Earth crust memory, earthquake remote triggering and self-organised criticality

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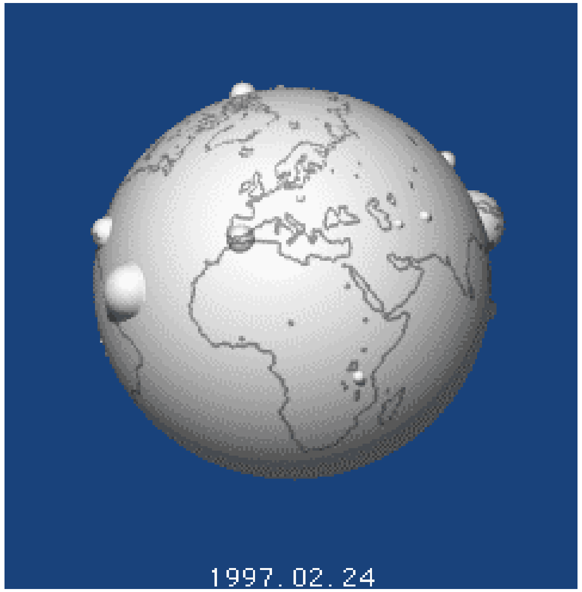
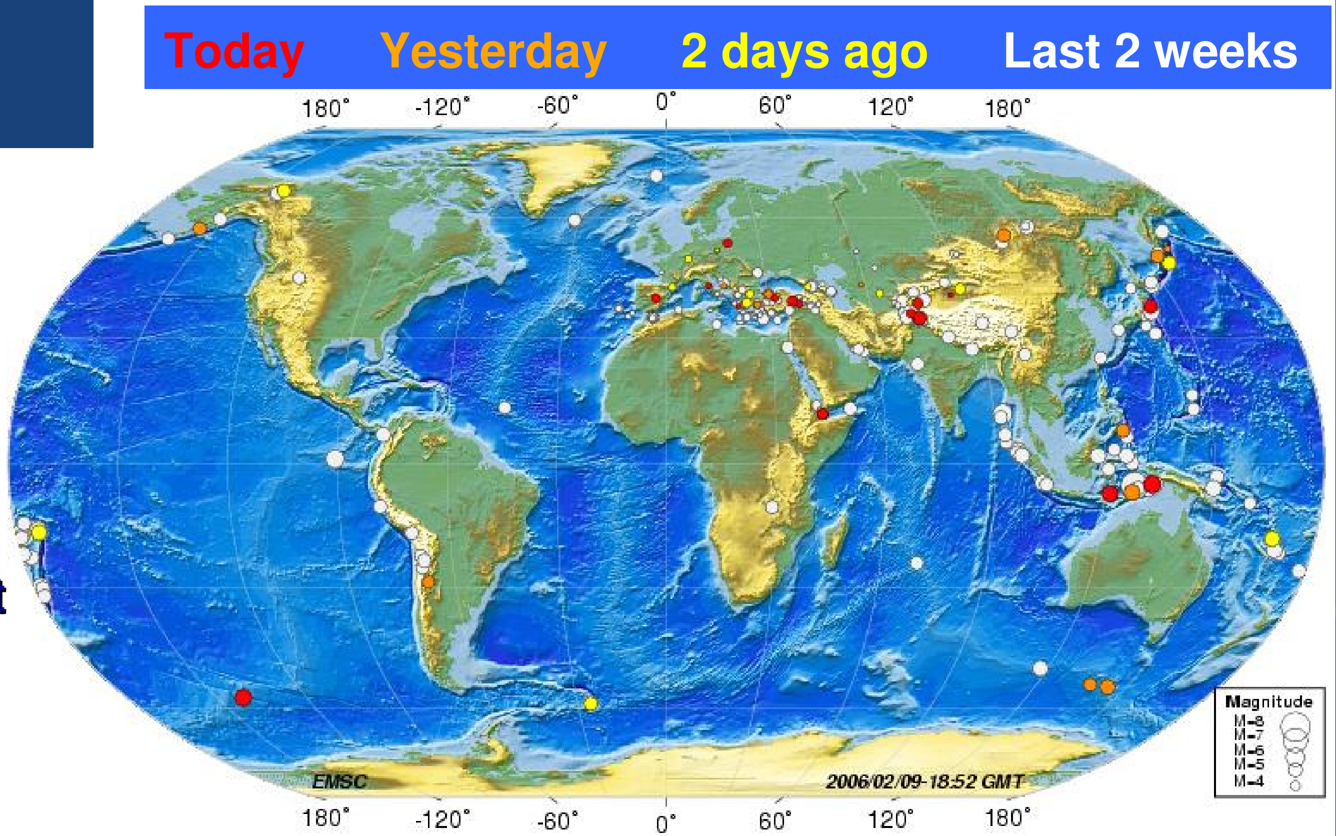


- Earth crust is broken into plates moving due to convection currents in the magma
- Most quakes occurs at plate boundaries (faults)
- Energy builds up in "locked" plates → QUAKE
- Fast energy release → P and S waves



Earthquakes exhibit self-similar scaling in energy, time and space distributions

Gutenberg-Richter Law

1954

$$P(>M) \sim 10^{-b\,M} \quad (b\sim1)$$

Kanamori,Anderson 1975

$$P(>M_0) \sim M_0^{-\alpha}$$

Seismic moment

$$M_0 = \mu A \Delta u \propto \frac{energy}{size}$$

$$M = (2/3)\log(M_0)-6$$

Universality of

$$\alpha \sim 0.7$$

Kagan 1994

Time distribution

Omori law

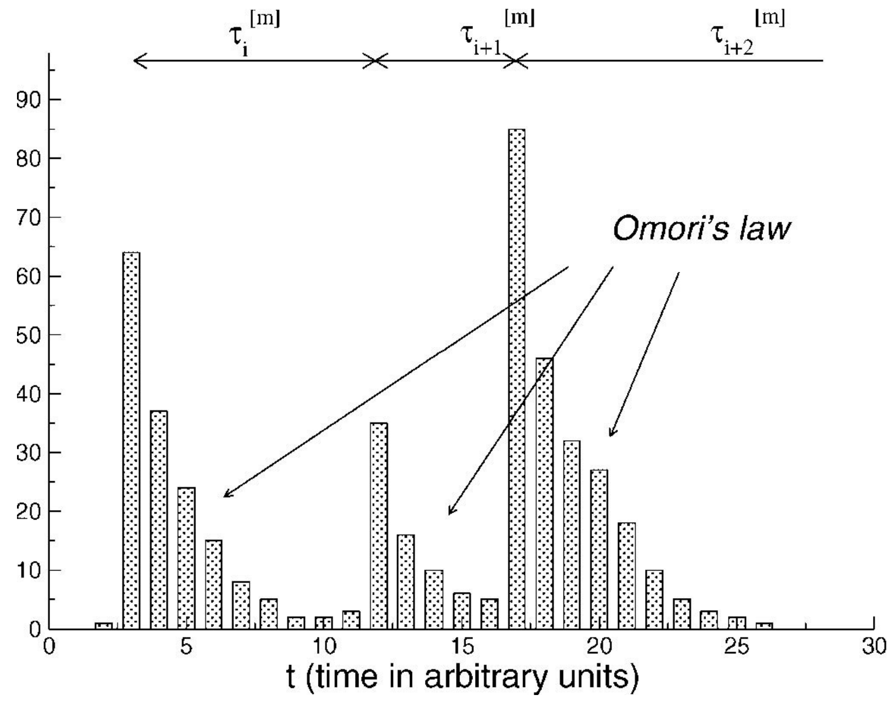
1894

At time t after a main shock at t=0

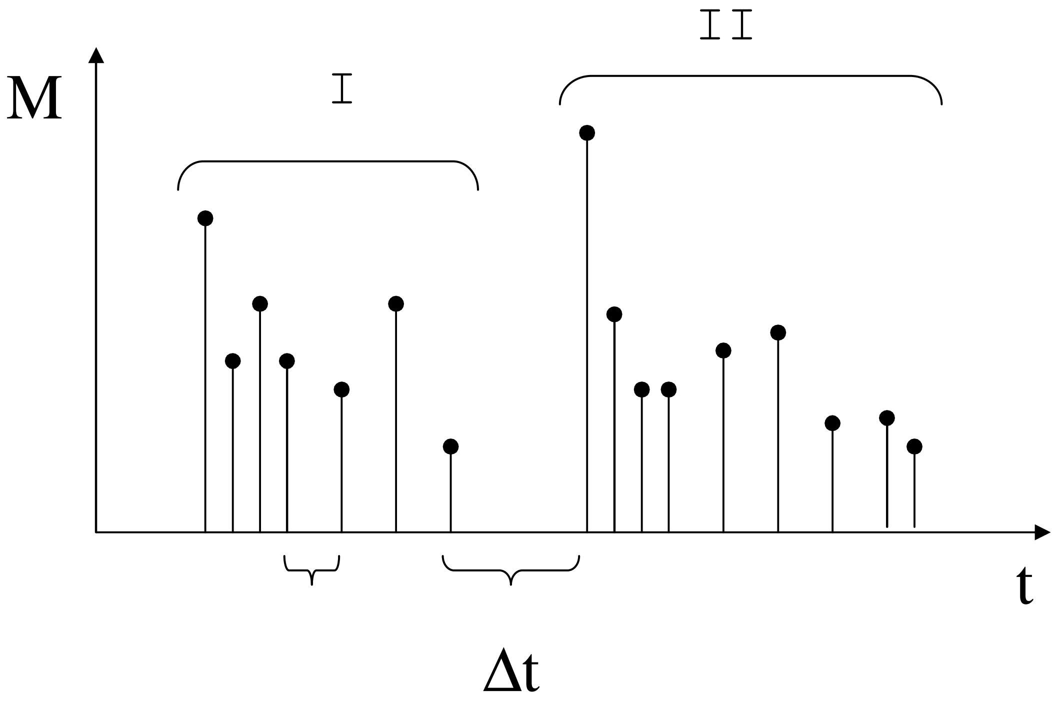
$$N_{AS}(t) \sim t^{-p}$$

$$N_{AS}(M) \sim 10^{\alpha M}$$

$p \sim 1 \quad \alpha < 1$



Inter-arrival time distribution N(Δt)



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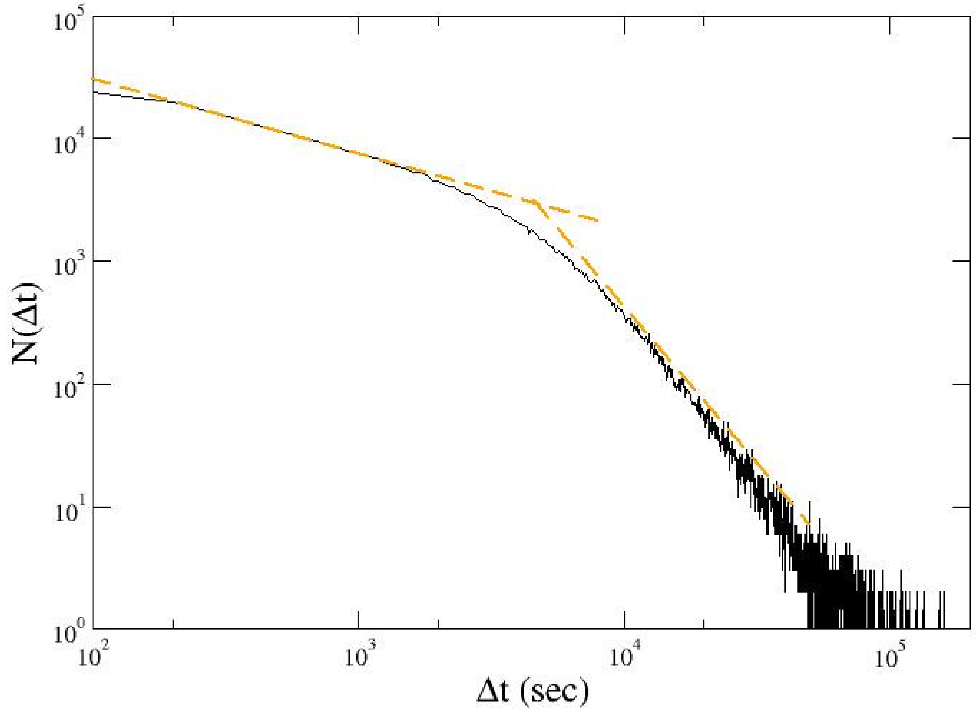
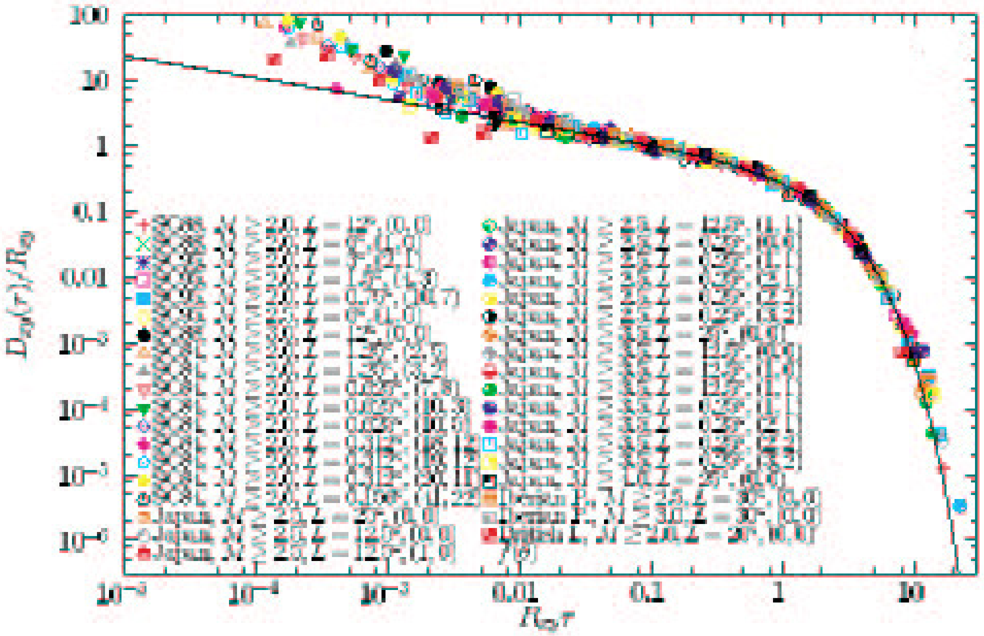


California catalog
356000 events with M>1
from 1967 to 2002

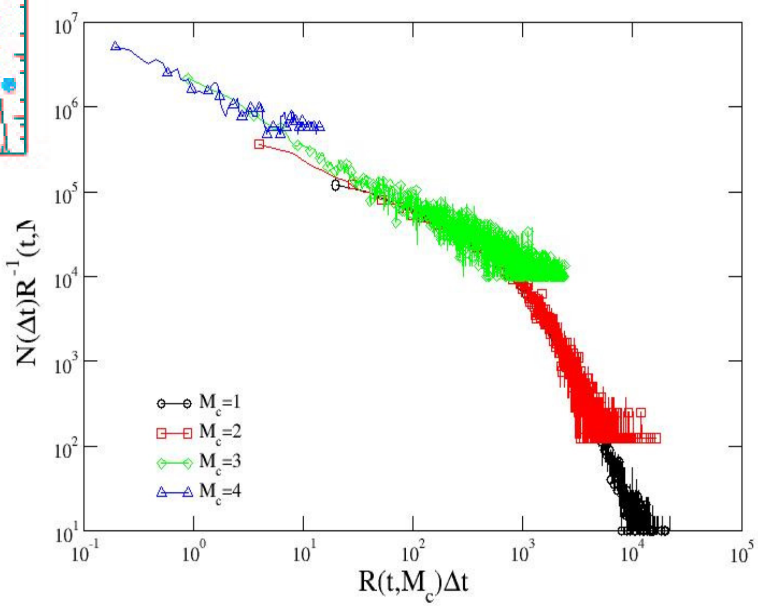
Scale behaviour of N(Δt)

N(Δt) independent of threshold M_c and geographic area



Corral, PRL 2004

Organization of big shocks follows same organization of small shocks



$R(t,M_C)$ is the rate of events with $M>M_C$ in a given geographic region

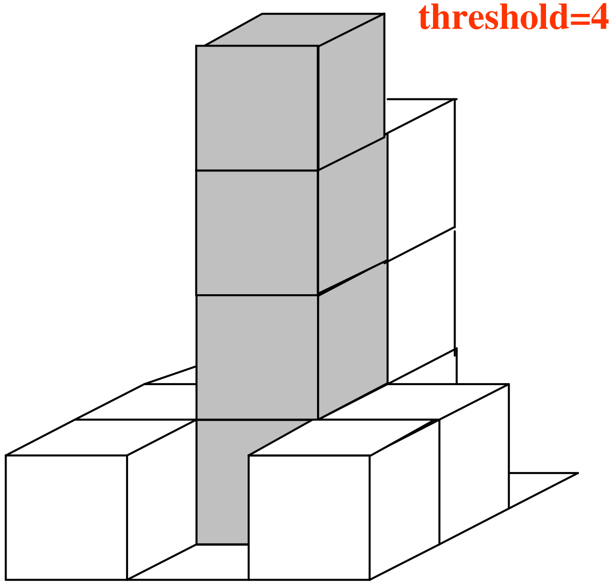
SELF-ORGANIZED CRITICALITY

Bak, Tang, Wiesenfeld, PRL 1987

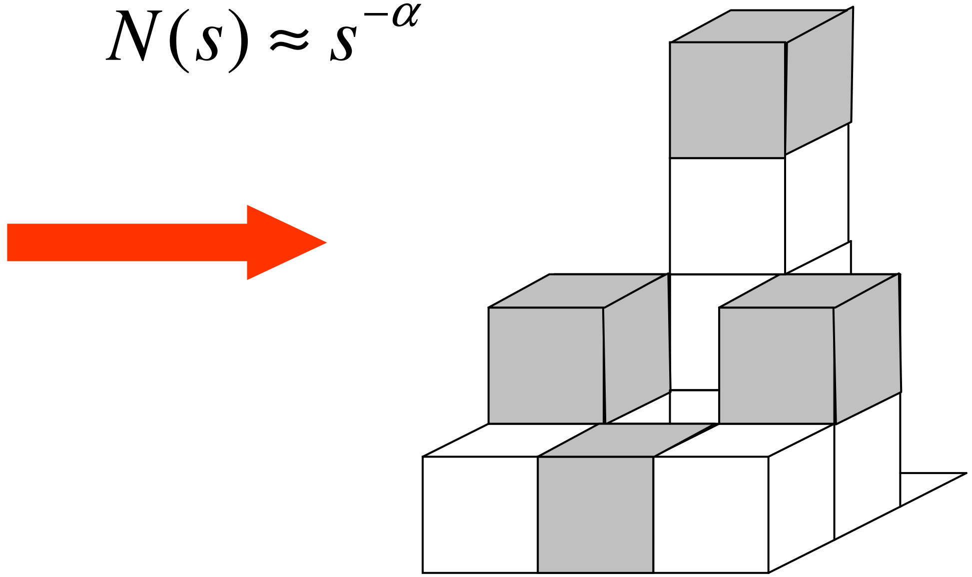
Dynamical systems spontaneously evolving toward a critical state without parameter tuning → no characteristic event size

Sand pile

by adding at random one grain…



$$N(s) \approx s^{-\alpha}$$



Fundamental ingredient: separation of time scales

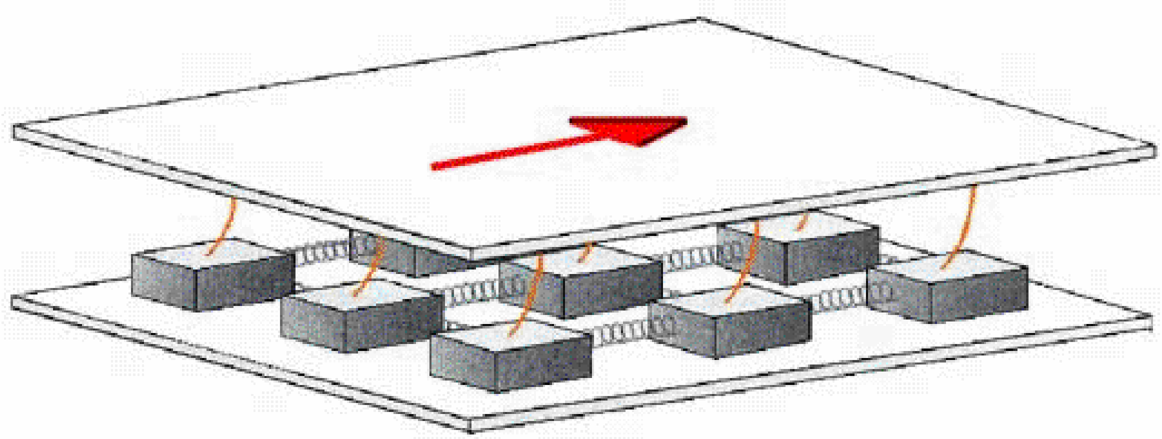
- **Slow scale: adding a grain**
- **Fast scale: propagation of an avalanche**

Soc applied to many natural phenomena

- Slides and avalanches
- Neural activity
- Solar flares
- Fluctuations in confined plasma
- Biological evolution
- Earthquakes

Burridge-Knopoff model

1967



Carlson-Langer
PRL 1989

Size distribution

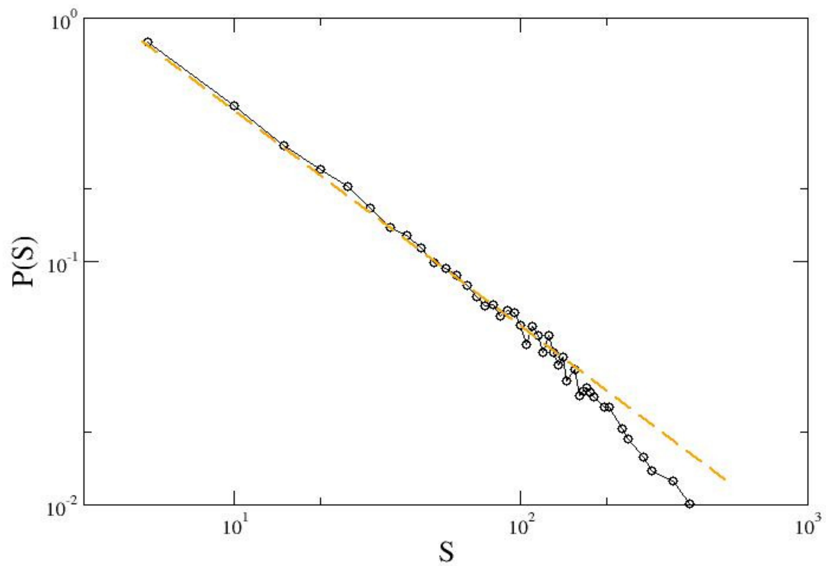
$P(S) \sim S^{-1}$

Parameter independent

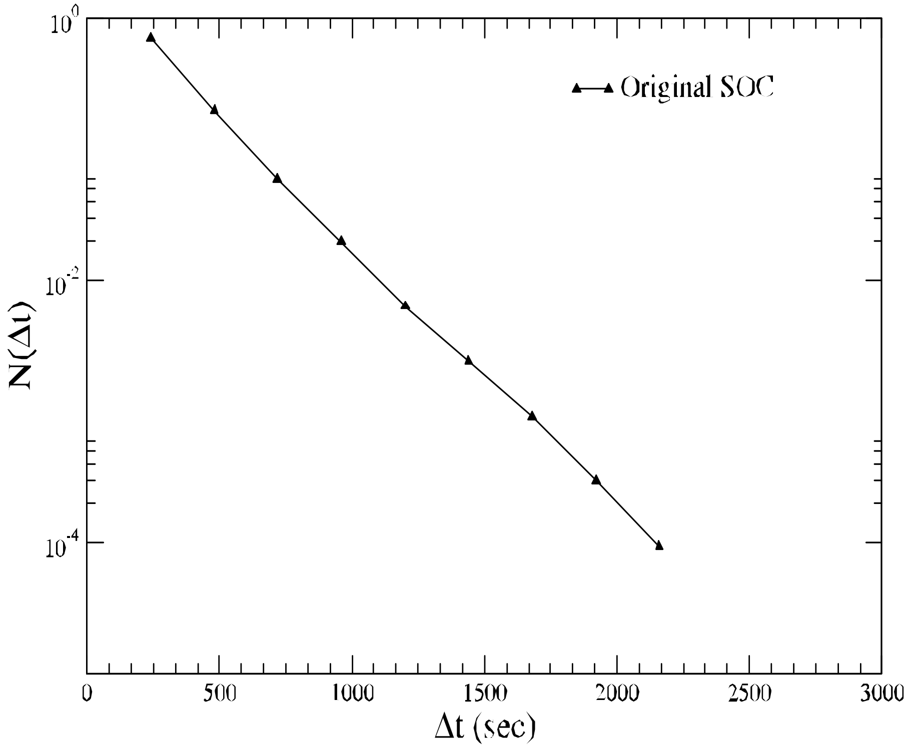
Similar to experimental distribution
for seismic moment

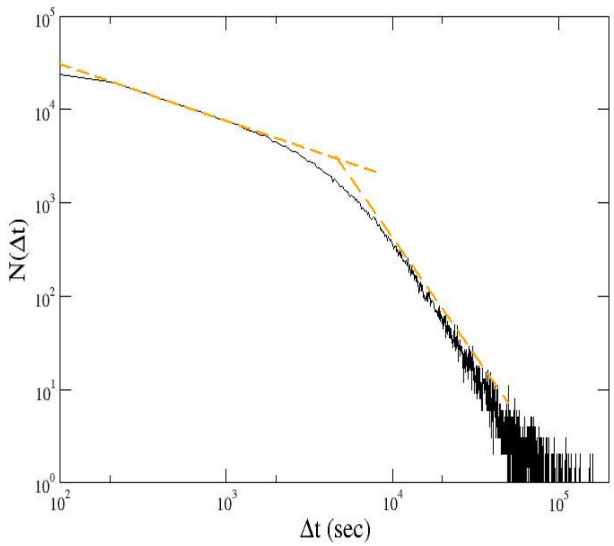
Nonconservative model, Olami Feder

Christensen PRL 1992



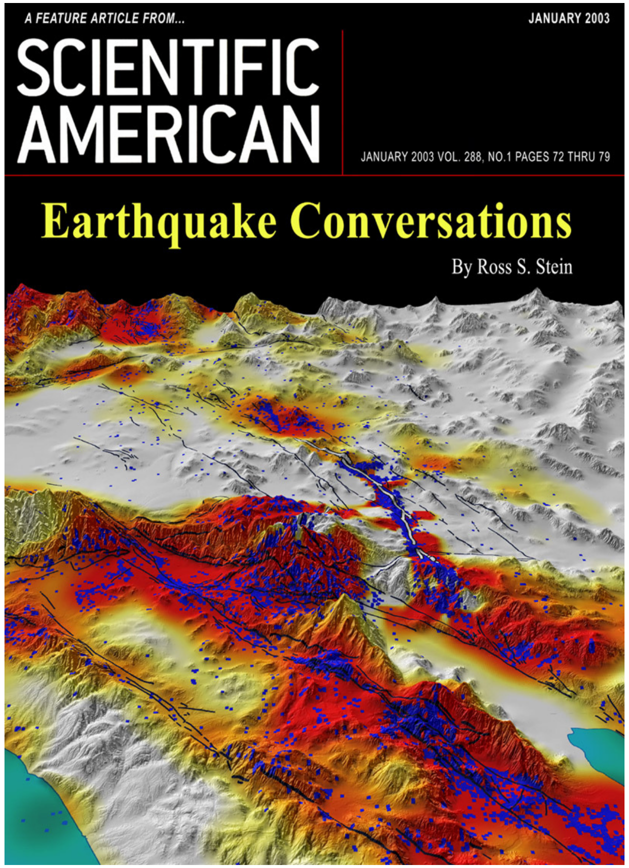
Intertime distribution in SOC:exponential behaviour





California catalog

Poisson process → Impredictibility



Earthquake interactions are a fundamental feature of seismicity leading to clustering and aftershocks:

- A large earthquake modifies sesmic activity in large areas
- In 1992 Landers event (m=7.3) triggered 3h later Big Bear event (m=6.5) on a different fault
- Static stress changes cannot explain this seismic activity increase (<0.01 bar for Landers) and remote triggering

Non linear interactions and Coulomb stress changes modify friction law on remote faults

State-rate formulation (J. Dieterich, JGR 1994) for frictional instability writes friction in terms of normal stress, slip velocity and the system ***state*** (temperature distribution, pore pressure variation, chemical reaction, etc..)

The seismicity rate

$$R(t) = f\,(\Delta\sigma, R(t-\varepsilon))$$

Memory within SOC

Time dependent friction law

Lippiello, LdA, Godano
EPL 2005

At each site we define

Local stress S_i and C_i counter

Herrmann, Kertesz LdA
EPL 1989

- **Counters contain entire history of applied loads**
- **Earthquake triggering is detemined by the combined effect of the increase in the local stress and the local weakening of the fault due to load global history**

Stress corrosion — Combined effect of mechanical stress and chemical agents

Fatigue — Material weakens under cyclic loads

Triggering mechanism

$$p_i = (s_i - s_c + z)/z$$

if $s_c - z \le s_i \le s_c$ **whereas** $p_i = 0$ **if** $s_i < s_c - z$ **and** $p_i = 1$ **if** $s_i > s_c$

Measures how close a site is to the critical stress

$$\alpha_i \equiv \frac{c_i}{p_i}$$

Seismic fracture probability depends on local instant stress and stress history contained in counters

$$0 \le c_i \le 1$$

Firing at threshold

$$\alpha_i < \alpha_c$$

Sites with $\alpha_i < \alpha_c$, are unstable because of the combined effect of high local stress p_i and entire history of loads stored in the counters c_i

ALGORITHM

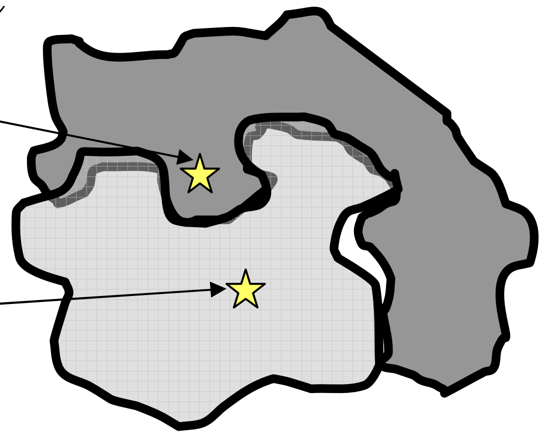
We start with a random initial configuration of s_i and c_i

- $s_i(t+1)=s_i(t)+1 \quad \forall i$
- evaluation of α_{min} (extreme value statistics)
- IF $\alpha_{min} > \alpha_c$
- $c_i(t+1)=c_i(t)-\alpha_{min}\, p_i \quad \forall i$
- IF $\alpha_{min} < \alpha_c$

long range correlations and remote triggering

2° epicenter

1° epicenter



Earthquake starts

➢The process goes on until no unstable ($\alpha_i < \alpha_c$) sites are present

➢The counters of all discharging sites are set to 1

Short time behaviour

Number of active sites at constant load → Rate of aftershocks

Define $q_i(t) = 1 - \alpha_i$ **for** $0 \le \alpha_i \le 1$ **, zero otherwise, where** $\alpha_i \equiv \frac{c_i}{p_i}$

At constant load $q_i(t+1) = q_i(t) - \alpha_{\min}$ **Or if site fires** $q_i(t+1) = 0$

→ **Ensemble average** $<q_i(t+1)> = (<q_i(t)> - <\alpha_{\min}>) P_i^s(t)$ **where**

$P_i^s(t)$ **probability for site *i* to be stable at time t**

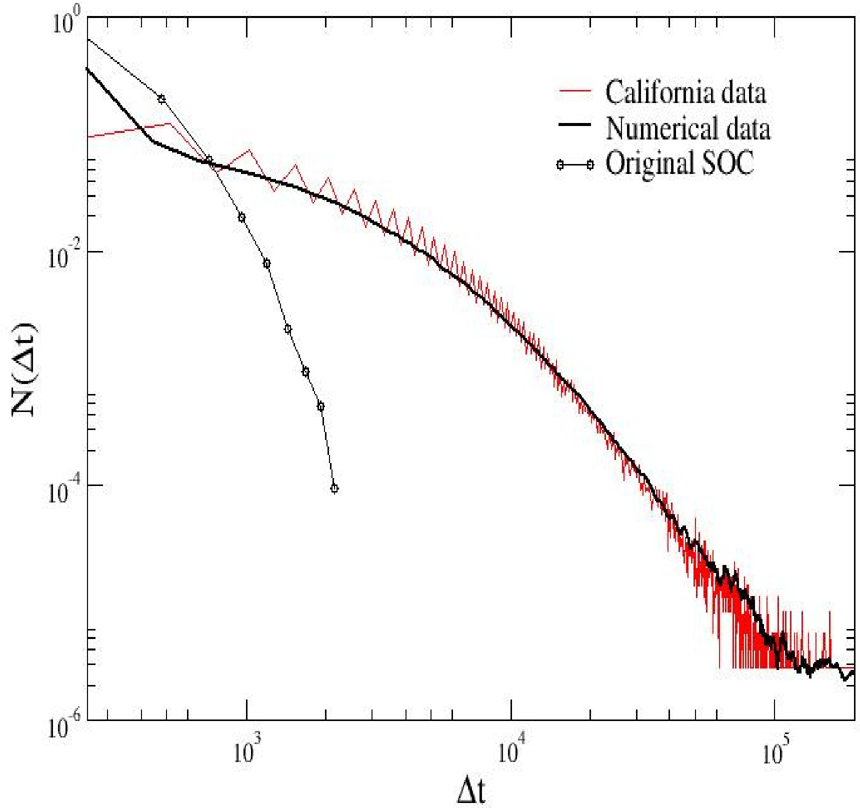
→ **Spatial average** $q(t) = \frac{1}{L^2} \sum_{i=1}^{L^2} q_i(t)$

- **fraction of active sites**
- **probability for a generic site to be unstable at time t**

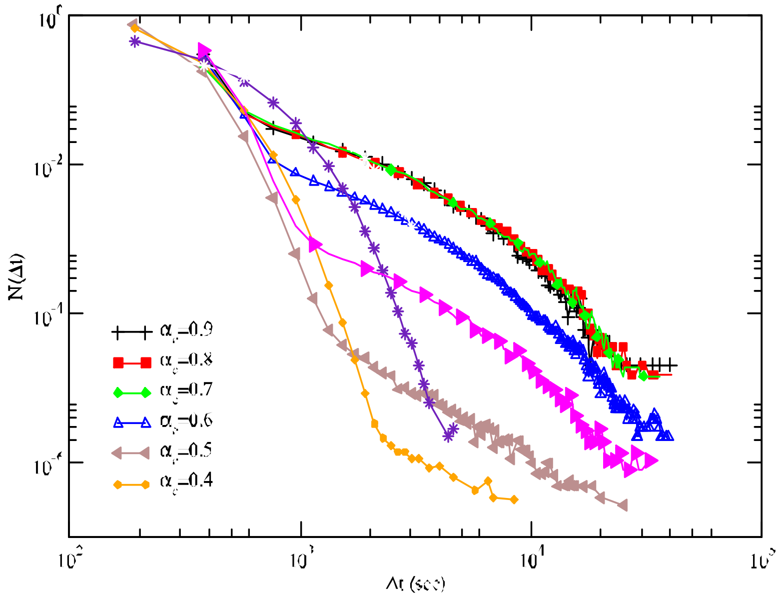
→ $q(t) \cong 1 - P_i^s(t)$ **for** $\alpha_{\min} << \alpha_i$ $q(t+1) \cong q(t)(1 - q(t))$

→ $q(t) \approx t^{-1}$ **Omori law**

Intertime distribution



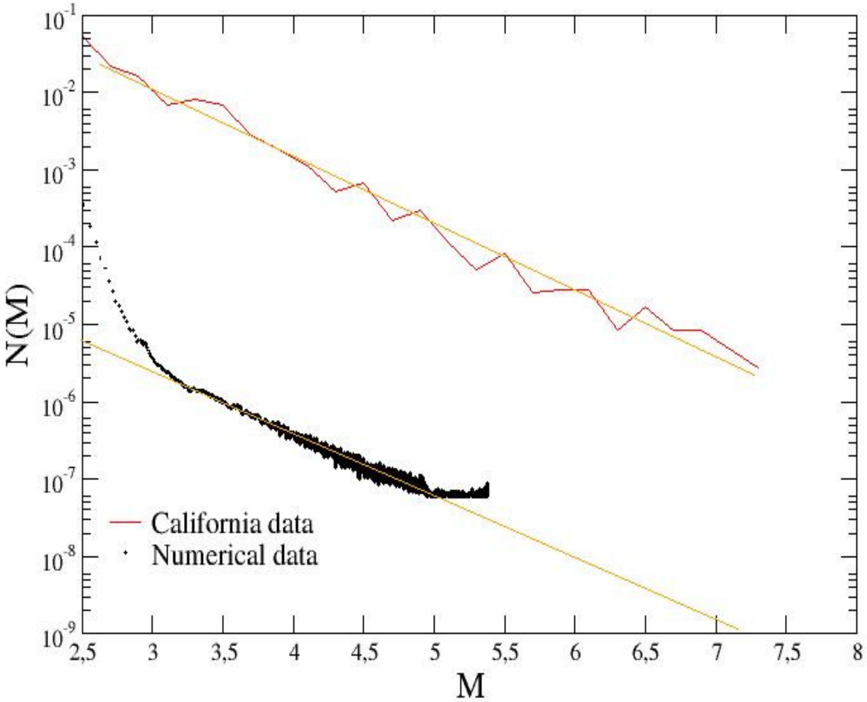
α_c=0.9   L=500



α_c irrelevant variable

for $0.7<\alpha_c<1$

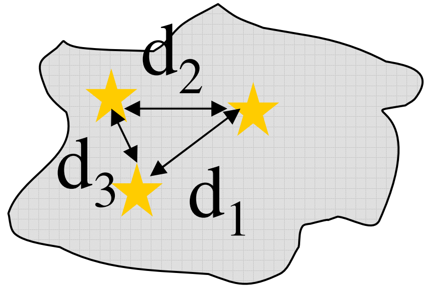
Magnitude and epicenter distance distribution



$N(M) \sim 10^{-bM}$

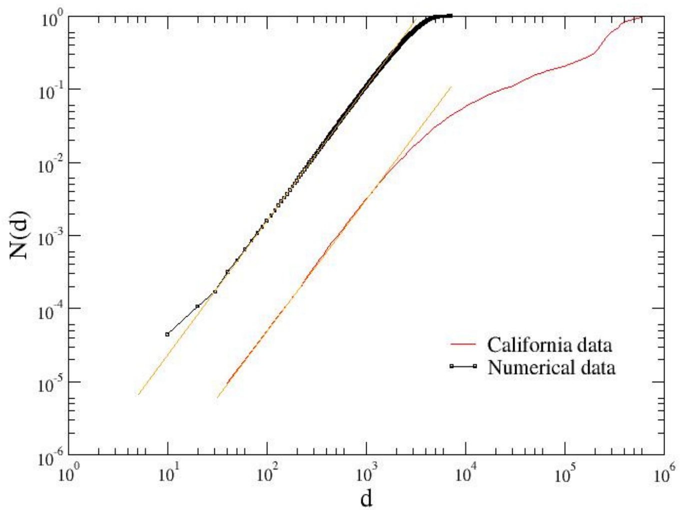
b=0.89 experimental

b=0.87 numerical



$N(d_i > d)$

exponent 1.84



Relation with the state-rate equation

State-rate formulation for stick-slip frictional instability

$$\tau = \Sigma_n \left[A \ln V + B \ln \gamma + C \right]$$

$$\frac{d\gamma}{dt} = 1 - D\gamma V$$

τ friction, V slip velocity, Σn normal stress, γ system state

A B C D phenomenological constants

In the model $\tau \approx s_c - z + z c_i / \alpha_c$ where $\Sigma_n \propto s_c$ is constant and the slip velocity $V = \theta(s_i - \tau)$

→ $c_i = \ln \gamma_i$ and from $c_i^{new} = c_i^{old} - \alpha_{min} p_i$

we recover

$$\frac{d\gamma}{dt} = -\gamma_i p_i \alpha_{min}$$

Summary

Complex seismic activity is controlled by:

- local stress redistribution
- long range correlations in the earth crust

These mechanisms are implemented in self-consistent local laws containing long range memory of stress history

- Large event may increase seismic activity by inducing global wekening,

  ……...or inihibit future earthquakes by resetting global memory

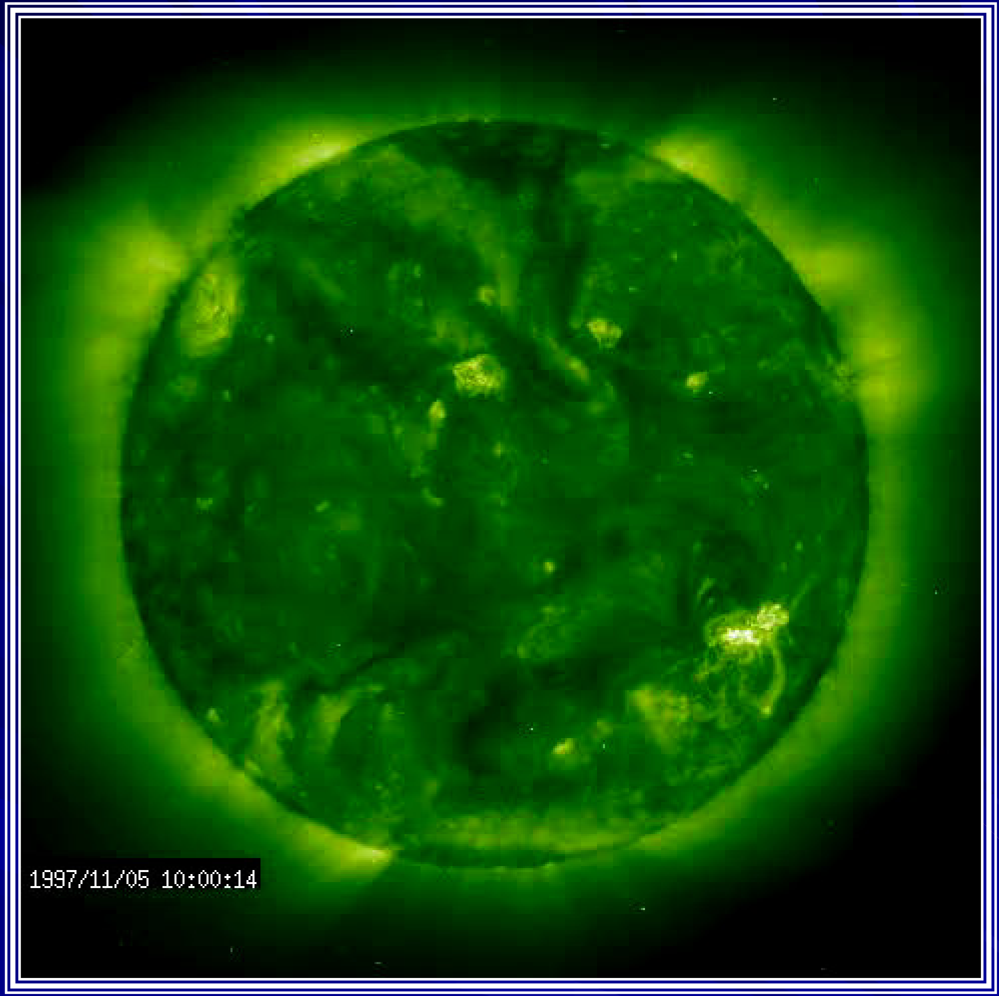
- Global memory → stress corrosion, pore pressure variation, fault gauge deterioration,….

Are there other phenomena with similar features?

Solar Flares

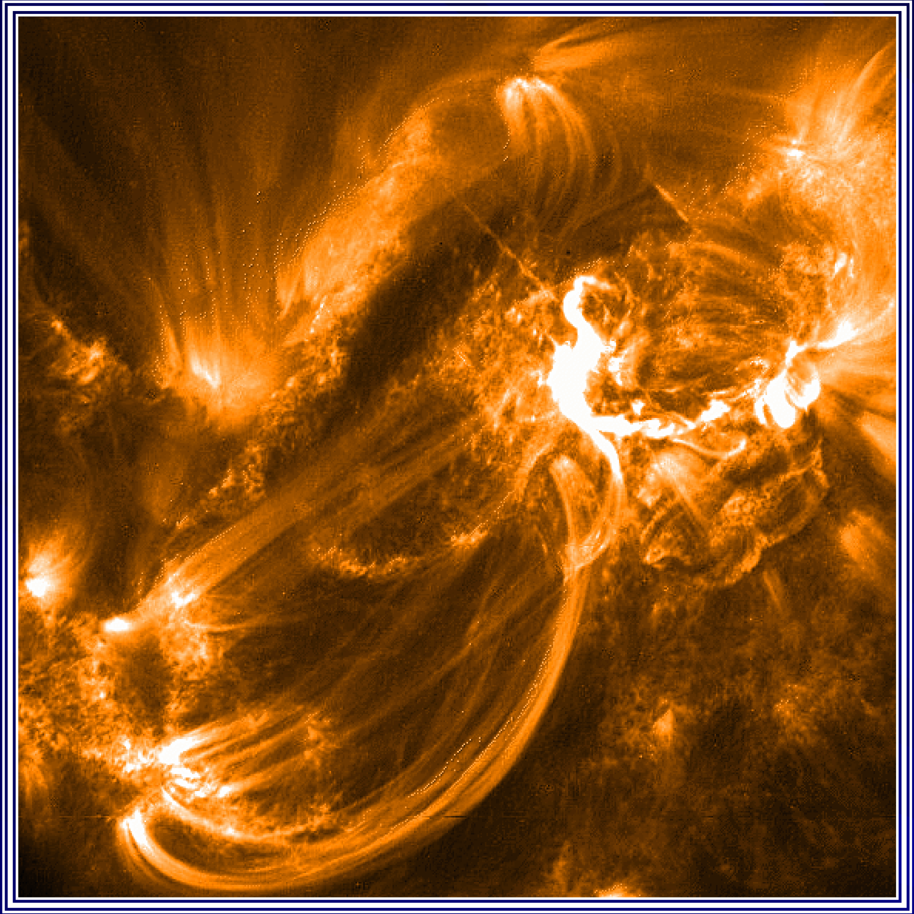
Solar flares are explosions of incredible power and violence that release energy equivalent to about 100 hurricanes in few minutes.

The strong and twisted magnetic fields in the vicinity of active sunspot groups are thought to provide the power that is released in the solar flares.



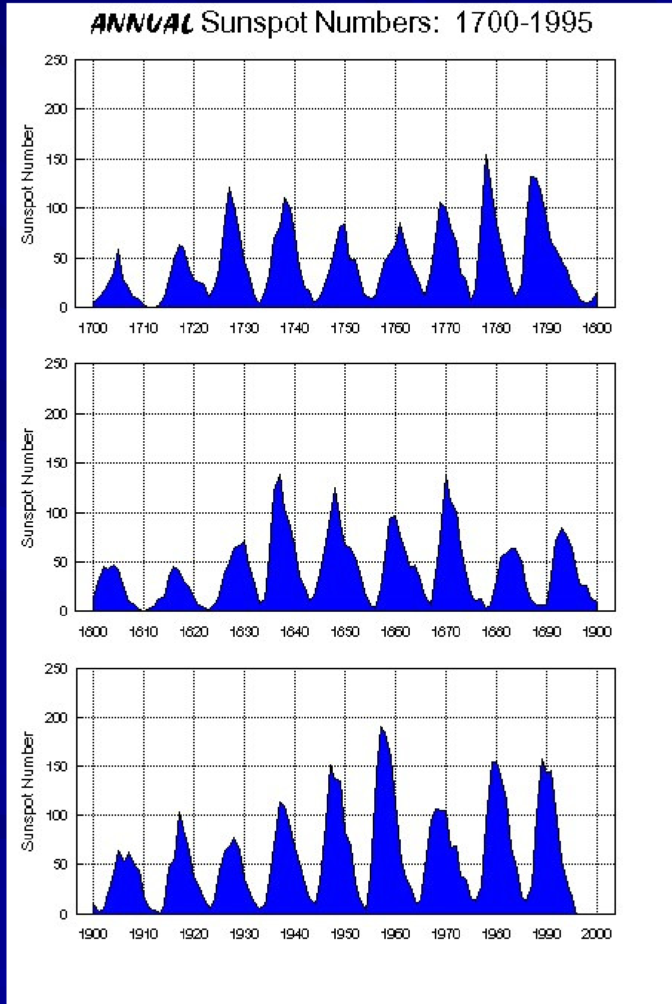
It is not known exactly how this occurs.

http://www.windows.ucar.edu/spaceweather/sun_earth4.html



Large quantities of Xrays and hot particles may strike the Earth following a solar flare event.

The Xrays arrive in just 8 minutes time and the hot particles follow several hours later.



The strongest flares occur only several times per year even in solar maximum.

Weaker flares are relatively common. Several tens of these weaker flares can occur in one day during active periods.

Analogy with earthquakes

- Impulsive localised release of energy
- Huge fluctuations
- Power law in the distribution of flare size

Various interpretations for power law behaviour:

- turbulence (Boffetta,Carbone,Giuliani,Veltri,Vulpiani PRL 1999)
- magnetohydrodynamics (Parker Physics Today 2000, Alpert Sci. Am. 2000)
- self-organised criticality (Lu, Hamilton PRL 1991, Hamon,Nicodemi, Jensen AA 2002)

A better understanding of flare time occurrence would improve knowledge of physical mechanism behind…

We consider different solar data catalogs

Name	*Catalog*	*Nevents*	*Energy*	*Period*
GOES	Soft X rays	21567	1.5-2.4 keV 3.1-24.8 keV	Jan 1992-Dec 2002
BATSE	Hard X rays	6658	>25 keV	Apr 1991-May2000
WATCH	Intermediate X rays	1551	10-30 keV	Jan 1990-Jul 1992

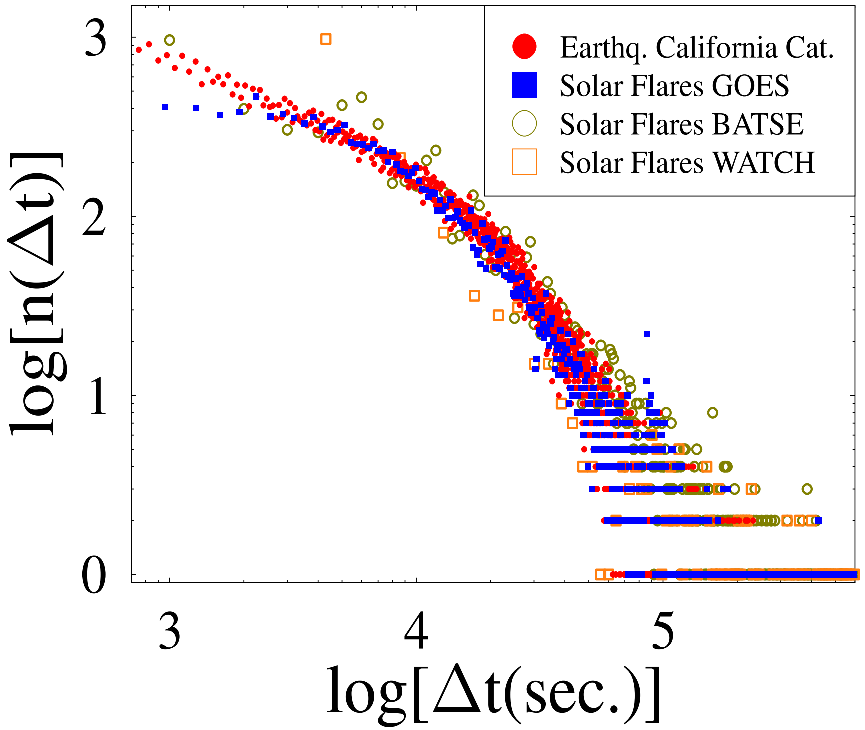
and compare them with the Southern California catalog...

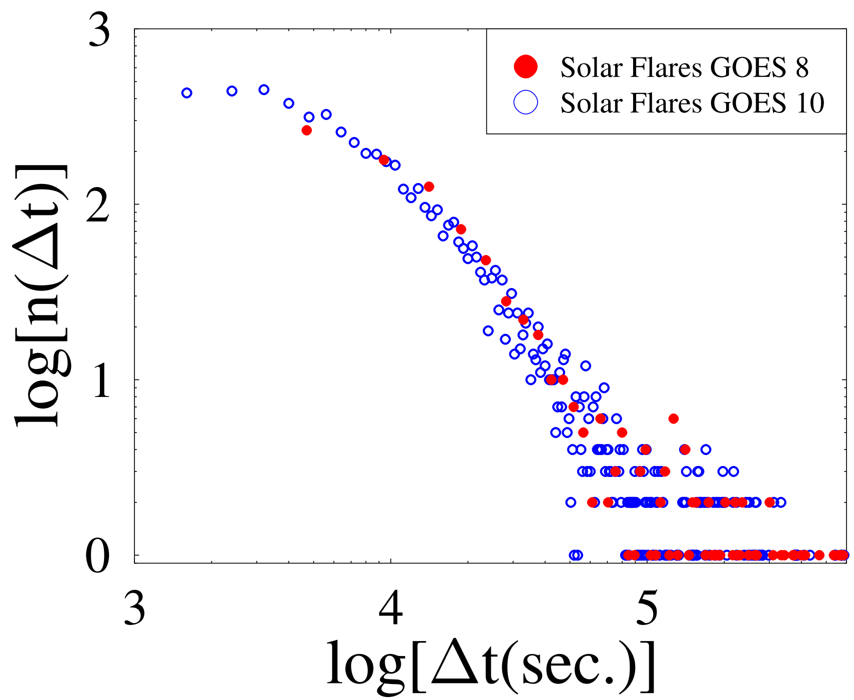
LdA, Godano, Lippiello, Nicodemi PRL 2006

Intertime distribution

We define Δt time between start of a flare and the next one

For a catalog with *Ne* events $n(\Delta t)$ counts number of events between Δt *e* $\Delta t+\lambda/Ne$ where λ sets the binning of raw data ($\lambda/Ne=75$ *sec* for the California catalog)





Events with peak flux > $1.4\cdot 10^{-6}Wm^{-2}$ (class C1) in maximum

> $10^{-7}Wm^{-2}$ (class B1) in minimum

Data collapse does not depend on solar phase

Size distribution

Number of events with size between s/s_0 and $s/s_0 + \lambda/Ne$

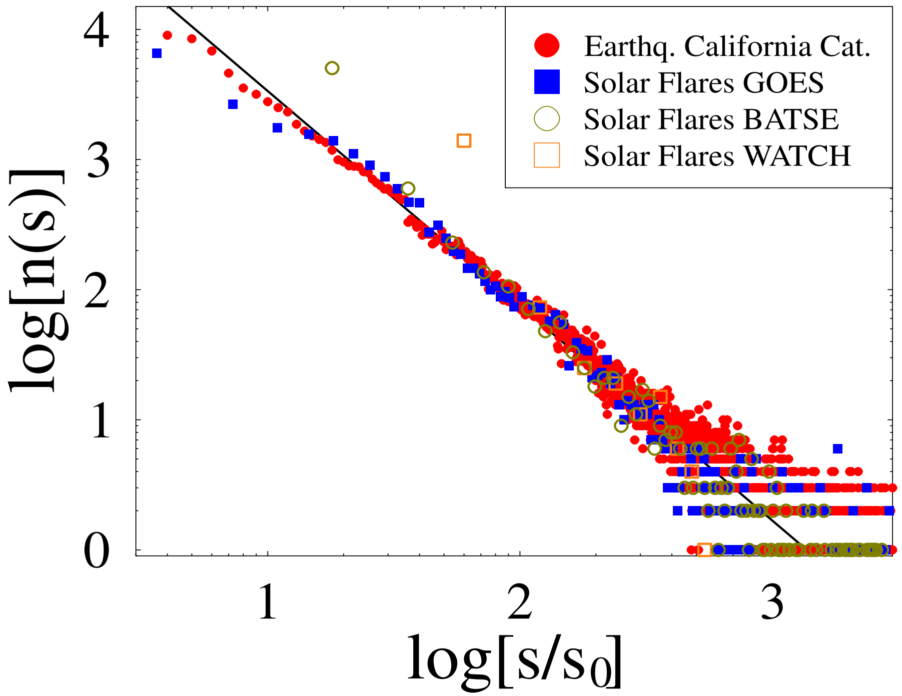
$\lambda/Ne = 1$ for the California catalog s_0 constant for each catalog

$$n(s/s_0) \approx (s/s_0)^{-\alpha}$$

$$\alpha = 1.65 \pm 0.1$$

Richter scale for flares

$$M_F = 2/3 \log(s) - K_F$$



Omori law

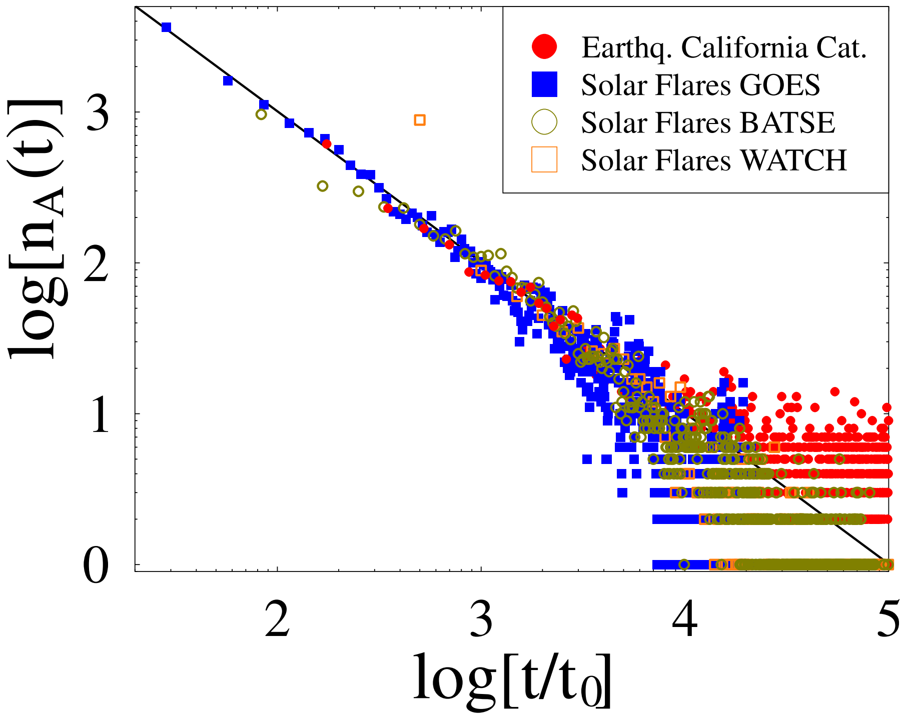
Main event → event with *M*>*M*main

After event → event with *M*cut<*M*<*M*main

with *M*cut = *M*main -2.5

$$n_A(t) \approx 1/t$$

Mainflares trigger sequences of afterflares



Common physical mechanism?

Faults → Active regions in the sun

Elastic energy → Magnetic energy

Slow drive → Emergence of new magnetic flux from sun's interior in active regions, shuffling of magnetic loops

Stick-slip → Complex magnetic field structures lead to energy dissipation via magnetic reconnections

Avalanche → One reconnection may trigger a cascade…..

- State-rate formulation for solar flares?
- Flare triggering depends on the entire history?
- Trigger models (ETAS) for solar flares?