DISCRETE SYMMETRIES OF LEPTON MIXING ANGLES

[based on hep-ph/0504165 and hep-ph/0512103 with Guido Altarelli]

Ferruccio Feruglio
Padova

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low-energy parameters

ν masses

[3 light active ν]

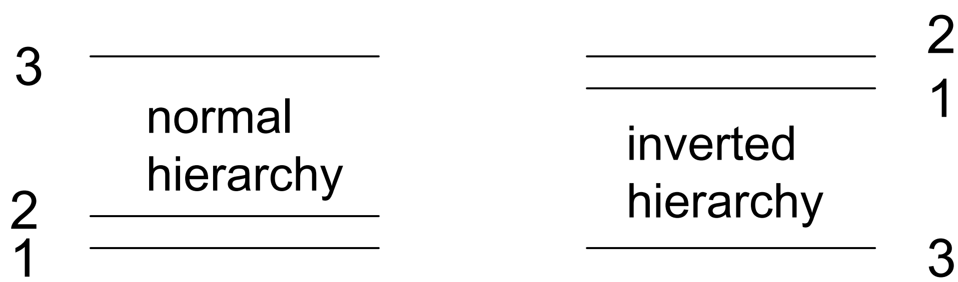
$$m_1, m_2, m_3$$

order $m_1 < m_2$

$$\Delta m^2_{21} < \left|\Delta m^2_{32}\right|, \left|\Delta m^2_{31}\right| \qquad [\Delta m^2_{ij} \equiv m_i^2 - m_j^2]$$

i.e. 1 and 2 are, by definition, the closest levels

two possibilities:



Mixing matrix (analogous to V_{CKM})

$$U_{PMNS} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{i\delta} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{-i\delta} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{-i\delta} & c_{13}s_{23} \\ -c_{12}s_{13}c_{23} + s_{12}s_{23}e^{-i\delta} & -s_{12}s_{13}c_{23} - c_{12}s_{23}e^{-i\delta} & c_{13}c_{23} \end{pmatrix} \times \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha} & 0 \\ 0 & 0 & e^{i\beta} \end{pmatrix}}$$

$$c_{12} \equiv \cos \vartheta_{12}, \ldots$$

- only if ν are Majorana
- drops in oscillations

Lepton Mixing Angles

[Fogli, Lisi, Marrone, Palazzo 0506083]
[Schwetz 0510331]

$[2\sigma \quad \text{errors (95\% C.L.)}]$

$\sin^2\vartheta_{23} = 0.44\,(1^{+0.41}_{-0.22})$ $\quad \sin^2\vartheta_{13} = 0.9^{+2.3}_{-0.9}\times 10^{-2}$ $\quad \sin^2\vartheta_{12} = 0.314\,(1^{+0.18}_{-0.15})$

$\vartheta_{23} = (41.6^{+10.4}_{-5.7})^0 \quad [2\sigma]$

[Hall, Murayama, Weiner 2000
De Gouvea, Murayama 0301050]

different viewpoints: - angles are all generically large [anarchy]
- angles reflect an underlying order

$\vartheta_{23} = 45^0$ $\quad \vartheta_{13} = 0$ $\quad \sin^2\vartheta_{12} = \frac{1}{3} \qquad \vartheta_{12} = 35.3^0$

[Harrison, Perkins and Scott (HPS) mixing pattern]

$\vartheta_{12} = (34.1\,^{+1.7}_{-1.6})^0 \quad [1\sigma]$

not a bad 1st order approximation!

θ_{12} right within $1\sigma \approx 2^0 \leq 0.04$ rad $\approx \lambda^2$, where λ=0.22
errors on θ_{23} and θ_{13} are still large…

future [< 10 yr] precision/sensitivity on θ_{23} and θ_{13} down to about λ^2
could confirm HPS mixing pattern

$\vartheta_{13} \approx \delta\vartheta_{23} \approx \lambda^2 \approx 0.04 \div 0.05\,\text{rad}\,(2.1^0 \div 2.9^0)$

[Gonzalez-Garcia, Maltoni, Smirnov 0408170]

$\sin^2\theta_{23}$

$$\vartheta_{23} \approx \frac{\pi}{4}$$

$$\delta\vartheta_{23} \approx \frac{\sqrt{\delta P_{\mu\mu}}}{2}$$

i.e. a small uncertainty on $P_{\mu\mu}$ leads to a large uncertainty on θ_{23}

$\delta(\sin^2\theta_{23})$ reduced by future LBL experiments from $\nu_\mu \to \nu_\mu$ disappearance channel

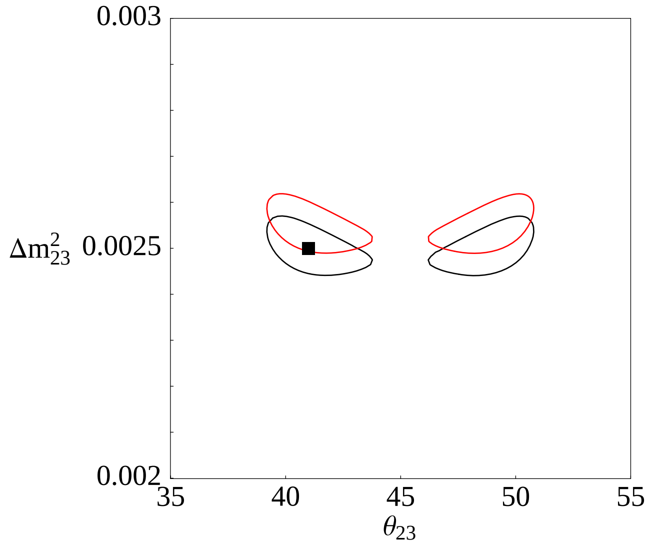
$$P_{\mu\mu} \approx 1 - \sin^2 2\vartheta_{23} \sin^2\left(\frac{\Delta m_{31}^2 L}{4E}\right)$$

- no substantial improvements from conventional beams
- superbeams (e.g. T2K in 5 yr of run)

$$\delta P_{\mu\mu} \approx 0.01$$

$$\delta\vartheta_{23} \approx 0.05\,\mathrm{rad} \leftrightarrow 2.9^0$$

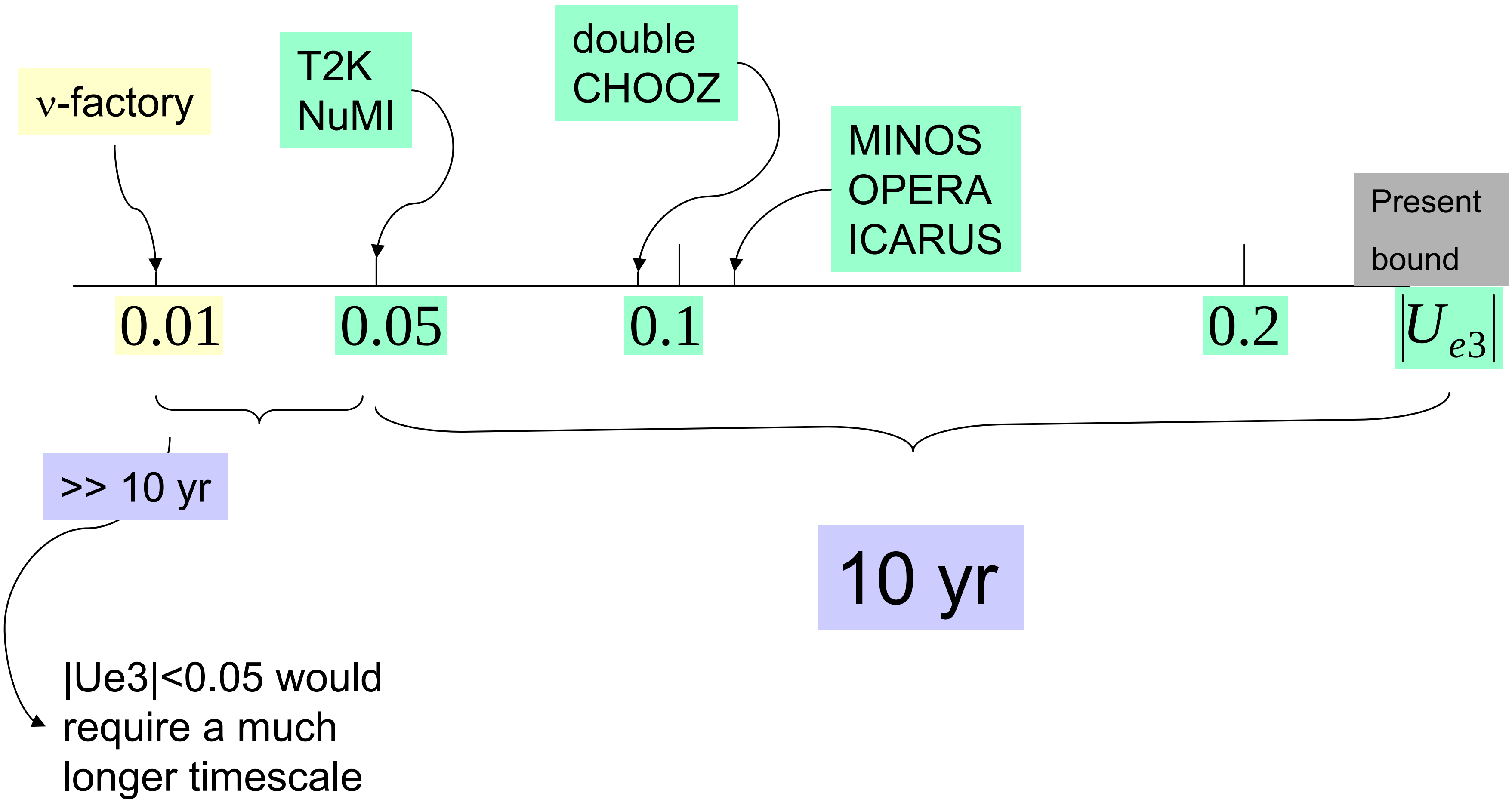
improvement by about a factor 2



T2K-1
90% CL
black = normal hierarchy
red = inverted hierarchy
true value 41^0
[courtesy by Enrique Fernandez]

$\sin\theta_{13}$

a similar sensitivity is expected on θ_{13} ($U_{e3}=\sin\theta_{13}$)



If future data will confirm HPS down to about λ^2 precision

$$U_{PMNS} = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} + O(\lambda^2)$$

quite symmetric!
also called
``tribimaximal''

reminiscent of

$$\pi^0 = \frac{|uu\rangle - |dd\rangle}{\sqrt{2}} \qquad \eta = \frac{|uu\rangle + |dd\rangle - 2|ss\rangle}{\sqrt{6}} \qquad \eta' = \frac{|uu\rangle + |dd\rangle + |ss\rangle}{\sqrt{3}}$$

theoretical challenges:
- how to derive HPS from a model?
more in general
- how to achieve exactly maximal θ_{23}
(eventually modified by small, $O(\lambda^2)$, corrections)?

many models predicts a large but not necessarily maximal θ_{23}

an example: abelian flavour symmetry group $U(1)_F$

$$F(l) = (\times,0,0) \qquad [\times \neq 0]$$

$$F(e^c) = (\times,\times,0)$$

$$m_e = \begin{pmatrix} \times & \times & \times \\ \times & \times & \times \\ \times & O(1) & O(1) \end{pmatrix} v_d \qquad m_\nu = \begin{pmatrix} \times & \times & \times \\ \times & O(1) & O(1) \\ \times & O(1) & O(1) \end{pmatrix} \frac{v_u^2}{\Lambda}$$

$$\vartheta_{23} \approx O(1)$$

maximal only by a fine-tuning!

similarly for all other abelian charge assignements

$$F(l) = (1,-1,-1)$$

$$m_\nu = \begin{pmatrix} \times & O(1) & O(1) \\ O(1) & \times & \times \\ O(1) & \times & \times \end{pmatrix} \frac{v_u^2}{\Lambda}$$

$$\vartheta_{23} \approx O(1) + \text{charged lepton contribution}$$

no help from the see-saw mechanism within abelian symmetries…

θ_{23} maximal by RGE effects?

[Ellis, Lola 1999
Casas, Espinoza, Ibarra, Navarro 1999-2003]

running effects important only for quasi-degenerate neutrinos

2 flavour case

$$m_2 \approx m_3 \approx m \qquad \text{or} \qquad \delta m \equiv m_2 - m_3 << m_2 + m_3 \approx 2m$$

boundary conditions at Q=Λ>> e.w. scale

$$m_2,\, m_3,\, \vartheta_{23} \qquad \frac{\delta m}{2m} << 1$$

at Q<Λ

$$\tan 2\vartheta_{23}(Q) = \frac{\delta m \sin 2\vartheta_{23}(1-\varepsilon)}{\delta m \cos 2\vartheta_{23}(1-\varepsilon) + 2m\varepsilon}$$

$$\varepsilon \approx \frac{1}{16\pi^2} y_\tau^2 \log\frac{\Lambda}{Q}$$

$$\vartheta_{23}(Q) \approx \frac{\pi}{4} \quad \Leftrightarrow \quad \varepsilon \approx -\frac{\delta m}{m}\cos 2\vartheta_{23}$$

gives the scale Q at which
θ_{23}(Q) becomes maximal

$\boxed{m_2,\, m_3,\, \vartheta_{23} \text{ fine tuned to obtain Q at the e.w. scale}}$

a similar conclusion also for the 3 flavour case:

$$\sin^2 2\vartheta_{12} = \frac{\sin^2\vartheta_{13}\sin^2 2\vartheta_{23}}{(\sin^2\vartheta_{23}\cos^2\vartheta_{13} + \sin^2\vartheta_{13})^2}$$

infrared stable fixed point

[Chankowski, Pokorski 2002]

$$\text{if} \quad \vartheta_{23} = \frac{\pi}{4}$$

$$\sin^2 2\vartheta_{12} = \frac{4\sin^2\vartheta_{13}}{(1+\sin^2\vartheta_{13})^2} < 0.2 \ \ \text{(Chooz)}$$

wrong!

θ_{23} maximal from non-abelian flavour symmetries ?

an obstruction: $\vartheta_{23} = 45^0$ can never arise in the limit of an exact realistic symmetry

charged lepton mass matrix:

$$m_l = m_l^0 + \delta m_l^0$$

symmetric limit

symmetry breaking effects: vanishing when flavour symmetry F is exact

realistic symmetry:

(1) $\left|\delta m_l^0\right| < \left|m_l^0\right|$

(2) m_l^0 has rank ≤1

$$m_l^0 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & m_\tau \end{pmatrix}$$

ϑ_{12}^e undetermined

$$U_{PMNS} = U_e^+ U_\nu$$

[omitting phases]

$$\tan\ \vartheta_{23}^0 = \tan\ \vartheta_{23}^\nu \cos\ \vartheta_{12}^e + \left(\frac{\tan\ \vartheta_{13}^\nu}{\cos\ \vartheta_{23}^\nu}\right) \sin\ \vartheta_{12}^e$$

undetermined

$\vartheta_{23} = 45^0$

determined entirely by breaking effects
(different, in general, for ν and e sectors)

requirements for a model based on a SB flavour symmetry

- spontaneous symmetry breaking → vacuum alignment problem $\langle\varphi_\nu\rangle, \langle\varphi_e\rangle, \ldots$

  should have specific magnitudes and relative directions in flavour space.

- alignment should be natural

  no ad-hoc relations: desired VEVs from most general V in a finite region of parameter space

- alignment not spoiled by sub-leading terms

in HPS

$$\vartheta_{13} = \underbrace{0}_{\text{leading order}} + \underbrace{a_1 \frac{\langle\varphi\rangle}{\Lambda} + a_2 \frac{\langle\varphi\rangle^2}{\Lambda^2} + \ldots}$$

$$\vartheta_{23} = \underbrace{\frac{\pi}{4}}_{\text{leading order}} + \underbrace{b_1 \frac{\langle\varphi\rangle}{\Lambda} + b_2 \frac{\langle\varphi\rangle^2}{\Lambda^2} + \ldots}_{\text{from higher-dimensional operators compatible with gauge and flavour symmetries}}$$

from higher-dimensional operators compatible with gauge and flavour symmetries

often $\frac{\langle\varphi\rangle}{\Lambda} \approx \lambda$

then $a_1 = b_1 = 0$ needed

leading order

- alignment compatible with mass hierarchies

$\frac{m_e}{m_\tau}, \quad \frac{m_\mu}{m_\tau}$ should vanish in the limit of exact symmetry

an example: spontaneously broken A_4 symmetry

[Ma, Rajasekaran 2001; Babu, Ma, Valle 2003; Hirsch, Romao, Skandage, Valle, Villanova de Moral 2003; Ma 0409075]

A_4 - group of even permutation of four objects
- subgroup of SO(3) leaving a tetrahedron invariant

it has 12 elements that can all be generated starting from 2 of them

$$S \quad T$$

``presentation''

$$S^2 = (ST)^3 = T^3 = 1$$

$$A_4 = \{1, S, T, ST, TS, T^2, ST^2, STS, TST, T^2S, TST^2, T^2ST\}$$

A_4 representations:

$$\omega \equiv e^{i\frac{2\pi}{3}}$$

1	$S=1$	$T=1$
$1'$	$S=1$	$T=\omega^2$
$1''$	$S=1$	$T=\omega$

$$3 \quad S = \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix} \qquad T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix}$$

S generates a Z_2 subgroup : G_S

T generates a Z_3 subgroup : G_T

patterns of symmetry breaking

A_4 triplet $\varphi = (\varphi_1, \varphi_2, \varphi_3)$

$$\langle \varphi \rangle \propto (1,1,1)$$

$$\langle \varphi \rangle \propto (1,0,0)$$

$$A_4 \rightarrow G_S$$

$$A_4 \rightarrow G_T$$

as we shall see

G_S is the low-energy symmetry in the ν sector

G_T is the low-energy symmetry in the charged lepton sector

basic structure (lepton sector)

	l	e^c	μ^c	τ^c	h_u	h_d	φ_T	φ_S	ξ_i
A_4	3	1	1''	1'	1	1	3	3	1

matter fields — Higgses — A_4 breaking sector

SU(2)xU(1)xA_4 invariant Lagrangian:

[Λ Is the cutoff]

$$L = y_e e^c(\varphi_T l) + y_\mu \mu^c(\varphi_T l)' + y_\tau \tau^c(\varphi_T l)'' + x_a \xi(ll) + x_b(\varphi_S ll) + \ldots$$

higher dimensional operators in $1/\Lambda$ expansion

- (…) denotes an A_4 singlet,…
- powers of $\left(\frac{h_{u,d}}{\Lambda}\right)$ have been set to 1
- some invariant is missing from L: [more on this later on…]

$$\begin{cases} \varphi_S \leftrightarrow \varphi_T \\ x(ll) \end{cases}$$

assume:

$$\langle \varphi_T \rangle = (v_T, 0, 0)$$
$$\langle \varphi_S \rangle = (v_S, v_S, v_S)$$
$$\langle \xi \rangle = u$$

$$v_T, v_S, u \leq \Lambda$$

$$[\langle h_{u,d} \rangle = v_{u,d} << v_T, v_S, u]$$

then:

$$m_l = \begin{pmatrix} y_e & 0 & 0 \\ 0 & y_\mu & 0 \\ 0 & 0 & y_\tau \end{pmatrix} v_d \left(\frac{v_T}{\Lambda} \right)$$

charged fermion masses

$$m_f = y_f v_d \left(\frac{v_T}{\Lambda} \right)$$

free parameters as in the SM
at this level

$$m_\nu = \begin{pmatrix} a + \frac{2}{3} b & -\frac{b}{3} & -\frac{b}{3} \\ -\frac{b}{3} & \frac{2}{3} b & a - \frac{b}{3} \\ -\frac{b}{3} & a - \frac{b}{3} & \frac{2}{3} b \end{pmatrix} \frac{v_u^2}{\Lambda}$$

$$a \equiv 2 x_a \frac{u}{\Lambda}$$
$$b \equiv 2 x_b \frac{v_S}{\Lambda}$$

2 complex
parameters in
ν sector
(overall phase unphysical)

mixing angles entirely from ν sector:

$$U_{PMNS}=\begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

independent from
|a|, |b|, $\Delta\equiv$arg(a)-arg(b) !!

ν masses:

$$m_1=|a+b|\frac{v_u^2}{\Lambda} \qquad m_2=|a|\frac{v_u^2}{\Lambda} \qquad m_3=|a-b|\frac{v_u^2}{\Lambda}$$

$m_2>m_1$ $\Rightarrow$ $-1\le\cos\Delta<-\left|\frac{b}{2a}\right|$ $\Rightarrow$ ν spectrum always of normal hierarchy type

$$\left|\frac{b}{2a}\right|\approx\begin{cases}1 & \text{[almost hierarchical spectrum]}\\ 0 & \text{[almost degenerate spectrum]}\end{cases}$$

$r\equiv\frac{\Delta m^2_{sol}}{\Delta m^2_{atm}}\approx\frac{1}{35}$ requires a (moderate) tuning prediction:

$$|m_3|^2 = |m_{ee}|^2 + \frac{10}{9}\Delta m^2_{atm}\left(1 - \frac{\Delta m^2_{sol}}{\Delta m^2_{atm}}\right)$$

range of VEVs:

$$m_\tau = y_\tau v_d \left(\frac{v_T}{\Lambda}\right)$$

$$y_\tau < 4\pi$$

$$\frac{v_T}{\Lambda} > 0.002(0.02)$$

$$\tan\beta = 2.5(30)$$

$$\tan\beta = \frac{v_u}{v_d}$$

from ν spectrum

$$\Lambda = 1.8 \times 10^{15}\left(\frac{v_S}{\Lambda}\right)\sin^2\beta \quad \mathrm{GeV}$$

assuming all VEVs of the same order

$$0.002 < \frac{v_T}{\Lambda} \approx \frac{v_S}{\Lambda} \approx \frac{u}{\Lambda} < 1$$

$$\Lambda < 0.25 \times 10^{15} \quad \mathrm{GeV}$$

natural vacuum alignment

$$\langle\varphi_T\rangle = (v_T,0,0)$$
$$\langle\varphi_S\rangle = (v_S,v_S,v_S)$$
$$\langle\xi\rangle = u$$

it is not a local minimum of the most general renormalizable scalar potential V depending on φ_S , φ_T , ξ and invariant under A_4

$$v_T \approx v_S \approx u$$

a simple solution in 1 extra dimension ≡ ED

[Altarelli, F. 0504165]

$$\langle\varphi_T\rangle = (v_T,0,0)$$

local minimum of V_0

e^c, μ^c, τ^c

$l, h_{u,d}$

0 y L

$$\langle\varphi_S\rangle = (v_S,v_S,v_S)$$
$$\langle\xi\rangle = u$$

local minimum of V_L

ν masses arise from local operators at y=L

$$\frac{(\varphi_S ll)h_u h_u}{\Lambda^2} \qquad \frac{\xi(ll)h_u h_u}{\Lambda^2}$$

this explains also the absence of the terms with $\varphi_S \leftrightarrow \varphi_T$

charged lepton masses from non-local operators

$$\left.\begin{array}{l} \dfrac{(f^c\varphi_T F)\delta(y)}{\sqrt{\Lambda}} \\ -MFF^c \\ \dfrac{(F^c l)h_d}{\sqrt{\Lambda}}\delta(y-L) \end{array}\right\} \quad E << M$$

bulk fermion Y=-1

$$\frac{(f^c\varphi_T l)h_d}{\Lambda} e^{-ML}$$

a 4D supersymmetric solution ≡ SUSY

[Altarelli,F. hep-ph/0512103]

L is identified with the superpotential w_{lepton} in the lepton sector

w_{lepton} is invariant under $A_4 \times Z_3 \times U(1)_R$

	l	e^c	μ^c	τ^c	$h_{u,d}$	φ_T	φ_S	ξ	$\tilde{\xi}$	φ_0^T	φ_0^S	ξ_0
A_4	3	1	1''	1'	1	3	3	1	1	3	3	1
Z_3	ω	ω^2	ω^2	ω^2	1	1	ω	ω	ω	1	ω	ω
$U(1)_R$	1	1	1	1	0	0	0	0	0	2	2	2

matter fields (l, e^c, μ^c, τ^c) — Higgses ($h_{u,d}$) — A_4 breaking sector ($\varphi_T, \varphi_S, \xi, \tilde{\xi}$) — ``driving fields'' ($\varphi_0^T, \varphi_0^S, \xi_0$)

absence of $\varphi_S \leftrightarrow \varphi_T$ $\quad x(ll)$ automatic

$$w = w_{lepton} + w_d + \ldots$$

$$w_d = M(\varphi_0^T \varphi_T) + g(\varphi_0^T \varphi_T \varphi_T) + g_1(\varphi_0^S \varphi_S \varphi_S) + g_2 \tilde{\xi}(\varphi_0^S \varphi_S) + g_3 \xi_0 (\varphi_S \varphi_S) + g_4 \xi_0 \xi^2 + g_5 \xi_0 \xi \tilde{\xi} + g_6 \xi_0 \tilde{\xi}^2$$

minimum of the scalar potential at:

$$\langle \varphi_T \rangle = (v_T, 0, 0)$$
$$\langle \varphi_S \rangle = (v_S, v_S, v_S)$$
$$\langle \xi \rangle = u$$
$$\langle \tilde{\xi} \rangle = 0$$

$$v_T = -\frac{3M}{2g} \qquad v_S^2 = -\frac{g_4}{3g_3} u^2$$

u undetermined

sub-leading corrections

arising from higher dimensional operators, depleted by additional powers of $1/\Lambda$. ⇨ they affect m_l , m_ν and they can deform the VEVs.

results

$$U_{PMNS} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} + \begin{cases} O(\frac{VEV}{\Lambda})^2 & \text{for ED case} \\ O(\frac{VEV}{\Lambda}) & \text{for SUSY case} \end{cases}$$

and similarly for neutrino masses

HPS pattern is preserved if corrections are $\leq \lambda^2 \approx 0.04$

given the range $0.002<(VEV/\Lambda)<1$, corrections can be kept below λ^2 in both ED and SUSY case.

in ED case, (VEV/Λ) can be as large as λ without spoiling HPS

alignment and mass hierarchies

$$m_l = \begin{pmatrix} y_e & 0 & 0 \\ 0 & y_\mu & 0 \\ 0 & 0 & y_\tau \end{pmatrix} v_d \left(\frac{v_T}{\Lambda} \right)$$

charged fermion masses are already diagonal

$$m_e << m_\mu << m_\tau$$

easily reproduced by U(1) flavour symmetry

$$Q(e^c) = 4 \quad Q(\mu^c) = 2 \quad Q(\tau^c) = 0$$
$$Q(l) = 0$$

compatible with A_4

$$y_e \approx \frac{\langle \vartheta \rangle^4}{\Lambda^4} \qquad y_\mu \approx \frac{\langle \vartheta \rangle^2}{\Lambda^2} \qquad y_\tau \approx 1$$

quark masses

simple and good first order approximation:

	q	u^c	c^c	t^c	d^c	s^c	b^c
A_4	3	1	1''	1'	1	1''	1'

same assignment as in the lepton sector

quark mass matrices diagonal in the leading order
mixing matrix $V_{CKM}=1$

unfortunately:
corrections induced by higher dimensional operators:
negligibly small

additional sources of A_4 breaking are needed in the quark sector

relation to the modular group

modular group PSL(2,Z): linear fractional transformation

complex variable

$$z \to \frac{a\,z+b}{c\,z+d} \qquad a,b,c,d \in Z \qquad ad-bc=1$$

discrete, infinite group generated by two elements

$$\underbrace{z \to -\frac{1}{z}}_{S} \qquad \underbrace{z \to z+1}_{T}$$

obeying

$$S^2 = (ST)^3 = 1$$

the modular group is present everywhere in string theory

A_4 is a finite subgroup of the modular group and

$$A_4 = \frac{PSL(2,Z)}{H} \quad \Rightarrow$$

representations of A_4 are representations of PSL(2,Z)

conclusion

mixing in the lepton sector is well described by the HPS pattern

$\vartheta_{23} = 45^0$ $\vartheta_{13} = 0$ $\sin^2 \vartheta_{12} = \frac{1}{3}$

errors on θ_{23} and θ_{13} are still large and future data are needed
to confirm HPS at the λ^2 level

most of existing models predict $\left| \frac{\pi}{4} - \vartheta_{23} \right| >> \lambda^2$

only in ``special'' models this condition is violated. If based on a SB flavour symmetry, special models should give rise to a

natural vacuum alignment

preserved by high-order effects

with a structure compatible with the observed charged fermion hierarchy

Here: an existence proof based on the discrete group A_4

vacuum alignment and stability is non-trivial and
the simplest solutions requires either an extra dimension
or a SUSY version

In both these models the neutrino spectrum is of normal hierarchy type
and the relation $|m_3|^2 = |m_{ee}|^2 + (10/9)\Delta m^2_{atm}\left(1 - \Delta m^2_{sol} / \Delta m^2_{atm}\right)$ is predicted