DARK ENERGY AND DARK MATTER AS CURVATURE EFFECTS?

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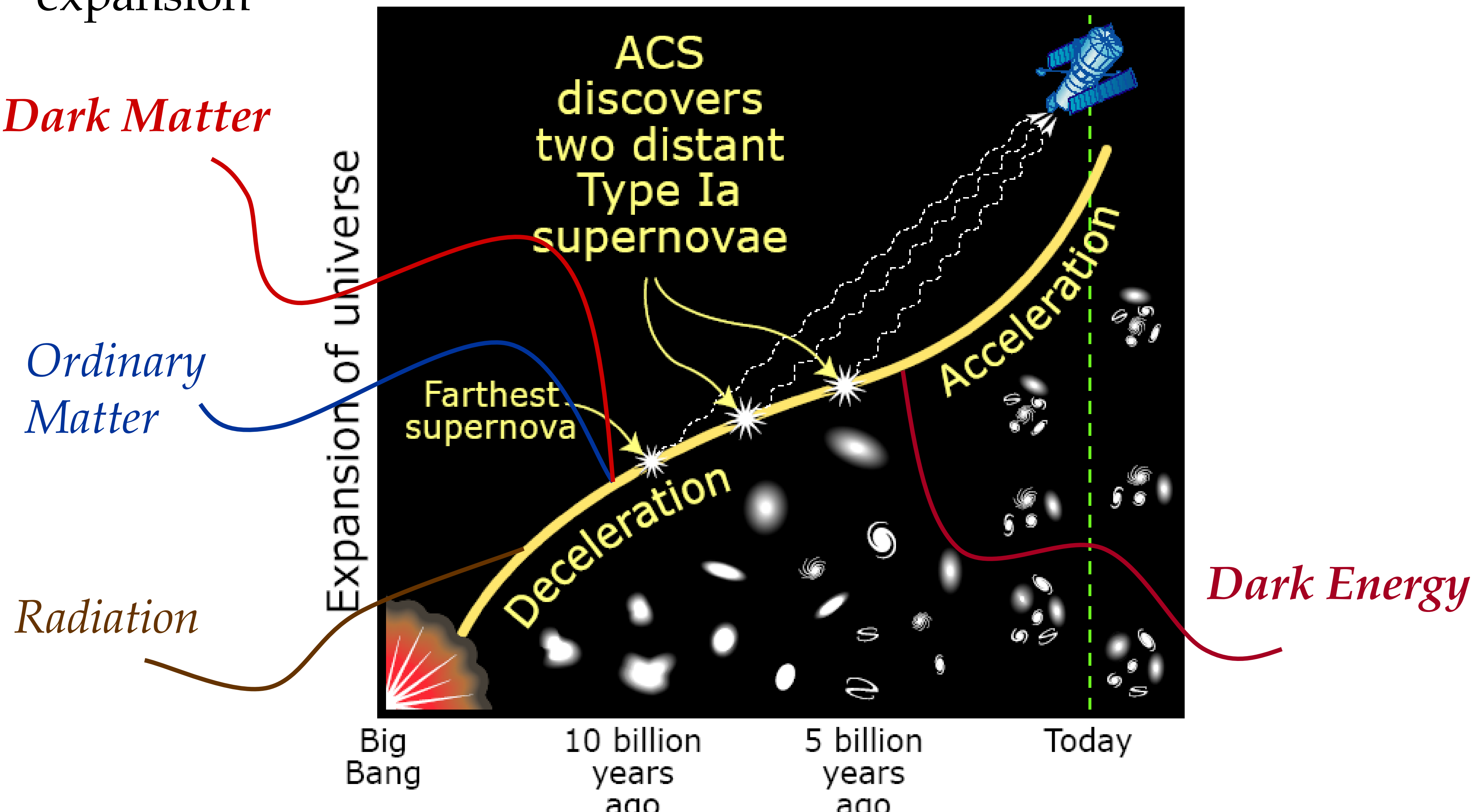
SUMMARY OF THE TALK

- **Dark Energy and Dark Matter from observations**
- **Higher Order Theories of Gravity**
- **Conformal Transformations**
- **Dark Energy as a Curvature Effect**
- **Dark Matter as a Curvature Effect**
- **The PPN limit, Solar System Experiments and VLBI**
- **Conclusions and Perspectives**

DE and DM from the Observations

- Universe evolution is characterized by different phases of expansion

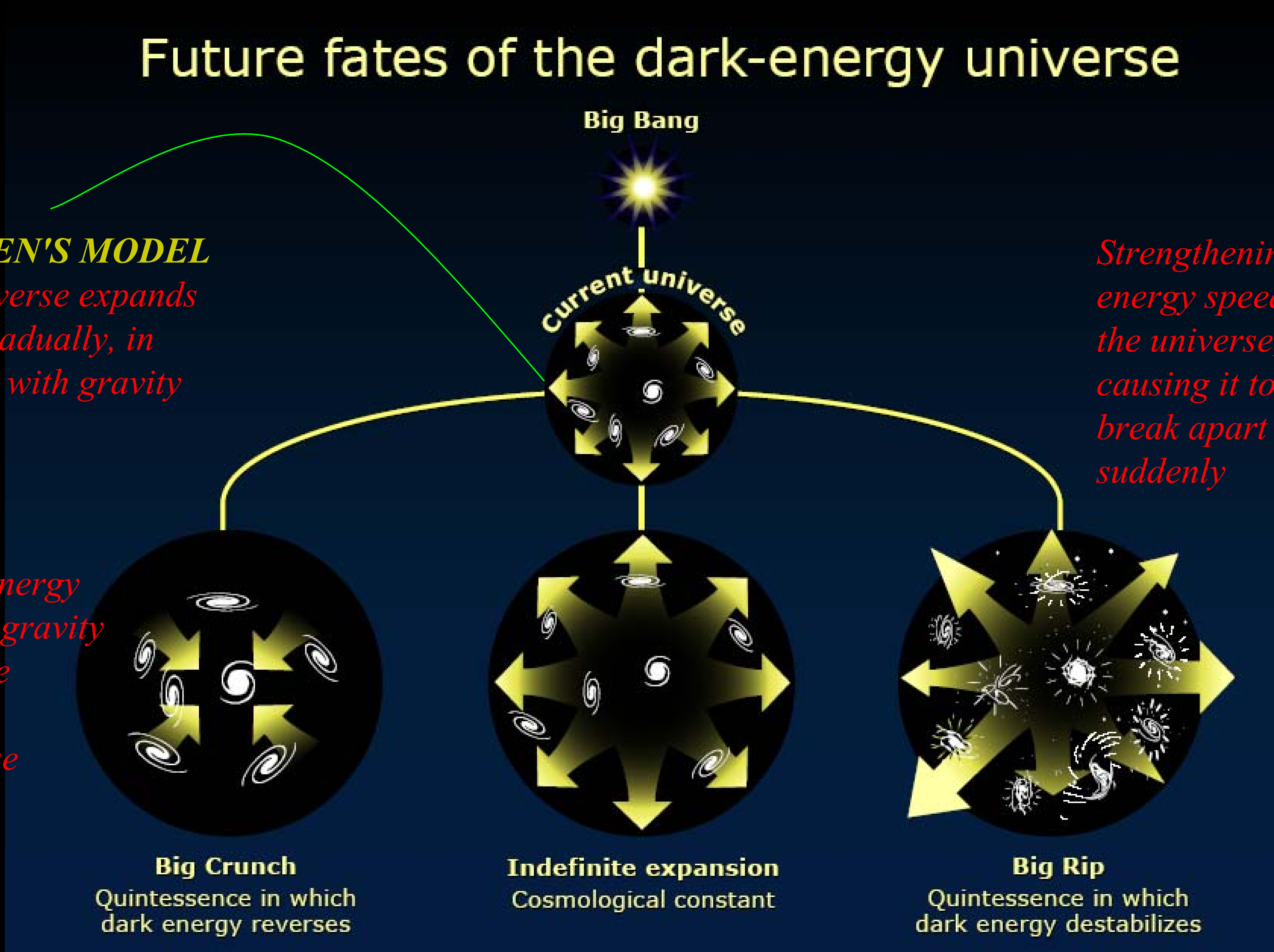


EINSTEN'S MODEL

The universe expands more gradually, in balance with gravity

Strengthening dark energy speeds up the universe, causing it to break apart suddenly

As dark energy weakens, gravity causes the universe to collapse



DE and DM from the Observations

COMPOSITION OF THE UNIVERSE

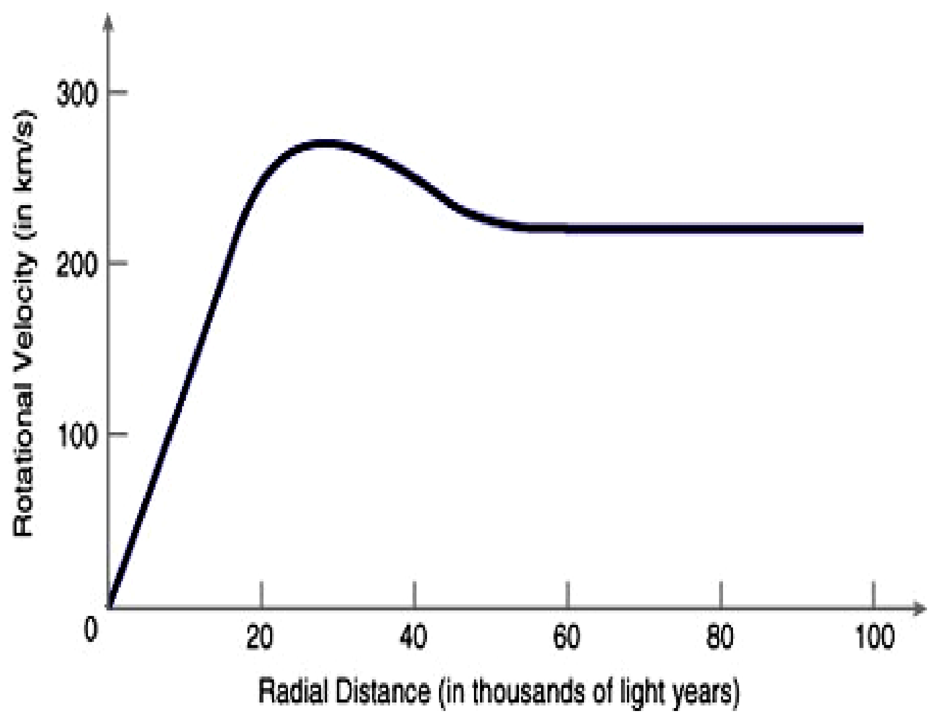
MATERIAL	REPRESENTATIVE PARTICLES	TYPICAL PARTICLE MASS OR ENERGY (ELECTRON VOLTS)	NUMBER OF PARTICLES IN OBSERVED UNIVERSE	PROBABLE CONTRIBUTION TO MASS OF UNIVERSE	SAMPLE EVIDENCE
Ordinary ("baryonic") matter	Protons, electrons	10^6 to 10^9	10^{78}	5%	Direct observation, inference from element abundances
Radiation	Cosmic microwave background photons	10^{-4}	10^{87}	0.005%	Microwave telescope observations
Hot dark matter	Neutrinos	≤ 1	10^{87}	0.3%	Neutrino measurements, inference from cosmic structure
Cold dark matter	Supersymmetric particles?	10^{11}	10[illegible]	25%	Inference from galaxy dynamics
Dark energy	"Scalar" particles?	10^{-33} (assuming dark energy comprises particles)	10[illegible]	70%	Supernova observations of accelerated cosmic expansion

95%

Unknown!!

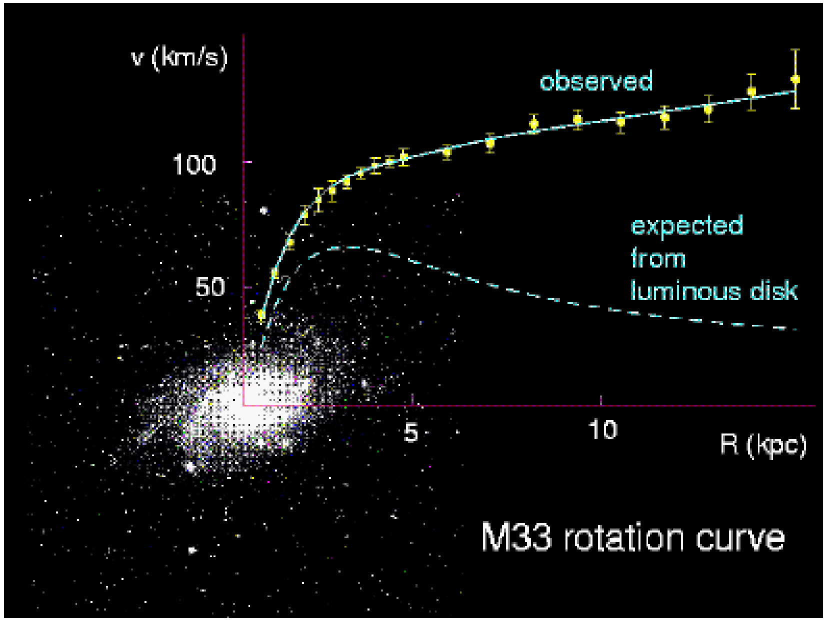
Dark Matter sector

- ✓ The presence of Dark Matter components has been revealed since 1933 by Zwicky as a lack in the mass content of galaxy clusters. The most peculiar effect of Dark Matter is the discover of a non-decaying velocity of rotation curves of galaxies



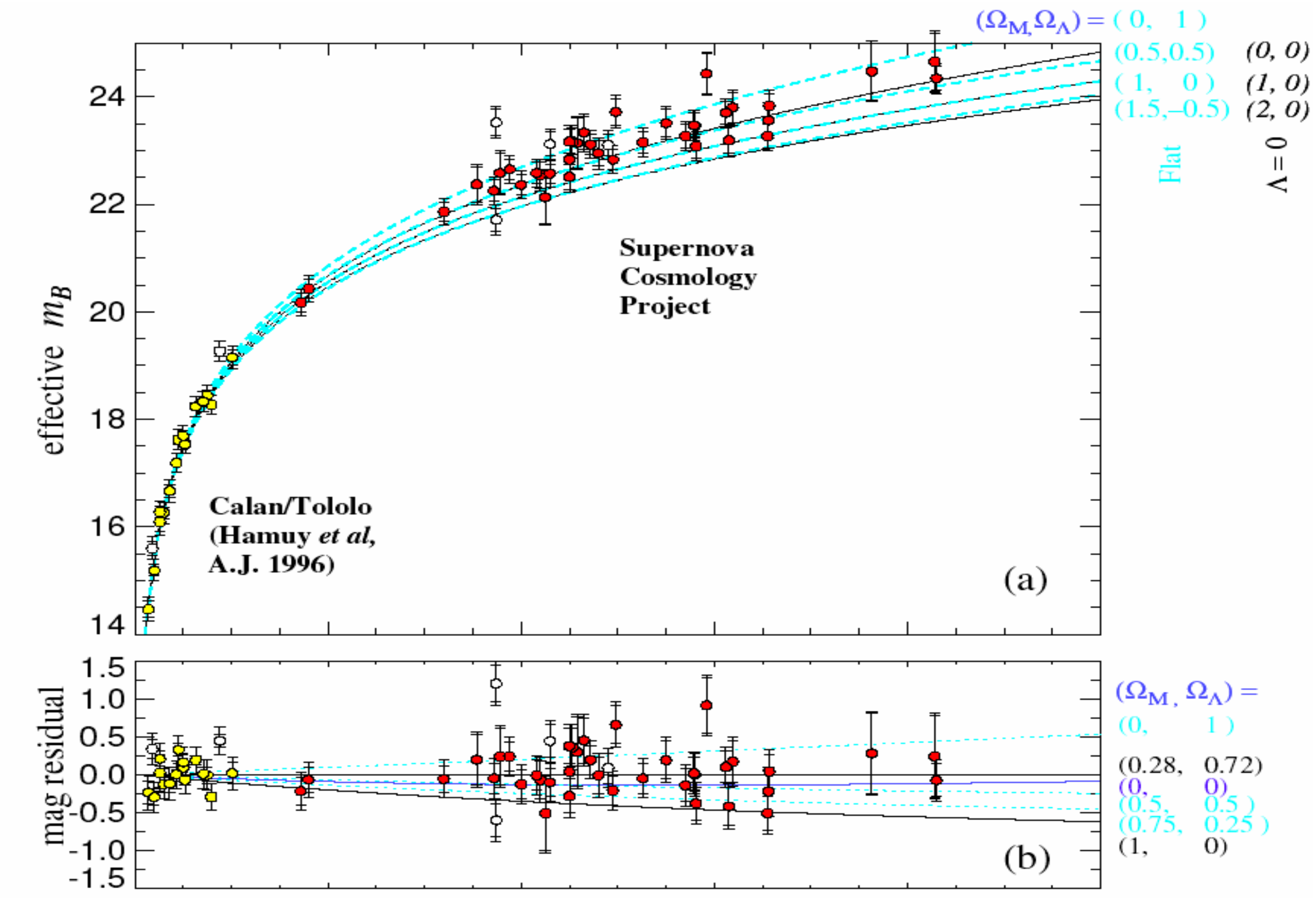
Dark matter differences in clusters and galaxies

An important difference between the distribution of dark matter in galaxies and clusters is that whereas dark matter seems to increase with distance in galaxies, it is just the opposite situation in clusters. Thanks to gravitational lensing effects, it is possible to estimate that the most of dark matter in clusters should be concentrated in the central regions (0.2-0.4 Mpc).



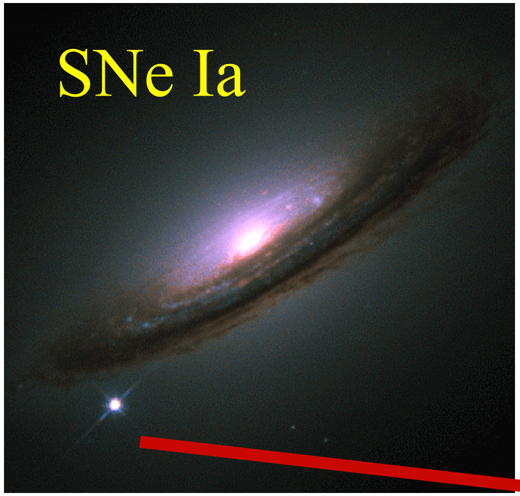
Dark Energy sector

- ✓ The presence of a Dark Energy component has been proposed after the results of SNeIa observations (HZT [Riess et al. (1998)]-SCP [Perlmutter et al. (1998)] collaborations).

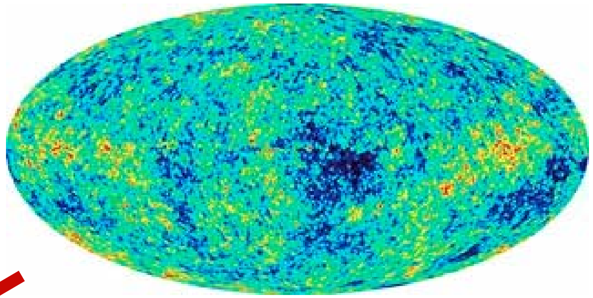


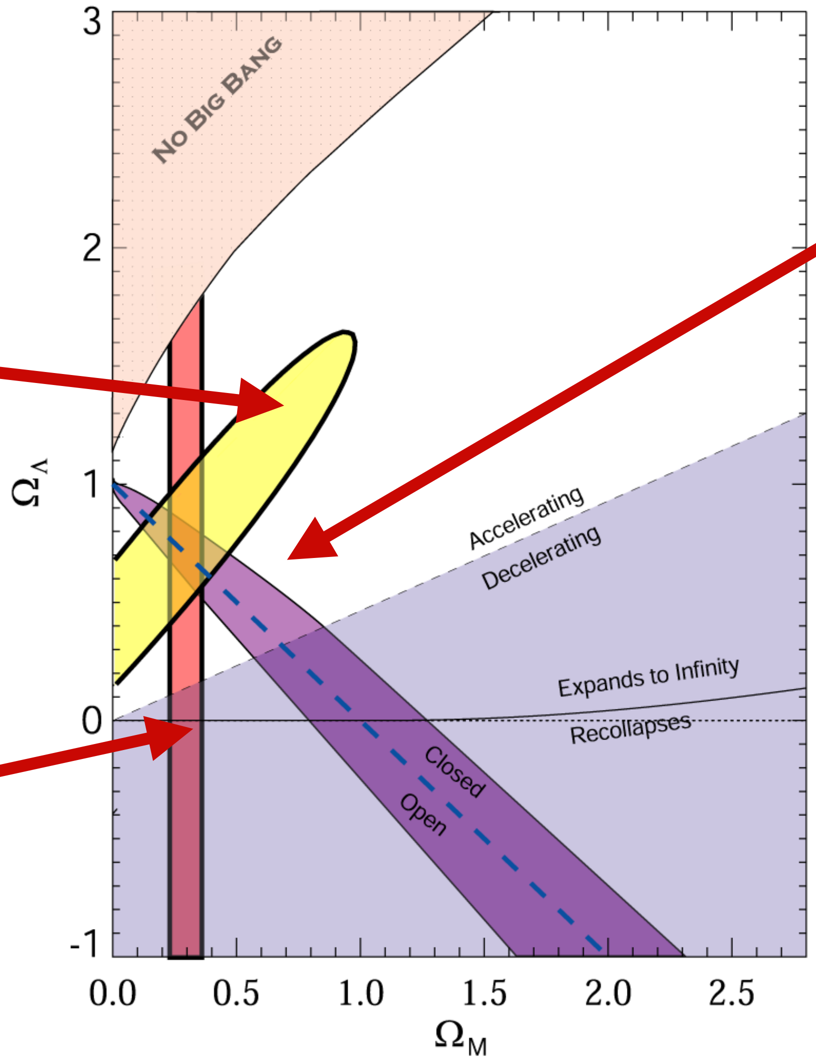
Main observational evidences for Dark Energy

After 1998, more and more data have been obtained confirming this result.

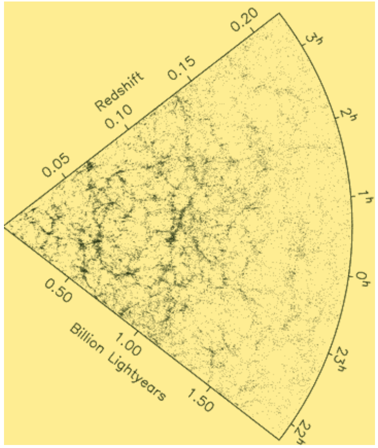


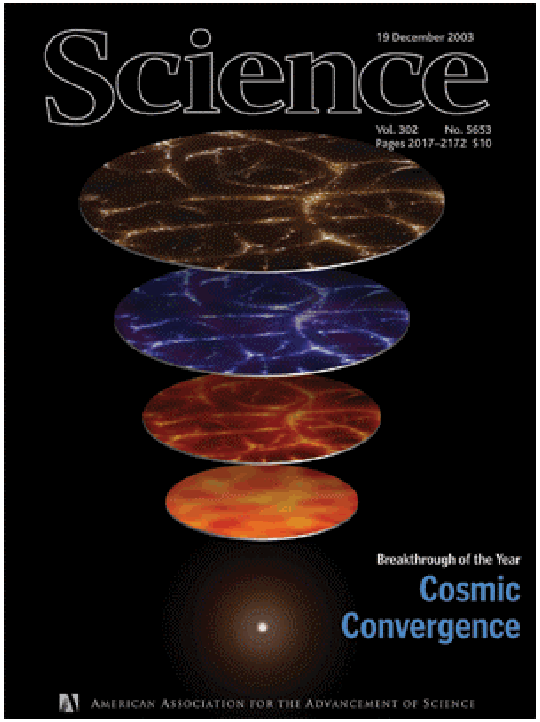
CMB(WMAP)





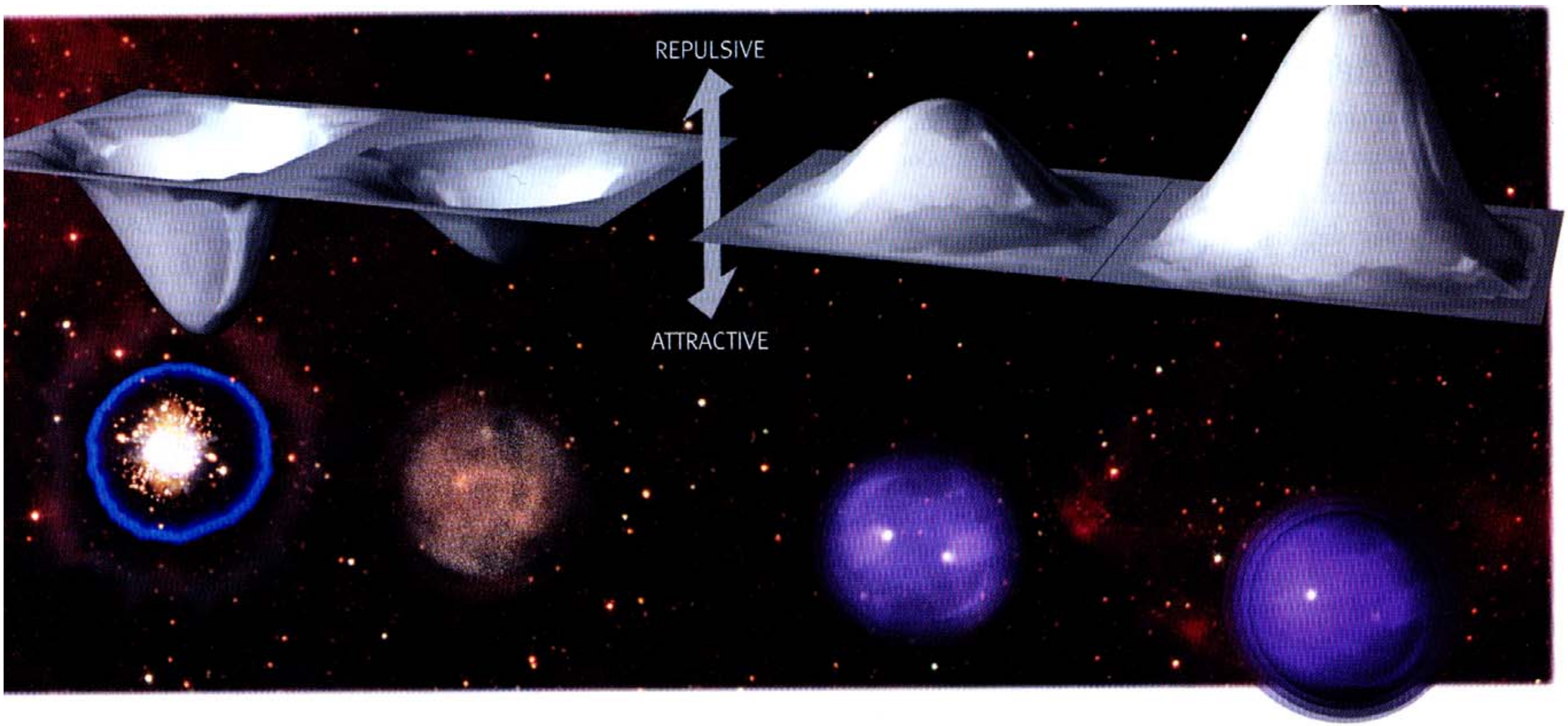
LSS





Dark Energy and *w*

In GR, force ∝ (ρ + 3p)



RADIATION

ORDINARY MATTER

QUINTESSENCE (MODERATELY NEGATIVE PRESSURE)

QUINTESSENCE (HIGHLY NEGATIVE PRESSURE)

(mini-inflation)

Cosmological Constant (vacuum)

$w = p/\rho = +1/3$ 0 $-1 < w < -1/3$ -1

If w < -1/3 the Universe accelerates, w< -1, phantom fields

Physical Effects of DE and DM

DE and DM affect the expansion rate of the Universe:

$$H^2 = \frac{8\pi G}{3}(\rho_M + \rho_X)$$

$$H(z)^2 = H_0^2 \Big[\Omega_M (1+z)^3 + \Omega_X \exp[3 \int_0^z (1+w(x)) d\ln(1+x)]\Big]$$

DE and DM may also interact: long-range forces, new laws of gravity?

Key Issues

- Are there Dark Energy and Dark Matter?

  Will the SNe and other results hold up?

- What is the nature of the DE and DM?

  Is it Λ, supersymmetric particles, or something else?

- How do $w = p_X/\rho_X$ and DM evolve?

  DE and DM dynamics, Λ theory, exotic particles..

.....Resume.....

A plethora of theoretical answers!

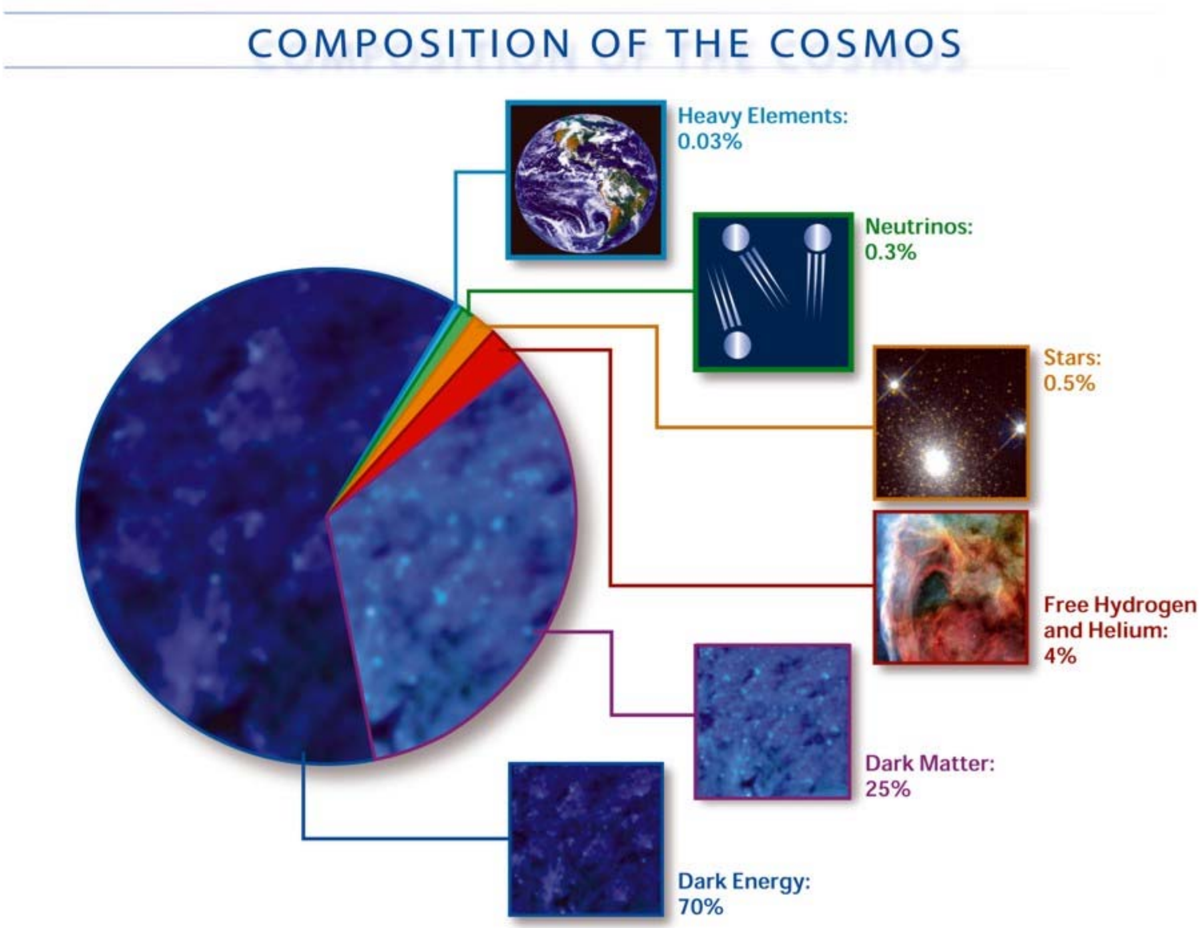
DARK MATTER

- ✓ Neutrinos
- ✓ WIMP
- ✓ Wimpzillas, Axions, the "particle forest".....
- ✓ MOND
- ✓ MACHOS
- ✓ Black Holes
- ✓ ......

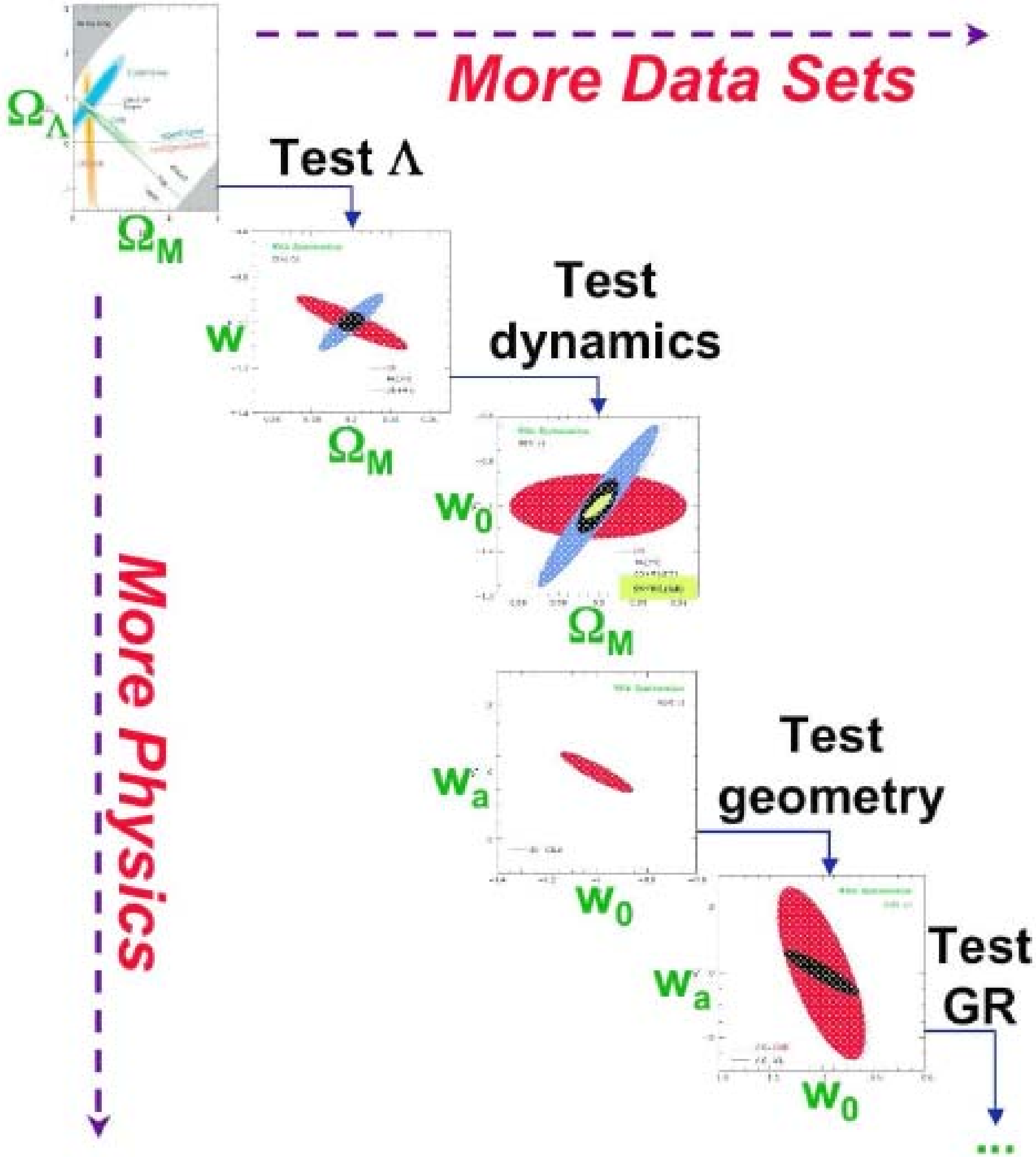
DARK ENERGY

- ➢ Cosmological constant
- ➢ Scalar field Quintessence
- ➢ Phantom fields
- ➢ String-Dilaton scalar field
- ➢ Braneworlds
- ➢ Unified theories
- ➢ ....

- In conclusion: The content of the universe is, up today, absolutely unknown for its largest part. The situation is very “DARK” while the observations are extremely good!
- Precision Cosmology without theoretical foundations??!



Incremental Exploration of the Unknown



there is a fundamental issue:

Are extragalactic observations and cosmology probing the breakdown of General Relativity at large (IR) scales?

The problem could be reversed

We are able to observe and test only baryons and gravity

Dark Energy and Dark Matter as “shortcomings” of GR.
Results of flawed physics!

The “correct” theory of gravity could be derived by matching the largest number of observations at ALL SCALES!

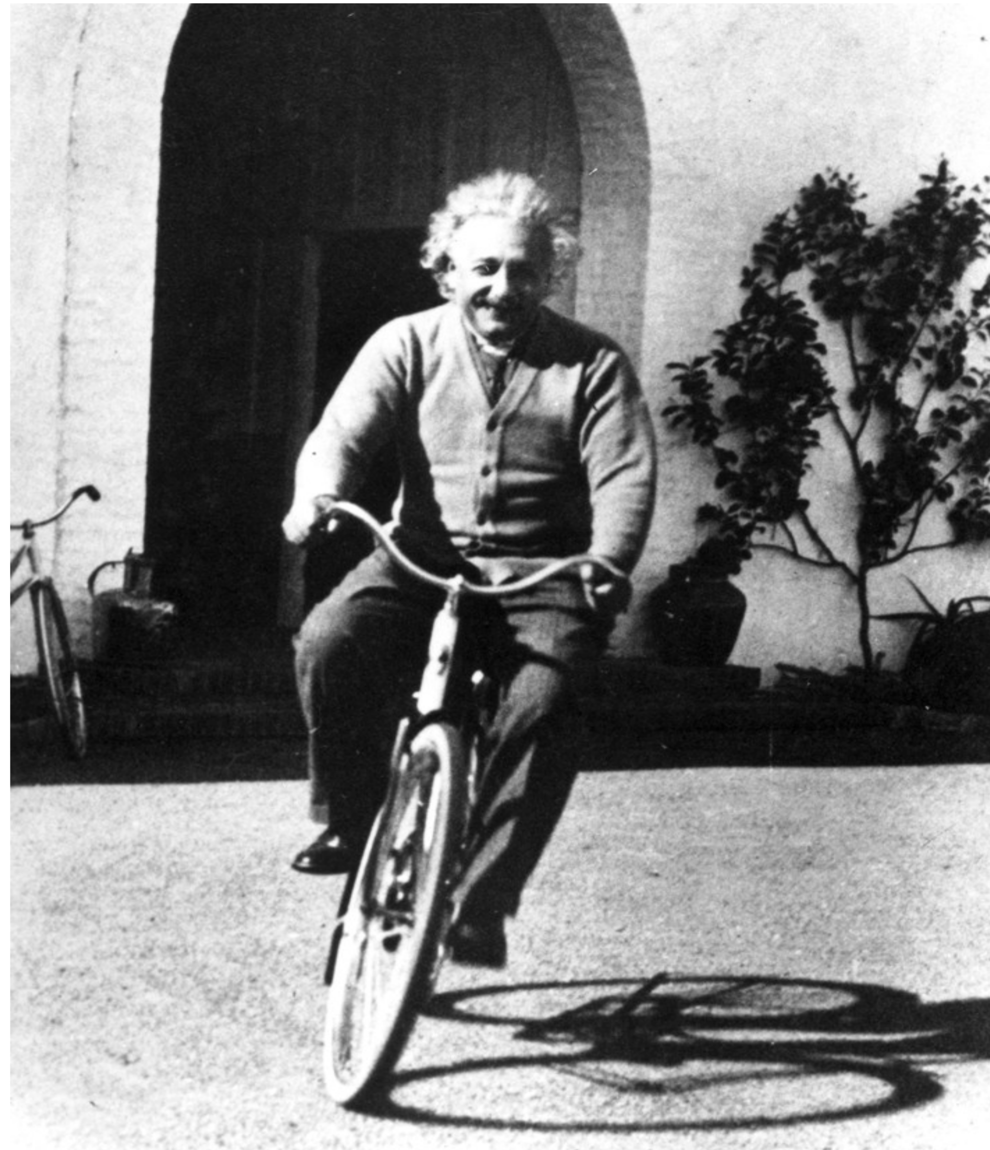
Accelerating behaviour (DE) and dynamical phenomena (DM) as ***CURVATURE EFFECTS***

Higher Order Theories of Gravity

- ✓ Generalization of the Hilbert-Einstein action to a generic (unknown) f(R) theory of gravity

$$\mathcal{A} = \int d^4x \sqrt{-g}\left[f(R) + \mathcal{L}_{(matter)}\right]$$

$$f'(R)R_{\alpha\beta} - \frac{1}{2}f(R)g_{\alpha\beta} = f'(R)^{;\mu\nu}(g_{\alpha\mu}g_{\beta\nu} - g_{\alpha\beta}g_{\mu\nu}) + \tilde{T}_{\alpha\beta}^{(matter)}$$



✓ *Theoretical motivations and features:*

- Quantization on curved space-time needs higher-order invariants corrections to the Hilbert-Einstein Action.
- These corrections are also predicted by several unification schemes as String/M-theory, Kaluza-Klein, etc.
- A generic action is $\mathcal{A} = \int d^4x\sqrt{-g}\left[F\left(R, \Box R, \Box^2 R, \ldots \Box^k R\right) + \mathcal{L}_m\right]$
- We can consider only fourth order terms f(R) which give the main contributions at large scales.
- DE under the standard of FOG inflationary cosmology (Starobinsky) but at different scales and late times.
- This scheme allows to obtain an "Einstein" two fluid model in which one component has a geometric origin

$$G_{\alpha\beta} = R_{\alpha\beta} - \frac{1}{2}g_{\alpha\beta}R = T^{(curv)}_{\alpha\beta} + T^{(matter)}_{\alpha\beta}$$

$$T^{(curv)}_{\alpha\beta} = \frac{1}{f'(R)}\left\{\frac{1}{2}g_{\alpha\beta}\left[f(R) - Rf'(R)\right] + f'(R)^{;\mu\nu}\left(g_{\alpha\mu}g_{\beta\nu} - g_{\alpha\beta}g_{\mu\nu}\right)\right\}$$

Conservation Properties of Higher Order Theories

The Hamiltonian derivative of any fundamental invariant is divergence free!!

- A key point in the above system is related to the **conservation equations**. Eddington proved in his book that every higher order correction to the Hilbert-Einstein Lagrangian produces terms that are divergence free: **the curvature fluid is conserved on his own** and **the matter follows the standard conservation equation**

$$\dot{\rho}_{curv} + 3H(\rho_{curv} + p_{curv}) = 0$$

In particular, the R^n Gravity:

Superstring Theory

Generalizations of Einstein gravity at higher dimensions (Lovelock gravity)

Renormalization of the matter stress energy tensor in QFT

Higher Order Theories of Gravity

$$A = \int \sqrt{-g}\left[\Lambda + c_0 R + c_1 R^2 + c_2 R_{\mu\nu} R^{\mu\nu} + \ldots + L_{mat}\right] d^4x$$

Fourth Order Gravity

R^n Gravity

$$\mathrm{A} = \int d^4x \sqrt{-\mathrm{g}}\left[-\chi R^n + L_{mat}\right]$$

The theoretical building:

Fourth order field equations

$$\mathcal{A} = \int d^4x\sqrt{-g}\left[f(R) + \mathcal{L}_{(matter)}\right]$$

$$f'(R)R_{\alpha\beta} - \frac{1}{2}f(R)g_{\alpha\beta} = f'(R)^{;\mu\nu}(g_{\alpha\mu}g_{\beta\nu} - g_{\alpha\beta}g_{\mu\nu}) + \tilde{T}_{\alpha\beta}^{(matter)}$$

This scheme allows to obtain an "Einstein" two fluid picture in which a component has a geometric origin

$$G_{\alpha\beta} = R_{\alpha\beta} - \frac{1}{2}g_{\alpha\beta}R = T_{\alpha\beta}^{(curv)} + T_{\alpha\beta}^{(matter)}$$

$$T_{\alpha\beta}^{(curv)} = \frac{1}{f'(R)}\left\{\frac{1}{2}g_{\alpha\beta}\left[f(R) - Rf'(R)\right] + f'(R)^{;\mu\nu}(g_{\alpha\mu}g_{\beta\nu} - g_{\alpha\beta}g_{\mu\nu})\right\}$$

$$\rho_{(curv)} = \frac{1}{f'(R)}\left\{\frac{1}{2}\left[f(R) - Rf'(R)\right] - 3\left(\frac{\dot{a}}{a}\right)\dot{R}f''(R)\right\}$$

(FRW)

$$p_{(curv)} = \frac{1}{f'(R)}\left\{2\left(\frac{\dot{a}}{a}\right)\dot{R}f''(R) + \ddot{R}f''(R) + \dot{R}^2f'''(R) - \frac{1}{2}\left[f(R) - Rf'(R)\right]\right\}$$

f(R) – Gravity

[SC, Nojiri, Odintsov, Troisi PLB 2006]

Ideal fluid description

Scalar tensor Gravity

Realistic cosmology can be realized via modified gravity

Conformal Transformation issue

- Mathematical equivalence between Jordan, Einstein and ”fluid” descriptions does not necessarily imply physical equivalence and solutions should be carefully studied into the frames in which they are obtained
- Quintessence potentials with different physical meaning into the two frames, cosmological solutions changing their shape, effective fluid equations of state with phantom-non phantom behaviour ….

Different views for the same problem:

$$S = \int d^4x \sqrt{-g} f(R)$$

$$g_{\mu\nu} \longrightarrow \mathrm{e}^{\sigma} g_{\mu\nu}$$

$$S_E = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} \left\{ R - \frac{3}{2} g^{\rho\sigma} \partial_\rho \sigma \partial_\sigma \sigma - U(\sigma) \right\}$$

$$\varphi = \pm \frac{1}{\kappa} \sqrt{\frac{3}{2}} \sigma$$

$$\tilde{V}(\varphi) = \frac{1}{2\kappa^2} U \left(\pm \kappa \sqrt{\frac{2}{3}} \varphi \right)$$

$$S = \int d^4x \sqrt{-g} \left\{ \frac{1}{2\kappa^2} R - \frac{1}{2} \partial_\mu \varphi \partial^\mu \varphi - \tilde{V}(\varphi) \right\}$$

$$g_{\mu\nu} \longrightarrow \mathrm{e}^{\pm \kappa \varphi \sqrt{\frac{2}{3}}} g_{\mu\nu}$$

But more……….

…..if one considers.. $\tilde{V}(\varphi)\colon\ W(\tilde{V}(\varphi)) = \varphi$

….and solves… $$-\frac{1}{\kappa}\sqrt{-2\frac{dh(\phi)}{d\phi}} = \frac{d}{d\phi}\left(W\left(\frac{1}{\kappa^2}\left(3h(\phi)^2 + \frac{dh(\phi)}{d\phi}\right)\right)\right)$$

It is possible to write $$S = \int d^4x\sqrt{-g}\left\{\frac{1}{2\kappa^2}R - \frac{1}{2}\omega(\phi)\partial_\mu\phi\partial^\mu\phi - V(\phi)\right\}$$

..with…. $\omega(\phi) = -\frac{2}{\kappa^2}h'(\phi)$ $\quad V(\phi) = \frac{1}{\kappa^2}\left(3h(\phi)^2 + h'(\phi)\right)$ $\quad \varphi = \int d\phi\sqrt{|\omega(\phi)|}$

So that one can define an effective fluid capable of mimicking a matter like component along the solution $\phi = t\ , \quad H = h(t)$

$$p = -\rho - \frac{2}{\kappa^2}h'\left(h^{-1}\left(\kappa\sqrt{\frac{\rho}{3}}\right)\right) \qquad \rho = \frac{1}{2}\omega(\phi)\dot{\phi}^2 + V(\phi)\ , \quad p = \frac{1}{2}\omega(\phi)\dot{\phi}^2 - V(\phi)$$

Hence, extended modified gravity may be presented in mathematically equivalent form as GR with ideal fluid!

Dynamical Behaviour:

$$f(R) = \frac{1}{2\kappa^2}R - \frac{c_1}{R^n}$$

$$H = 3\left(\frac{n+1}{n+2}\right)^2 \frac{1}{t} \quad \text{or} \quad a \propto t^{3\left(\frac{n+1}{n+2}\right)^2}$$

Einstein Frame, always
dH/dt<0
NO PHANTOM PHASE

$$\sigma = \pm 2\left(\frac{n+1}{n+2}\right)^2 \ln\frac{t}{t_0}$$

$$dt_J = \mathrm{e}^{\sigma/2}dt \propto t^{\pm\left(\frac{n+1}{n+2}\right)^2}dt$$

$$a_J = \mathrm{e}^{\sigma/2} \propto t^{\pm\frac{n+1}{n+2}+3\left(\frac{n+1}{n+2}\right)^2}$$

$$a_J = t_J^{h_J^\pm}\ ,$$

$$h_J^\pm = \frac{(n+1)(4n+5)}{(2n+3)(n+2)}\ , \quad \frac{(n+1)(2n+1)}{n+2}$$

$$H_J \equiv \frac{\dot{a}_J}{a_J} = \frac{(n+1)(2n+1)}{n+2}\frac{1}{t}$$

Jordan Frame, dH/dt > 0, if
-1<n<-1/2
PHANTOM PHASE

$$
\begin{array}{ccccc}
EoS & \longleftrightarrow & \mathcal{L}_{ST} & \longleftrightarrow & \mathcal{L}_{F(R)} \\
\updownarrow & & \updownarrow & & \updownarrow \\
\text{Einstein Eqs.} & \longleftrightarrow & \text{ST Eqs.} & \longleftrightarrow & F(R)\ \text{Eqs.} \\
\updownarrow & & \updownarrow & & \updownarrow \\
\text{E-frame Sol.} & \longleftrightarrow & \text{E-frame Sol.}+\phi & \longleftrightarrow & \text{J-frame Sol.}
\end{array}
$$

Summary of the three approaches compared at Lagrangian, Field Equations and Solutions level. See (Allemandi, Capone, S.C., Francaviglia 2006) for the Palatini approach and (S.C., Nojiri, Odintsov 2006) for the metric approach.

We have to choose the frame carefully!

Dark Energy as a curvature effect

Starting from the above considerations, it is possible to write a **curvature pressure** and a **curvature energy density** in the FRW metric (**curvature EoS**)

$$p_{(curv)} = \frac{1}{f'(R)}\left\{2\left(\frac{\dot{a}}{a}\right)\dot{R}f''(R) + \ddot{R}f''(R) + \dot{R}^2 f'''(R) - \frac{1}{2}\left[f(R) - Rf'(R)\right]\right\}$$

$$\rho_{(curv)} = \frac{1}{f'(R)}\left\{\frac{1}{2}\left[f(R) - Rf'(R)\right] - 3\left(\frac{\dot{a}}{a}\right)\dot{R}f''(R)\right\}$$

$$w_{curv} = -1 + \frac{\ddot{R}f''(R) + \dot{R}\left[\dot{R}f'''(R) - Hf''(R)\right]}{\frac{1}{2}\left[f(R) - Rf'(R)\right] - 3H\dot{R}f''(R)}$$

As a simple choice, we can assume a power law function for *f(R)* and for the scale factor *a(t)*

$$f(R) = f_0 R^n\,, \qquad a(t) = a_0\left(\frac{t}{t_0}\right)^{\alpha}$$

- The power law *f(R)* function is interesting since each analytical Lagrangian in R can be locally approximated by a Taylor polynomial expansion. See also Starobinsky Lagrangian (S.C., Francaviglia 2006)

$$f_0|R|^{(1+\epsilon)} \simeq f_0|R|\left(1+\epsilon\ln|R|+\frac{\epsilon^2\ln^2|R|}{2}+\dots\right)$$

- Unstable FRW matter-dominated solutions evolving into accelerated solutions at late times are particularly interesting for LSS formation and DE issues (Allemandi, Borowiec, Francaviglia 2004; Carloni, Dunsby, S.C., Troisi 2005; S.C., Nojiri, Odintsov, Troisi 2006) .
- Effective "true" Lagrangian has to recover the positive results of this approach (S.C., Rubano, Troisi 2006; Nojiri, Odintsov 2006).
- And further motivations…..

- R^2 - *Gravity* → Inflationary Scenarios (i.e. $R^2 Ln\ R$) [Starobinsky, Mukhanov et al. 1980, 1982…] – good agreement with quantum primordial perturbations (COBE) [Hwang & Noh, 2001]
- *R^n – Gravity* as Dark Matter source (LSB rotation curves, Effective DM halos, Pointlike Lensing Phenomenology) [SC, Cardone, Troisi, 2006]
- *R^n+R^(-m) – Gravity* describing both Inflationary and Dark Energy scenarios [Odinstov, Nojiri 2004,2005]
- *R+f(R) – Gravity* → Dark Energy [Carroll et al. 2003] - Matter perturbations growth (f(R)= A Exp(B R)) [Peng, 2006]
- Several others approaches based on *R^n – Gravity* ….

Matching with data

SNeIa data [SC, Cardone, Carloni, Troisi 2003,2004,2005,2006]

The test with the SNeIa data has been done considering the so called "distance modulus"

$$\mu(z) = 5 \log \frac{c}{H_0} d_L(z) + 25$$

comparing its theoretical estimate with the observed one.

The luminosity distance *d_L(z)* is defined in relation to the considered cosmological model. In our case, it is

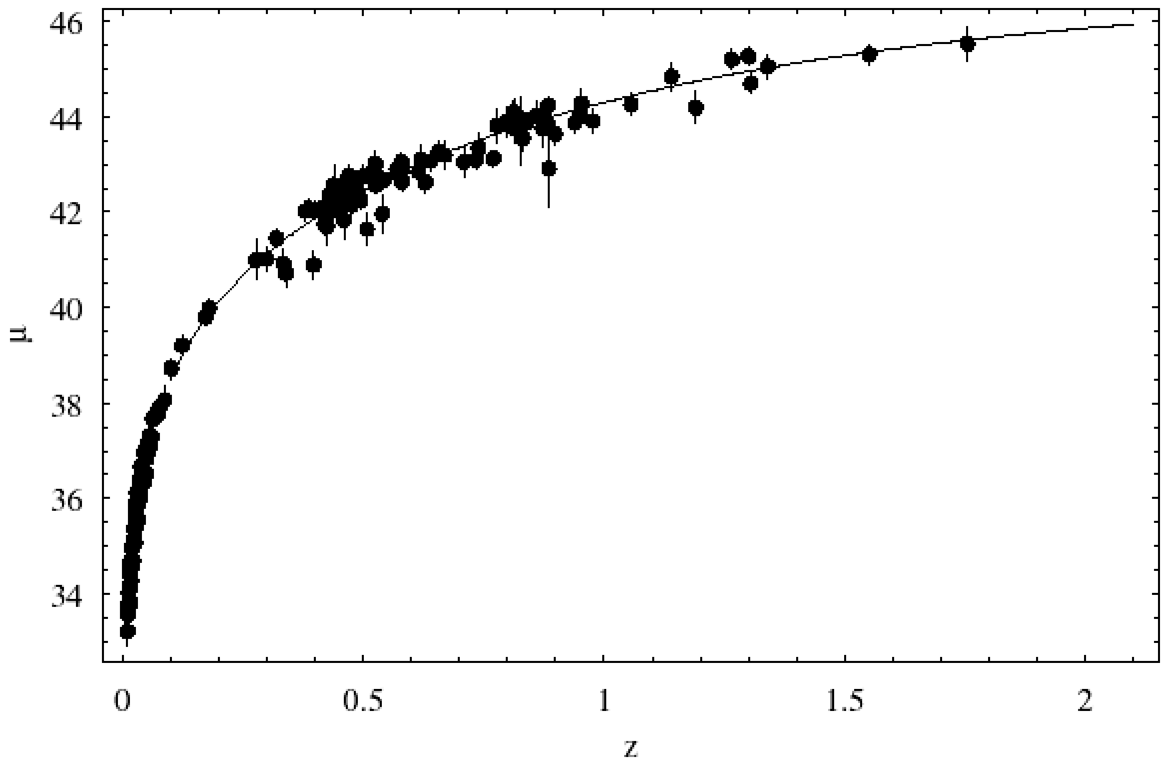
$$d_L(z, H_0, n) = \frac{c}{H_0} \frac{\alpha}{\alpha - 1} (1+z) \left[(1+z)^{\frac{\alpha}{\alpha-1}} - 1 \right]$$

The analysis is performed minimizing the

$$\chi^2(H_0, n) = \sum_i \frac{[\mu_i^{theor}(z_i|H_0, n) - \mu_i^{obs}]^2}{\sigma^2_{\mu_0,i} + \sigma^2_{mz,i}}$$

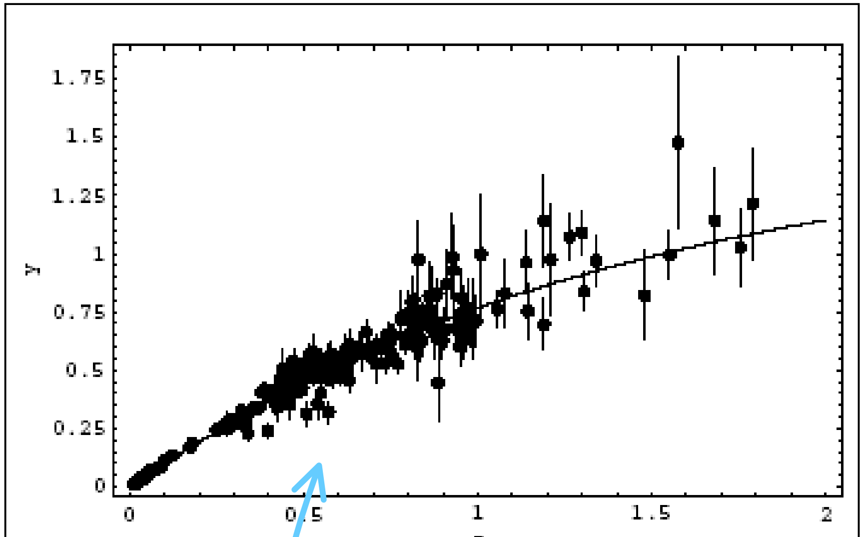
Range	$H_0^{best}(km\,s^{-1}Mpc^{-1})$	n^{best}	χ^2
$-100 < n < 1/2(1-\sqrt{3})$	65	-0.73	1.003
$1/2(1-\sqrt{3}) < n < 1/2$	63	-0.36	1.160
$1/2 < n < 1$	100	0.78	348.97
$1 < n < 1/2(1+\sqrt{3})$	62	1.36	1.182
$1/2(1+\sqrt{3}) < n < 3$	65	1.45	1.003
$3 < n < 100$	70	100	1.418

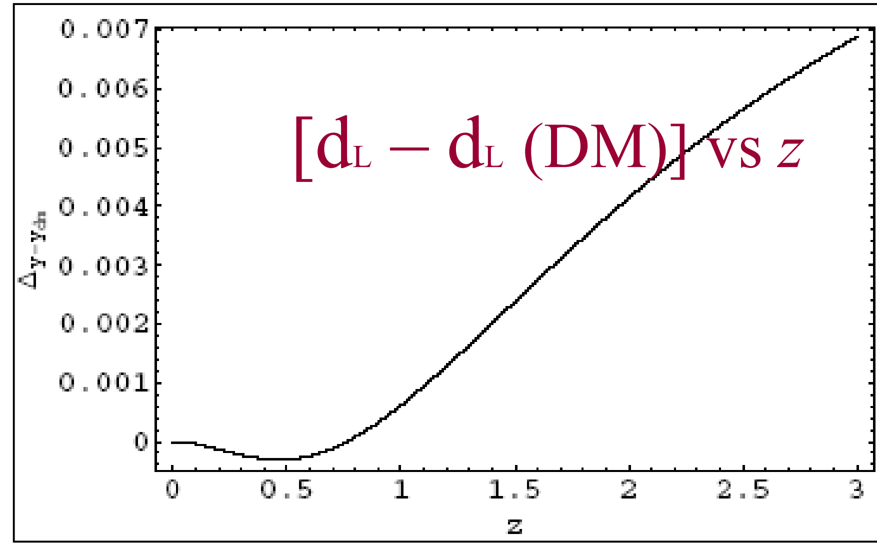
f(R) solutions fitted against SNeIa



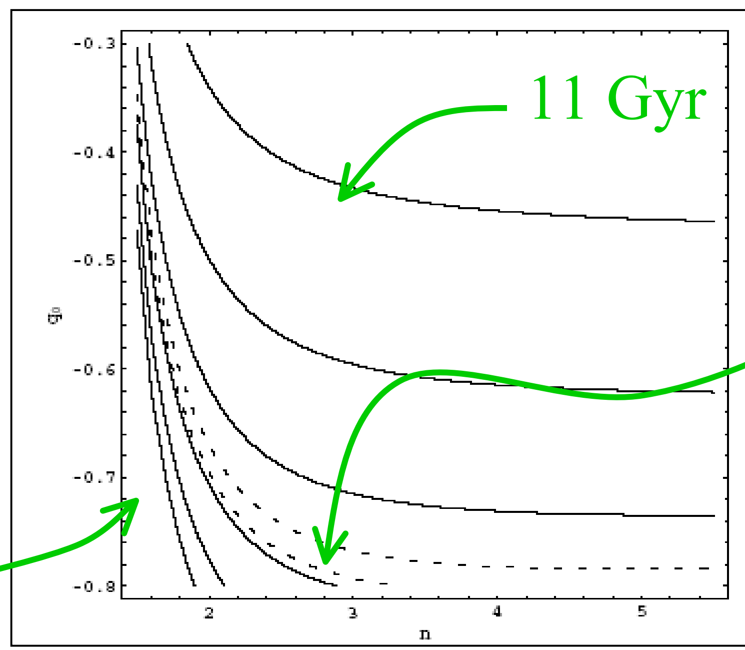
Very promising results in metric and Palatini approaches!
We used GOODS survey (S.C., Cardone, Francaviglia 2006)

R^n – Gravity as Dark Energy source (SNeIa fitted with n=3.42 (No DM), n= 3.52 (with DM)) [SC, Cardone, Troisi 2006]





Hubble diagram of 20 radio-galaxies + SNeIa Gold sample



WMAP 1σ region

16 Gyr

The Age test

The age of the universe can be theoretically calculated if one is capable of furnishing the Hubble parameter. We have:

$$t = \left(\frac{-2n^2 + 3n - 1}{n - 2}\right) H^{-1}$$

For the experimental value, we have considered both data coming from globular cluster observations and WMAP measurements of CMBR (this last estimate is very precise 13.7 ± 0.02 *Gyr*) [SC, Cardone, Troisi 2005]

Result → A fourth order theory $f(R) = f_0 R^n$ is able to fit SNeIa data and WMAP age prediction with

$Range$	$\Delta H (km\,s^{-1} Mpc^{-1})$	Δn	q_0
$-100 < n < 1/2(1-\sqrt{3})$	$50 - 80$	$-0.450 \leq n < -0.370$	< 0
$1/2(1-\sqrt{3}) < n < 1/2$	$57 - 69$	$-0.345 < n \leq -0.225$	> 0
$1 < n < 1/2(1+\sqrt{3})$	$56 - 70$	$1.330 \leq n < 1.360$	> 0
$1/2(1+\sqrt{3}) < n < 2$	$54 - 78$	$1.366 < n \leq 1.376$	< 0

f (R) theories (extensions of Einstein Gravity) seem in good agreement with DARK ENERGY!

Inverse scattering procedure: observational H(z) gives f(R)

[SC, Cardone, Troisi 2005]

From field equations

$$\frac{dH}{dt} = -\frac{1}{2f'(R)}\left\{3H_0^2\Omega_{M,0}(1+z)^3 + f''(R)\ddot{R} + \left[f'''(R)\dot{R} - Hf''(R)\right]\dot{R}\right\}$$

$$\rho_m = \Omega_{M,0}\,\rho_{crit}\,a^{-3}$$

Fourth order equation for a(t)

Considering the relation between ***R*** and ***H*** (FRW-metric) and changing time variabile *t* with redshift *z*, we get a ***third order differential equation for H(z) and, as a direct consequence, for f(z)***

$$R = -6\left[2H^2 - (1+z)H\frac{dH}{dz}\right]$$

$$\mathcal{H}_3(z)\frac{d^3f}{dz^3} + \mathcal{H}_2(z)\frac{d^2f}{dz^2} + \mathcal{H}_1(z)\frac{df}{dz} = -3H_0^2\Omega_{M,0}(1+z)^3$$

with $\mathcal{H}_i$ functions defined in term of *R(H(z),z)* and its *z* derivatives

Two level approach:

1) H(z) can be inferred by observations and phenomenological considerations. 2) One can deal with the Hubble flow of a certain cosmological model and deduce the corresponding *f(R)-gravity model* capable of providing the *same* cosmic dynamics.

$$\ln(-f) = l_1 \left[\ln(-R)\right]^{l_2} \left[1 + \ln(-R)\right]^{l_3} + l_4$$

Q-essence,

$$(l_1, l_2, l_3, l_4) = (2.6693, 0.5950, 0.0719, -3.0099)$$

Chaplygin,

$$(l_1, l_2, l_3, l_4) = (1.9814, 0.5558, 0.2665, -2.5337)$$

Unified Model

$$\rho(z) = A \left(1 + \frac{1+z}{1+z_s}\right)^{\beta-\alpha} \left[1 + \left(\frac{1+z}{1+z_b}\right)^{\alpha}\right]$$

$$\frac{f(R)}{R} = 1.02 \times \frac{R}{R_0}\left[1 + \left(-0.04 \times (\frac{R}{R_0})^{0.31} +0.69 \times (\frac{R}{R_0})^{-0.53}\right) \times \ln(\frac{R}{R_0})\right]$$

Exponential Quintessence

$$V(\varphi) \propto \exp\left(-\sqrt{\tfrac{3}{2}}\varphi\right)$$

$$f(R) \simeq \eta_s f_0 \left(\frac{R}{R_0}\right)\left\{1 + \left[\eta_1 \left(\frac{R}{R_0}\right)^{\alpha} + \eta_2 \left(\frac{R}{R_0}\right)^{-\beta}\right] \ln\left(\frac{R}{R_0}\right)\right\}$$

$$(\eta_s, \eta_1, \eta_2, \alpha, \beta) = (1.07, 0.029, 0.810, 0.232, 0.221)$$

H(z) is inferred from cosmological data. The goal is numerically finding *f(z)* and then back tranforming this thanks to *z=z(R)* in *f(R)*. *A* similar approach can be pursued using also deceleration, snap and jerk parameters.

$$D_L(z) = (1+z)\int_0^z \frac{d\zeta}{H(\zeta)} \quad \Longrightarrow \quad R(z) = \frac{6(1+z)^6 D_L''(z)}{[D_L(z) - (1+z)D_L'(z)]^3}$$

If $(\delta_0, \delta_1, \delta_2) = (D_L, D_L', D_L'')$ $\quad \sigma_R = \sqrt{\left|\frac{\partial R}{\partial \delta_0}\right|^2 \sigma_0^2 + \left|\frac{\partial R}{\partial \delta_1}\right|^2 \sigma_1^2 + \left|\frac{\partial R}{\partial \delta_2}\right|^2 \sigma_2^2}$

σ_i corresponding uncertainties

$$\sigma_R = \left|\frac{6(1+z)^6}{[\delta_0 - (1+z)\delta_1]^3}\right| \times \Delta_R \qquad \Delta_R = \sqrt{\left|\frac{3\delta_2\sigma_0}{(1+z)\delta_1 - \delta_0}\right|^2 + \left|\frac{3(1+z)\delta_2\sigma_1}{\delta_0 - (1+z)\delta_1}\right|^2 + \sigma_2^2}$$

Polinomial fit, $H(z) = H_0\sqrt{\Omega_M x^3 + A_1 + A_2 x + A_3 x^2}$ *where* $(1+z) = x$

Can f(R)-theories reproduce also Dark Matter dynamics?

Main research interests:

1) Galactic dynamics (rotation curves of spiral galaxies)
2) DM in the Ellipticals
3) Galaxy cluster dynamics

The problem: we search for f (R)-solutions capable of fitting consistently the data. A nice feature could be that the same f(R) – theory works for Dark Energy (very large, unclustered scales) and Dark Matter (small and clustered scales).

DM in the galaxies as a curvature effect

Theoretical issues

Spiral galaxies are ideal candidates to test DM models. In particular, LSB galaxies are DM-dominated so that fitting their rotation curves WITHOUT DM is a good test for any alternative gravity theory

A further test is trying to fit Milky Way rotation curve WITHOUT DM. In this case, the approach could be problematic

In general, is it possible to define an *effective dark matter halo* induced by cuvature effects?

What about *baryonic Tully-Fischer relation* without DM?

DM as a curvature effect

Let us consider again: $f(R) = f_0 R^n$

- In the low energy limit, the spacetime metric can be written as

$$ds^2 = A(r)dt^2 - B(r)dr^2 - r^2 d\Omega^2$$

- A physically motivated hypothesis is $$A(r) = \frac{1}{B(r)} = 1 + \frac{2\Phi(r)}{c^2}$$

- An exact solution is $$\Phi(r) = -\frac{Gm}{2r}\left[1 + \left(\frac{r}{r_c}\right)^{\beta}\right]$$

with $$\beta = \frac{12n^2 - 7n - 1 - \sqrt{36n^4 + 12n^3 - 83n^2 + 50n + 1}}{6n^2 - 4n + 2}$$

- ***For n=1, $\beta = 0$. The approach is consistent with GR!***

- The pointlike rotation curve is

$$v_c^2(r) = \frac{Gm}{2\,r}\left[1 + (1-\beta)\left(\frac{r}{r_c}\right)^{\beta}\right]$$

- The galaxy can be modeled considering a thin disk and a bulge component.

- The potential can be splitted in a Newtonian and a correction part

$$\Phi(r) = \Phi_N(r) + \Phi_c(r) = \frac{GM(r)}{r} + \Phi_c(r)$$

- Integration can be performed considering a spherical symmetry for the bulge and a cylindrical symmetry for the disk.

$$\Phi_i(\tilde{r}, z) = \int_0^{\tilde{r}} \tilde{r}'\, d\tilde{r}' \int_0^{2\pi} d\phi \int_{-z}^{+z} dz'\, \rho(\tilde{r}', z')\, r^{\eta_i} \qquad \eta_i = \begin{cases} -1 & \text{i = N} \\ \beta(n) & \text{i = c} \end{cases}$$

- The rotation curve is given by

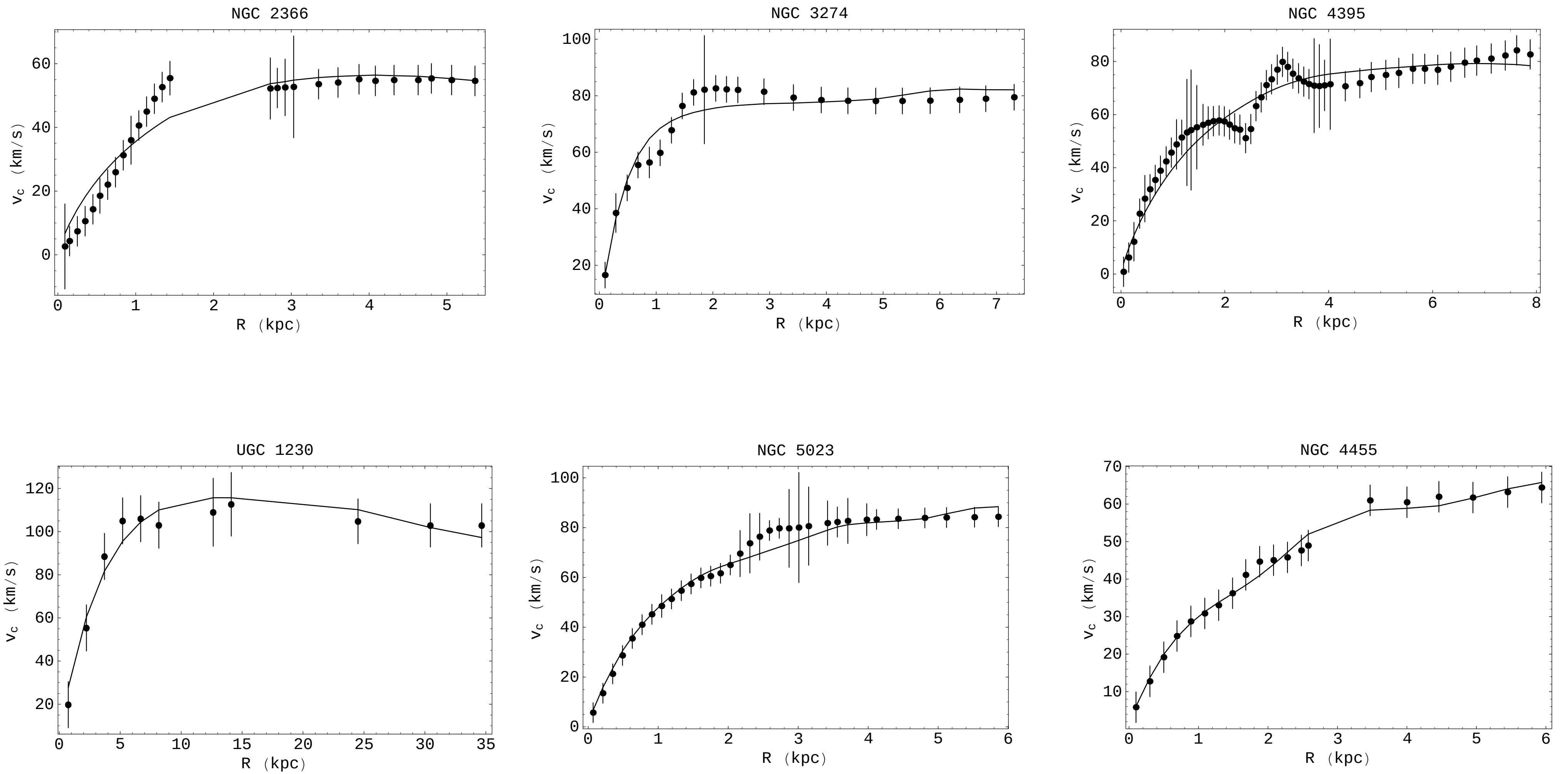
$$v_{circ}{}^2(\tilde{r}) = \tilde{r}\left.\frac{\partial \Phi_i}{\partial \tilde{r}}\right|_{z=0}$$

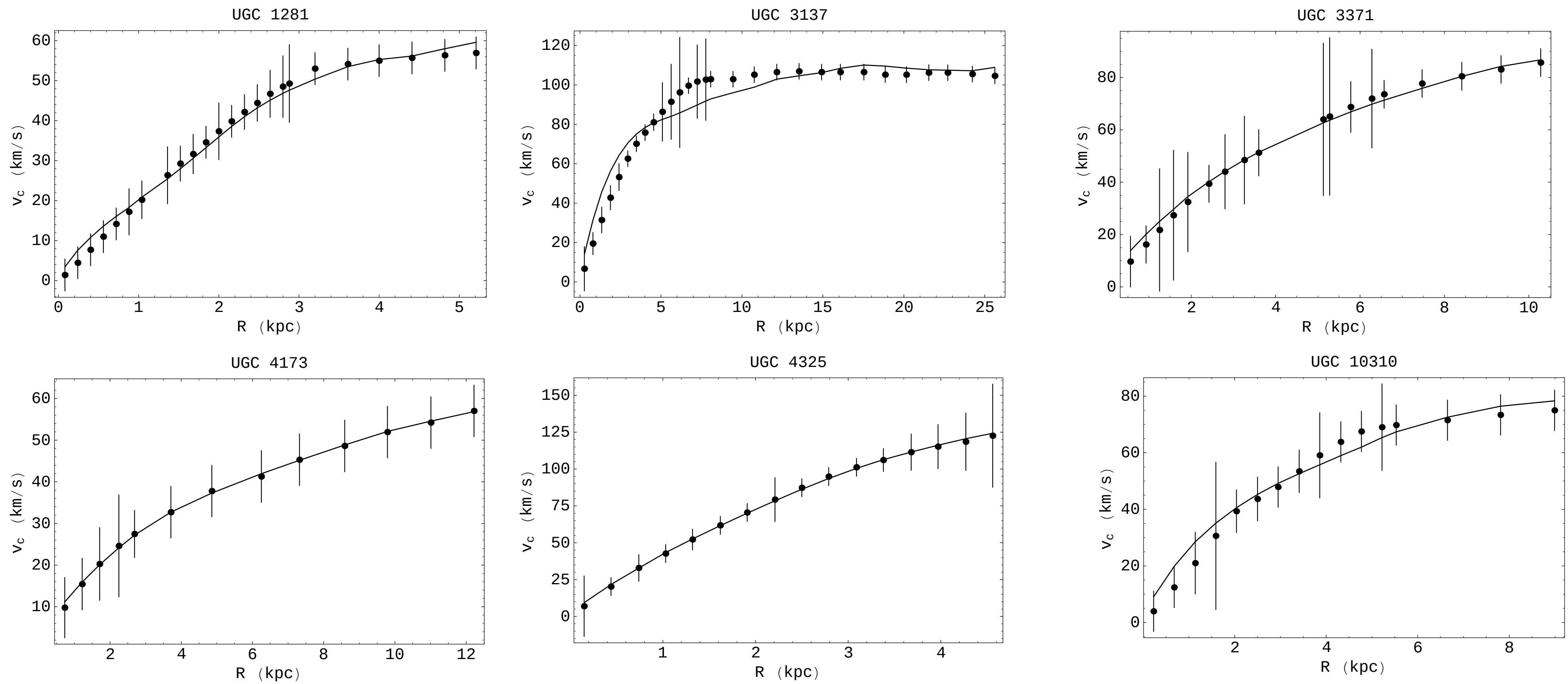
DM in LSB galaxies as a curvature effect: results of fits on a sample of 15 galaxies [sample from de Blok, Bosma 2002]

Id	β	$\log r_c$	f_g	$\Upsilon_\star$	χ^2/dof	σ_{rms}
UGC 1230	0.83 ± 0.02	-0.39 ± 0.09	0.15 ± 0.01	15.9 ± 0.5	2.97/8	0.96
UGC 1281	0.38 ± 0.01	-3.93 ± 0.80	0.65 ± 0.08	0.64 ± 0.33	3.48/21	1.05
UGC 3137	0.72 ± 0.03	-1.86 ± 0.06	0.65 ± 0.02	9.8 ± 0.9	48.1/26	1.81
UGC 3371	0.78 ± 0.05	-1.85 ± 0.01	0.41 ± 0.01	3.3 ± 0.2	0.48/15	1.30
UGC 4173	0.94 ± 0.02	-0.97 ± 0.22	0.34 ± 0.01	9.37 ± 0.04	0.12/10	0.52
UGC 4325	0.79 ± 0.07	-2.85 ± 0.44	0.70 ± 0.02	0.50 ± 0.05	0.09/13	1.19
NGC 2366	0.96 ± 0.14	-0.58 ± 0.42	0.64 ± 0.01	14.5 ± 0.9	28.6/25	1.10
IC 2233	0.42 ± 0.01	-3.50 ± 0.05	0.64 ± 0.01	1.29 ± 0.06	6.1/22	2.10
NGC 3274	0.71 ± 0.03	-2.30 ± 0.19	0.55 ± 0.03	2.3 ± 0.3	17.6/20	2.7
NGC 4395	0.13 ± 0.02	-3.68 ± 0.31	0.14 ± 0.01	7.6 ± 0.3	37.7/52	1.40
NGC 4455	0.87 ± 0.05	-2.32 ± 0.07	0.83 ± 0.01	0.42 ± 0.04	3.3/17	1.12
NGC 5023	0.81 ± 0.02	-2.54 ± 0.05	0.53 ± 0.02	0.91 ± 0.06	8.9/30	2.50
DDO 185	0.92 ± 0.10	-2.75 ± 0.35	0.90 ± 0.03	0.21 ± 0.07	5.03/5	0.81
DDO 189	0.54 ± 0.08	-2.40 ± 0.61	0.63 ± 0.04	4.2 ± 0.7	0.44/8	1.08
UGC 10310	0.72 ± 0.04	-1.87 ± 0.04	0.59 ± 0.02	1.39 ± 0.04	2.90/13	1.02

Table 1. Best fit values of the model parameters from minimizing $\chi^2(\beta, \log r_c, f_g)$. We also report the value of $\Upsilon_\star$, the χ^2/dof for the best fit parameters (with $dof = N - 3$ and N the number of datapoints) and the root mean square σ_{rms} of the fit residuals. Errors on the fitting parameters and the M/L ratio are estimated through the jacknife method hence do not take into account parameter degeneracies.

Experimental points vs. Fourth Order Gravity induced theoretical curves





Data:

- Rotation curves
- Mass surface density
- Gas component
- Disk photometry

fit →

- M/L ratio $\Upsilon_\star$
- ß =universal parameter
- r_c = characteristic parameter related to the total mass of every galaxy

Results → Best Fit

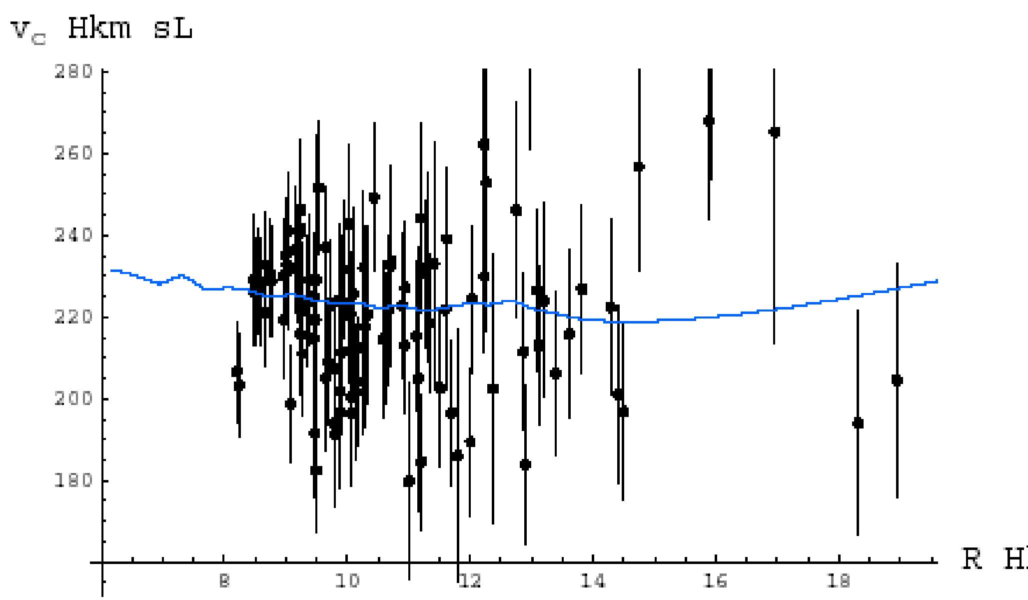
$$\beta = 0.80 \pm 0.08$$

↓

$$2.3 \leq n \leq 5.3$$

No DARK component has been used!

Milky Way data fit



Result (SC, Cardone, Carloni, Troisi 2004)

The upper points are affected by systematic errors as discussed in Pont et al. 1997

Also in this case, NO Dark Matter is needed

At this point, it is worth wondering whether a link can be found between Fourth Order Gravity and the standard approach, based on *effective dark matter haloes,* since both approaches fit equally well the data

Considering $v_c^2(r) = v_{c,N}^2(r) + v_{c,corr}^2(r)$ **due to modified gravity**

and $v_c^2(r) = v_{c,disk}^2(r) + v_{c,DM}^2(r)$ **due to effective DM halo**

being $v_{c,DM}^2(r) = GM_{DM}(r)/r$

Equating the two expressions, we get $M_{DM}(\eta) = 2^{\beta-5}\eta_c^{-\beta}\pi(1-\beta)\Sigma_0 R_d^2 \eta^{\frac{\beta+1}{2}} \mathcal{I}_0(\eta,\beta)$

where $\eta = r/R_d,\ \Sigma_0 = \Upsilon_\star I_0$ and $\mathcal{I}_0(\eta,\beta) = \int_0^\infty \mathcal{F}_0(\eta,\eta',\beta) k^{3-\beta} \eta'^{\frac{\beta-1}{2}} e^{-\eta'} d\eta'$

this means that the mass profile of an effective spherically symmetric DM halo, which provides corrected disk rotation curves, can be reproduced. For $\beta = 0.80 \pm 0.08$ ***Burkert models are exactly reproduced!*** [Burkert 1995]

Baryonic Tully-Fisher relation

From Virial theorem: $V_{vir} = GM_{vir}/R_{vir}$

$$M_d \propto \frac{(3/4\pi\delta_{th}\Omega_m\rho_{crit})^{\frac{1-\beta}{4}} R_d^{\frac{1+\beta}{2}} \eta_c^{\beta}}{2^{\beta-6}(1-\beta)G^{\frac{5-\beta}{4}}} \frac{V_{vir}^{\frac{5-\beta}{2}}}{\mathcal{I}_0(V_{vir},\beta)}$$

where $M_d \propto V_{vir}^a$ is the disk mass. For $\beta = 0.80 \pm 0.08$

empirical baryonic Tully-Fisher relation is reproduced!

[SC, Cardone, Troisi 2006]

In summary: Dark Matter dynamics can be reproduced by f(R)-gravity

Results:

1) Galactic dynamics (rotation curves of LSB spiral galaxies)

2) Milky Way rotation curve

3) Burkert haloes

4) Baryonic Tully-Fisher relation

Further issues: HSB galaxies, Elliptical galaxies, Clusters.

A nice feature, as said, could be that the same f(R) – theory works for Dark Energy (very large scales, unclustered) and Dark Matter (small and medium scales, clustered structures)

Last issue: Parametrized Post-Newtonian limit

Considering the post-Newtonian approximation of the metric

$$ds^2 \simeq \left[1 + 2\frac{U}{c^2} + 2\beta\left(\frac{U}{c^2}\right)^2\right] c^2dt^2 - \left[1 - 2\gamma\frac{U}{c^2} + \frac{3}{2}\epsilon\left(\frac{U}{c^2}\right)^2\right] d\mathbf{x}^2$$

One can attempt to evaluate deviations from Newtonian gravity (and then GR) via astrophysical experiments or ground based tests

Mercury Perihelion Shift	$\lvert 2\gamma_0^{\rm PPN} - \beta_0^{\rm PPN} - 1\rvert < 3 \times 10^{-3}$
Lunar Laser Ranging	$4\beta_0^{\rm PPN} - \gamma_0^{\rm PPN} - 3 = -(0.7 \pm 1) \times 10^{-3}$
Very Long Baseline Interferometry	$\lvert\gamma_0^{\rm PPN} - 1\rvert = 4 \times 10^{-4}$
Cassini spacecraft	$\gamma_0^{\rm PPN} - 1 = (2.1 \pm 2.3) \times 10^{-5}$

GR prescriptions: $\gamma = 1$; $\beta = 1$!!!

Parametrized Post-Newtonian limit of fourth order gravity inspired by scalar-tensor gravity

- Exploiting the fourth order gravity – scalar tensor gravity analogy, the previous PPN formalism can be generalized to *f(R)* Lagrangians

- The PPN parameters are recovered through *R* dependent quantities *via* the relations

$$\gamma_R^{\mathrm{PPN}} - 1 = -\frac{f''(R)^2}{f'(R) + 2f''(R)^2}$$

$$\beta_R^{\mathrm{PPN}} - 1 = \frac{1}{4}\frac{f'(R)\cdot f''(R)}{2f'(R) + 3f''(R)^2}\frac{d\gamma_R^{\mathrm{PPN}}}{d\varphi}$$

(S.C., Troisi 2005)

- Considering the above definitions of PPN parameters as two differential equations and recasting β in terms of γ, one obtains the general solution

$$f_{\pm}(R) = \frac{1}{12}\left|\frac{1-\gamma}{2\gamma-1}\right|R^3 \pm \frac{1}{2}\sqrt{\left|\frac{1-\gamma}{2\gamma-1}\right|}R^2 + R + \Lambda$$

-which is a cubic function where deviations with respect to GR emerge if γ is different from 1. NOTE: FOR γ=1, GR IS RESTORED!

-if γ fulfils the condition |γ-1|<10^(-4) given by the experimental bounds, deviations from GR are possible inside Solar System and could be tested by ground based experiments!

(SC, Stabile, Troisi 2006)

Solutions and Observational constraints

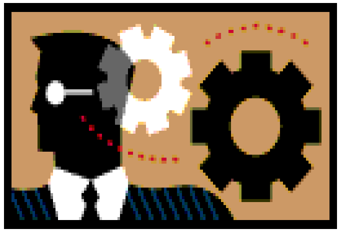
- As a first check, one can consider Lunar Laser Ranging and Cassini spacecraft constraints which give tight bounds on the PPN-parameters. It has been observed that, in relation to the experimental errors, if these requirements are matched **->** even Pulsar upper limits on the ratio $\frac{\beta^{\mathrm{PPN}}-1}{\gamma^{\mathrm{PPN}}-1} < 1.1$ are fulfilled.

-To achieve more significant checks of fourth order gravity with respect to PPN-observational bounds, the upper limits coming from the perihelion shift of Mercury and the **very long baseline interferometry** (or VIRGO interferometer) have to be considered.

-Other than the above solution, several f(R) models of physical relevance can be confronted with these experimental bounds

Lagrangian	Parameters constraints
$f_0 R^2$	$R_0 < 0, \frac{R_0}{4996} < f_0 < -\frac{R_0}{5004}$ $R_0 > 0, -\frac{R_0}{5004} < f_0 < \frac{R_0}{4996}$
$f_0 R^3$	$-\frac{1}{30024} < f_0 < \frac{1}{29976}$
$R + aR^2$	$a = 0$ $\{a > 09992a < \frac{1}{a} + 2R_0\}$ $\{a > 0, \frac{1}{a} + 10008a + 2R_0 < 0\}$ $\{a < 0, \frac{1}{a} + 10008a + 2R_0 > 0\}$ $\{a < 09992a > \frac{1}{a} + 2R_0\}$
$A \log[R]$	$A \leq 0, R_0 < 13.5685 A^{1/3}$ $R_0 > -13.5757 A^{1/3}$ $A > 0, R_0 < -13.5757 A^{1/3}$ $R_0 > 13.5685 A^{1/3}$

- Similar results hold also for generic $f(R)=f_0 R^n$ and $f(R)=R+\alpha/R$. It seems that there is room for alternative theories, other than GR, in Solar System!!! (S.C., Troisi 2005; Allemandi, Francaviglia, Ruggiero, Tartaglia 2005). This statement is not conclusive at this stage, but it is promising.



Summarizing the results

- **Higher Order Curvature Theories give solutions consistent with Dark Energy dynamics in metric and Palatini approaches** [SC (2002), SC, Cardone, Troisi (2003, 2004), Allemandi, Borowiec, Francaviglia (2004,2005), Nojiri, Odintsov (2004,2005), Multamaki, Vilja (2005, 2006)].
- **Solutions agree with SNeIa e WMAP data (distance measurements)** [SC, Cardone, Troisi (2003,2004), S.C., Cardone, Francaviglia (2006)].
- **Good agreement also with lookback time methods (time measurements)** [SC, Cardone, Funaro, Andreon (2004)]
- **Luminosity distance in f(R)** [SC, Cardone, Troisi (2004)]
- **f(R)- dynamical systems and stability analysis** [Carloni, SC, Dunsby, Troisi (2005)]
- **Rotation curves of LSB galaxies** [SC, Cardone, Troisi (2006)]
- **Baryonic Tully-Fisher, Burkert haloes** [SC, Cardone, Troisi (2006)]
- **Gravitational Lensing in f(R) gravity** [SC, Cardone, Troisi (2006)]
- **Room for f(R) in Solar System?** [SC, Troisi (2005), Allemandi, Francaviglia, Ruggiero, Tartaglia (2005)]
- **GW and f(R)** [SC, Corda, De Laurentis (2006)]
- **f(R) and torsion fields** [SC, Stornaiolo(2006)]

Conclusions (1)

- Higher Order Gravity seems a viable approach to describe the Dark Side of the Universe. It is based on a straightforward generalization of Einstein Gravity and does not account for exotic fluids, fine-tuning problems or unknown scalar fields (*R* can be considered a "geometric" scalar field!).
- Comfortable results are obtained by matching the theory with data (SNeIa, Radio-galaxies, Age of the Universe, CMBR).
- Transient dust-like Friedman solutions (LSS) evolving in de Sitter- like expansion (DE) at late times are particularly interesting.
- Generic quintessential and DE models can be easily "mimicked" by *f(R)* through an inverse scattering procedure where *H(z)* is phenomenologically given by observations. Deatailed models can be achieved using also deceleration, jerk and snap parameters.
- Conformal transformations should be carefully considered (it seems that physical solutions in Jordan Frame match the data).
- A comprehensive cosmological model from early to late epochs should be achieved by *f(R)*. LSS issues have to be carefully addressed.

Conclusions (2)

- Rotation curves of galaxies can be naturally reproduced, without huge amounts of DM, thanks to the corrections to the Newton potential, which come out in the low energy limit.
- The baryonic Tully- Fisher relation has a natural eplanation in the framework of *f(R)* theories.
- Effective haloes of spiral galaxies are reproduced by the same mechanism. Also in this case, no need of DM.
- Good evidences also for galaxy clusters (work in progress)
- The PPN limit of these models falls into the experimental bounds of Solar System experiments. It seems that there is room for alternative theories also at small scales. Final answer from VLBI or VIRGO interferometer?

Perspectives:

DE & DM as curvature effects

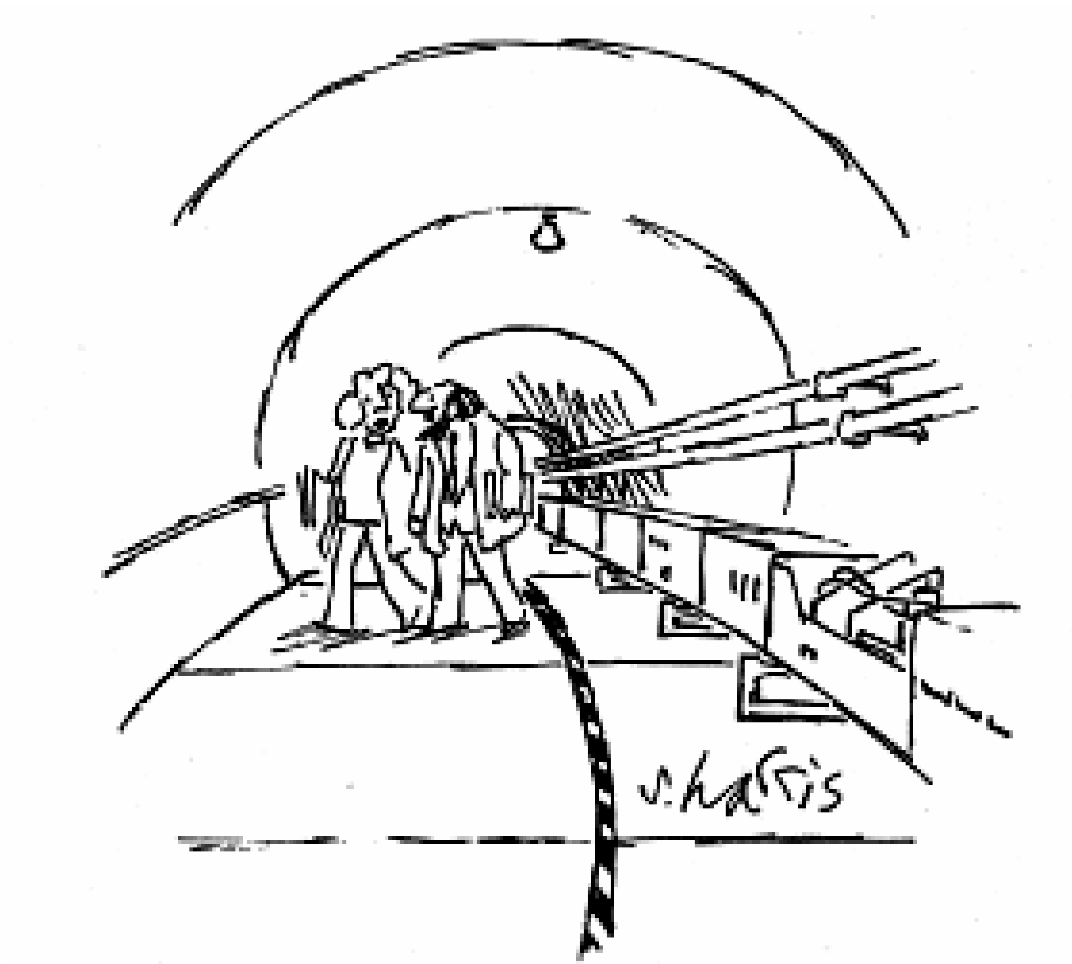
- Matching other DE models
- Jordan Frame and Einstein Frame
- Systematic studies of rotation curves for other galaxies
- Galaxy cluster dynamics (virial theorem, SZE, etc.)
- Luminosity profiles of galaxies in f(R).
- Faber-Jackson & Tully-Fisher

Weak Fields, GW, Further results

- Systematic studies of PPN formalism
- Relativistic Experimental Tests in f(R)
- Gravitational waves and lensing
- Birkhoff 's Theorem in f (R)-gravity
- f(R) with torsion

WORK in PROGRESS! (suggestions are welcome!)

Ending with a joke: Dark Matter in the Lab!



....and if, after spending all this money,

no further particles remain to discover?