Giunzioni Josephson Anulari ed intrappolamento di quanti di flusso

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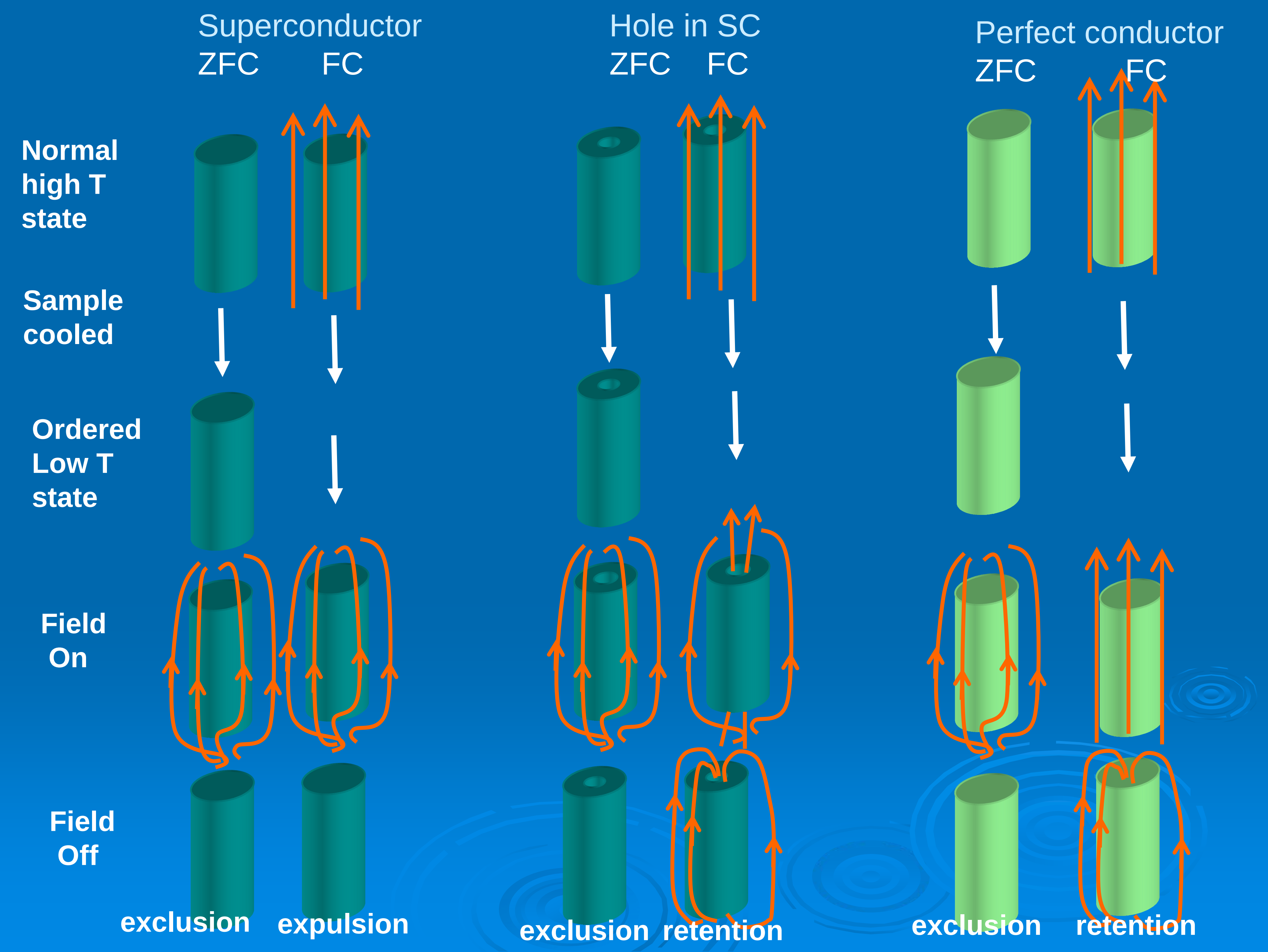
Sommario

- Multiply connected Superconductors. Meissner effect.
- Flux conservation. Flux quantization
- Josephson effect (also in magnetic field)
- Josephson vortices and other kind of vortices
- Annular junctions
- sine-Gordon equation and long annular Josephson junctions
- How to insert vortices in long annular junctions and how count them (with a brief cosmological parenthesis)
- small annular junctions: how to count vortices in small a.j.
- Resonances in small annular junctions
- Detectors
- How to put all together to solve a “device problem”
- When there is not room enough


Superconductor
ZFC
FC
Hole in SC
ZFC
FC
Perfect conductor
ZFC
FC
Normal high T state
Sample cooled
Ordered Low T state
Field On
Field Off
exclusion
expulsion
exclusion
retention
exclusion
retention

London equations

Fritz and Heinz London (1935)

$$\lambda_L = \left(\frac{m^*}{\mu_0 n_s e^{*2}} \right)^{1/2}$$

$$\mathbf{E} = \mu_0 \lambda_L{}^2 \frac{d}{dt} \mathbf{J}$$

$$\mathbf{B} = -\mu_0 \lambda_L{}^2 \nabla \times \mathbf{J}$$

Using Maxwell's Equations we get:

$$\nabla^2 \mathbf{B} = \frac{\mathbf{B}}{\lambda_L{}^2}$$

$$\nabla^2 \mathbf{J} = \frac{\mathbf{J}}{\lambda_L{}^2}$$

Fluxoid conservation

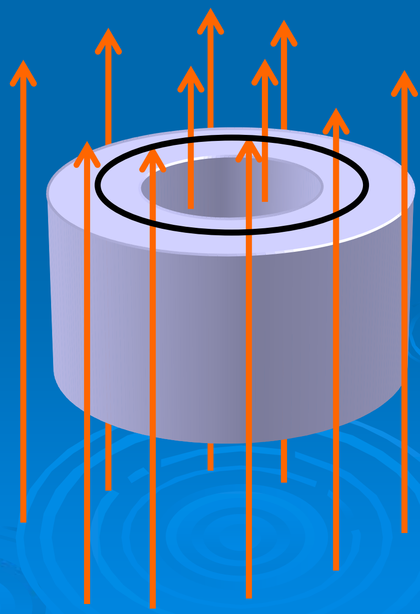
Integrating the first London's equation along one of the possible closed path C surrounding the cavity

$$\mathbf{E} = \mu_{\mathbf{0}} \lambda_L{}^2 \frac{d}{dt} \mathbf{J}$$

$$\frac{\partial}{\partial t} \int_C \mu_0 \lambda_L{}^2 \mathbf{J} \cdot d\mathbf{l} = \int_C \mathbf{E} \cdot d\mathbf{l} = \int_S \nabla \times \mathbf{E} \cdot d\mathbf{S} = -\int_S \frac{\partial}{\partial t} \mathbf{B} \cdot d\mathbf{S}$$

We get for the "fluxoid" (indipendent from C):

$$\int_C \mu_0 \lambda_L{}^2 \mathbf{J} \cdot d\mathbf{l} + \int_S \mathbf{B} \cdot d\mathbf{S} = const. \quad in\ time$$



Since the current can flows only in a surface layer Taking C in the bulk gives J=0. So for the "flux":

You can turn off the field but Φ does not change: Is a trapping!

$$\Phi = \int_S \mathbf{B} \cdot d\mathbf{S} = const$$

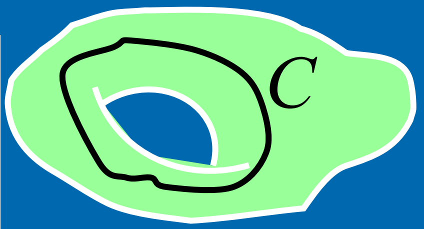
Flux quantization in a multiply-connected superconductor

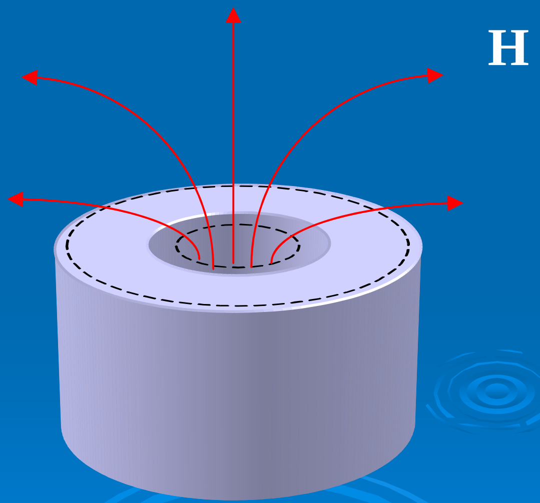
- **The previous constant cannot take any value. Simply apply the Bohr-Sommerfeld quantum condition:**

$$\Phi' = \oint_C \mu_0 \lambda^2 \mathbf{J} \cdot d\mathbf{l} + \int_S \mathbf{B} \cdot d\mathbf{S} =$$

$$= \oint_C \left(\mu_0 \lambda^2 \mathbf{J} + \mathbf{A}\right) \cdot d\mathbf{l} = \frac{1}{e^*} \oint_C \left(m^* \mathbf{v}_\mathbf{s} + e\mathbf{A}\right) \cdot d\mathbf{l} =$$

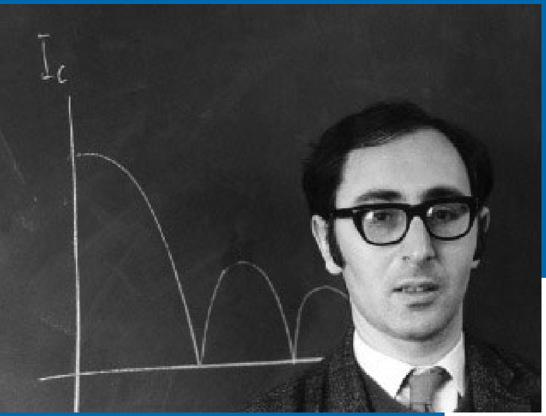
$$= \frac{1}{e^*} \oint_C \mathbf{p} \cdot d\mathbf{l} = \frac{nh}{e^*} = n\Phi_0; \quad \Phi_0 = \frac{h}{e^*}$$





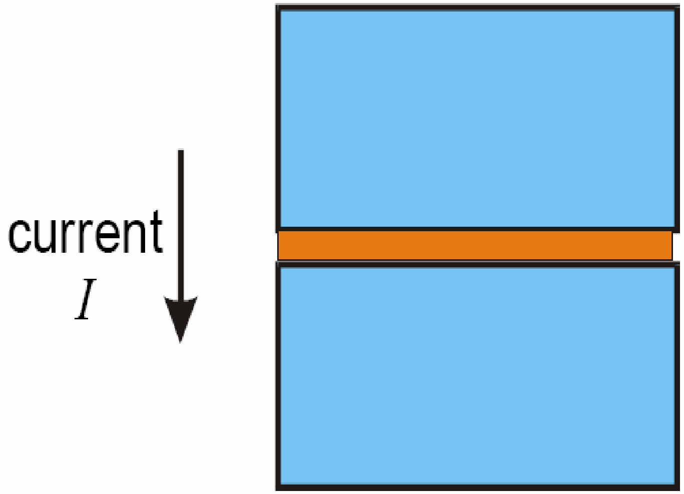
- This may also be seen as a very general requirement of single valuedeness of the order parameter.

Josephson effect



Nobel Prize in Physics 1973 with
Leo Esaki
Ivar Giaever

Obtained by B. Josephson in 1962



Wavefunctions of superconducting electrodes:

$$\Psi_1 = |\Psi_1|\exp(i\theta_1), \;\; \Psi_2 = |\Psi_2|\exp(i\theta_2)$$

Phase difference: $\varphi = \theta_2 - \theta_1$

Josephson equations

dc Josephson effect:

$$I_s(\varphi) = I_c \sin\varphi \quad (1)$$

ac Josephson effect:

$$\frac{d\varphi}{dt} = \frac{2e}{\hbar} V \qquad (2)$$

$$\Phi_0 = 2.068 \times 10^{-15}\,\mathrm{Wb}$$

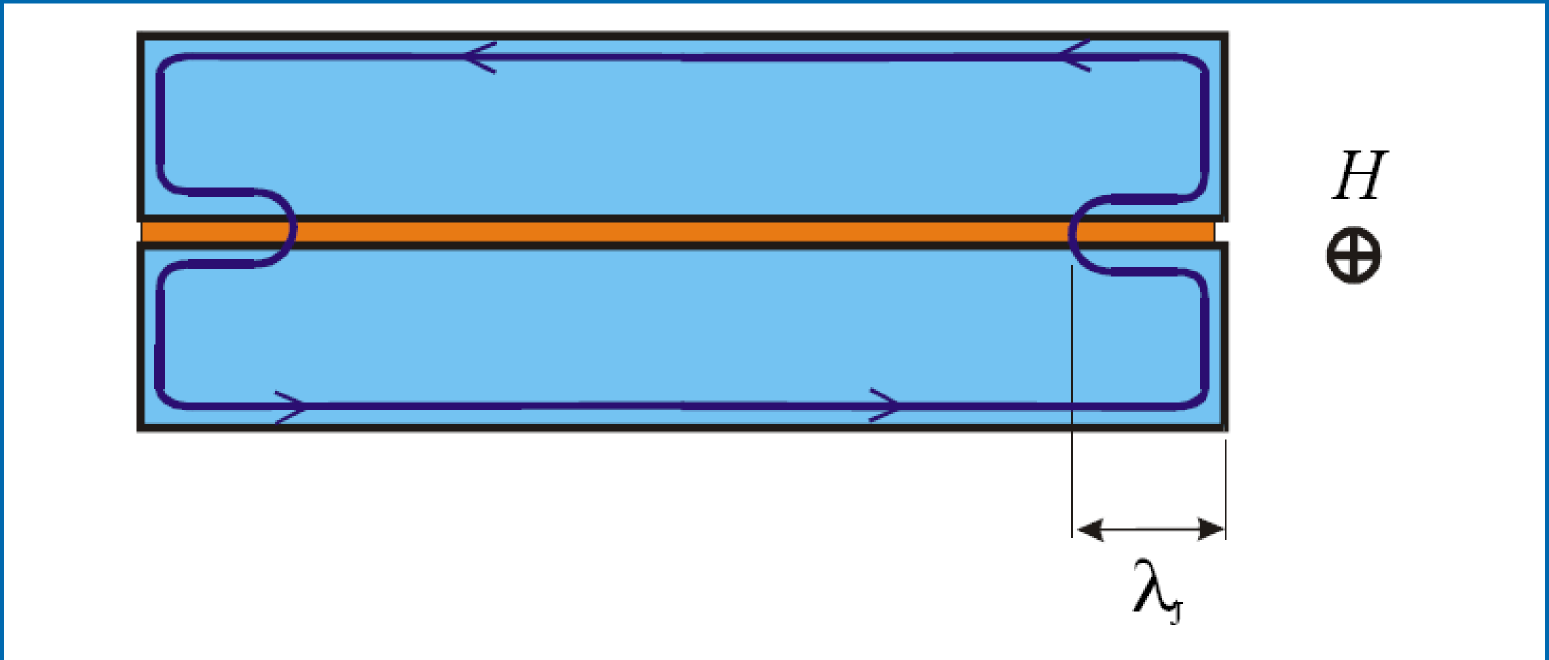
$$\varphi = \frac{2e}{\hbar} V t + \varphi_0$$

$$f = \frac{2e}{2\pi\hbar} V = \frac{1}{\Phi_0} V$$

Josephson junction is a *quantum dc voltage - to - frequency converter*

$$1\,\mu\mathrm{V} \leftrightarrow 483.59767\,\mathrm{MHz}$$

Josephson junction in magnetic field

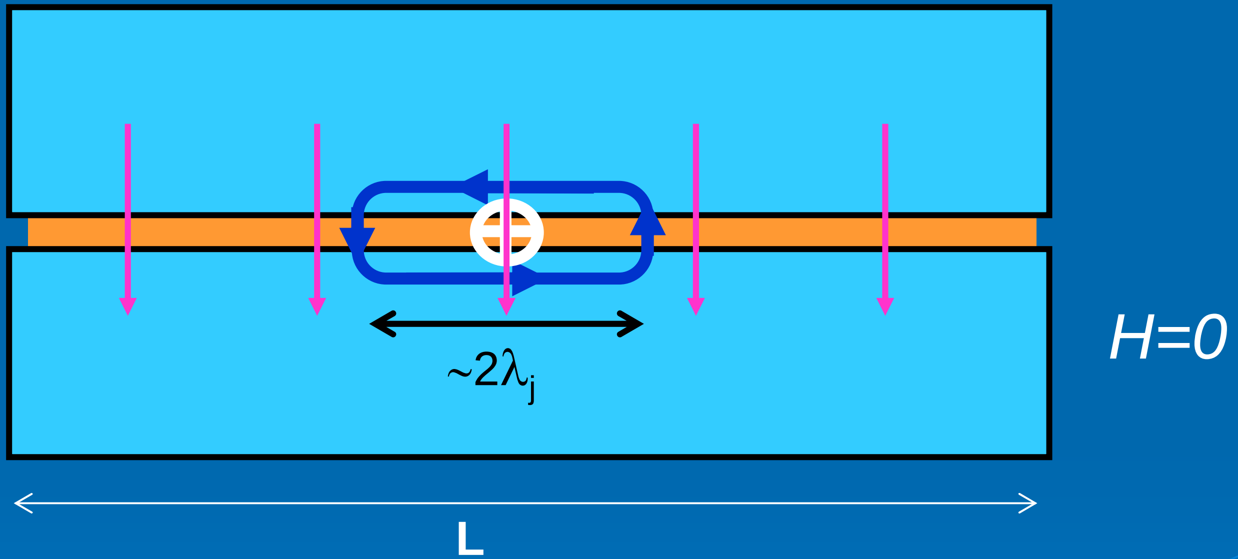


$$\lambda_J = \sqrt{\frac{\Phi_0}{2\pi j_c \mu_0 (2\lambda_L + t_0)}}$$

typical value: 10μm

Josephson vortex

- A stable self-sustained tunneling current structure
- It needs $L > \lambda j$



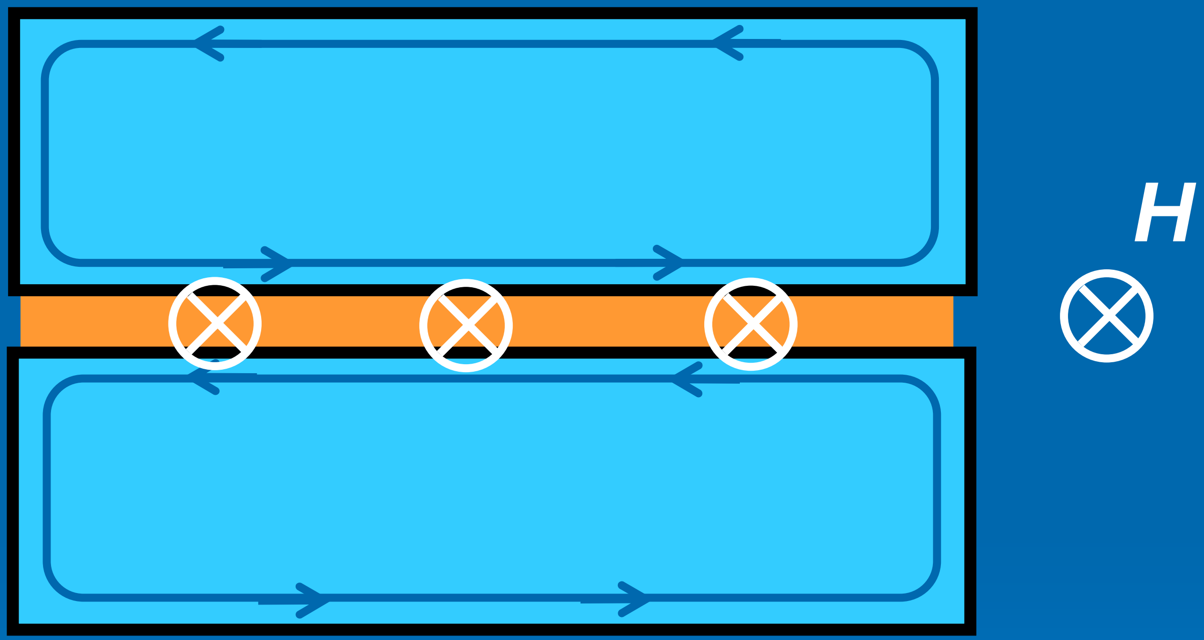
- Associated flux is the elementary flux quantum:

$\Phi_0 = 2.07 \times 10^{-15}$ tesla • m^2
$= 2.07 \times 10^{-15}$ V • s

magnetic field in the center of static fluxon

$H_F \sim 10^{-4}\,T = 1\,Oe$

Small junction in m.f.



The magnetic field penetrates completely
Because shielding is ineffective if L<<λj

Summary of the relevant relationships

dc Josephson effect:

$$I_s(\varphi) = I_c \sin\varphi \quad (1)$$

ac Josephson effect:

$$\frac{d\varphi}{dt} = \frac{2e}{\hbar} V \qquad (2)$$

effect of the magnetic field on the phase

$$\nabla_{xy}\varphi = \left(\frac{2\pi\mu_o d}{\Phi_o}\right)\mathbf{H}\times\mathbf{n}$$

(Third Josephson equation)

….plus Maxwell's equations…

Small junction in magnetic field

$$\frac{d\varphi}{dx} = \frac{2\pi\mu_0 d}{\Phi_0} H_y \qquad -L/2 < x < L/2;$$

$$k = \frac{2\pi\mu_0 d H_y}{\Phi_0}; kL = 2\pi\frac{\Gamma}{\Phi_0}; \Phi = dL\mu_0 H_y$$

$$\varphi = kx + \varphi_0$$

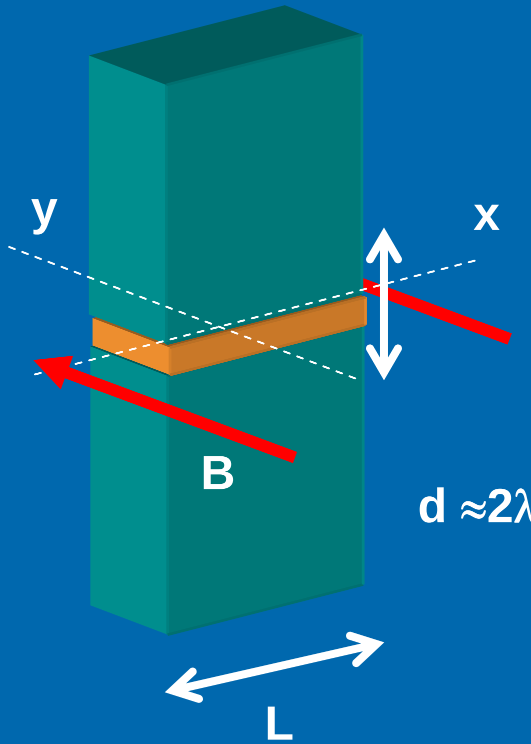
$$j = j_1 \sin\varphi = I_1 \sin(kx + \varphi_0)$$

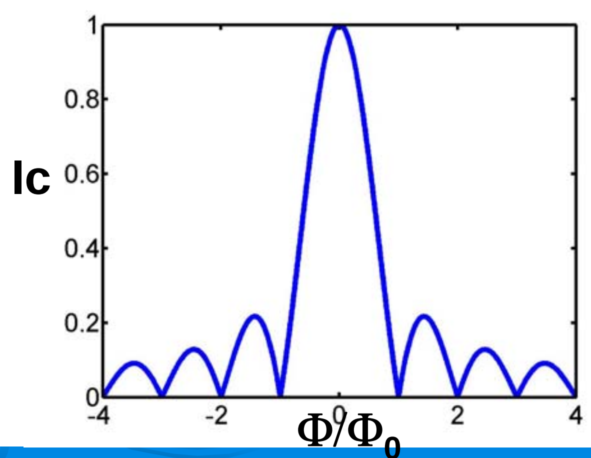
$$I_c = \overset{\max}{\{\varphi_0\}} j_1 \int_{-L/2}^{L/2} \sin(kx + \varphi_0) dx = \overset{\max}{\{\varphi_0\}} j_1 L \int_{-1/2}^{1/2} \sin(kL\bar{x} + \varphi_0) d\bar{x}$$

$$I_c(0) = j_1 L$$

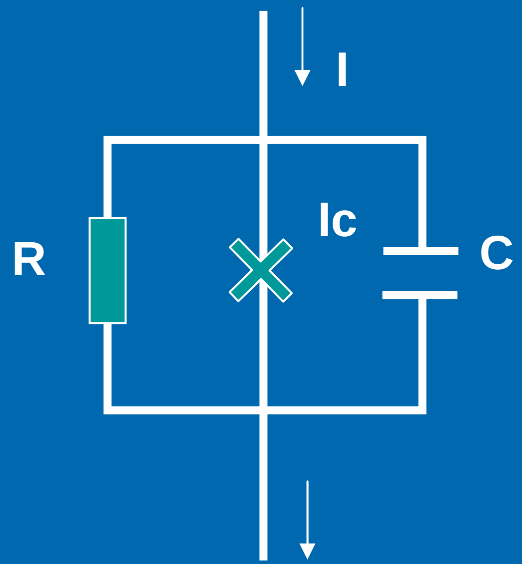
$$\frac{I_c(\Phi)}{I_c(0)} = \overset{\max}{\{B\}} \left(\sin B \frac{\sin\left(\pi\frac{\Phi}{\Phi_0}\right)}{\pi\frac{\Phi}{\Phi_0}} \right) = \left| \frac{\sin\left(\pi\frac{\Phi}{\Phi_0}\right)}{\pi\frac{\Phi}{\Phi_0}} \right|$$

Fraunhofer pattern



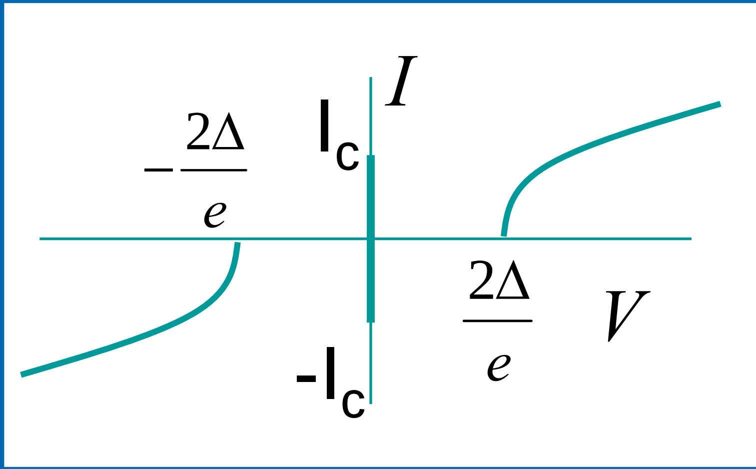


RCSJ



$$\dot{\varphi} = \frac{2\pi}{\Phi_0} V$$

$$I = I_c \sin\varphi$$



$$I = I_c \sin\varphi + \frac{V}{R} + C\frac{dV}{dt}$$

$$I = I_c \sin\varphi + \frac{\Phi_0}{2\pi R}\dot{\varphi} + \frac{\Phi_0 C}{2\pi}\ddot{\varphi}$$

$$\tau = \omega_p t; \quad \omega_p = \left(\frac{2\pi}{\Phi_0}\frac{I_c}{C}\right)^{1/2}$$

$$\beta_J = \frac{1}{\omega_p}\frac{1}{RC}; \quad \alpha = \frac{I}{I_c}$$

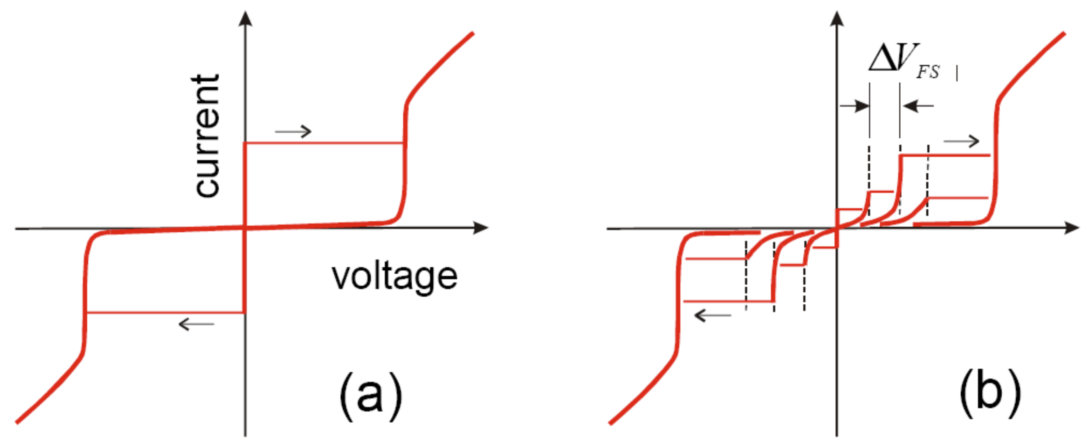
$$\alpha = \frac{d^2\varphi}{d\tau^2} + \beta_J \frac{d\varphi}{d\tau} + \sin\varphi$$

In this approximation the phase depends only on time

Short junction dynamics and Fiske steps

Fiske (1964)

$I - V$ curve: (a) for $H = 0$, (b) for $H \neq 0$.



Voltage spacing between Fiske steps:

$$\Delta V_{FS} = \frac{\Phi_0 \bar{c}}{2L}$$

$j_c = 50\,A/cm^2$

$\lambda_j \approx 50\,\mu m$

$\omega_p^{-1} \approx 5\,ps$

Perturbed sine-Gordon models

$$\varphi_{xx} - \varphi_{tt} = \sin\varphi + \alpha\varphi_t - \beta\varphi_{xxt} - \gamma$$

Boundary conditions:

$$(\varphi_x + \beta\varphi_{xt})\Big|_{x=0} = \eta_1$$

$$(\varphi_x + \beta\varphi_{xt})\Big|_{x=\ell} = \eta_2$$

Space and time normalized to:

$$\lambda_J = \sqrt{\frac{\Phi_0}{2\pi j_c \mu_0 (2\lambda_L + t_0)}}$$

$$\omega_p = \sqrt{\frac{2\pi j_c}{\Phi_0 C}}$$

Sine-Gordon Equation

$$\frac{\partial^2 \varphi}{\partial x^2} - \frac{\partial^2 \varphi}{\partial t^2} = \sin \varphi$$

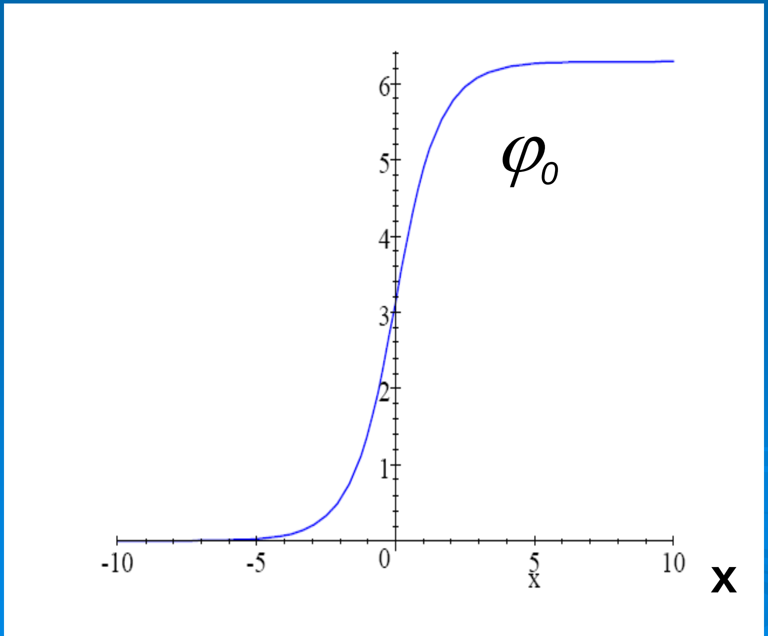
Nonlinearity may compensate dispersion:
Permanent profile wave or “solitary” wave
solution possibility

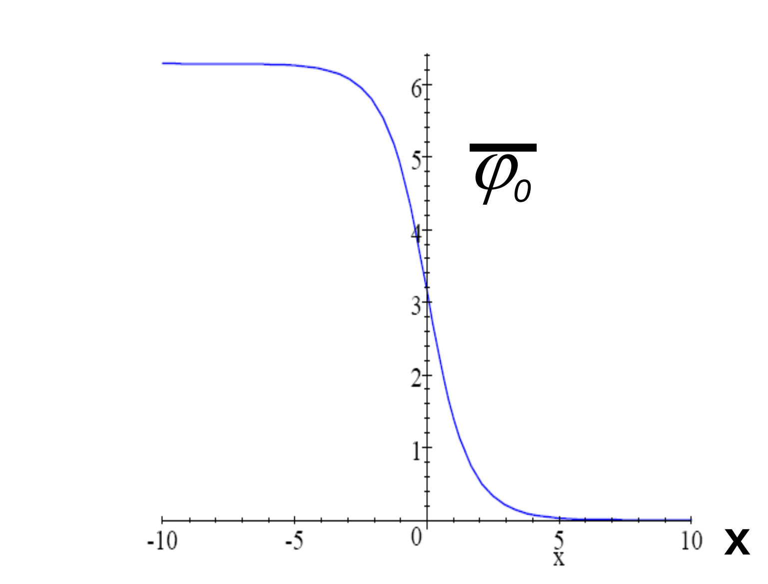
A Barone, F Esposito, C J Magee, A C Scott *Theory and Applications of sine-Gordon Equation* Rivista del Nuovo Cimento **1** p. 227-267 (1971)

"Kink" solution or a "soliton"

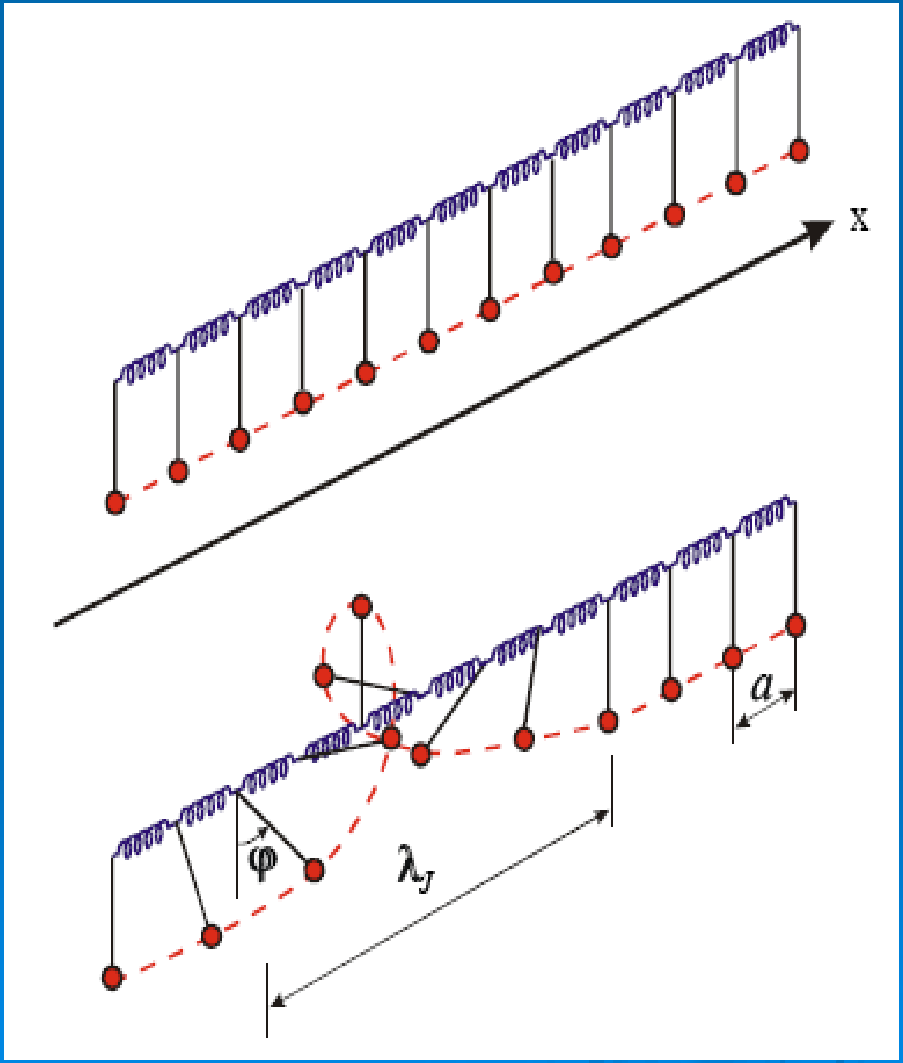
$$\varphi_0(x,t) = 4\arctan[\exp\frac{x - vt - x_0}{\sqrt{1 - v^2}}]$$

$$\overline{\varphi_0}(x,t) = 4\arctan[\exp(-\frac{x-vt-x_0}{\sqrt{1-v^2}})]$$



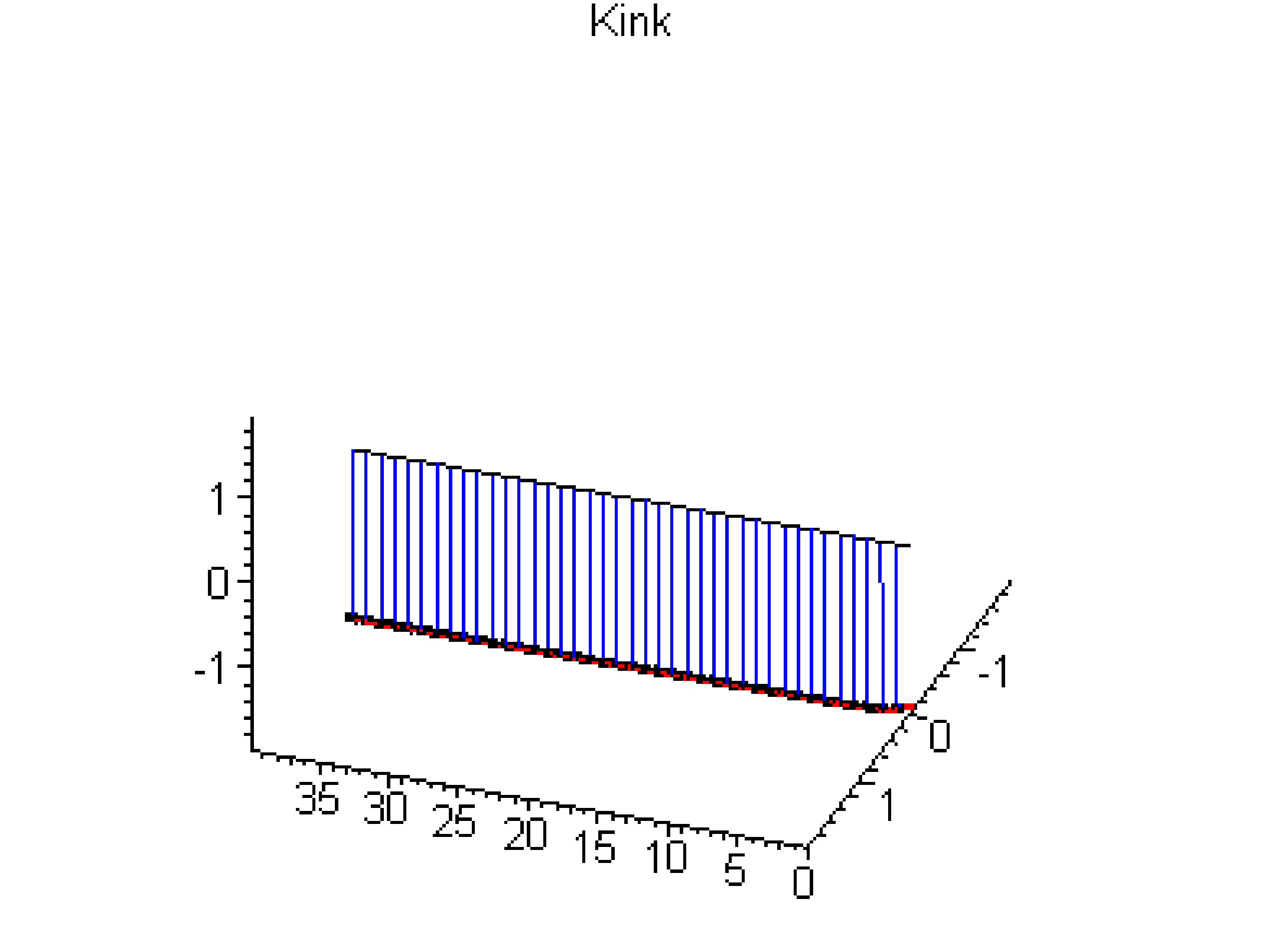


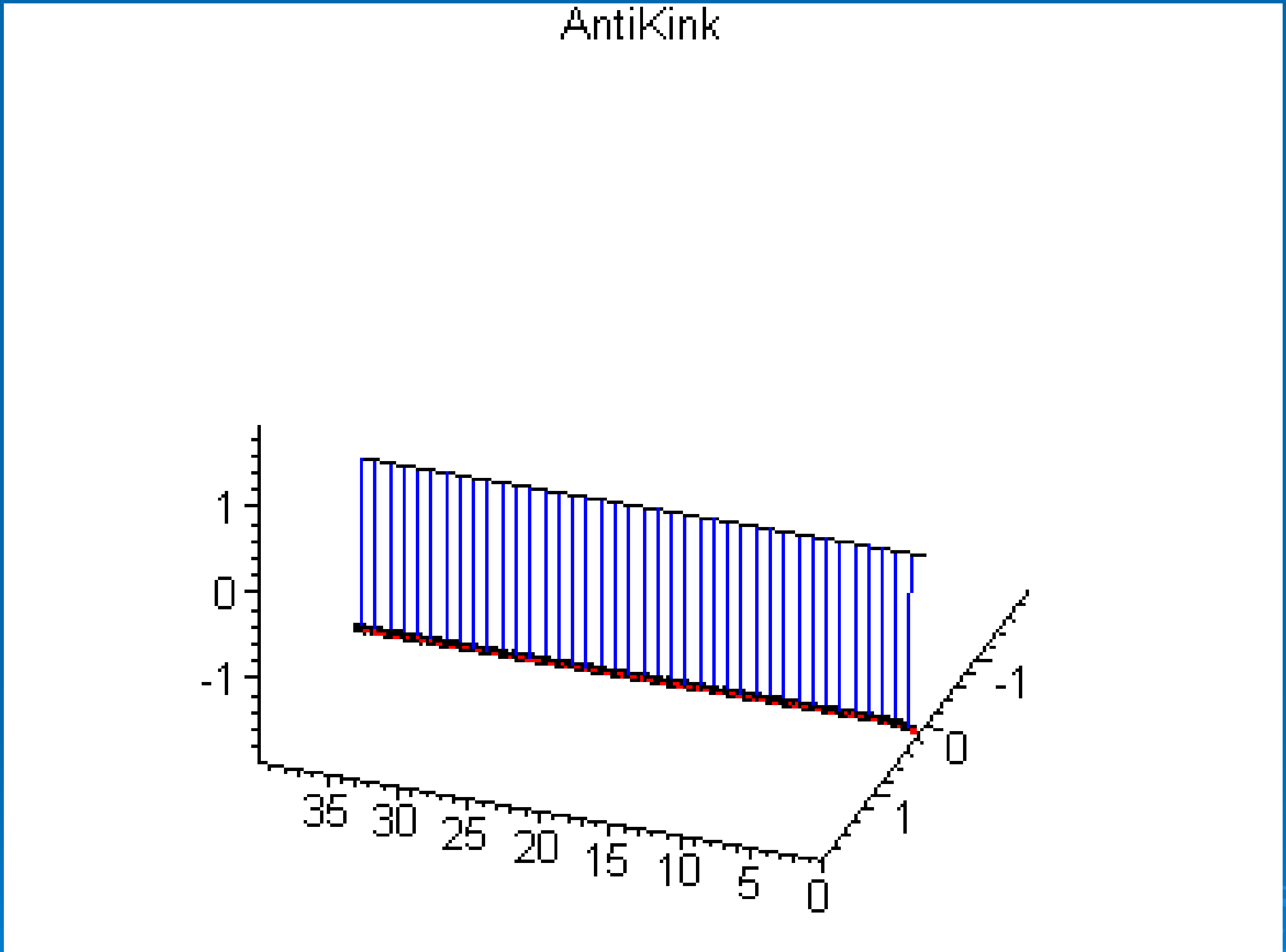
Mechanic analog: chain of pendula



See for example Barone & Paternò
Physics and Applications
of the Josephson Effect
N Y Wiley 1982

(In the book a 'pocket ' version
is also shown)


Kink
1
0
-1
35
30
25
20
15
10
5
0
-1
0
1

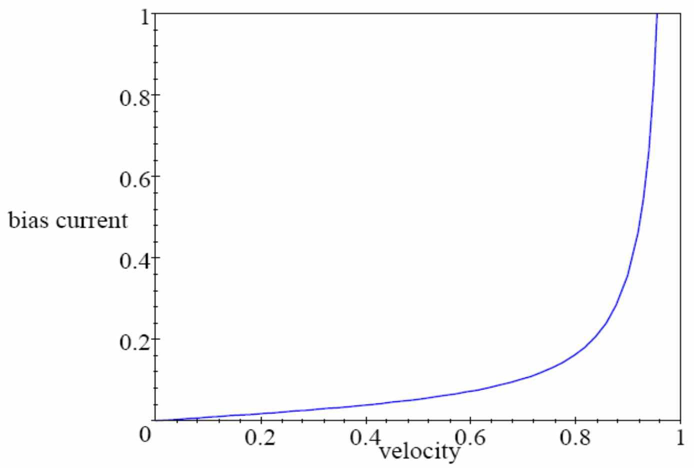

AntiKink
1
0
-1
35
30
25
20
15
10
5
0
-1
0
1

Energy and perturbation analysis for a single fluxon

D.W.McLaughlin and A.C.Scott, Phys.Rev.B **18**, 1652 (1978)

In this approximation a fluxon behaves as a relativistic "particle"

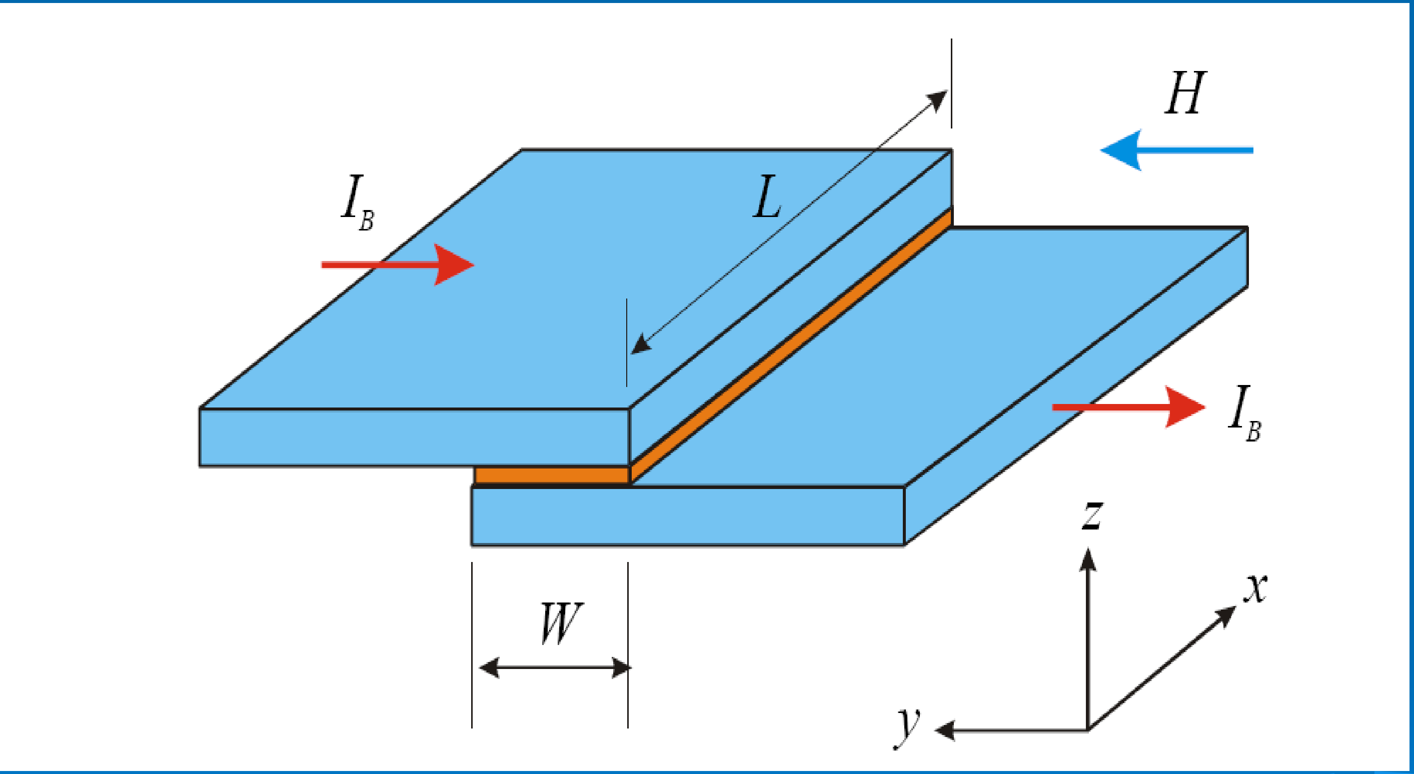
$$E_0 = \frac{8}{\sqrt{1-v^2}}$$

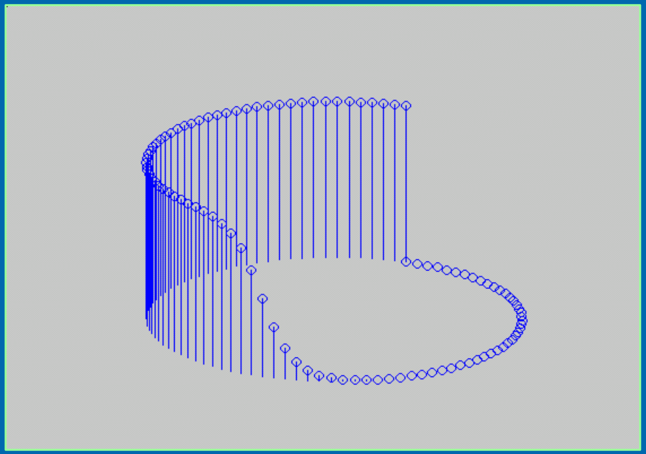


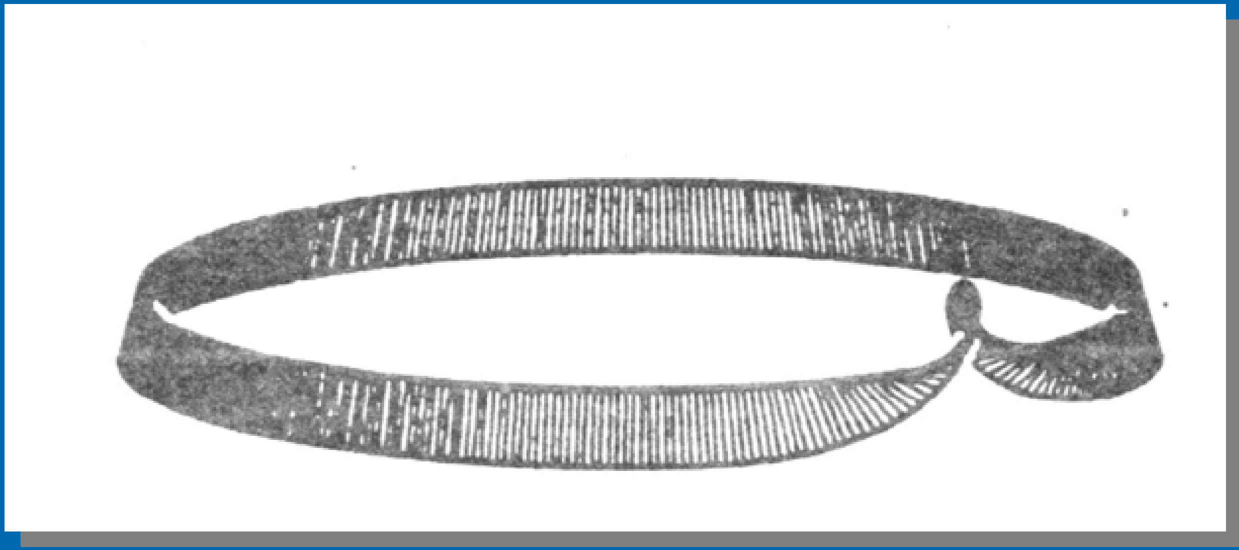
at $\alpha = \beta = 0.05$

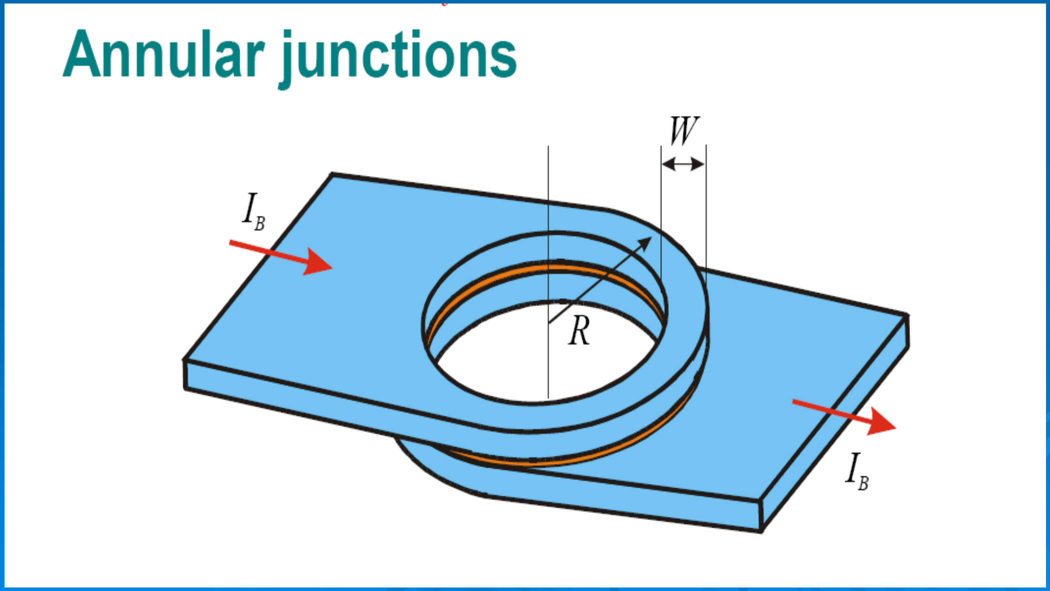
$$\frac{dv}{dt} = -\alpha v(1-v^2) - \frac{1}{3}\beta v - \frac{1}{4}\pi\gamma(1-v^2)^{3/2}$$

Overlap geometry

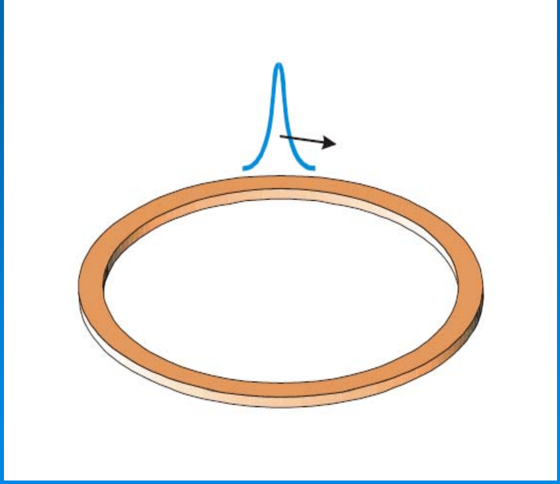


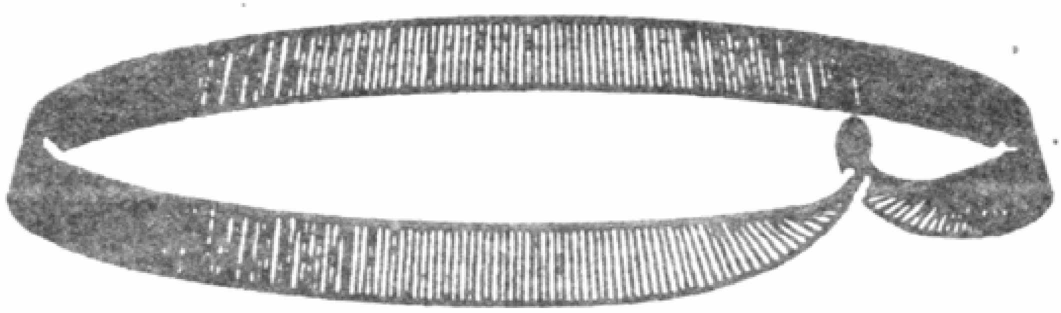





Annular junctions
W
I_B
R
I_B

- In the mechanical analog: start with an open chain, introduce a 2π kink and then close the loop on itself.
- Once extablished the kink so introduced can never disappear. Additional solitons can only be created in soliton-antisoliton pairs (if the chain is sufficiently long) with no net twist
- Like the analog the annular junction is also a closed loop. If upon cooling through the transition temperature one or more flux quanta are trapped in the junction they can never disappear as long as the electrodes remain superconducting





From Davidson, Dueholm, and Pedersen J.Appl. Phys. **60** (1986)1447

How to insert fluxons in an annular junction

- Perform a normal-superconducting transition (Field cooling repeated trials), then monitor I-V characteristic. (spontaneous defect formation)

(Notably this mechanism of production of fluxons during the phase transition is related to the so called Kibble-Zurek mechanism introduced in cosmology to predict cosmic string density at phase transition in the early Universe). Experiments are at moment in course on annular junctions to this end. (Roberto Monaco)

Using Annular Josephson Tunnel Junctions to Monitor Causal Horizons

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I-84081 Baronissi, Italy
b) Blackett Laboratory, Imperial College London,
London SW7 2AZ, U.K.
(Dated: March 2, 2005)

PHYSICAL REVIEW B **67**, 104506 (2003)

Spontaneous fluxon formation in annular Josephson tunnel junctions

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(Received 23 July 2002; published 13 March 2003)

It has been argued by Zurek and Kibble that the likelihood of producing defects in a continuous phase transition depends in a characteristic way on the quench rate. In this paper we discuss our experiment for measuring the Zurek-Kibble (ZK) scaling exponent σ for the production of fluxons in annular symmetric Josephson tunnel junctions. The predicted exponent is $\sigma=0.25$, and we find $\sigma=0.27\pm0.05$. Further, there is agreement with the ZK prediction for the overall normalization.

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Zurek-Kibble Domain Structures: The Dynamics of Spontaneous Vortex Formation in Annular Josephson Tunnel Junctions

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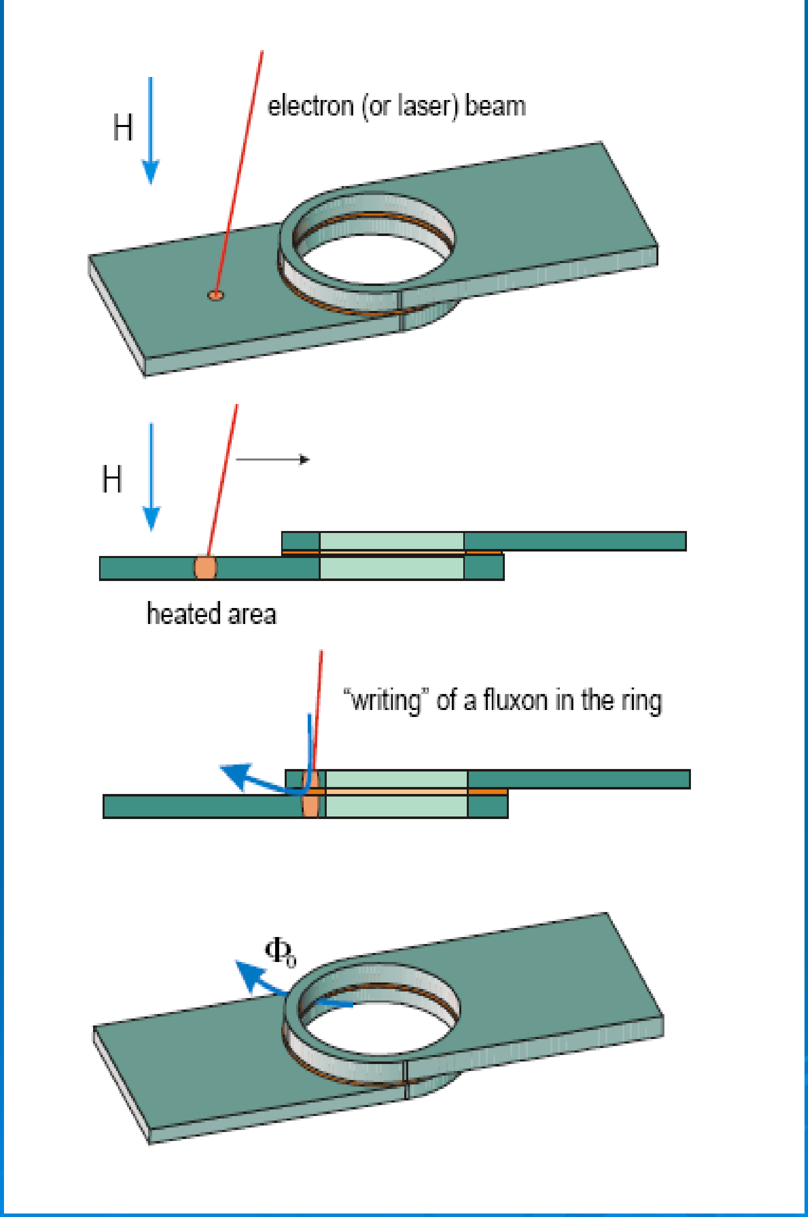
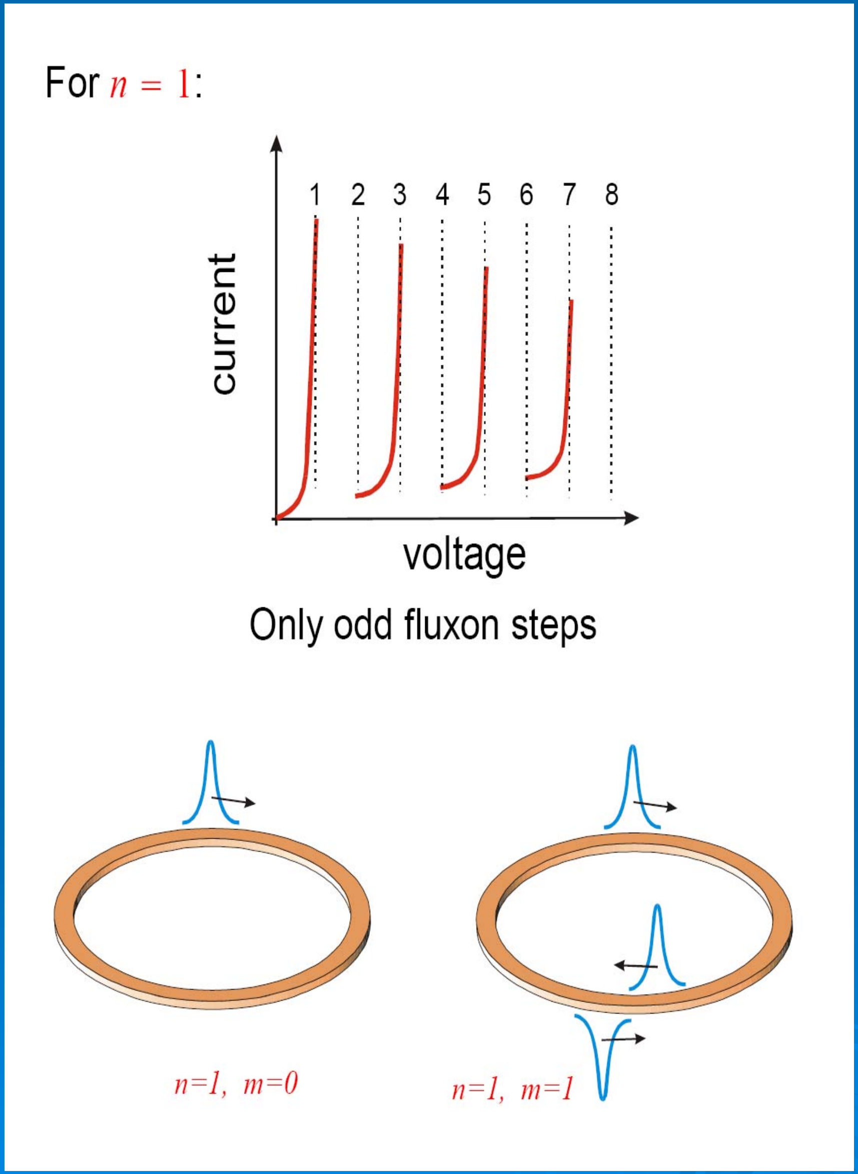
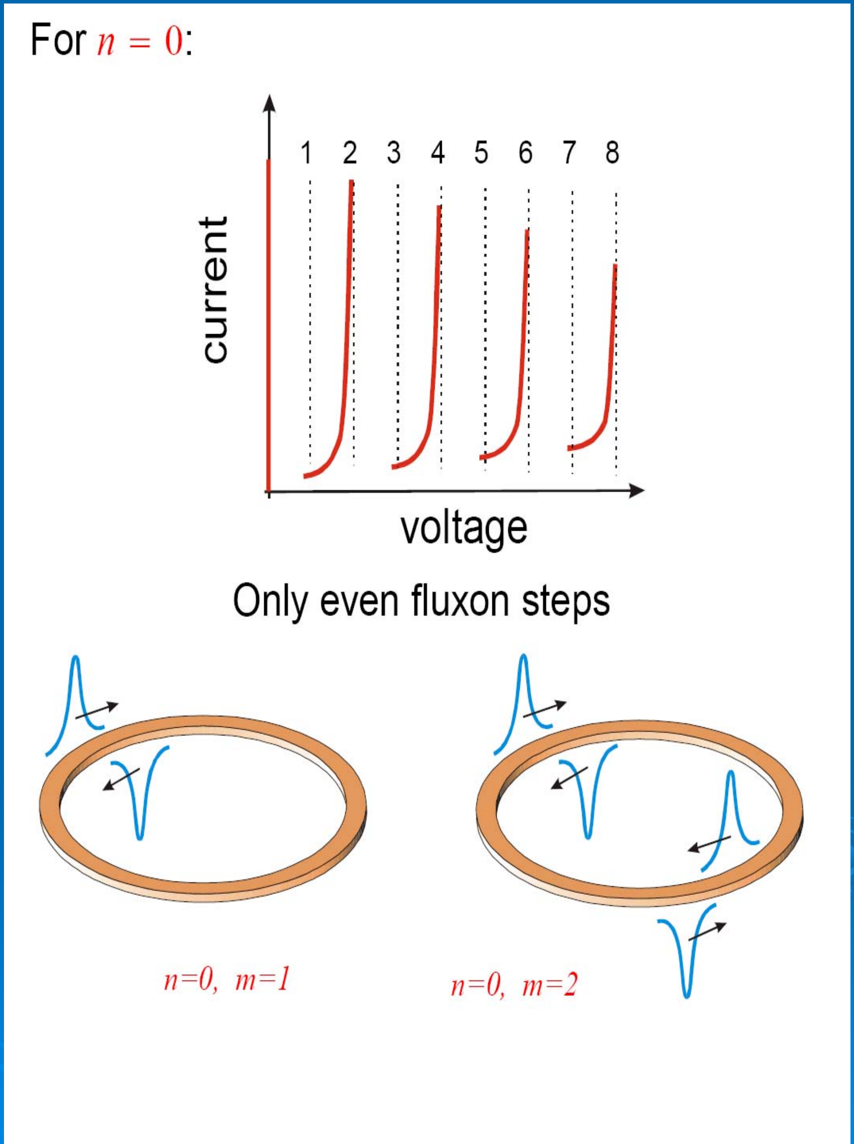
(Received 18 December 2001; published 6 August 2002)

Ustinov's group
Method
Break superconductivity
With a laser or an e. beam
Force the magnetic flux into

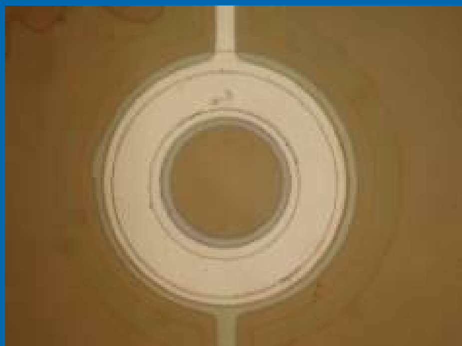


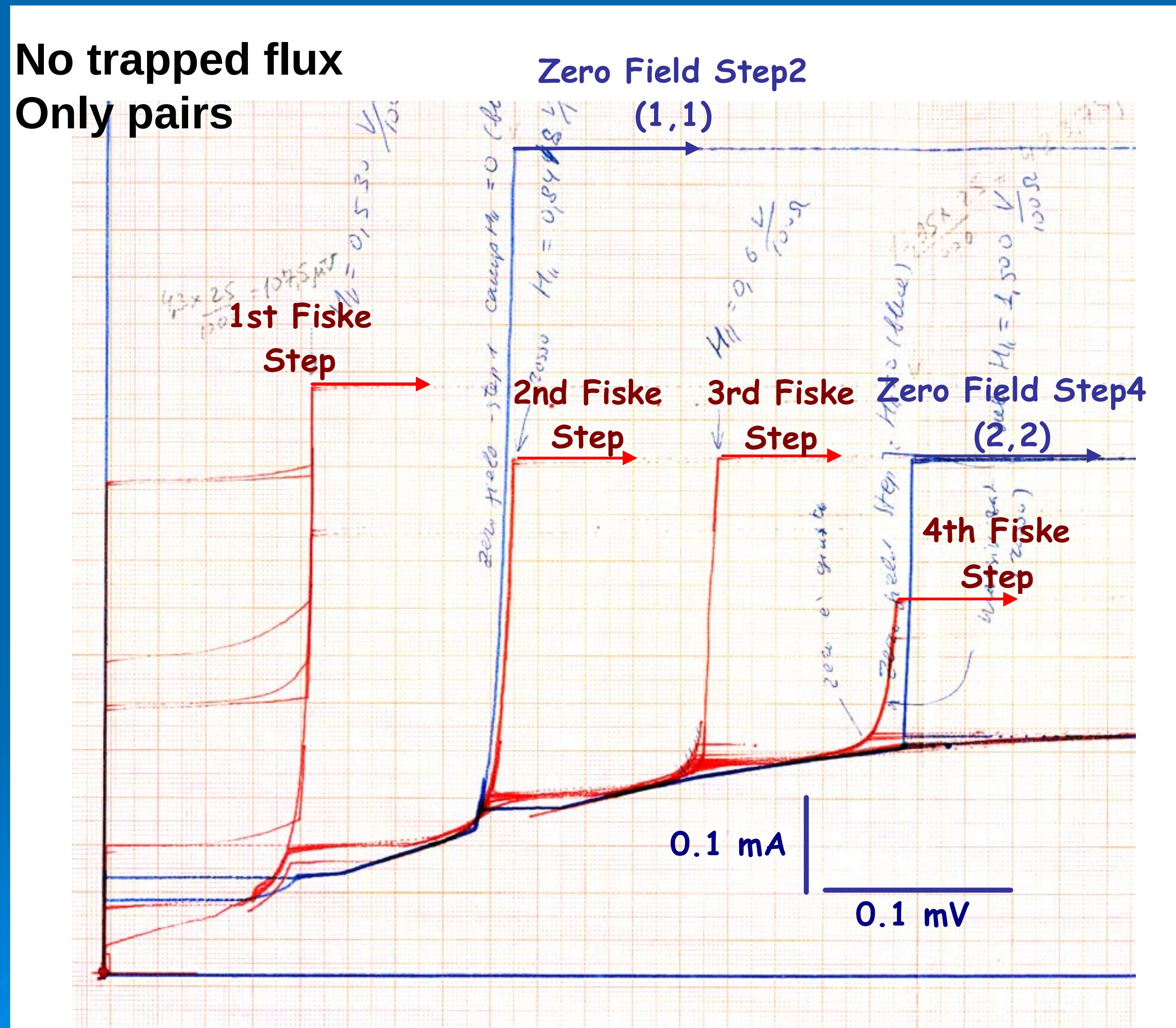
Indirect observation of fluxons





Current Voltage characteristic





R=45μm. R_{int}=30μm, λ_j~ 45μm
j_c=54 A/cm^2
$Nb/Al/Al_2O_3/Al/Nb$

A long JJ clock based on a rotating fluxon

Supercond. Sci. Technol. **12** (1999) 769–772. Printed in the UK

PII: S0953-2048(99)04955-6

Low-jitter on-chip clock for RSFQ circuit applications

Yongming Zhang† and Deepnarayan Gupta‡

† Conductus, Inc., 969 West Maude Avenue, Sunnyvale, CA 94086, USA
‡ HYPRES, 175 Clearbrook Road, Elmsford, NY 10523, USA

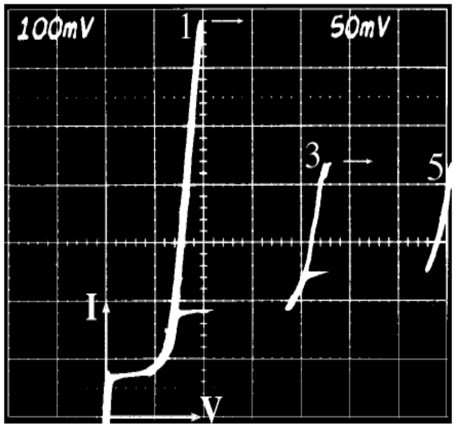


Figure 3. Current against voltage for various numbers of solitons at 4.2 K. There is one trapped soliton. The number next to each curve is the total number of solitons and antisolitons moving to produce that curve.

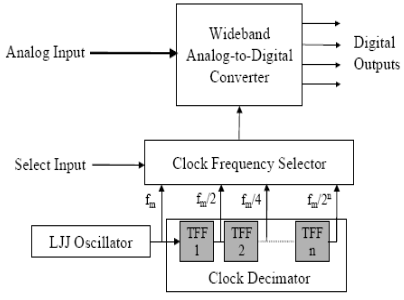


Figure 1. A proposed flash ADC with integrated LJJ clock.

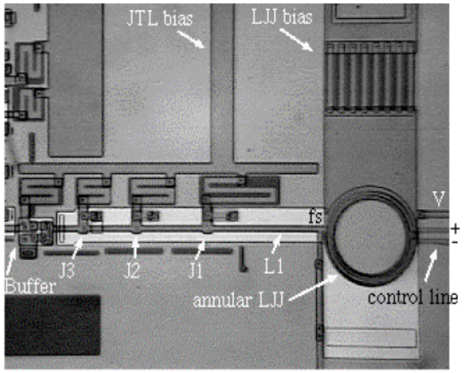
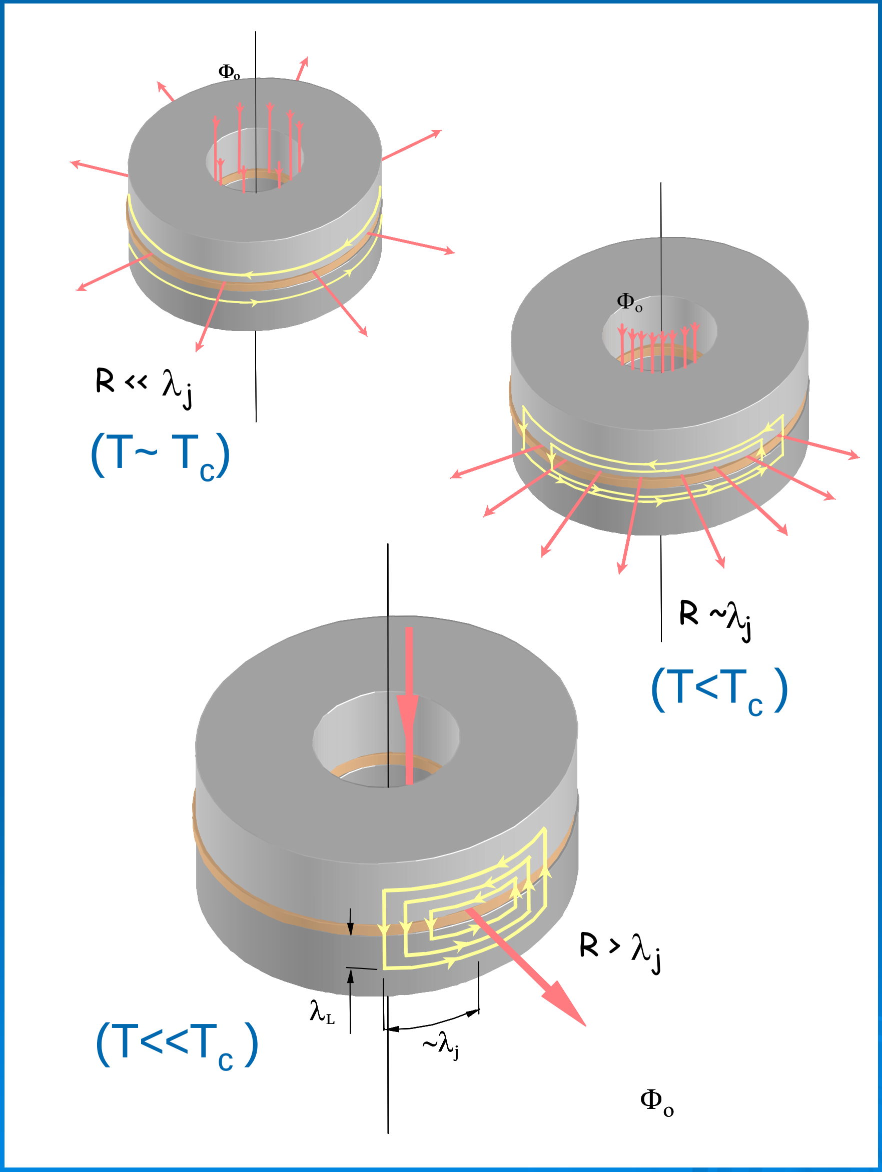


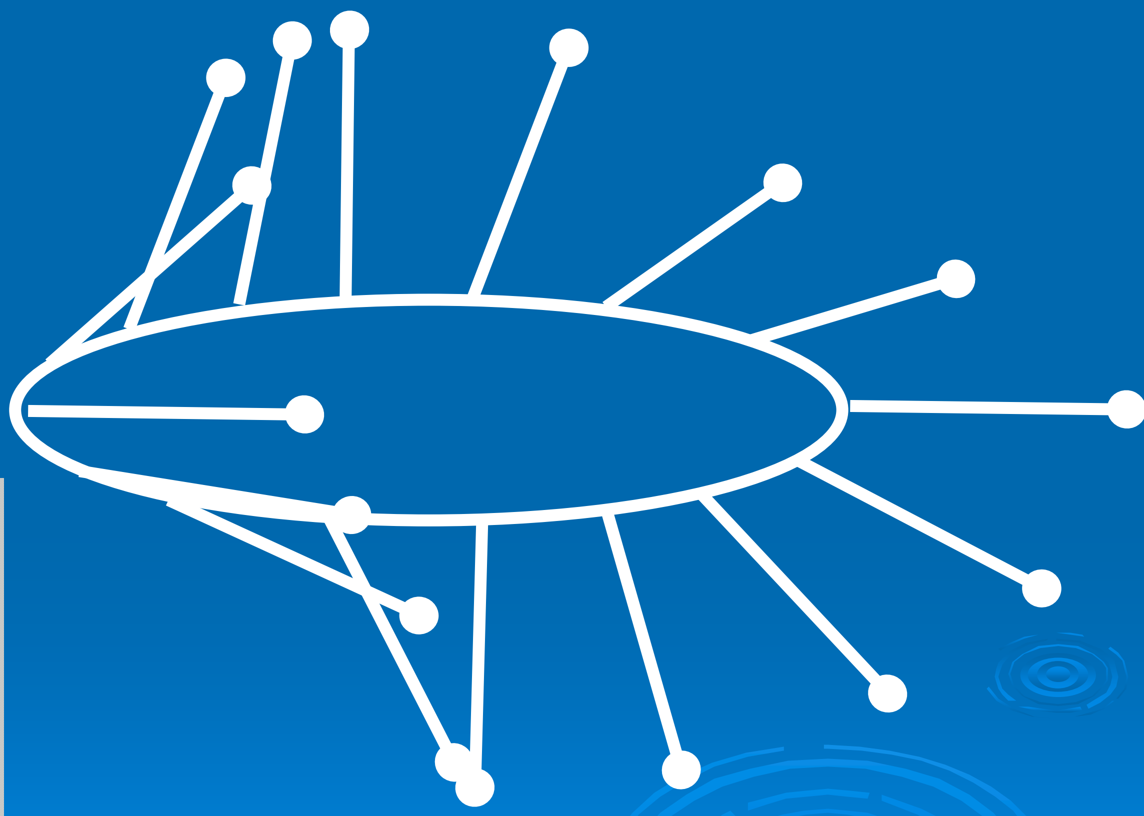
Figure 2. An optical micrograph of a fabricated annular long junction that is coupled to the SFQ circuit. The length of the junction is 230 μm, and the width is 5 μm. The circuit was fabricated using a HYPRES 1 kA cm^{-2} Nb/AlO$_x$/Nb junction fabrication process.

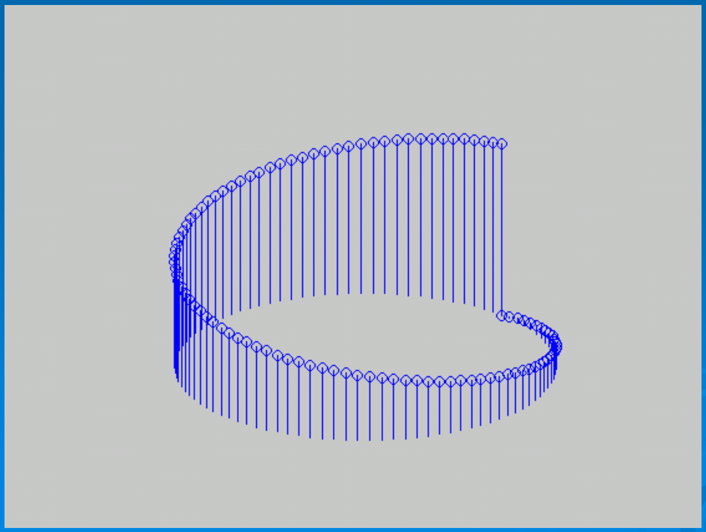
Josephson junction size affect on fluxons structure and dynamics

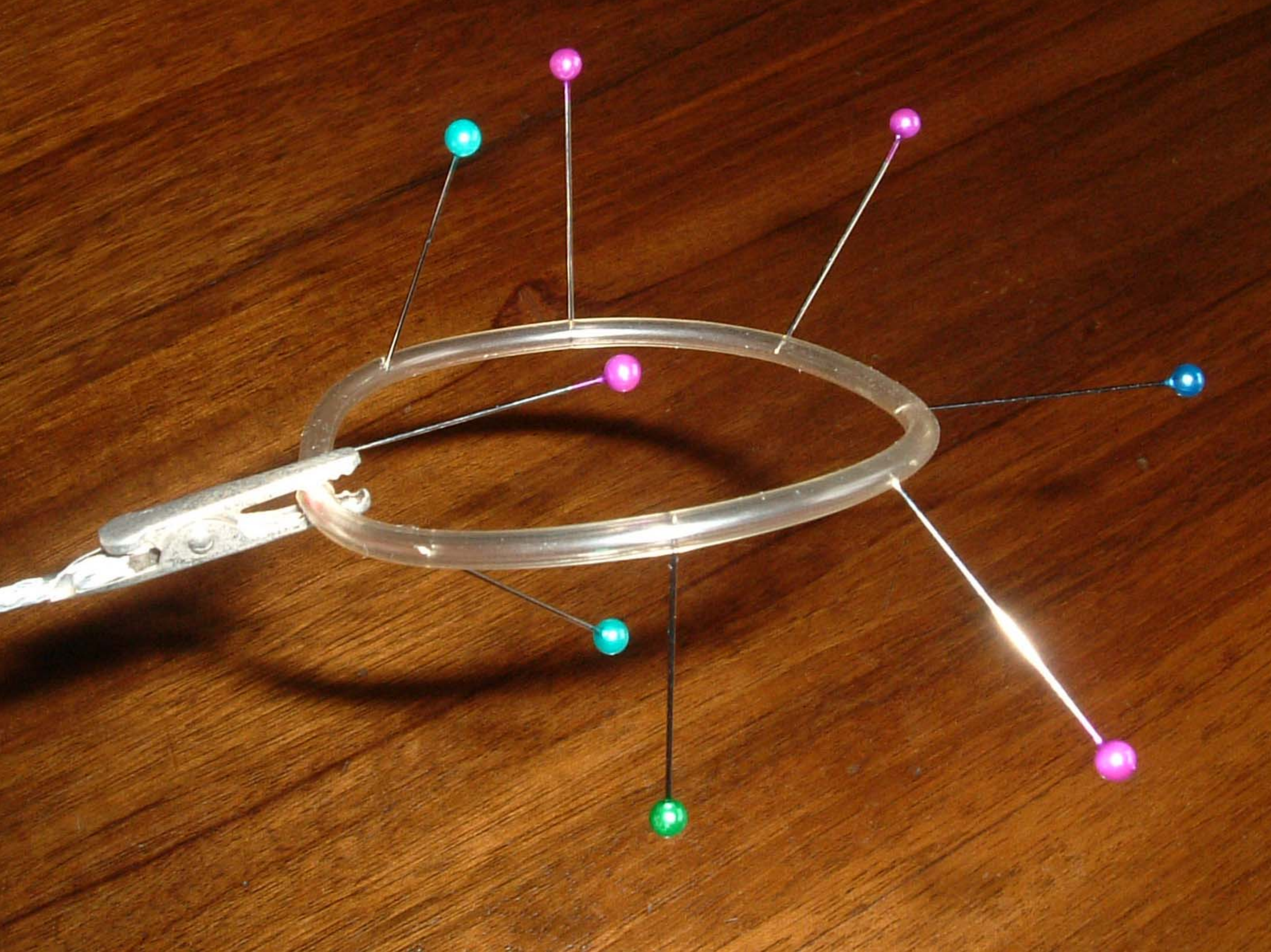


- The more the coupling the larger the screening effect of the tunneling supercurrents
- The main effect of this screening on the magnetic structure is a localization of the magnetic field lines
- In the presence of trapped flux stationary tunneling currents exist even in the absence of external bias current.
- When a bias current is feeded in, a rotation of the magnetic field (and current) structure occurs: a finite voltage is developed across the junction.

"Small" chain of pendula for a 1-D small annular junction displaying a "trapped fluxon"







1-D or 2-D 'small' case

$$\nabla \equiv \hat{\rho}\frac{\partial}{\partial \rho} + \hat{\vartheta}\frac{1}{\rho}\frac{\partial}{\partial \vartheta}$$

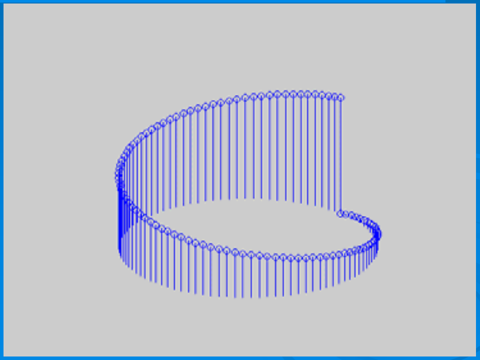
$$\nabla \varphi = \left(\frac{2\pi\mu_o d}{\Phi_o}\right)\mathbf{H}\times\mathbf{n}$$

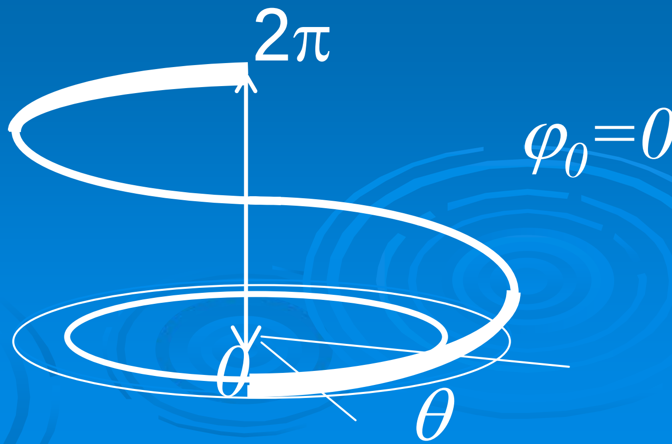
Neglecting all self-fields we simply take:

$$\mathbf{H} = \mathbf{H_v} = \frac{n\Phi_o}{2\pi\,\rho\mu_o d}\hat{\rho}$$

Obtain for the static phase φ :

$$\varphi = \varphi_o + n\vartheta$$





If the external magnetic field is also considered, we get

$$\varphi = \varphi_o - n\vartheta - \kappa\rho \sin\vartheta$$

$$\kappa = \frac{2\pi\mu_o H d}{\Phi_o}$$

From this, the critical-current diffraction pattern, i.e. the dependece of the critical current on the m.f., is recovered

$$I_c(H) = \max_{\varphi_o} \int_S j_c \sin\varphi \, dS$$

S the surface of the annular tunnelling barrier

'max' is the maximum with respect to φ_o

Effect of trapped magnetic flux quanta on the Josephson critical current

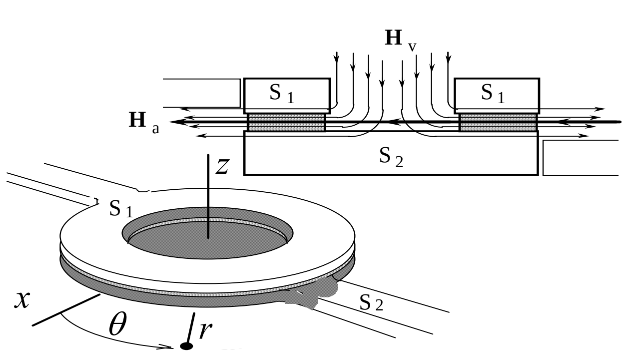
$$I_c = I_0 \left| \frac{2}{(1-\delta^2)} \int_\delta^1 x J_n\left(x \frac{H}{H_0} \right) dx \right|$$

C. Nappi, Phys. Rev. B **55**, 15117 (1997)

n is the number of trapped magnetic flux quanta

$\delta = Ri/Re \quad H_0 = \Phi_0/(2\pi R_e \mu_0 deff)$

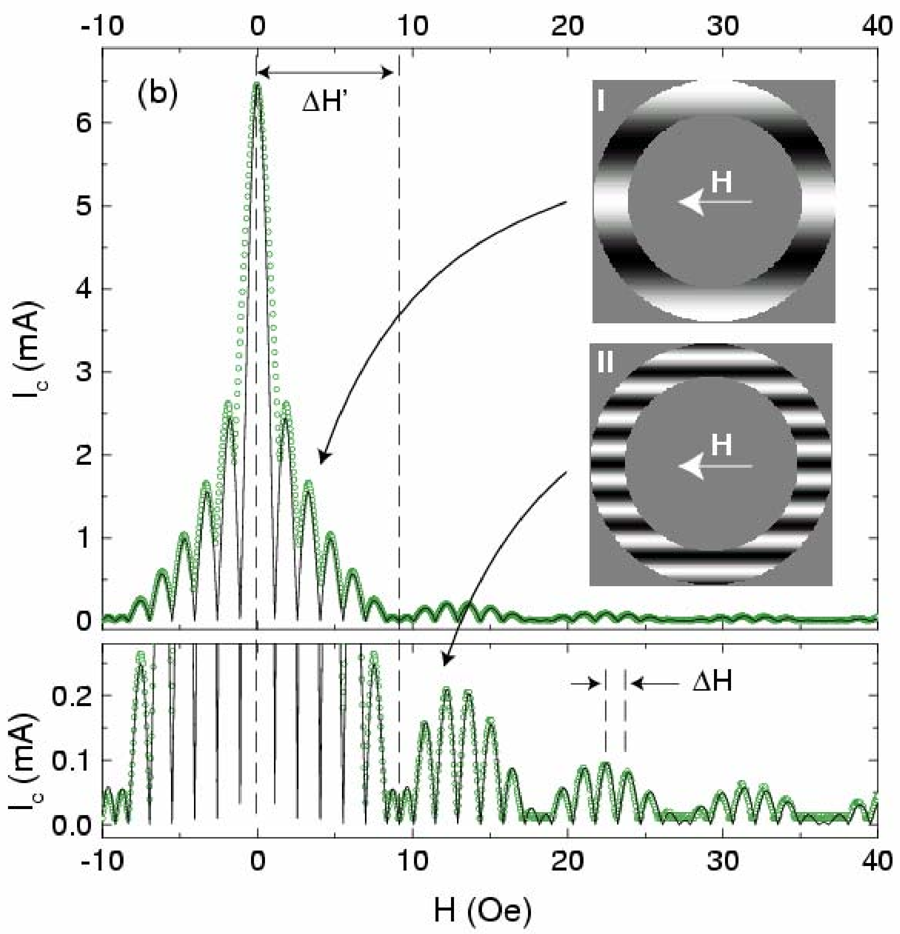
Main prediction is that, with $n \neq 0$, $I_c(0)=0$



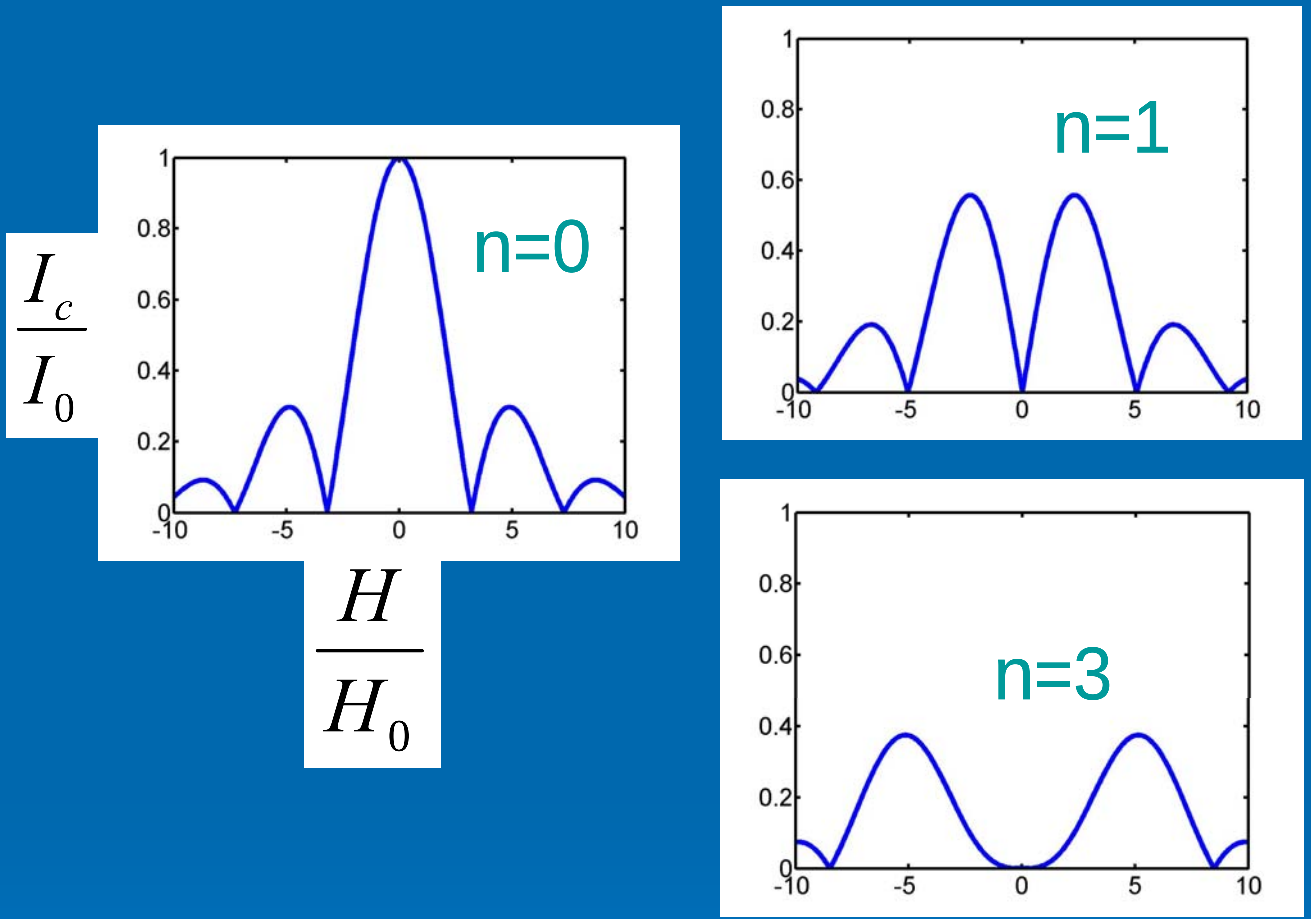
Critical-current diffraction pattern with no trapping (n=0)

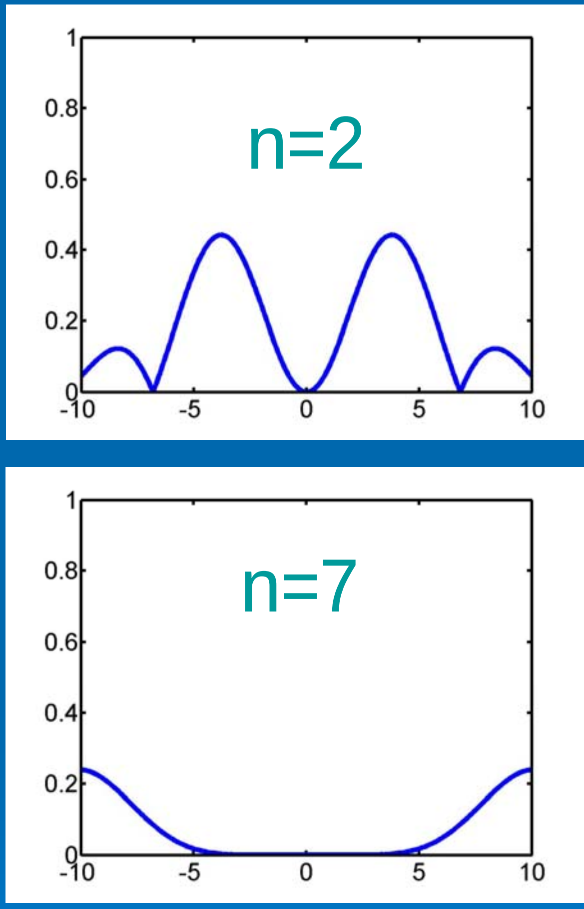
Nb, R_i=35μm, R_i=50μm, λ_j ~30μm @ 4.2K

(A.Franz, A. Wallraff, A.V. Ustinov PRB 62, 119 (2000))



Dots, experimen tal data, solid line, theoy





(For this case $\delta = Ri/Re = 0.47$)

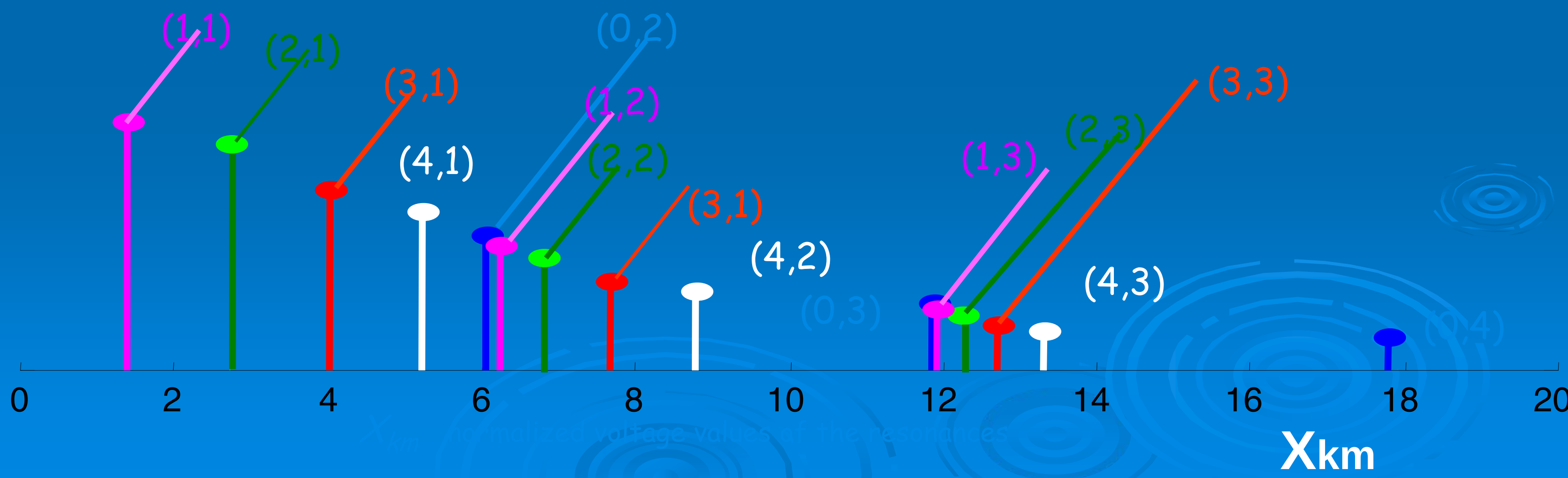
- The critical current diffraction pattern in the Parallel magnetic field can reveal the number of trapped flux quanta

Fiske positions in annular junctions

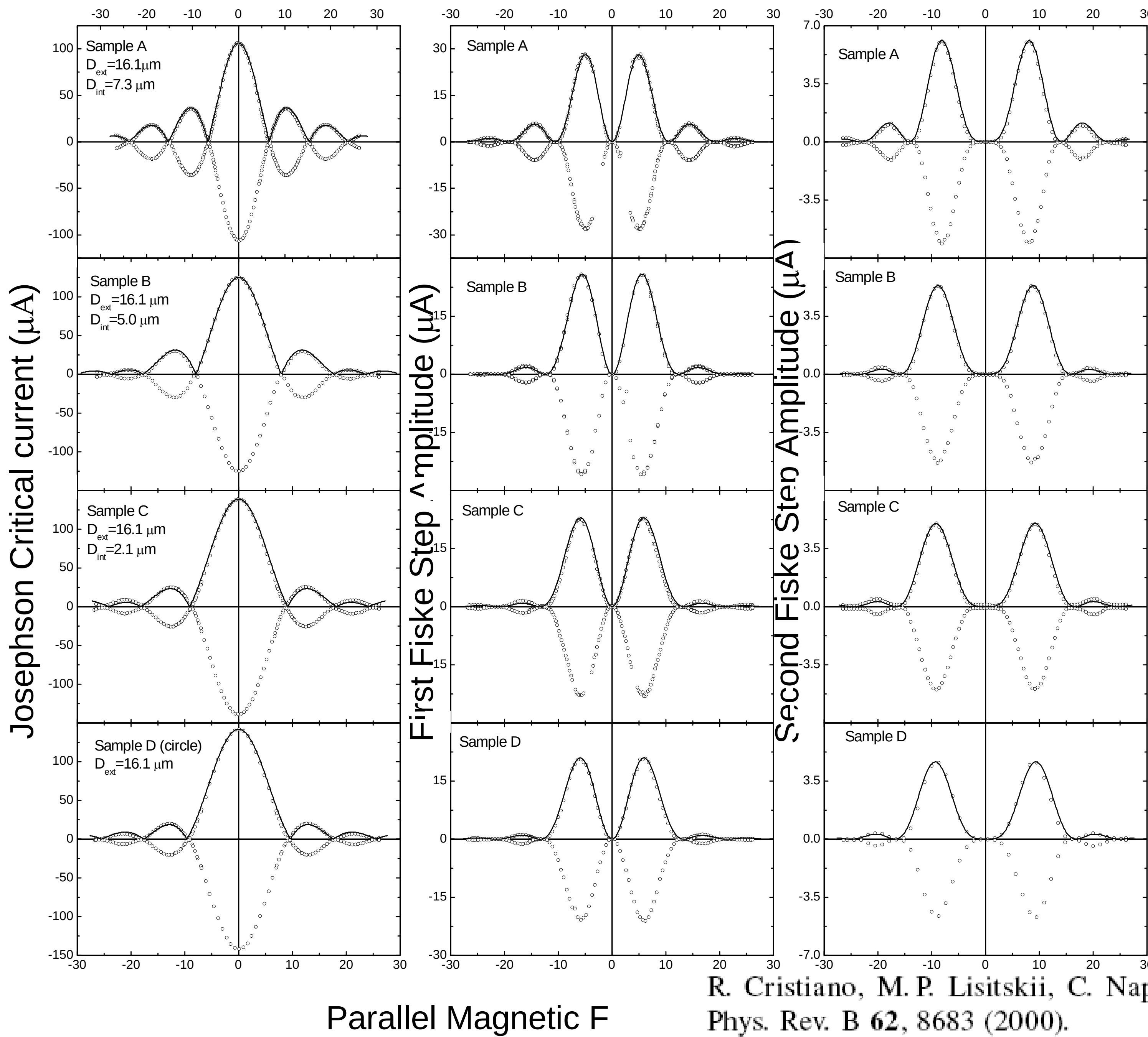
R_i/R_e=0.47. In the presence of k fluxons trapped and zero external field only the corresponding single color raw is excited

$V = X_{km}\, c'\, \Phi_o / 2\pi R_e$

k \ m	1	2	3
0	6.03	11.89	17.76
1	1.38	6.23	11.98
2	2.73	6.80	12.26
3	4.01	7.67	12.71
4	5.22	8.77	13.32



X_{km} normalized voltage values of the resonances



Small junctions, <u>no trapped fluxons</u>

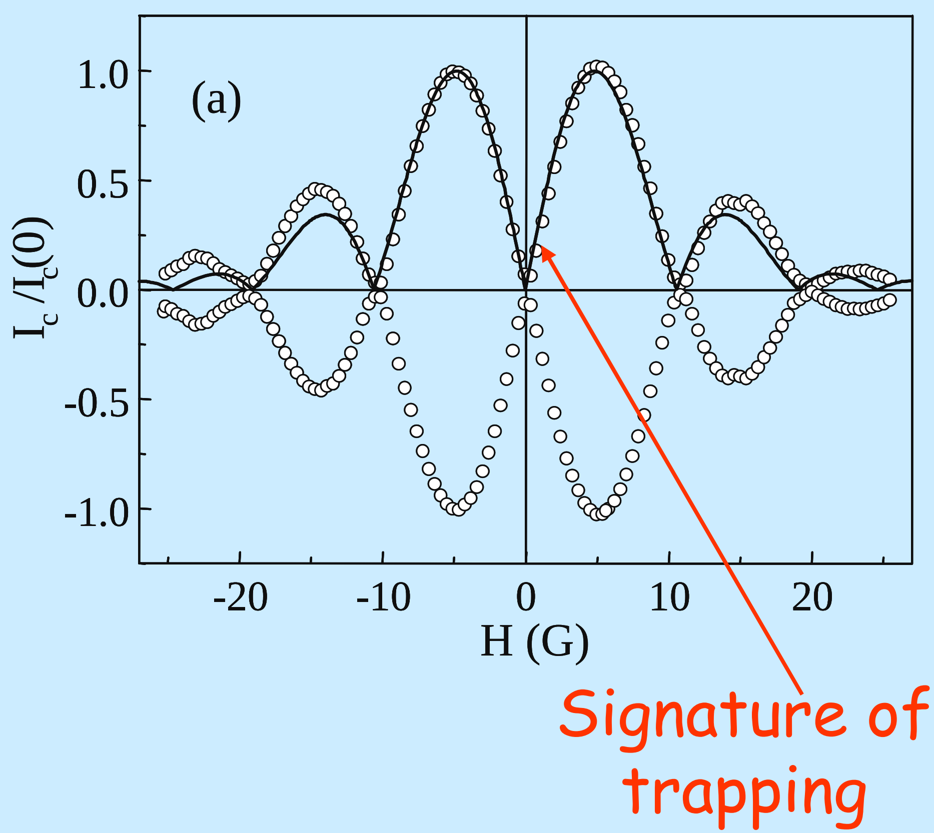
Critical Current, first and second Fiske step-Current dependences on the external magnetic Field.

The diameter of the inner hole decreases from top to bottom as indicated

Solid line: theory, circles: experiments

R. Cristiano, M. P. Lisitskii, C. Nappi, and A. Barone, Phys. Rev. B **62**, 8683 (2000).

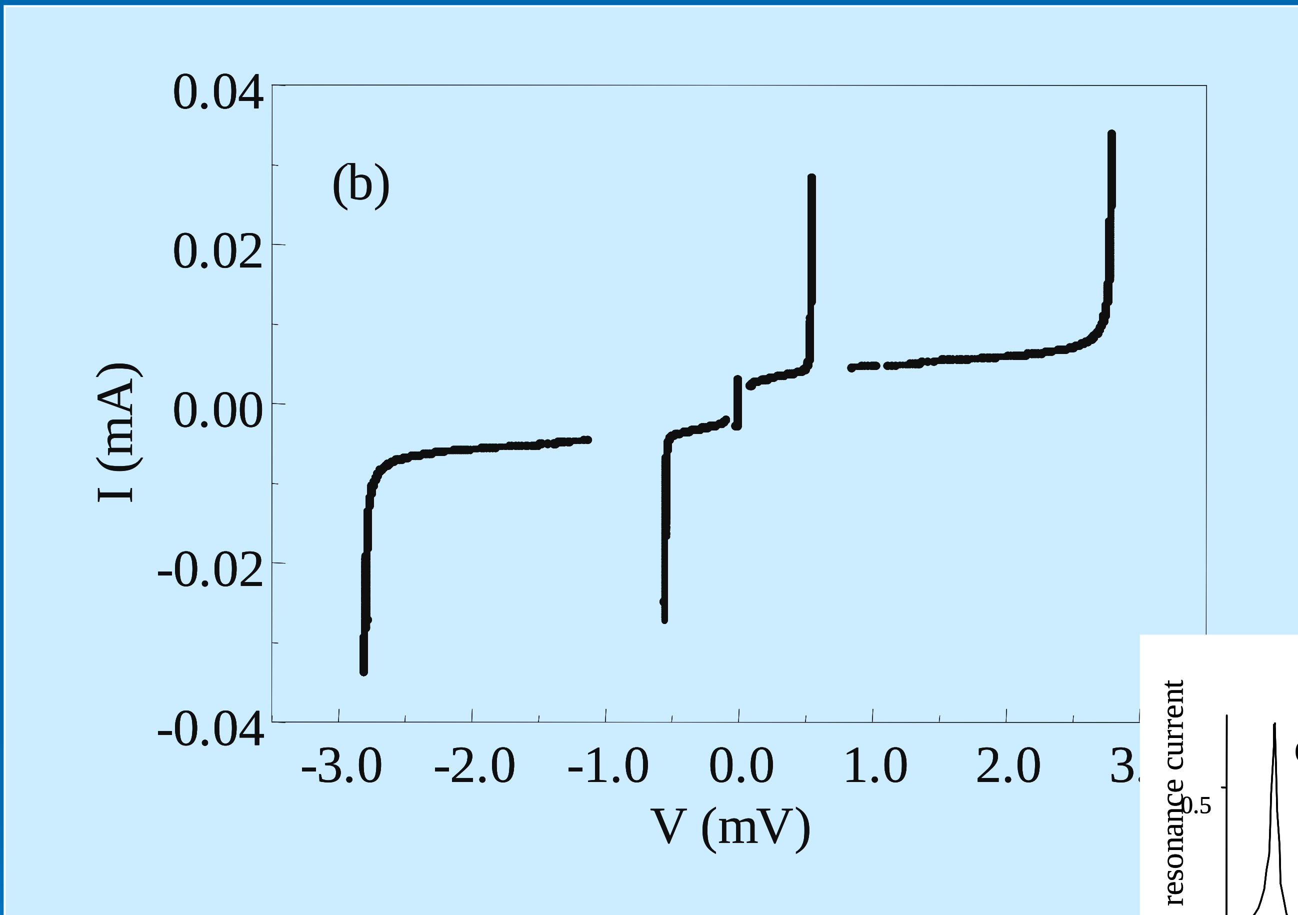
Trapping of one fluxon in a small junction

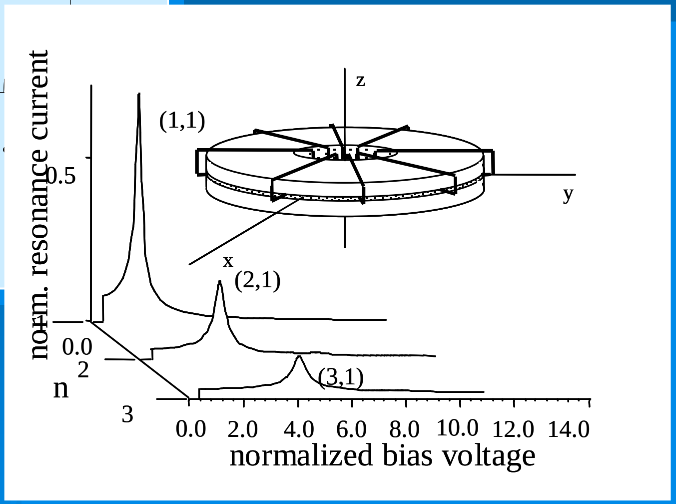


2Re = 16 μm
2Ri = 7.5 μm
Nb λ_j = 50 μm @4.2K
(ξo=38nm)
ICIB-C.N.R.

R. Cristiano et al. APL **74**, (1999) 3389

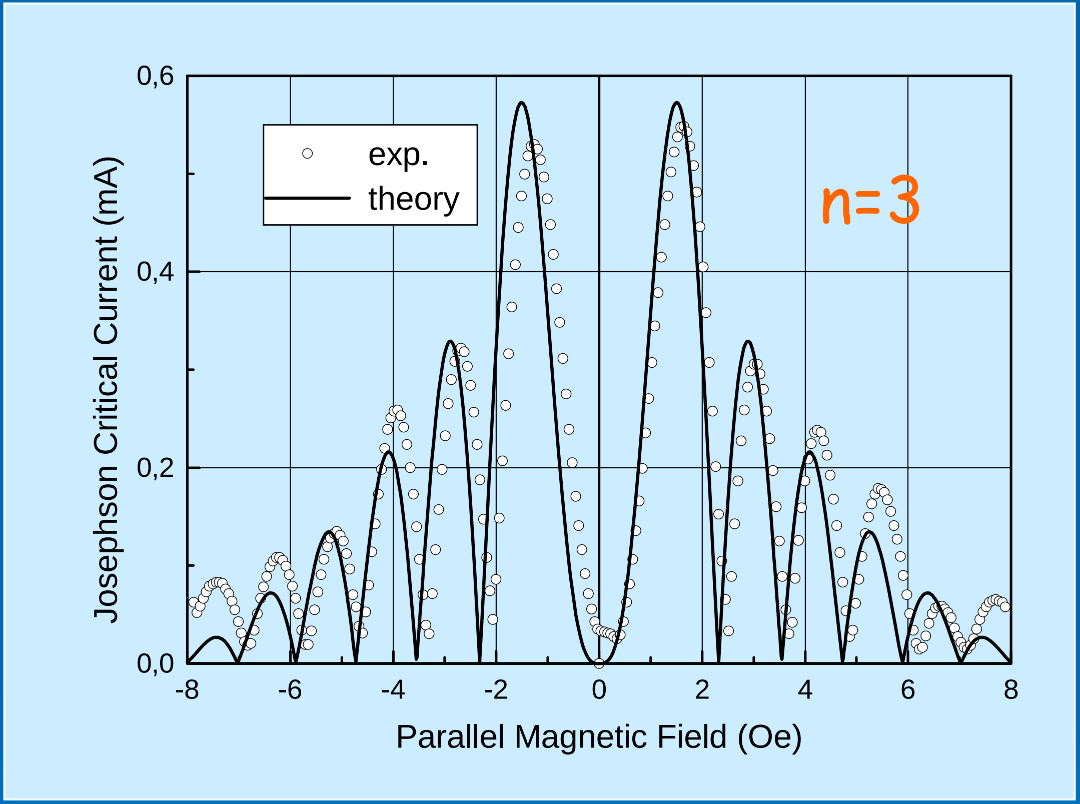
Trapping of a fluxon in a small junction

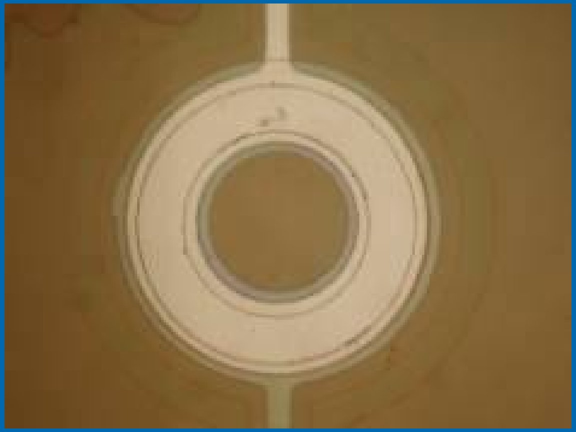




R. Cristiano et al. APL **74**, (1999) 3389

Ic with three trapped fluxons





Photograph annular junction

R=45μm. R_{int}=30μm,

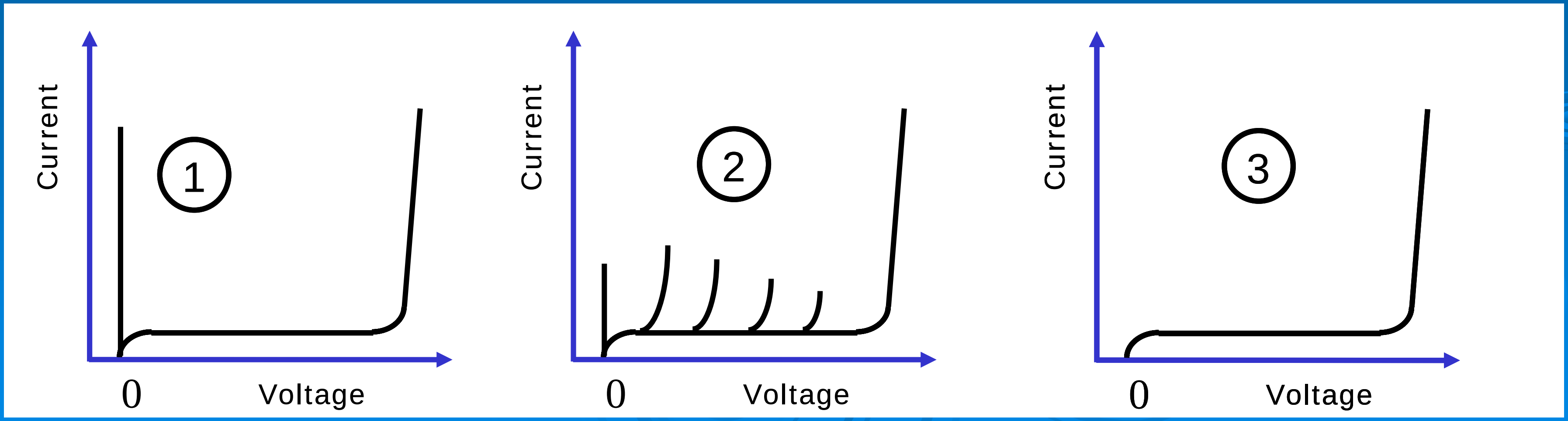
λ_j~ 45μm

IC-C.N.R.

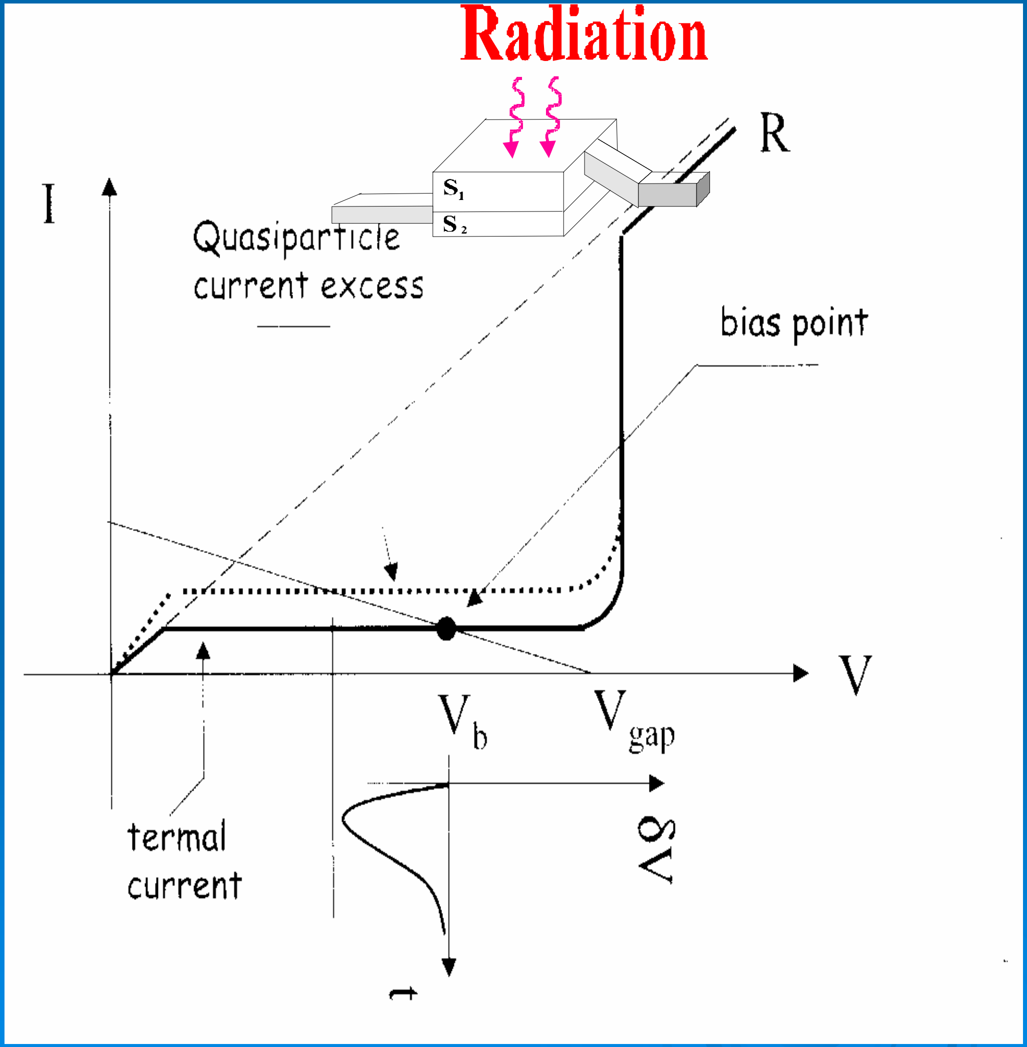
C. Nappi, R. Cristiano, M.P. Lissitski, R. Monaco, A. Barone, Physica C **367** (2002), 241

A high magnetic field suppresses Josephson current and “steps”

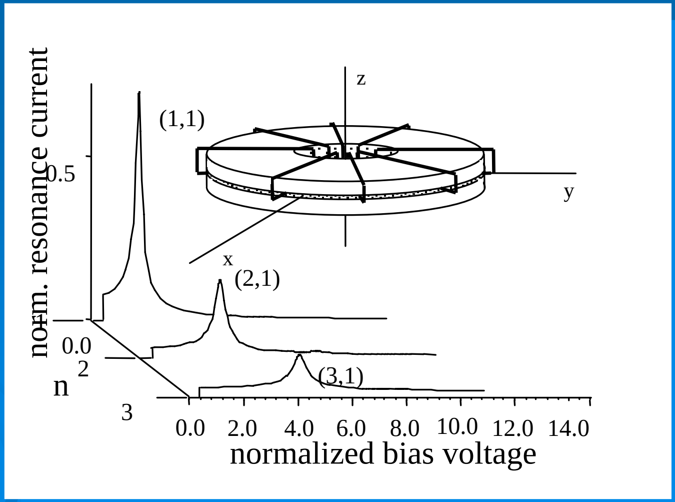
As the external magnetic field increases, the Josephson critical current lowers. In the same time a number of resonances (known as Fiske steps) are excited in the I-V characteristic of the junction as a consequence of the a.c. Josephson effect (2) . At fields relatively high (50 – 100 Gauss) both the Josephson critical current and the Fiske resonances are suppressed. Under this condition (3) it is possible to polarize the junction in the subgap region to make it operate as radiation detector.



Superconducting tunnel junction detector: Operation principle

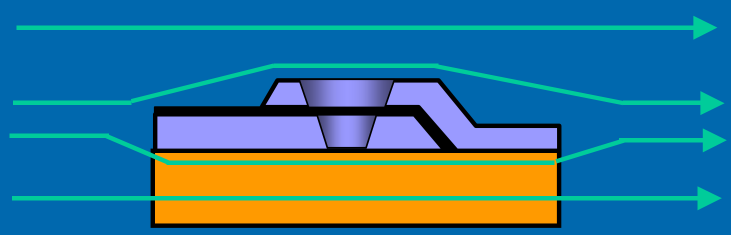


The Josephson branch has to be suppressed for a stable operation point is obtained

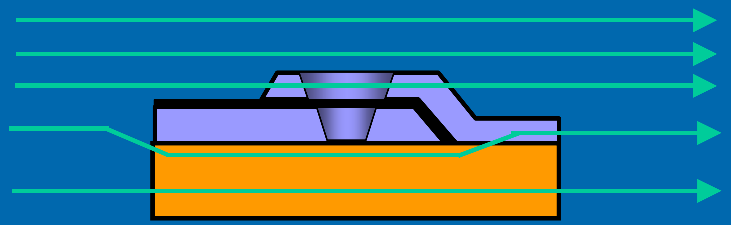


Experimental results under X-ray irradiation from 55Fe X-ray calibration source

Trapping of magnetic flux quanta by applying parallel magnetic field

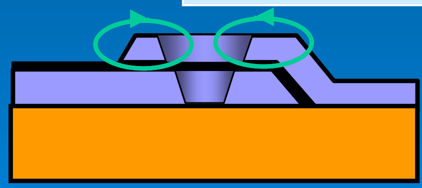


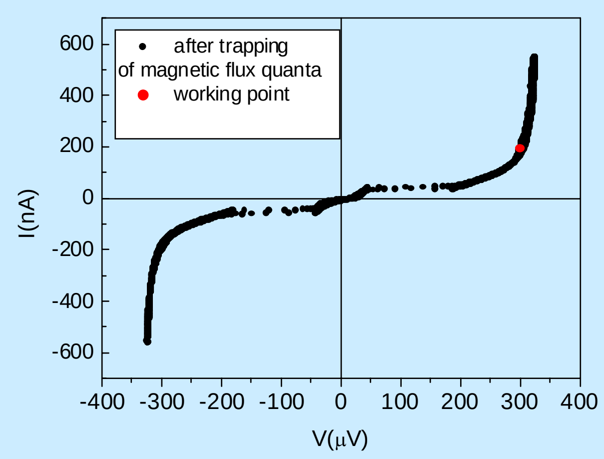
$H < H_c^{top}$



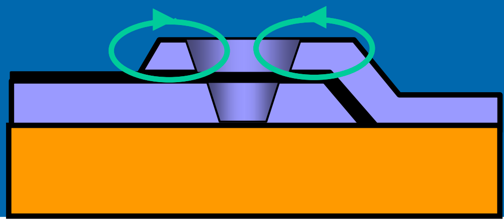
$H_c^{top} < H < H_c^{bottom}$

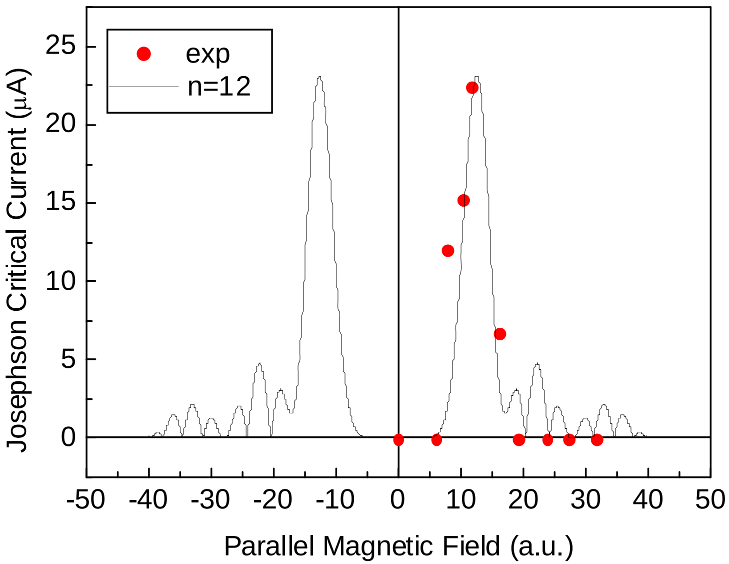
H is turned off





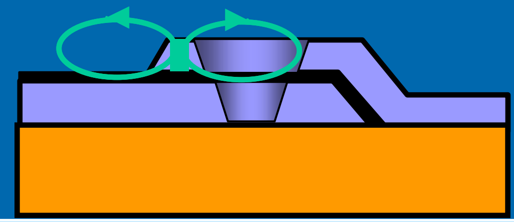
Estimation of the number of trapped magnetic flux quanta

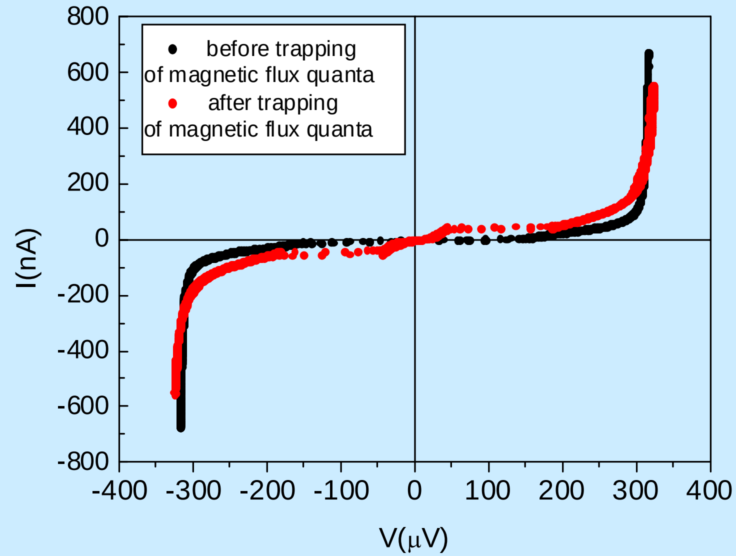




n=12 was obtained

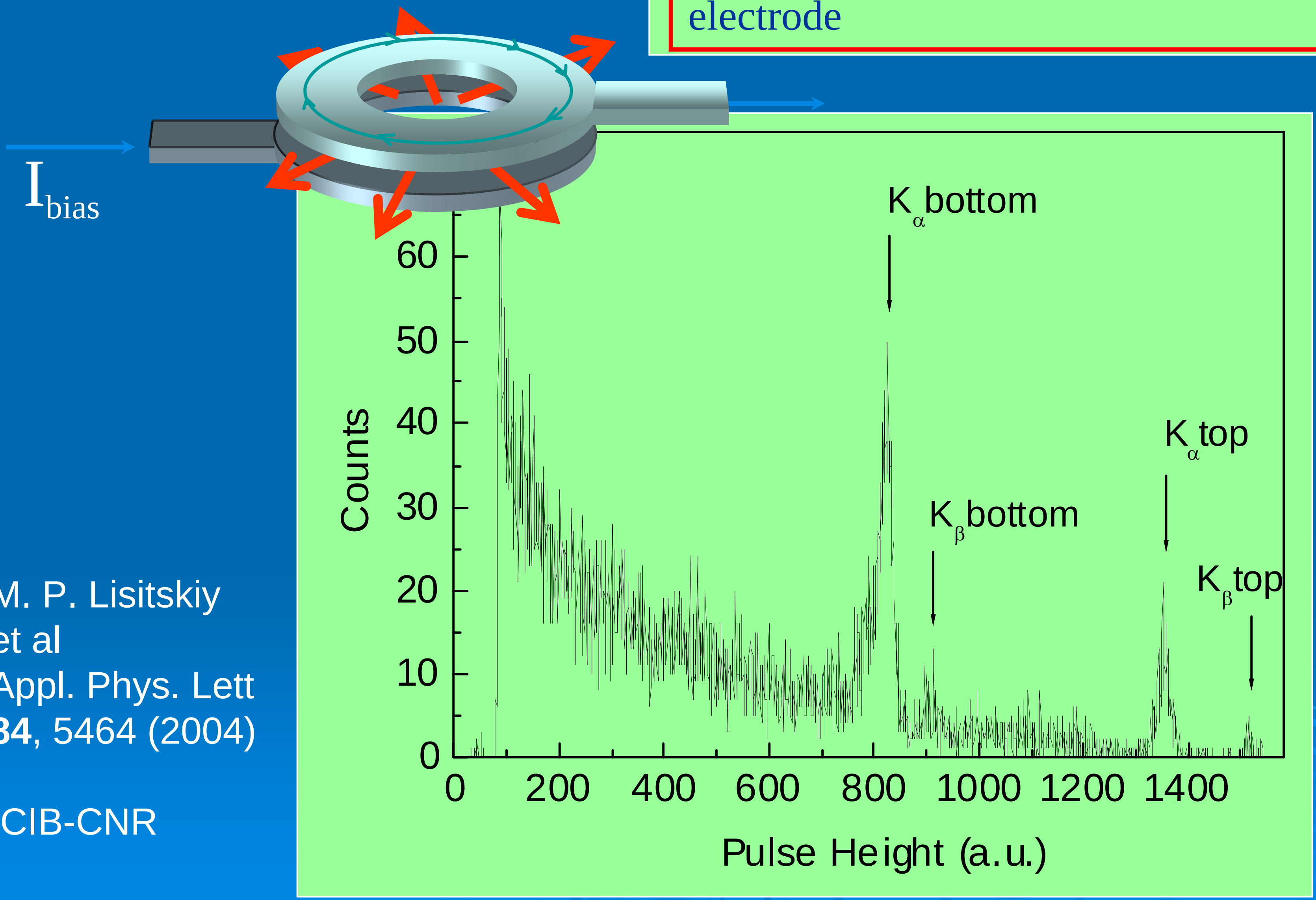
Situation with trapped Abrikosov vortices





The increase of quasiparticle current is due to the presence of one Abrikosov vortex

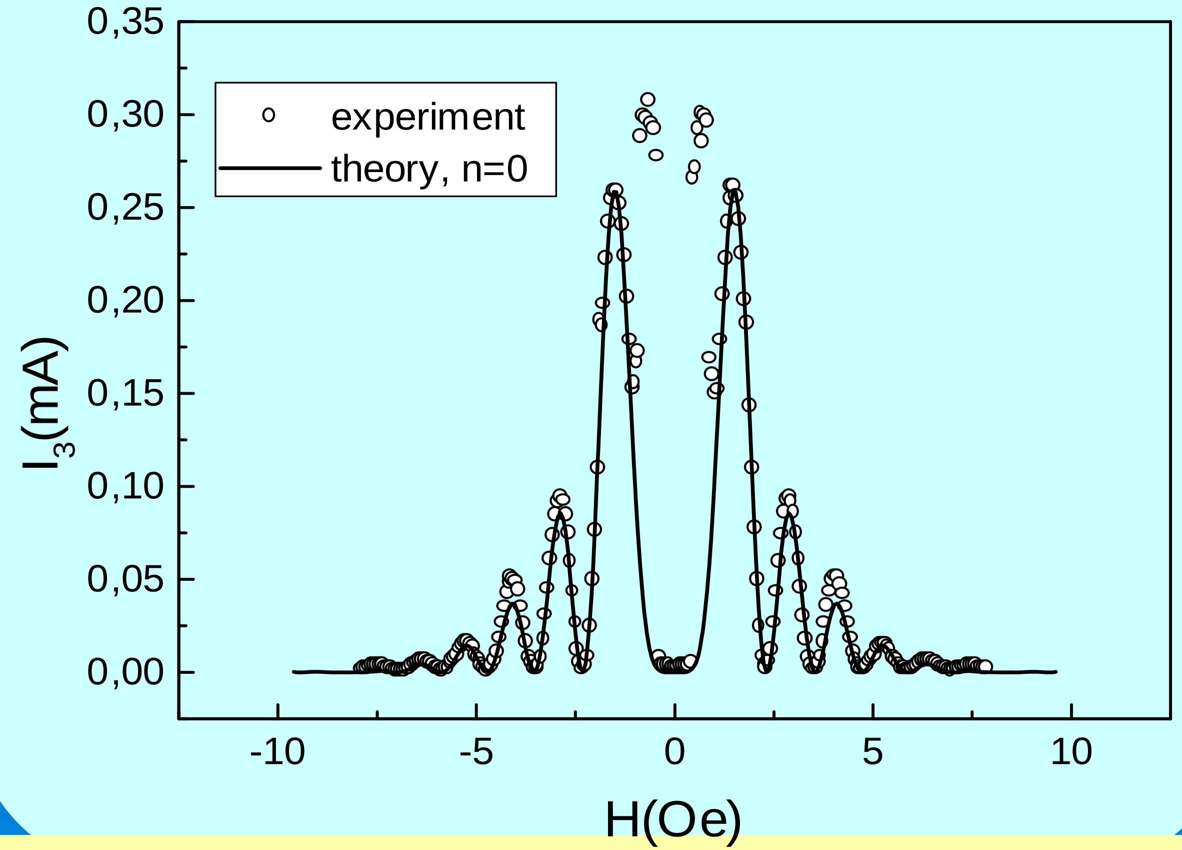
An energy resolution of about 100 eV was obtained for the K_α line of the top electrode



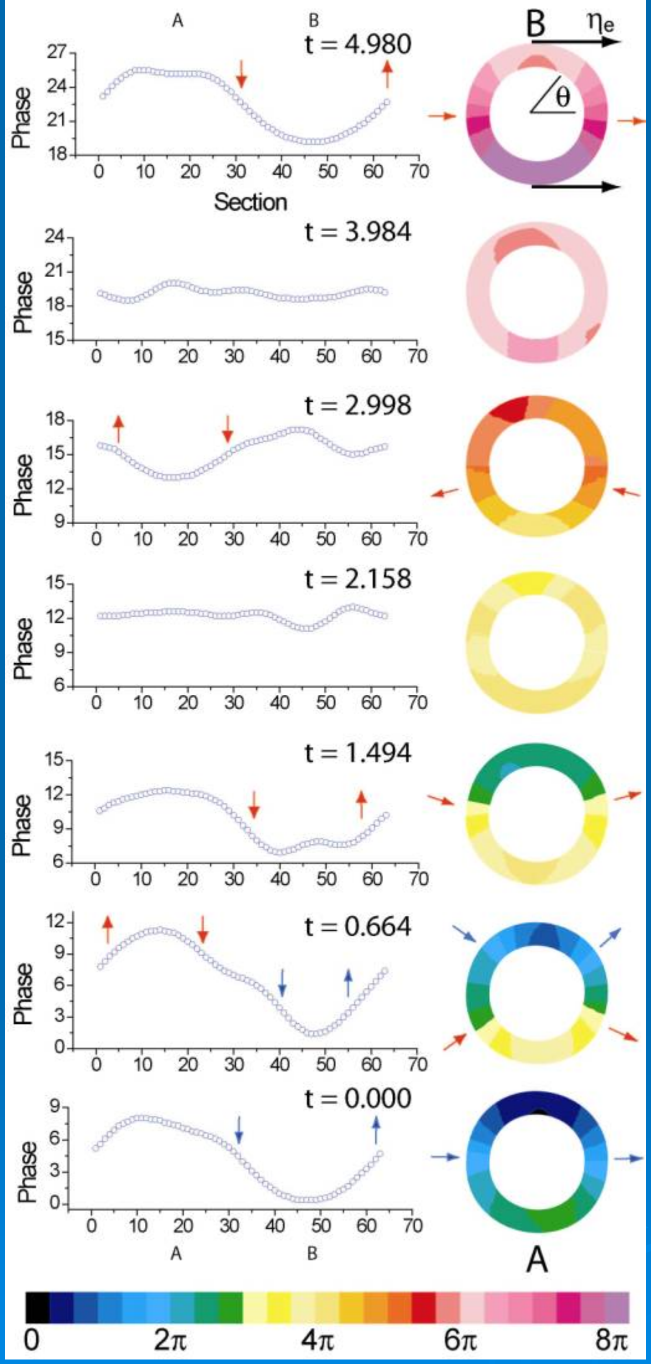
M. P. Lisitskiy et al
Appl. Phys. Lett **84**, 5464 (2004)

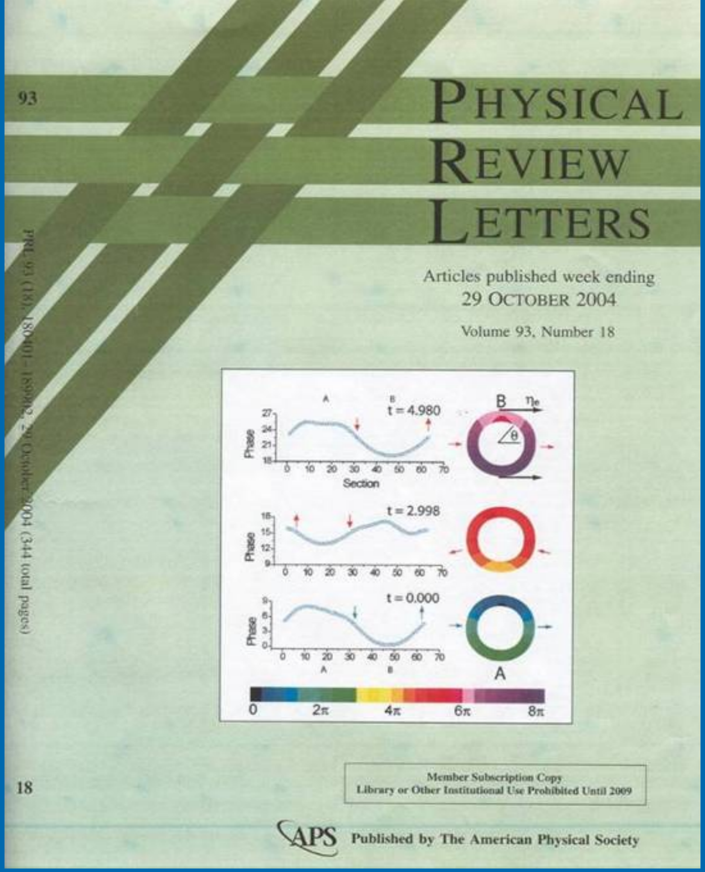
ICIB-CNR

The failure of the theory based on the cavity modes serves to detect and enlighten the mechanism of onset of fluxon sustained resonaces at small fields



Amplitude of the 3rd resonance vs magnetic field





C. Nappi, M. Lisitskiy
G.Rotoli, R. Cristiano,
A. Barone
Phys.Rev. Lett.
93, 187001(2004)

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New Fluxon Resonant Mechanism in Annular Josephson Tunnel Structures

C. Nappi,[1] M. P. Lisitskiy,[1] G. Rotoli,[2,3] R. Cristiano,[1] and A. Barone[2,4]

Part of the material of this seminar has been taken from the web sites:

For the animations:

- http://homepages.tversu.ru/~s000154/collision/solpen_m/SOLPEN1.html
- http://www.math.h.kyoto-u.ac.jp/~takasaki/soliton-lab/gallery/solitons/sg-e.html

Figure and other, A. Ustinov group web site:

- http://fluxon group.physik.uni-erlangen.de/