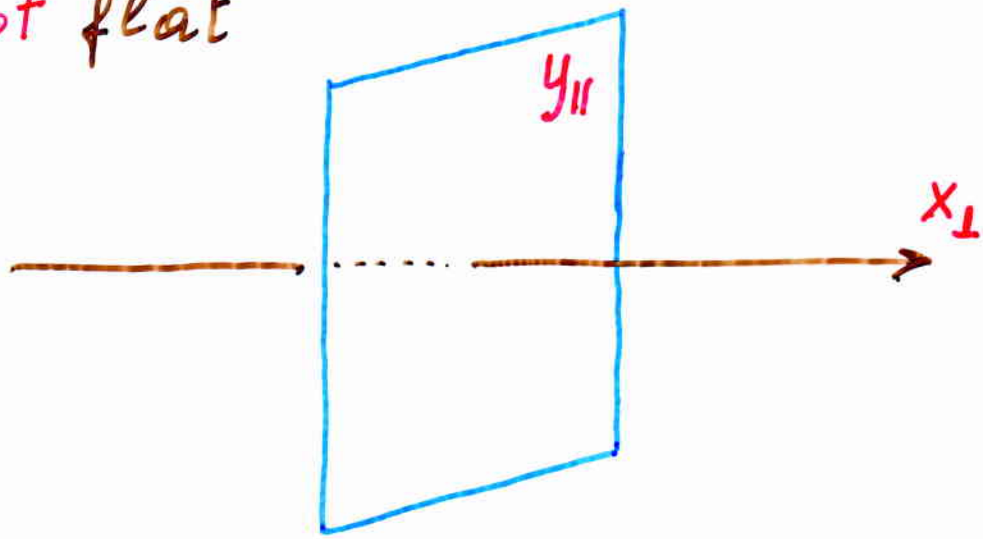
Low energy ($\ell_s \to 0$) effective theory of (type II) string th. 10-D SUGRA

p-branes: classical ($g_s \ll 1$) solutions of SUGRA extended in p space directions

(p spacelike transl. Killing vectors)

$g_s \ll 1$ to suppress string loops

space time
not flat



p-branes: charged under antisym. p+1 form
i.e. couples minimally to a p+1 potential

$$q \int_{W_1} A_\mu dx^\mu \longrightarrow q_p \int_{W_{p+1}} C_{p+1}$$

q_p charge (per unit p-volume) of the brane

Type IIA $\Rightarrow$ p even
Type IIB $\Rightarrow$ p odd

Tension $\parallel$ Charge $\Rightarrow$ SUSY i.e. EXTREMAL sol. (BPS)

SUSY STATIC EXTREMAL p-branes in D=10

Symmetry: $SO(1,p)\otimes SO(9-p)$

ANSATZ:

$$ds_{10}^2 = H(r)^{2a}\eta_{\alpha\beta}\,dy^\alpha dy^\beta + H(r)^{2b}\,d\vec{x}_\perp\cdot d\vec{x}_\perp$$

$$r^2 = \vec{x}_\perp\cdot\vec{x}_\perp$$

String th. (& SUGRA) $\Rightarrow$ dilaton field: ϕ

$$g_s = e^\phi$$

Solution (p<7):

$$ds_{10}^2 = H_p(r)^{-1/2}(-dt^2+dy_{\parallel}^2) + H_p(r)^{1/2}\,dx_\perp^2$$

$$e^\phi = H_p(r)^{(3-p)/4}$$

$$C_{01\cdots p} = C_0 = \frac{1}{g_s}\left\{1 - H_p^{-1}(r)\right\}$$

H_p : harmonic in $\vec{x}_\perp$ $\qquad H_p(r) = 1 + \dfrac{c_p}{r^{7-p}}$

$$c_p = N_p\, g_s\, \ell_s^{7-p}$$

charge quantization

Horizon at $r=0 \Rightarrow$ black p-branes

$(g_{rr}(0) \to \infty)$

Remind of R-N isotropic coordinates:

$$ds^2_{RN} = -\left(1+\frac{r_H}{r}\right)^{-2} dt^2 + \left(1+\frac{r_H}{r}\right)^2 \left(dr^2 + r^2 d\Omega_2^2\right)$$

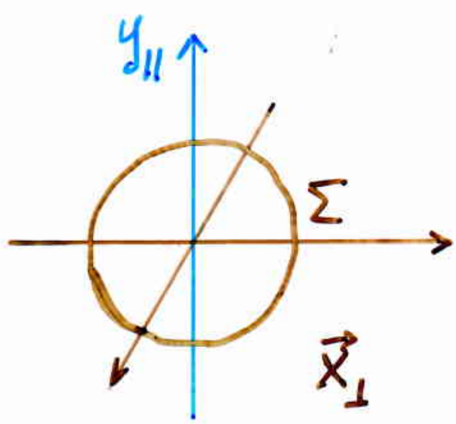
good "outside" the B-H. Horizon $r=0$

Why go at $r<0$?

$$A(S^2) = \int \sqrt{g_{\theta\theta}\, g_{\varphi\varphi}}\, d\theta\, d\varphi$$

$$\xrightarrow[r\to 0]{} 4\pi r_H^2 \neq 0$$

There is an "interior"!



Analogously for a p-brane

$$A(\Sigma) \underset{r\to 0}{\sim} r^{(3-p)(8-p)/4}$$

So:
$$A(\Sigma) \underset{r\to 0}{\sim} r^{(p-3)(8-p)/4}$$

$$g_s^{\text{eff}} = e^{\phi} \underset{r\to 0}{\sim} r^{(p-3)(7-p)/4}$$

- $p < 3$ $\quad g_s^{\text{eff}} \xrightarrow[r\to 0]{} \infty$ $\quad$ Classical sol. unreliable
- $p = 3$
  - Constant dilaton
  - $A(S^5) \xrightarrow[r\to 0]{}$ const. i.e. there is an "inside"

  just as for the R-N black hole
- $p = 4, 5, 6$ $\quad A(\Sigma^{8-p}) \xrightarrow[r\to 0]{} 0$

  $\Rightarrow$ Coordinates are "complete"

  No "interior": horizon at $r=0$

  (null singularity)

No problem: compactify the "parallel" dir.'s $\Rightarrow$ Black hole in 10-p dim.!

NO!

$$g_{\perp}(r) = \left(1 + \frac{c_p}{r^{7-p}}\right)^{1/2} = g_{\parallel}^{-1}$$

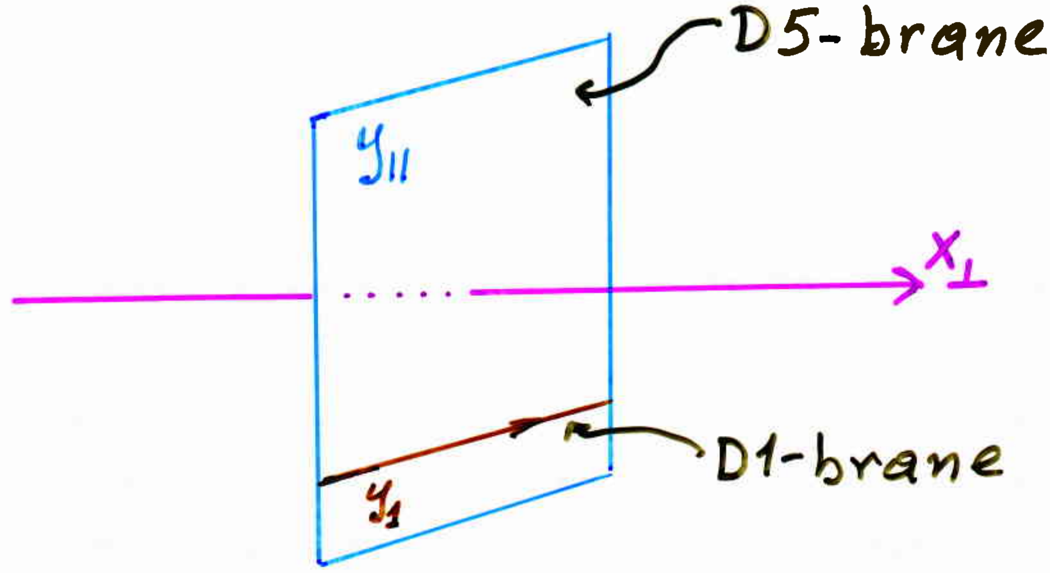
As $r \to 0$

- transverse dir.'s expand $(g_{\perp} \to \infty)$
- "parallel" " shrink $(g_{\parallel} \to 0)$

No matter how big the parallel extension of the brane is at $r \to \infty$ it will vanish at the horizon $(r=0)$ and contains no mass!

Black holes of zero volume.

*



Directions y_2, y_3, y_4, y_5

"parallel" to D5-brane $\Rightarrow$ contracted

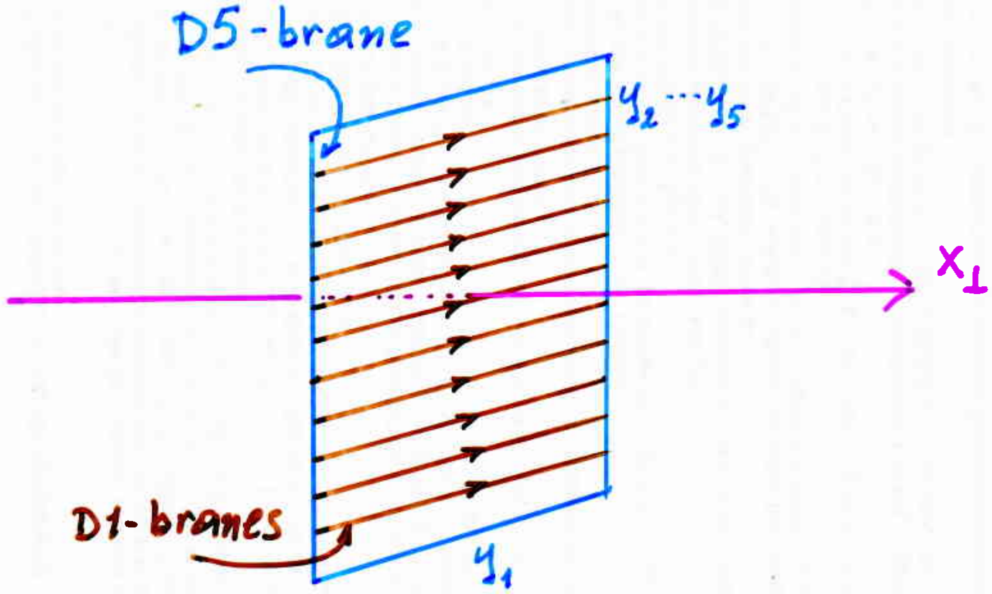
"transverse" to D1-brane $\Rightarrow$ expanded

Different behavior as $r \to 0$, but

Arrays of BPS branes

H_p : harmonic $\Rightarrow$ $H_p(\vec{x}_\perp) = 1 + \sum_i \frac{c_p^i}{|\vec{x}_\perp - \vec{x}_\perp^{\,i}|^{7-p}}$

* No force also among Dp-branes and D(p-4)-branes parallel to the Dp



For a single D1-brane along y_1

$$H_1(r) = 1 + \frac{c_1}{r^6} \qquad r^2 = \sum_{i=2}^{5} y_i^2 + \vec{x}_\perp \cdot \vec{x}_\perp$$

For D1-branes smeared on the D5

$$\hat{H}_1(r) = 1 + \frac{\hat{c}_1}{r^2} \qquad r^2 = \vec{x}_\perp^{\,2}$$

$$\hat{c}_1 = c_1 \, N_1 / V_4 \qquad c_1 = g_s \, \ell_s^6$$

V_4 : asymp. vol. of D5 along $y_2, \cdots, y_5$

N_1 : total n. of D1-branes

BLACK HOLES from BLACK BRANES

Harmonic func.'s rule:

for a superposition of branes the solution is just given by the "superposition" of the harmonic func.'s

E.g. N_5 D5-branes along $y_1, \cdots, y_5$

N_1 smeared D1-branes along y_1

metric:
$$ds_{10}^2 = \left[H_1(r) H_5(r)\right]^{-1/2} \left(-dt^2 + dy_1^2\right) + \left(\frac{H_1}{H_5}\right)^{1/2} dy^2 + \left[H_1 \cdot H_5\right]^{1/2} \left(dr^2 + r^2 d\Omega_3^2\right)$$

dilaton
$$e^{\Phi} = \left(H_1(r) / H_5(r)\right)^{1/2}$$

gauge fields:
$$C_0^{(1)} = \frac{1}{g_s}\left(1 - \frac{1}{H_1(r)}\right) \quad ; \quad C_0^{(5)} = \frac{1}{g_s}\left(1 - \frac{1}{H_5(r)}\right)$$

$$r^2 = \vec{x}_\perp^{\,2} \qquad H_i(r) = 1 + c_i / r^2$$

$$c_1 = g_s N_1 \ell_s^6 / V_4 \qquad c_5 = g_s N_5 \ell_s^2$$

metric:

$$ds_{10}^2 = \left[H_1(r)H_5(r)\right]^{-1/2}(-dt^2 + dy_1^2) +$$

$$+ \left(\frac{H_1(r)}{H_5(r)}\right)^{1/2} dy_{2,\dots,5}^2 \quad +$$

$$+ \left[H_1 \cdot H_5\right]^{1/2}(dr^2 + r^2 d\Omega_3^2)$$

dilaton:

$$e^{\Phi} = \left(H_1(r)/H_5(r)\right)^{1/2}$$

gauge fields:

$$C_0^{(1)} = \frac{1}{g_s}\left(1 - \frac{1}{H_1(r)}\right) \quad ; \quad C_0^{(5)} = \frac{1}{g_s}\left(1 - \frac{1}{H_5(r)}\right)$$

where:

$$r^2 = \vec{x}_\perp^{\,2} \quad ; \quad H_i(r) = 1 + c_i/r^2$$

$$c_1 = g_s N_1 \ell_s^6/V_4 \quad ; \quad c_5 = g_s N_5 \ell_s^2$$

Wrapping $y_2, \dots, y_5$ on T^4

$\Rightarrow$ black string with 2 charges in $d=6$

- Dilaton regular at horizon

$$e^{\phi} \xrightarrow[r \to 0]{} (c_1/c_5)^{1/2} = \left(\frac{N_1 \, \ell_s^4}{N_5 \, V_4} \right)^{1/2}$$

- Volume of T^4:

$$V_4(r) = \frac{1 + c_1/r^2}{1 + c_5/r^2} V_4 \xrightarrow[r \to 0]{} \frac{N_1}{N_5} \ell_s^4 \quad \Big\} \text{independ. of } V_4$$

Attractor mechanism! (S. Ferrara et al. 1995)

- To get a Black Hole: compactify y_1 on a circle of asymptotic length $2\pi R_1$

$\Rightarrow$ near horizon: $L(r) \to 2\pi R_1 \dfrac{r}{(c_1 c_5)^{1/4}} \to 0$

- Event horizon of zero area

$\Rightarrow$ zero entropy

B-H of FINITE MACROSCOPIC ENTROPY

To puff up the horizon add gravit. waves along the D1-branes all in the same direction to preserve SuSy.

Then:

$$ds_5^2 = -(H_1 \cdot H_5 \cdot H_w)^{-2/3} dt^2 + (H_1 \cdot H_5 \cdot H_w)^{1/3} (dr^2 + r^2 d\Omega_3^2)$$

$$H_i = 1 + c_i / r^2$$

$$c_1 = g_s \frac{N_1 \ell_s^6}{V_4} \; ; \quad c_5 = g_s N_5 \ell_s^2 \; ; \quad c_w = g_s^2 N_w \frac{\ell_s^8}{R_1^2 \, V_4}$$

Near horizon:

$$(H_1 \cdot H_5 \cdot H_w)^{1/3} \sim \frac{(c_1 c_5 c_w)^{1/3}}{r^2} \Rightarrow$$ Finite horizon area

Schw. radius: $r_s^2 = (c_1 c_5 c_w)^{1/3} = \left(\frac{g_s^4 \ell_s^{16}}{R_1^2 V_4^2} N_1 N_5 N_w \right)^{1/3}$

B-H mass: $M = \frac{N_w}{R_1} + \frac{N_1 R_1}{g_s \ell_s^2} + \frac{N_5 R_1 V_4}{g_s \ell_s^6}$

B-H entropy:

$$S^{BH}_{macro} = \frac{A(S^3)}{4\,G_5}$$

$$G_5 = g_s^2 \frac{l_s^8}{R_1\,V_4} \Rightarrow$$

$$S^{BH}_{macro} = 2\pi\sqrt{N_1 N_5 N_w}$$

independent of asymptotic moduli.

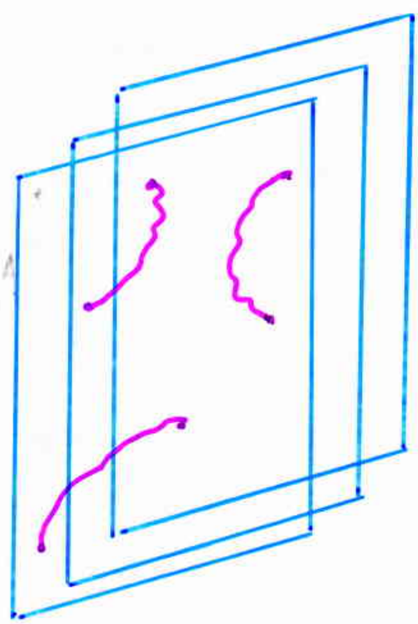
THE RECOVERY of LOST MICROSTATES

Alternative description of D-branes in string theory:

extended surfaces where open strings can end. (Dirichlet b.c.)

In the perturbative regime ($g_s N_i \ll 1$)

low energy d.o.f. : massless states ~~those~~ of the open strings ending on the stack of branes



in flat spacetime.

This is the opposite of the regime in which SUGRA is valid ($g_s N_i \gg 1$)

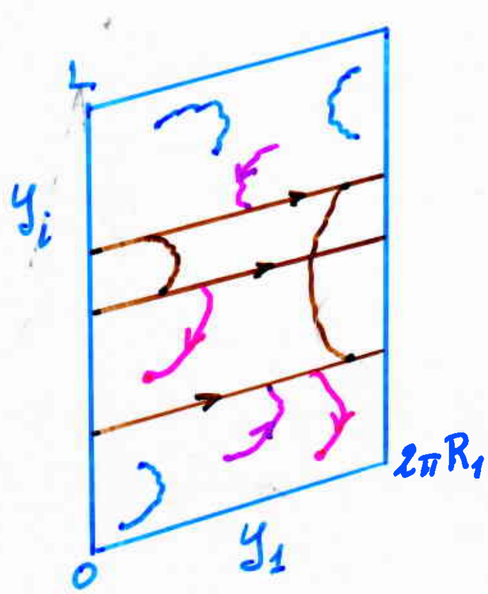
A config. of branes becomes a B-H going from weak to strong coupling

SuSy $\Rightarrow$ n. of d.o.f. unchanged

Let us count this number.

Four kinds of open strings $\quad (5,5) \; ; \; (1,1)$

$\qquad (5,1) \neq (1,5)$



$$y_i = y_i + L$$

$$y_1 = y_1 + 2\pi R_1$$

B-H D-brane config.

N_5 coincident D5-branes + N_1 smeared D1-branes wrapped on $T^4 \otimes S^1$

N_w quanta of light-like ("left-mooving") momentum travelling along the D1-branes.

In how many different ways can one distribute the total momentum among the massless excitations of the system?

Only open strings which start and end on branes ~~coinc.~~ on top of each other can have massless excitations.

Strominger & Vafa (1996)

B-H entropy does not depend on V_4 $\Rightarrow$ take $V_4 \ll l_s^4 \ll R_1^4$ to have the D1-branes on top of each other

⇒ Effective 1+1 dim. superconformal theory of which we know how to count the number of massless d.o.f.

• Massless excit. of (5,5) and (1,1) strings: vector multiplet of SuSy Yang-Mills in the adjoint of $U(N_5)$ and $U(N_1)$

• Massless excit. of (5,1) and (1,5) strings: matter multiplet in the bifundamental rep. $N_5 \otimes \overline{N_1}$ and $N_1 \otimes \overline{N_5}$

But roughly speaking the scalars in the matter multiplets acquire a V.E.V.

⇒ Higgs phase: all vector multiplets become massive

We are left with $2 \cdot N_1 \cdot N_5$ massless states due to (5,1) and (1,5) strings

NO!

"Spinorial" vacuum $\Rightarrow$ $4 \cdot N_1 \cdot N_5$ dof carrying N_W units of quantized energy ($E = cp$, $p = n/R_1$) along the D1-branes.

Dimensional analysis:

For N massless (bose) fields in a volume V_D, for given total energy $\mathcal{E}$, the entropy is:

$$S_D = \kappa \frac{D+1}{D}\left(\eta_D \frac{N V_D}{(hc)^D}\right)^{1/D+1} \mathcal{E}^{\frac{D}{D+1}}$$

where η_D can be easily obtained by Planck formula.

In our case $D = 1$, $\eta_1 = \pi^2/6$

$$\Rightarrow \quad S_{micro} = 2\pi\sqrt{N R_1 \mathcal{E}/6}$$

$$\mathcal{E} = N_w / R_1 \quad , \quad N = 4 \cdot N_1 \cdot N_5$$

SuSy: equal # of fermi fields

$$\Rightarrow \quad N_{eff} = 6 \cdot N_1 \cdot N_5$$

and

$$S_{micro} = 2\pi\sqrt{N_1 N_5 N_w} = S_{macro}$$

CONCLUSIONS

- d.o.f. of B-H due to their brane nature and to extra dim's at micro. level ?

- location and nature of d.o.f unclear: the regime in which we can count # of d.o.f. different from the regime in which we have a B-H

- Attractor mech.: in the extremal ($\neq$ SuSy) case the values of "moduli" at horizon get attracted to values independent of values at infty, and depend only on the quantized charges of the B-H

- Entropy independent of moduli at infty
  $\Rightarrow S_{micro} = S_{macro}$ obtained in several cases

- Beyhond extremality: Hawking rad.