The rich structures of a simple hamiltonian: critical points and phase transitions in atomic nuclei.

J. Jolie, Universität zu Köln

1. A simple hamiltonian in the interacting boson model.

2. Shape phase transitions in the atomic nucleus.

3. Level dynamics and phase transitions

4. Chaos and regularity in the Casten triangle

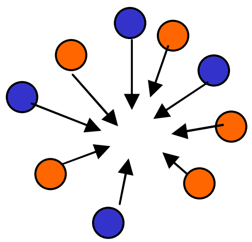
How do complex systems emerge from simple ingredients

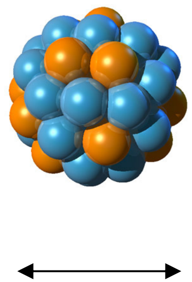
<u>Basic ingredients:</u>

two sets of indistinguishable fermions

a complex short range force (Van der Waals typ)

the possibility that one kind of fermions becomes the other kind



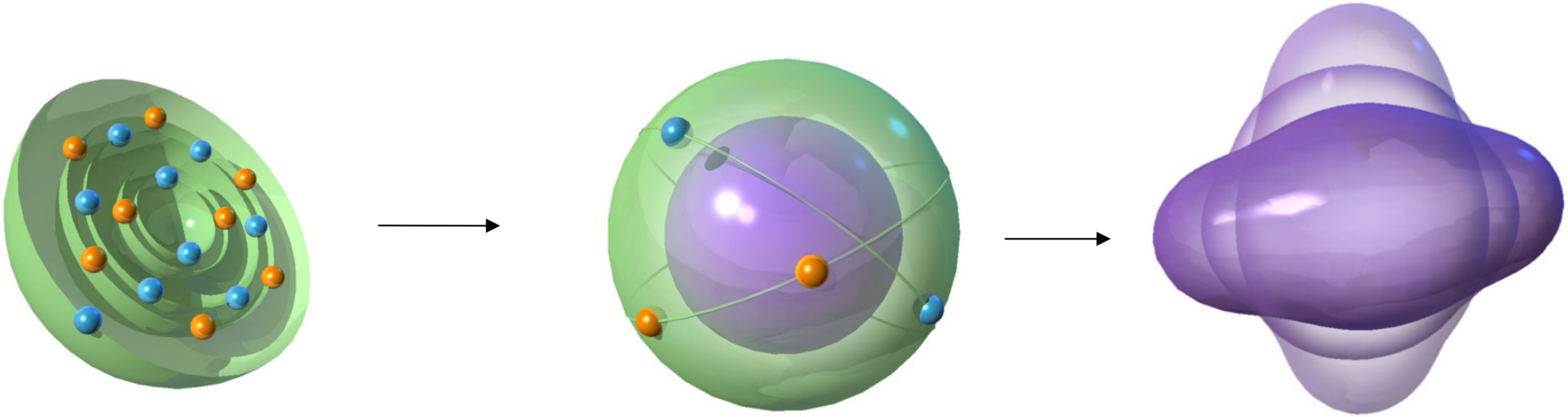


+ Binding energy

3fm

The atomic nucleus forms a unique two-component mesoscopic system, which is hard to manipulate but genereous in the number of observables it emits.

Once the atomic nucleus is formed effective (in-medium) forces generate simple collective motions.



shell structure:
valence nucleons

Cooper pairing:
N s,d boson system

Collective motion:
nuclear shapes

The interacting boson approximation (IBA)
(A. Arima and F.Iachello)

N s,d Boson System (A. Arima& F. Iachello)

$$E\,\Psi = \left[\sum_{k,l} b_k^\dagger \langle k|T|l\rangle b_l + \frac{1}{2}\sum_{k,l,m,n} b_k^\dagger b_l^\dagger \langle kl|V|mn\rangle b_m b_n \right]\Psi$$

N=cte

$$\hat{H} = \varepsilon \hat{n}_d + c_1 \hat{L}.\hat{L} + c_2 \hat{Q}.\hat{Q} + c_3 \hat{T}_3.\hat{T}_3 + c_4 \hat{T}_4.\hat{T}_4$$

Dynamical symmetries of a N s,d boson system

$\supset U(5) \supset O(5) \supset SO(3) \supset SO(2)$ vibrational nuclei

$\{nd\} \quad (v) \quad L \quad M$

$U(6) \supset O(6) \supset O(5) \supset SO(3) \supset SO(2)$ γ-unstable nuclei

$[N] \quad \langle\Sigma\rangle \quad (\tau) \quad L \quad M$

$\supset SU(3) \supset SO(3) \supset SO(2)$ prolate rotor

$(\lambda,\mu) \quad L \quad M$

$\supset \overline{SU(3)} \supset SO(3) \supset SO(2)$ oblate rotor

$(\lambda,\mu) \quad L \quad M$

1. A simple hamiltonian in the interacting boson model.

Most nuclei are very well described by a very simple IBA hamiltonian:

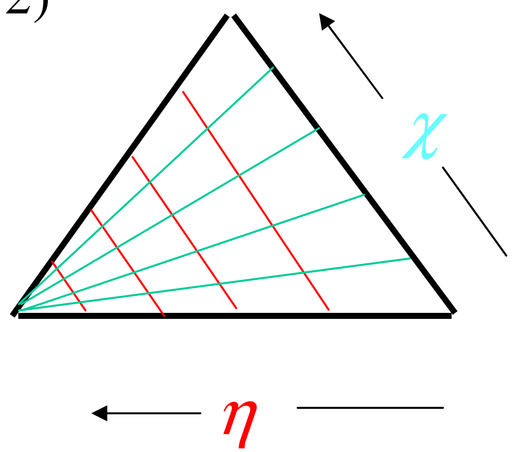
generates spherical shape

generates deformed shape

$$\hat{H} = a\left[\eta \hat{n}_d - \frac{(1-\eta)}{N}\hat{Q}_\chi \cdot \hat{Q}_\chi\right]$$

with $$\hat{Q}_\chi = (s^+\tilde{d} + d^+ s)^{(2)} + \chi (d^+ \tilde{d})^{(2)}$$

with two structural parameters η and χ and a scaling factor *a*.



The simple hamiltonian has four dynamical symmetries

$$\hat{H} = a\left[\eta \hat{n}_d - \frac{(1-\eta)}{N}\hat{Q}_\chi \cdot \hat{Q}_\chi\right]$$

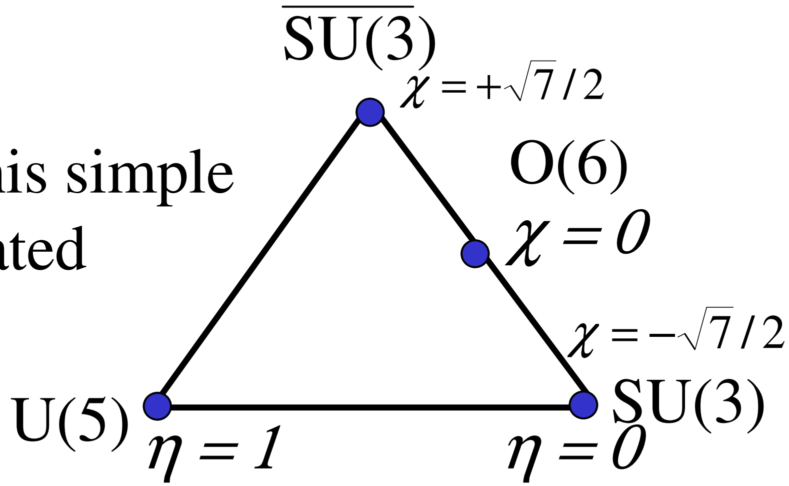
$\eta = 1$ U(5) limit $U(6) \supset U(5) \supset O(5) \supset SO(3)$

$\eta = 0, \chi = 0$ O(6) limit $U(6) \supset O(6) \supset O(5) \supset SO(3)$

$\eta = 0, \chi = -\sqrt{7}/2$ SU(3) limit $U(6) \supset SU(3) \supset SO(3)$

$\eta = 0, \chi = +\sqrt{7}/2$ $\overline{SU(3)}$ limit $U(6) \supset \overline{SU(3)} \supset SO(3)$

The rich structure of this simple hamiltonian are illustrated by the Casten triangle



Nuclear shapes associated with the four dynamical symmetries

The shapes can be studied using the coherent state formalism.

$$\left|N,\eta,\chi;\alpha_\mu\right\rangle = \frac{1}{\sqrt{N}}\left(s^+ + \sum_\mu \alpha_\mu d_\mu^+\right)^N \left|0\right\rangle$$

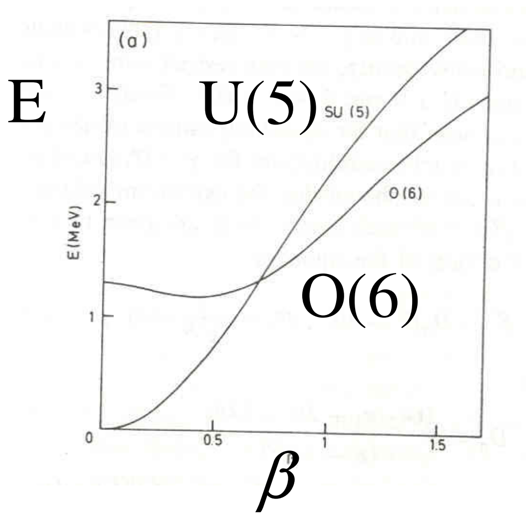
using the intrinsic state (Bohr) variables:

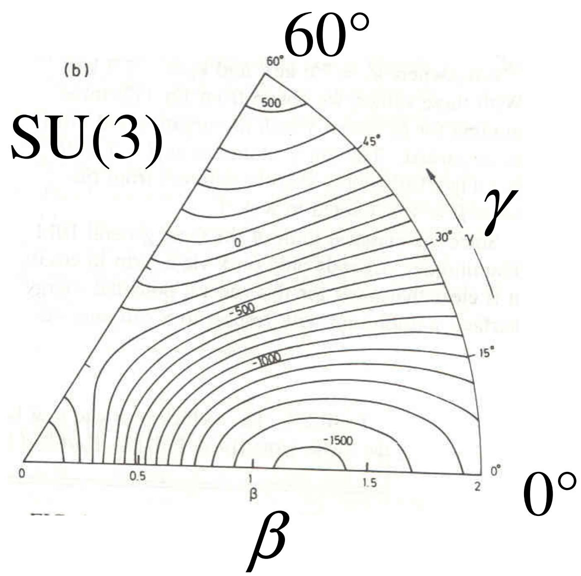
$$\left|N,\eta,\chi;\beta,\gamma\right\rangle = \frac{1}{\sqrt{N}}\left(s^+ + \beta\cos\gamma\, d_0^+ + \frac{\beta\sin\gamma}{\sqrt{2}}(d_{+2}^+ + d_{-2}^+)\right)^N \left|0\right\rangle$$

Then the energy functional:

$$E(N,\eta,\chi;\beta,\gamma) = \frac{\left\langle N,\eta,\chi;\beta,\gamma\right|\hat{H}\left|N,\eta,\chi;\beta,\gamma\right\rangle}{\left\langle N,\eta,\chi;\beta,\gamma \middle| N,\eta,\chi;\beta,\gamma\right\rangle}$$

can be evaluated for each value of β and γ.





U(5) limit: $\beta_0 = 0; \gamma_0$ irrelevant: spherical vibrator

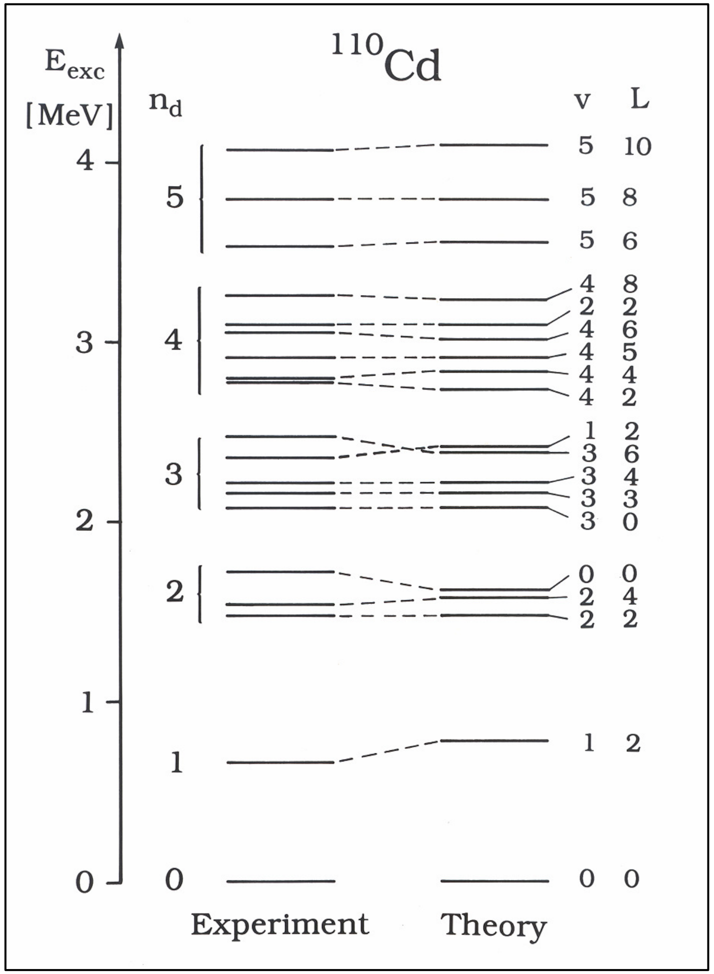
O(6) limit: $\beta_0 \neq 0; \gamma_0$ flat: γ-unstable rotor

SU(3) limit: $\beta_0 \neq 0; \gamma_0 = 0°$ prolate rotor

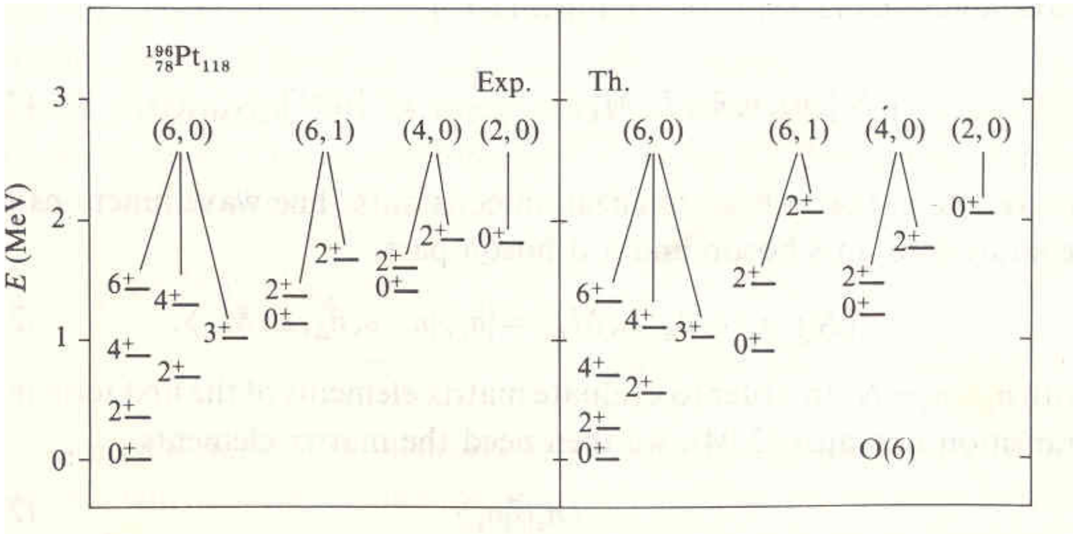
$\overline{\mathrm{SU(3)}}$ limit: $\beta_0 \neq 0; \gamma_0 = 60°$ oblate rotor

Experimental example:

^{110}Cd a U(5) nucleus



^{196}Pt the first O(6) nucleus

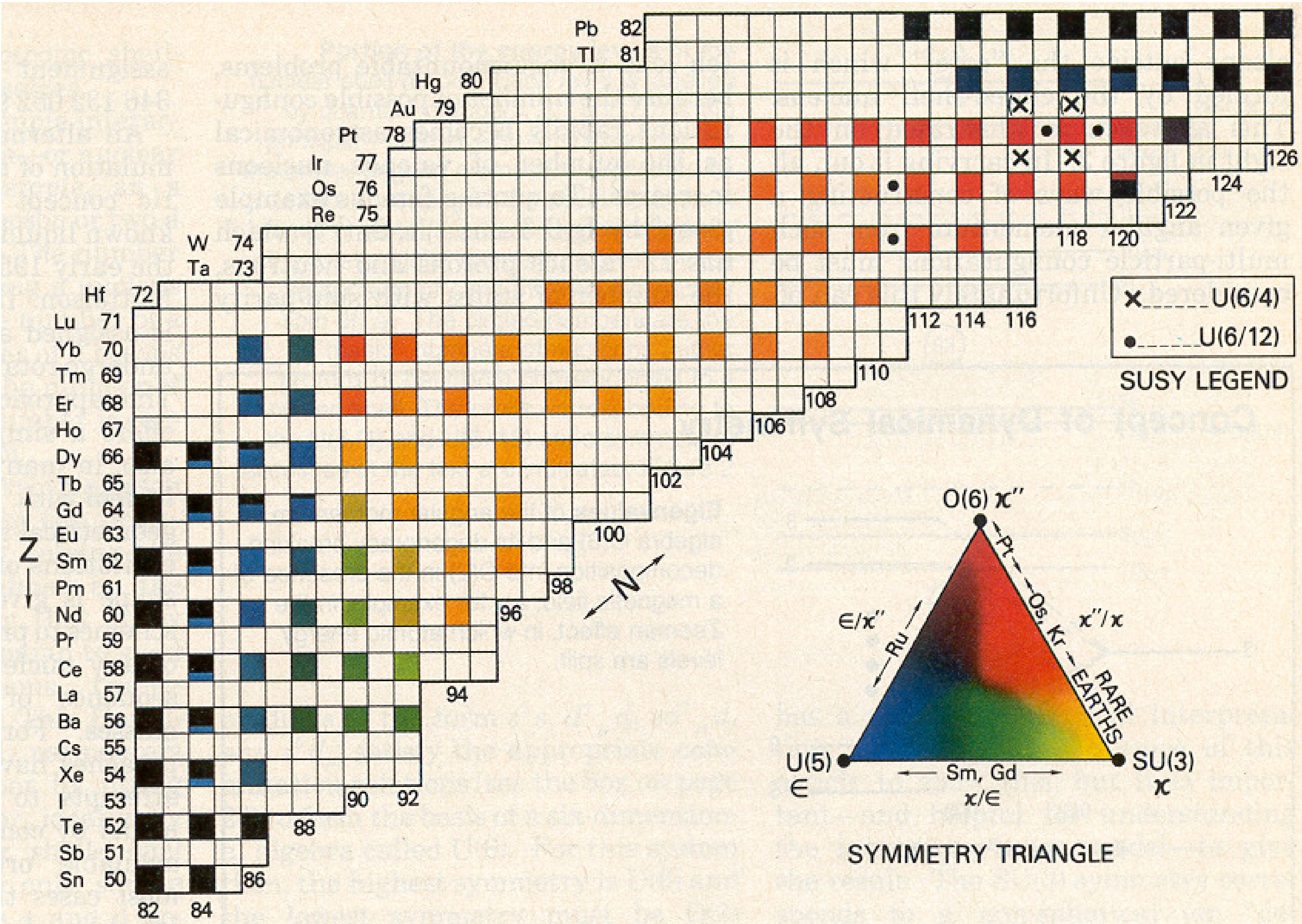


Discovered in 1978 using GAMS2/3 data.

Cizewski, Casten, Smith, Stelts, Kane, Börner, Davidson,
Phys. Rev. Lett. 40 (1978) 167

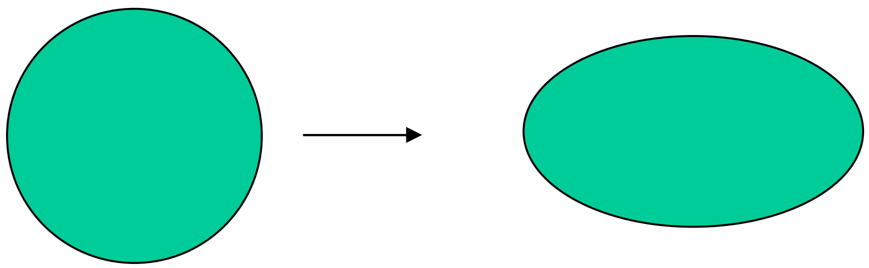
M. Bertschy, S. Drissi, P.E. Garrett, J. Jolie, J. Kern,
S.J. Manannal, J.P. Vorlet, N. Warr, J. Suhonen,
Phys. Rev C 51 (1995) 103

Besides the atomic nuclei representing a dynamical symmetry, the IBA is also able to describe transitional nuclei.

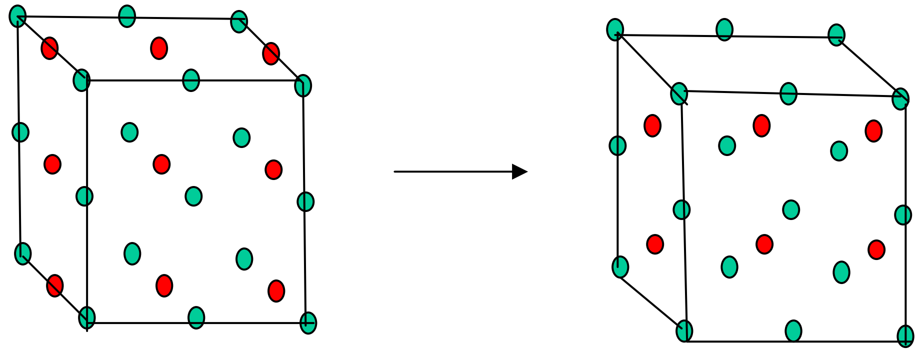


2. Shape phase transitions in the atomic nucleus.

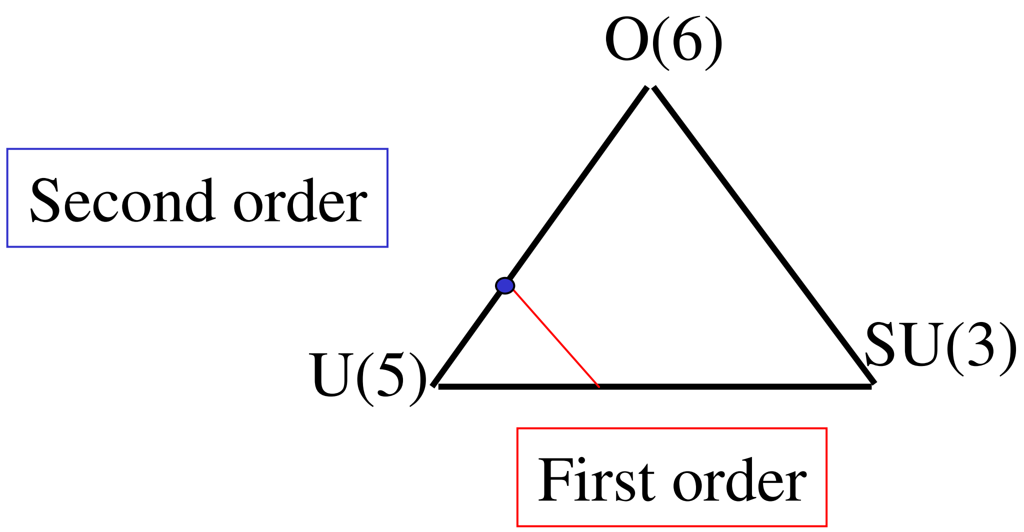
When studying the changes of the nuclear shape one might observe shape phase transitions of the groundstate configuration.



They are analogue to phase transitions in crystals



Feng, Gilmore and Deans have studied the shape phase transitions in detail in the early eighties using the coherent states (Phys. Rev. C23 (1981)1254).

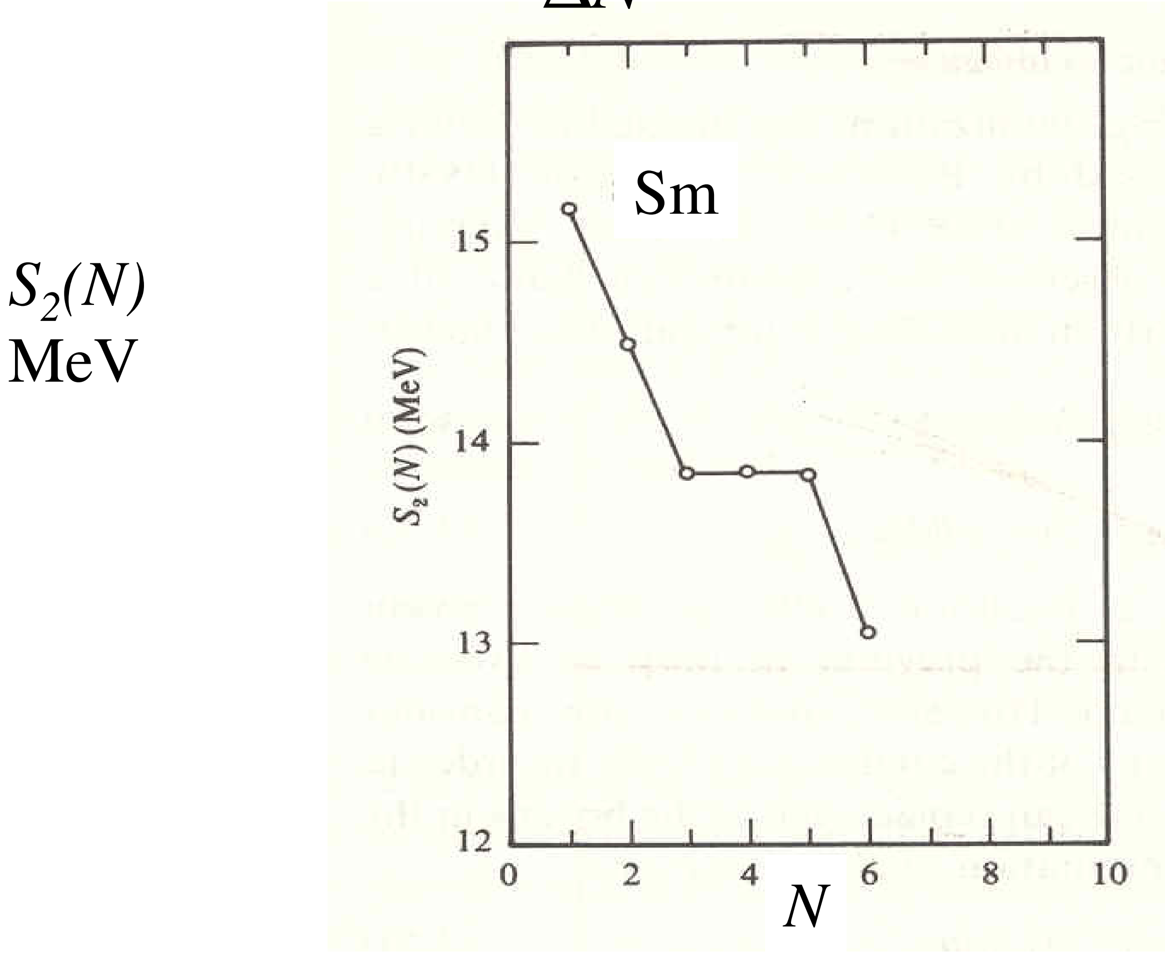


What are the experimental observables of shape phase transitions?

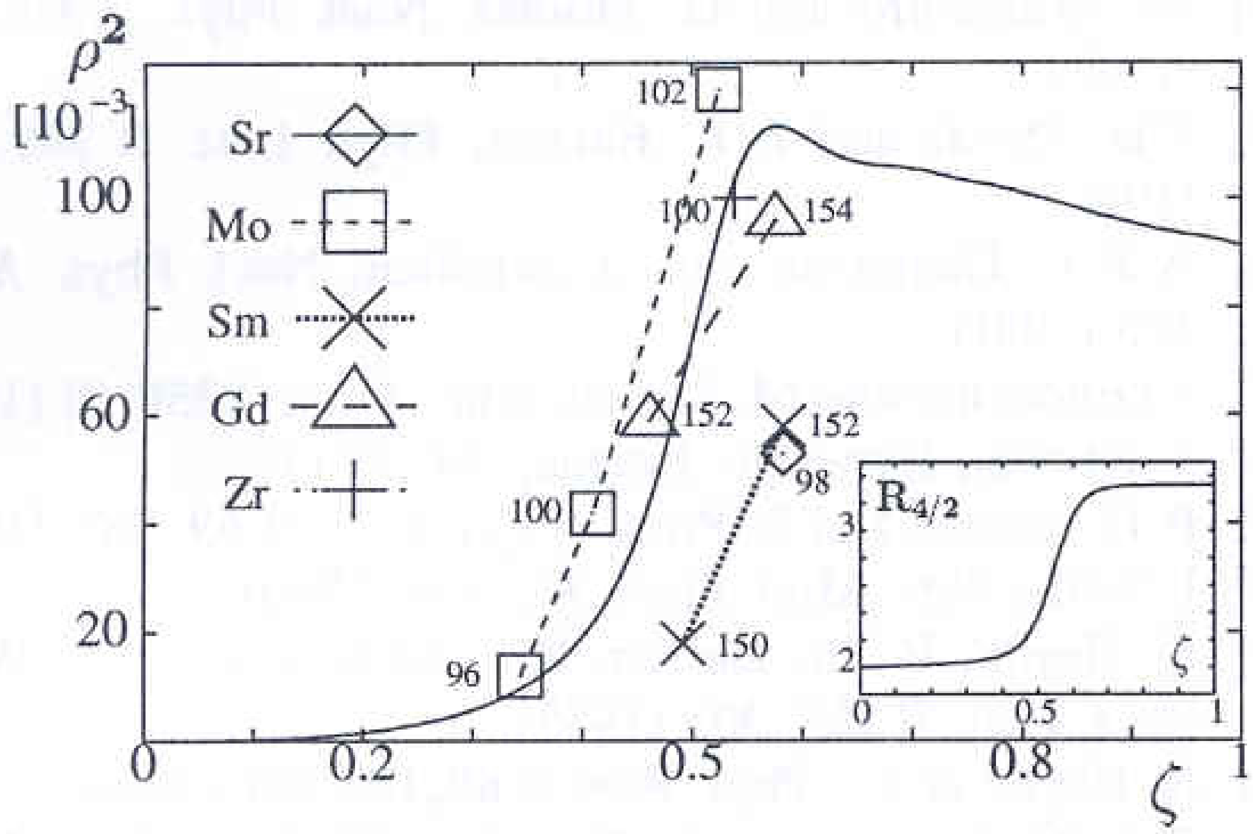
Two-neutron separation energies have been used before:

$$S_2(N) = E_B(N+1) - E_B(N) = \frac{\Delta E_B}{\Delta N}$$

First order transition in U(5) to SU(3)

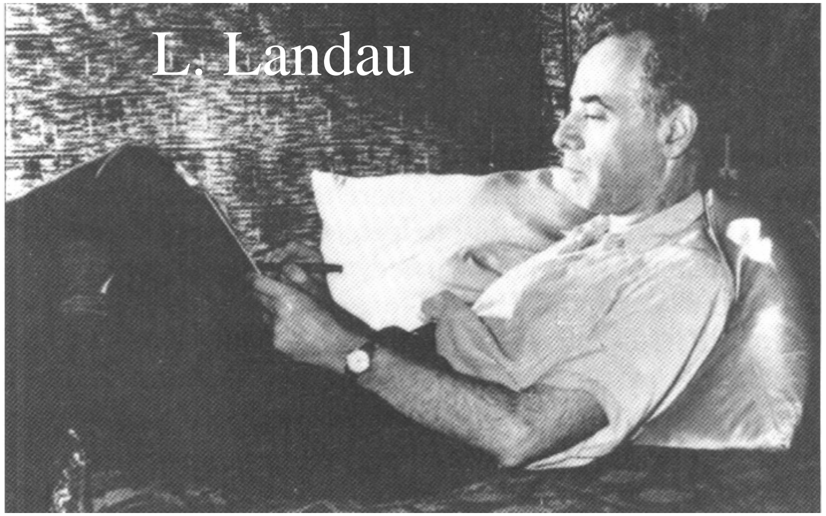


Recently, the importance of E0 transitions was stressed.
P. von Brentano et al. Phys. Rev. Lett. 93 (2004) 152502



Landau theory of continuous phase transitions (1937) describes these shape phase transitions.

J.Jolie, P. Cejnar, R.F. Casten, S. Heinze, A. Linnemann, V. Werner, Phys. Rev. Lett. 89 (2002) 182502.
P. Cejnar, S. Heinze, J.Jolie, Phys. Rev. C 68 (2003) 034326

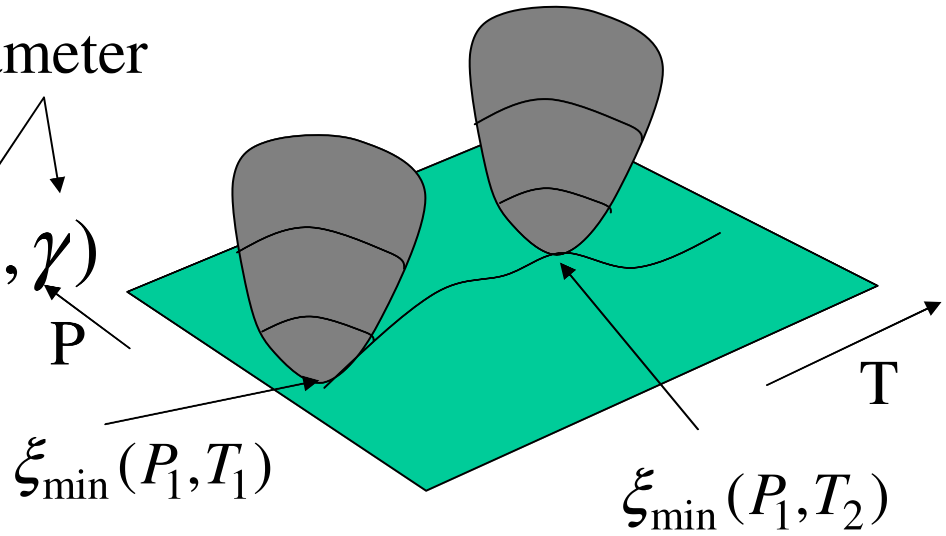


Thermodynamic potential: $\Phi(P,T;\xi)$

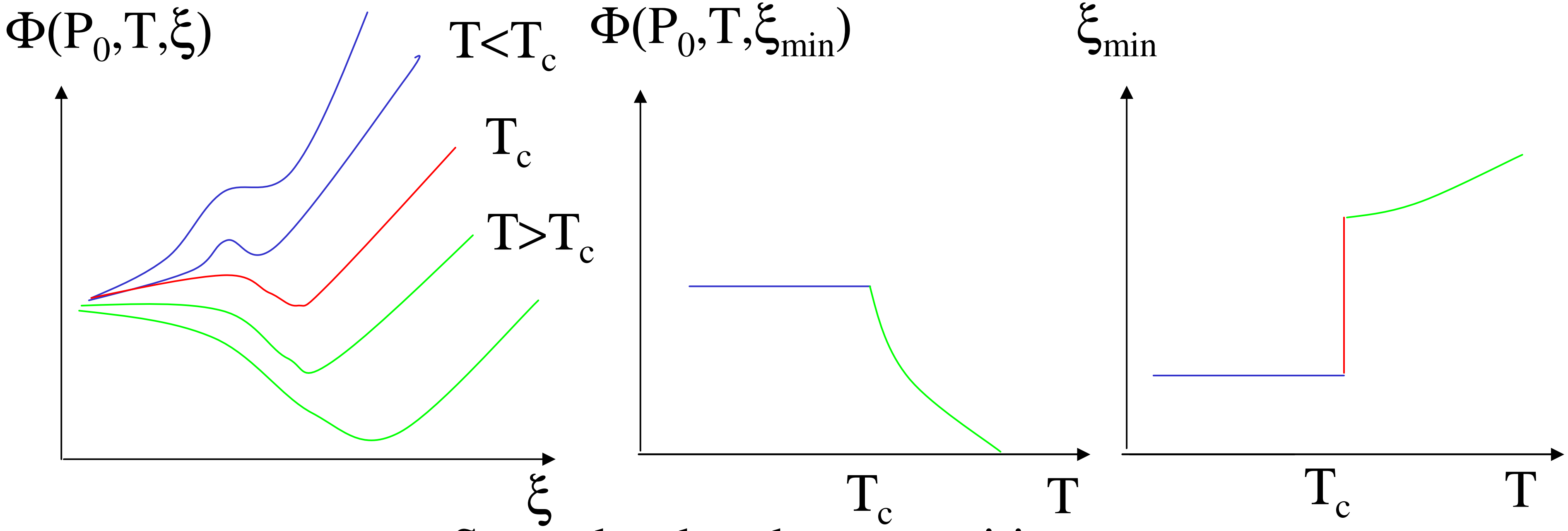
External parameters → P, T; Order parameter → ξ

External parameters → N, η, χ; Order parameter → β, γ

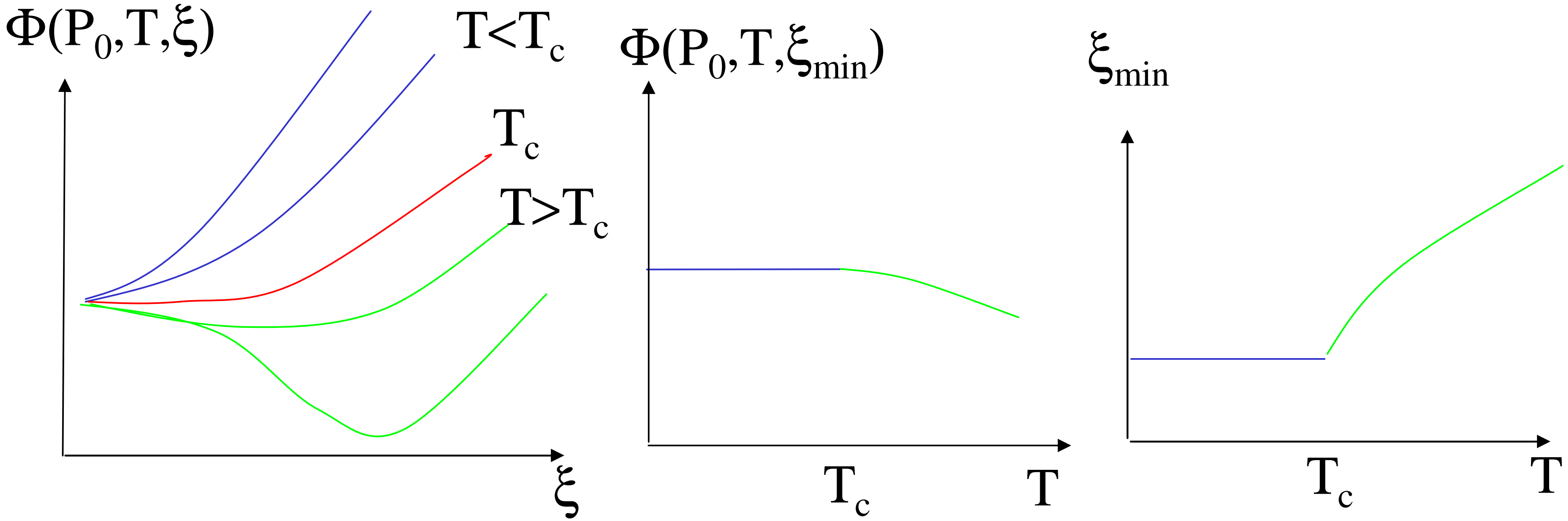
Energy functional: $E(N,\eta,\chi;\beta,\gamma)$



First order phase transition with $P = P_0 = const$



Second order phase transition



$$\Phi(P,T;\xi) = \Phi_0 + A(P,T)\xi^2 + B(P,T)\xi^3 + C(P,T)\xi^4 + \ldots$$

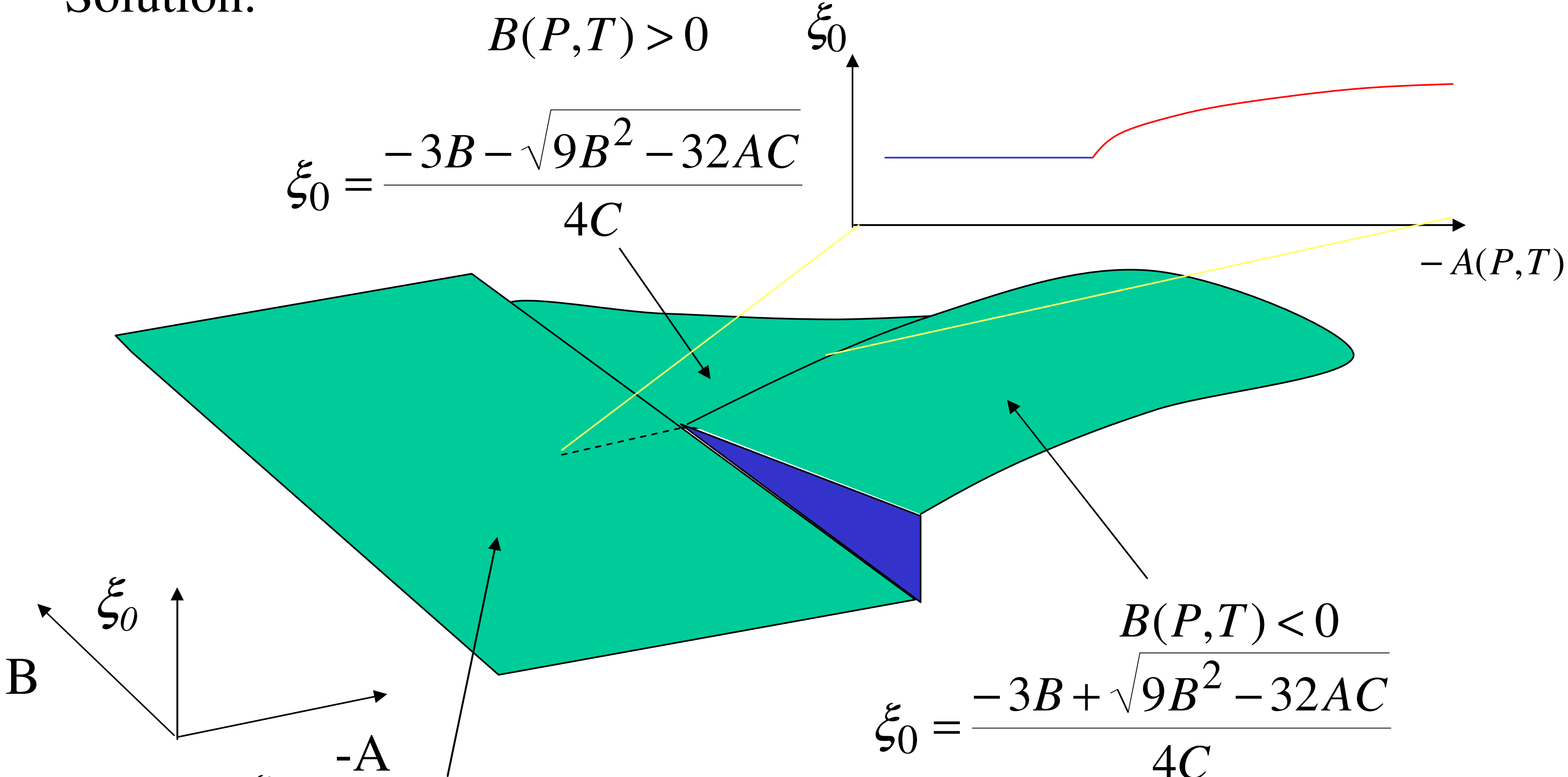
with $B(P,T) \neq 0; \quad \forall P,T$

$\Phi(P,T;\xi_0)$ should be continuous everywhere.

$\dfrac{\partial \Phi(P,T;\xi)}{\partial \xi}$ if discontinuous at ξ_0 : first order phase transition.

$\dfrac{\partial^2 \Phi(P,T;\xi)}{\partial \xi^2}$ if discontinuous at ξ_0: second order phase transition.

Solution:



$B(P,T) > 0$

$$\xi_0 = \frac{-3B - \sqrt{9B^2 - 32AC}}{4C}$$

$B(P,T) < 0$

$$\xi_0 = \frac{-3B + \sqrt{9B^2 - 32AC}}{4C}$$

$\xi_0 = 0$

$$A(P,T) > \frac{B(P,T)^2}{4C(P,T)}$$

First order phase transitions at:

$$A(P,T) = \frac{B(P,T)^2}{4C(P,T)} \quad \text{or} \quad B(P,T) = 0$$

Second order at:

$$A(P,T) = B(P,T) = 0$$

Energy functional in coherent state formalism

$$E(N,\eta,\chi;\beta,\gamma) = -5(1-\eta) + \frac{1}{(1+\beta^2)^2}\Bigg[\{N\eta-(1-\eta)(4N+\chi^2-8)\}\beta^2 + 4N(1-\eta)\sqrt{\frac{2}{7}}\chi\,\beta^3\cos 3\gamma + \left\{N\eta-(1-\eta)\left(\frac{2N+5}{7}\chi^2-4\right)\right\}\beta^4\Bigg],$$

$$\beta^3\cos(3\gamma): \chi<0 \rightarrow \gamma_0 = 0° \qquad \chi>0 \rightarrow \gamma_0 = 60°$$

and $\frac{1}{(1+\beta^2)^2} = 1-2\beta^2+3\beta^4-4\beta^6+\ldots$

So we can absorb it by allowing negative β values !

$$\gamma_0 = 0° \rightarrow \beta>0 \qquad \gamma_0 = 60° \rightarrow \beta<0$$

One obtains then: $$E(N,\eta,\chi;\beta,\gamma) = E_0(N,\eta) + A(N,\eta,\chi)\beta^2 + B(N,\eta,\chi)\beta^3 + C(N,\eta,\chi)\beta^4 + ...$$

when we fix N: $$\Phi(P,T;\xi) = \Phi_0 + A(P,T)\xi^2 + B(P,T)\xi^3 + C(P,T)\xi^4 + ...$$

$$B(N,\eta,\chi) = 4(N-1)(1-\eta)\chi\sqrt{\frac{2}{7}}$$

$$A(N,\eta,\chi) = \{N\eta - (1-\eta)(4N+\chi^2-8)\}$$

The first order phase transitions should occur when

$$A(N,\eta,\chi) = 0 \rightarrow \eta_0 = \frac{4N+\chi^2-8}{5N+\chi^2-8} \xrightarrow{N\to\infty} \frac{4}{5}$$ spherical-deformed

$$B(N,\eta,\chi) = 0 \rightarrow \chi_0 = 0$$ prolate-oblate

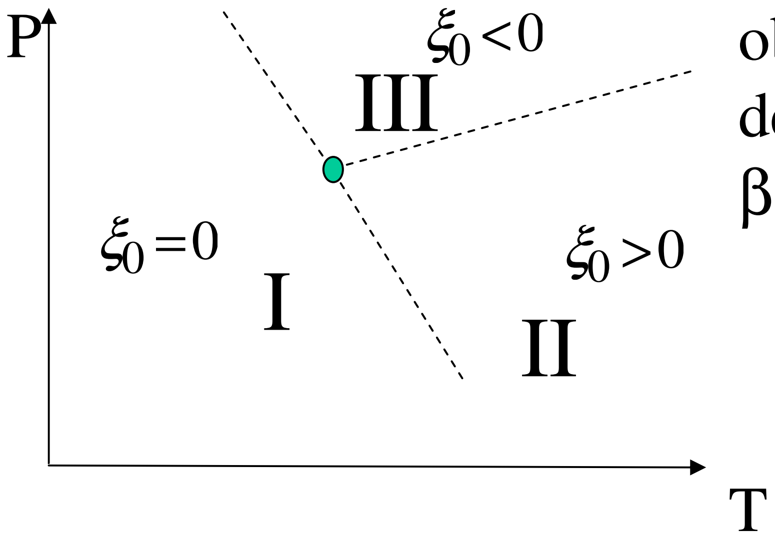
The isolated second order transition at:

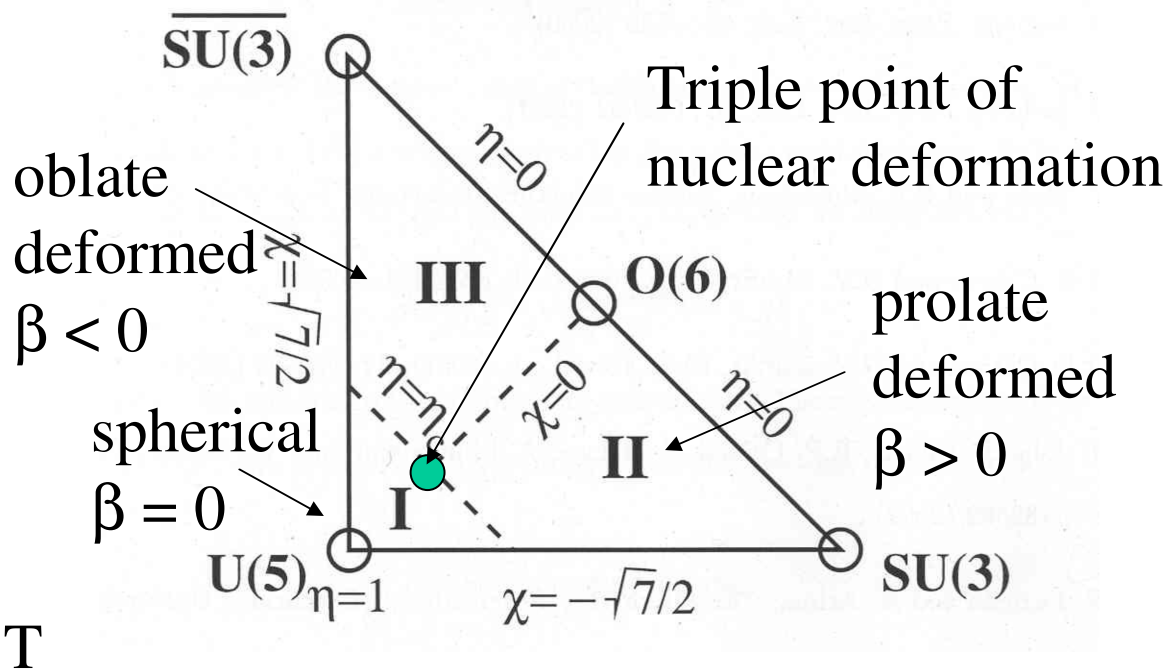
$$A(N,\eta,\chi) = B(N,\eta,\chi) = 0, \rightarrow \chi_0 = 0 \quad and \quad \eta_0 = \frac{4N-8}{5N-8}$$

Landau theory and nuclear shapes.

- - - - - - - - : first order transition

● : isolated second order transition





Thermodynamic potential: $\Phi(P,T;\xi)$

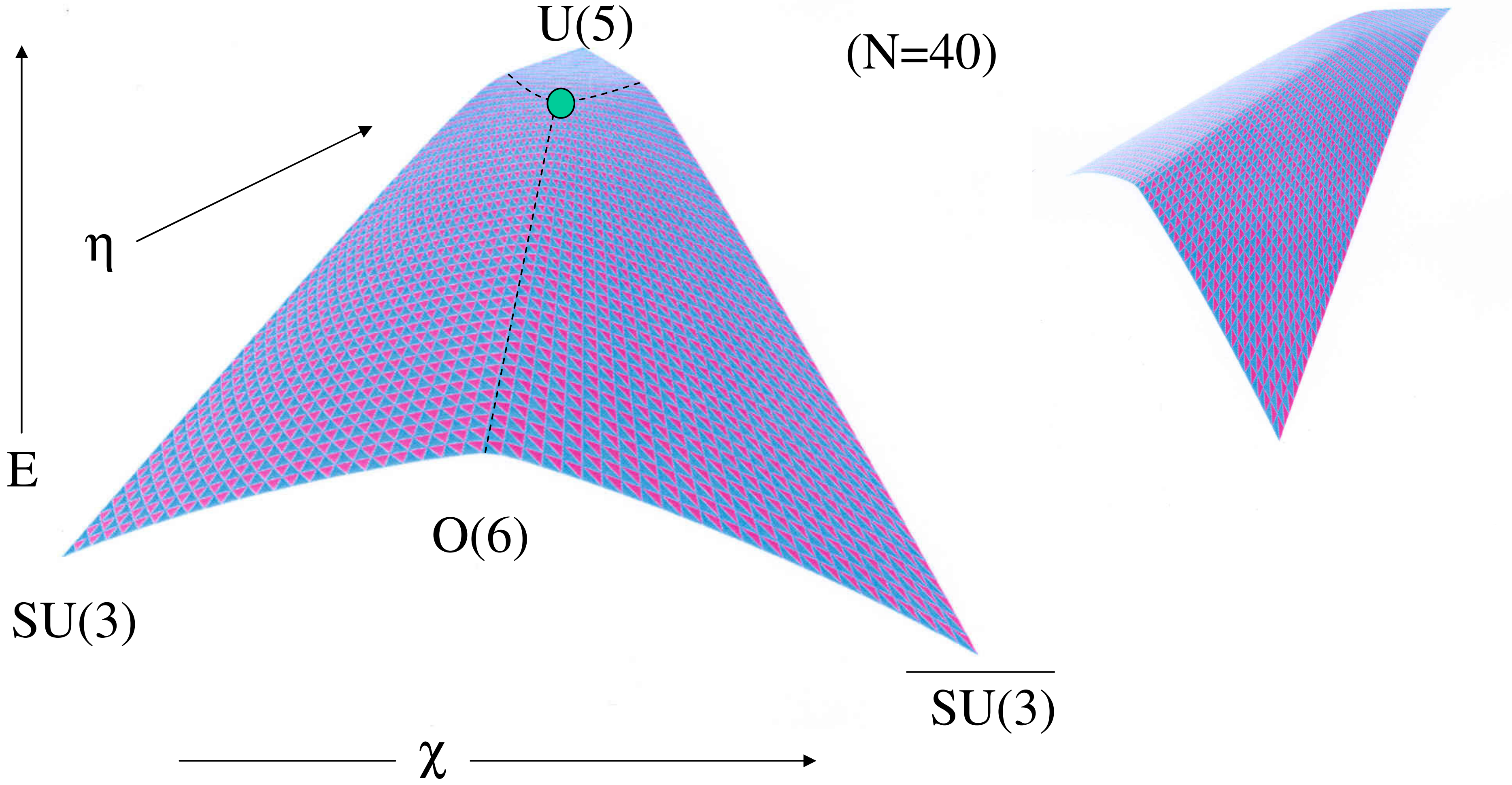
Energy functional: $E(N,\eta,\chi;\beta,\gamma)$

Order parameter

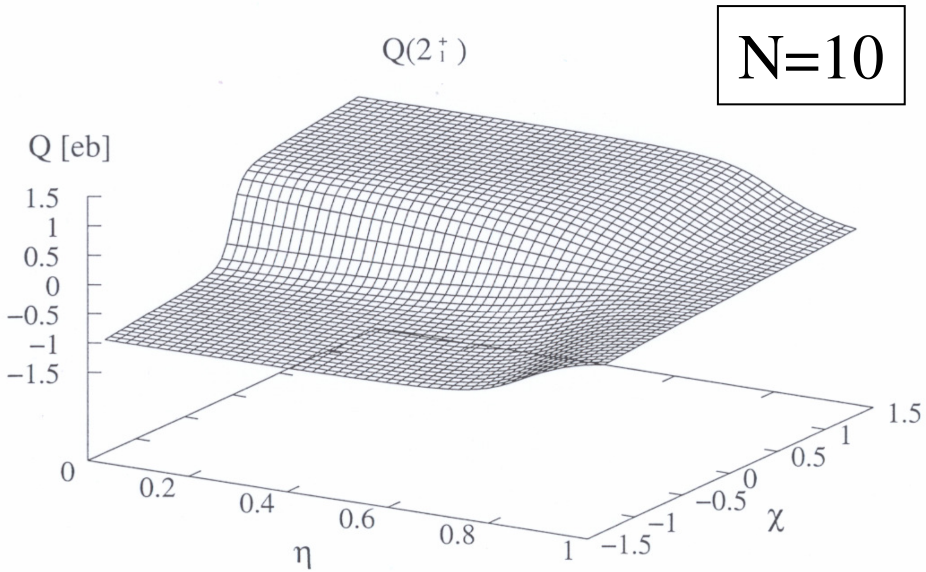
External parameters

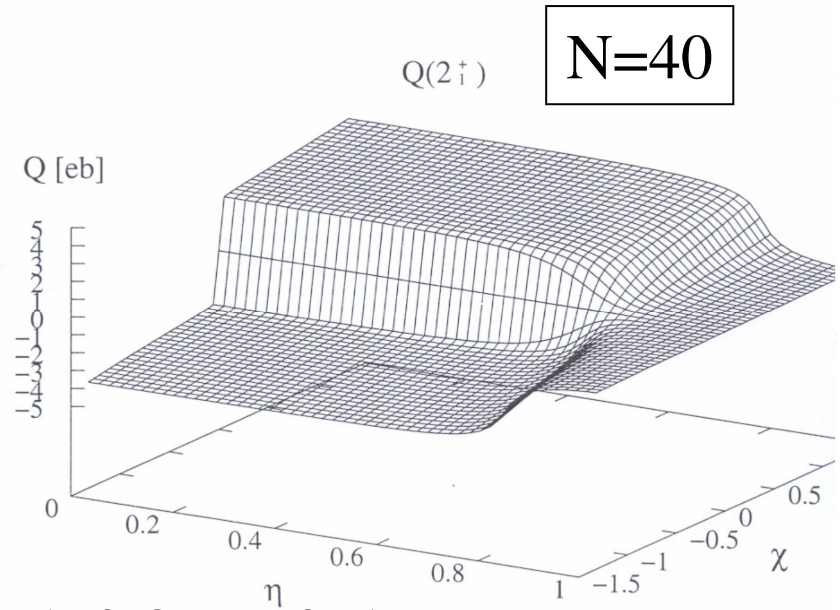
J.Jolie, P. Cejnar, R.F. Casten, S. Heinze, A. Linnemann, V. Werner, Phys. Rev. Lett. 89 (2002)182502

The shape phase transitions can be seen by the groundstate energies.

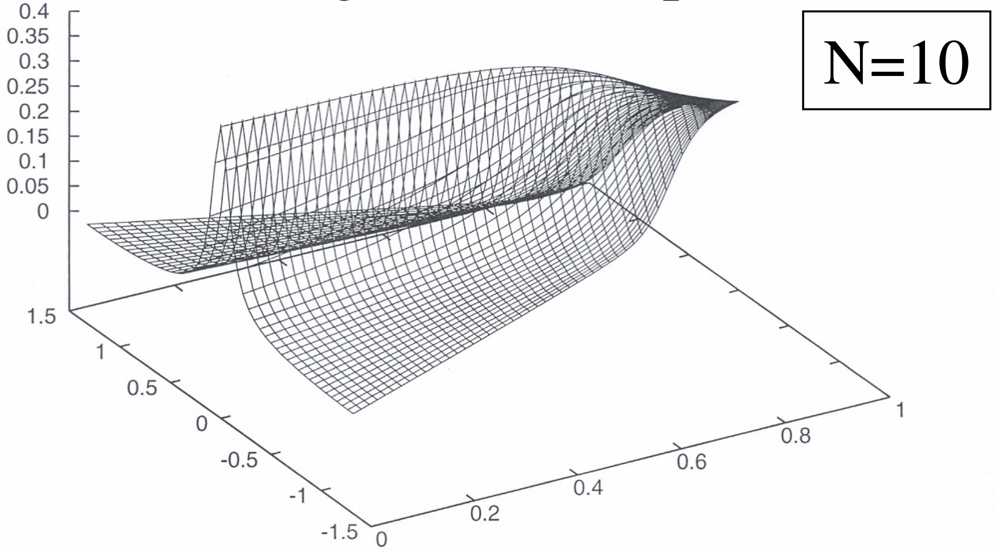


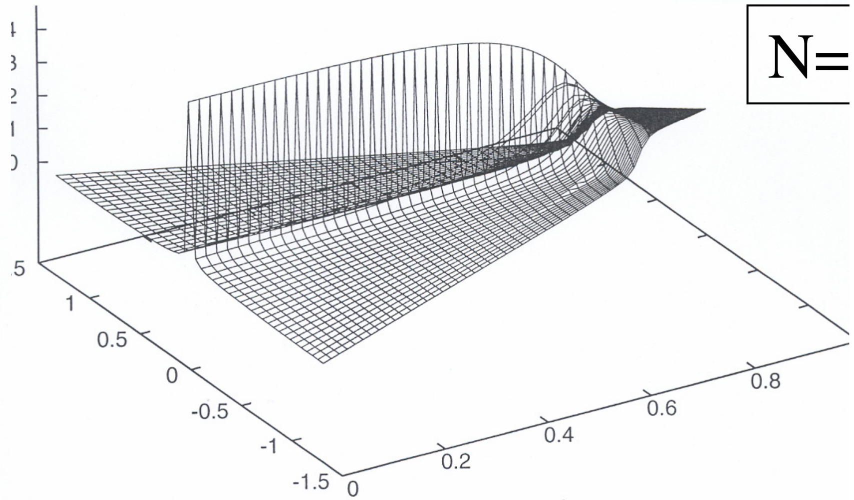
The quadrupole moment corresponds to the control parameter β_0:





A sensitive signature is in particular the B(E2;2_2^+-> 2_1^+)



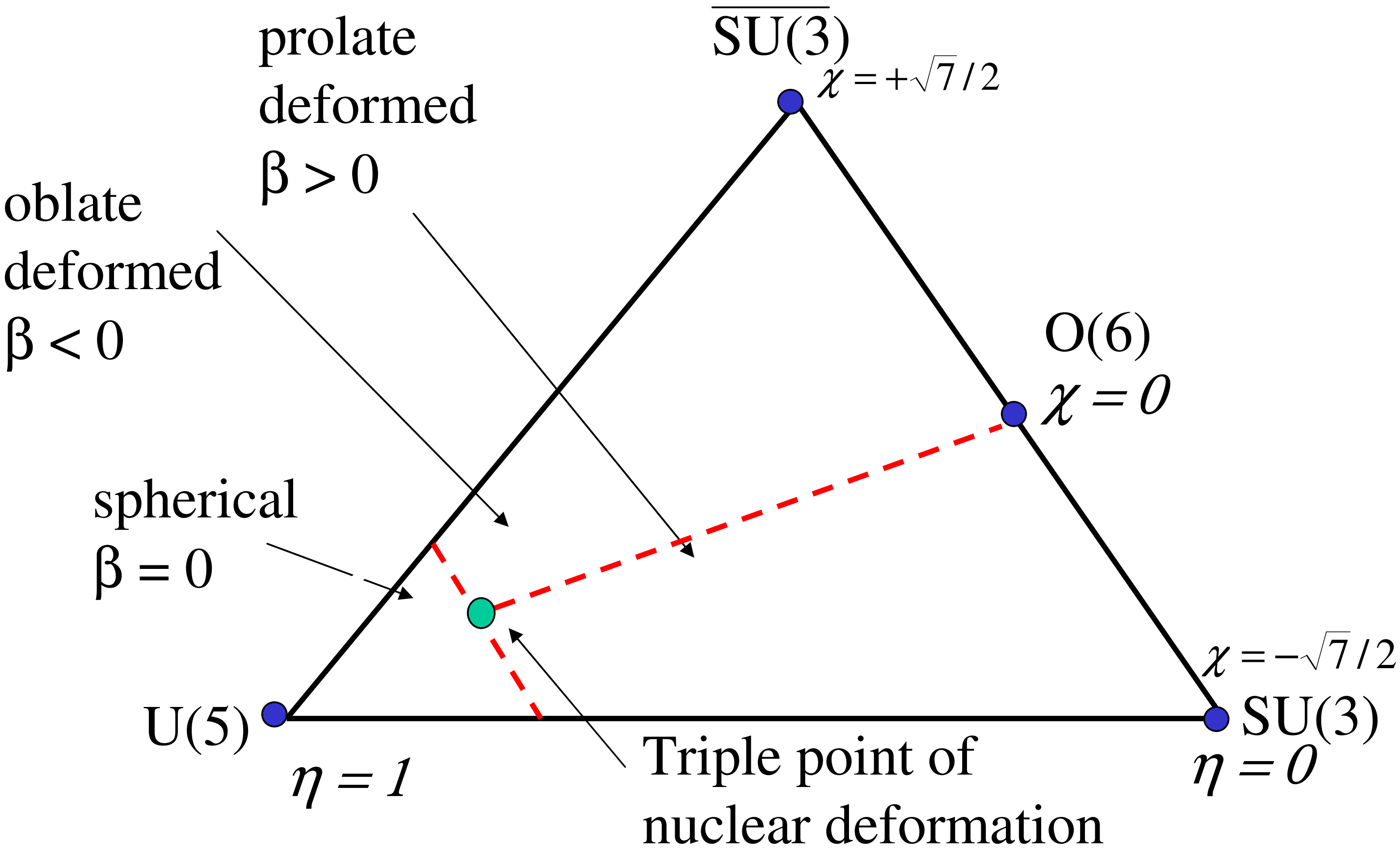


The new nuclear shape phase diagram

J. Jolie, R.F. Casten, P. von Brentano, V. Werner, Phys.Rev.Lett.87 (2001)162501

- - - - first order transition

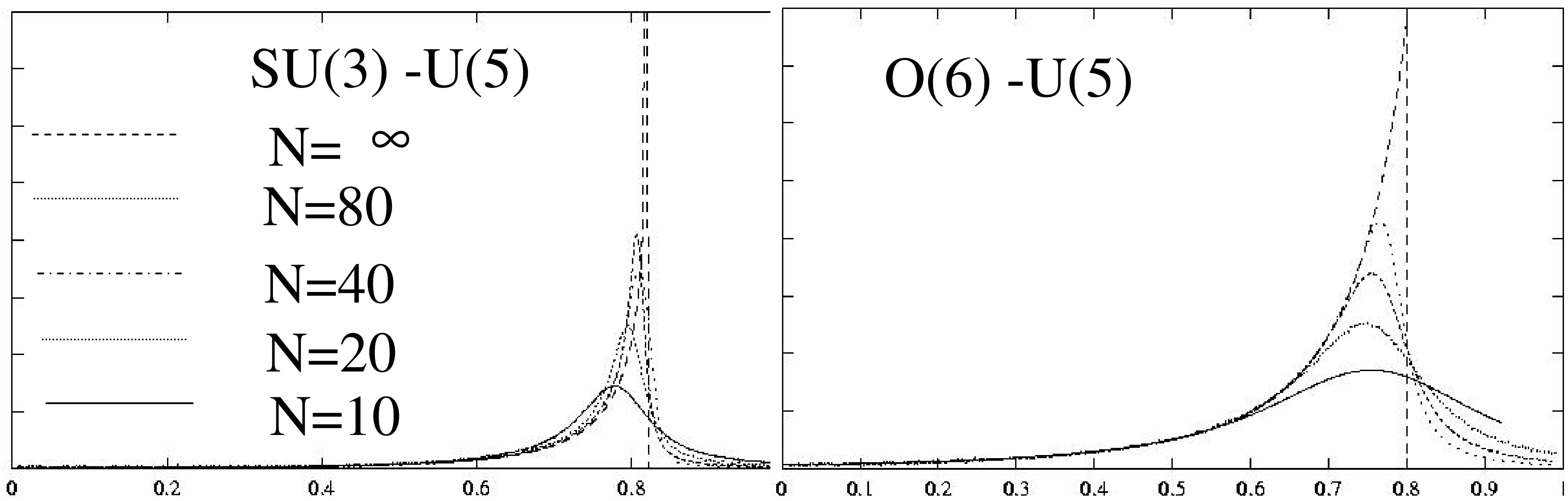
● second order transition



The analogy with thermodynamics can be further investigated.

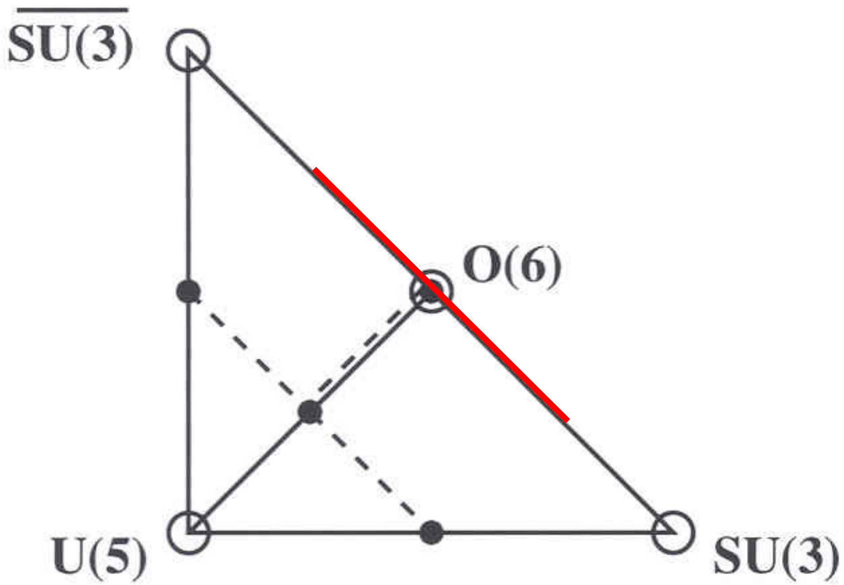
Specific heat:

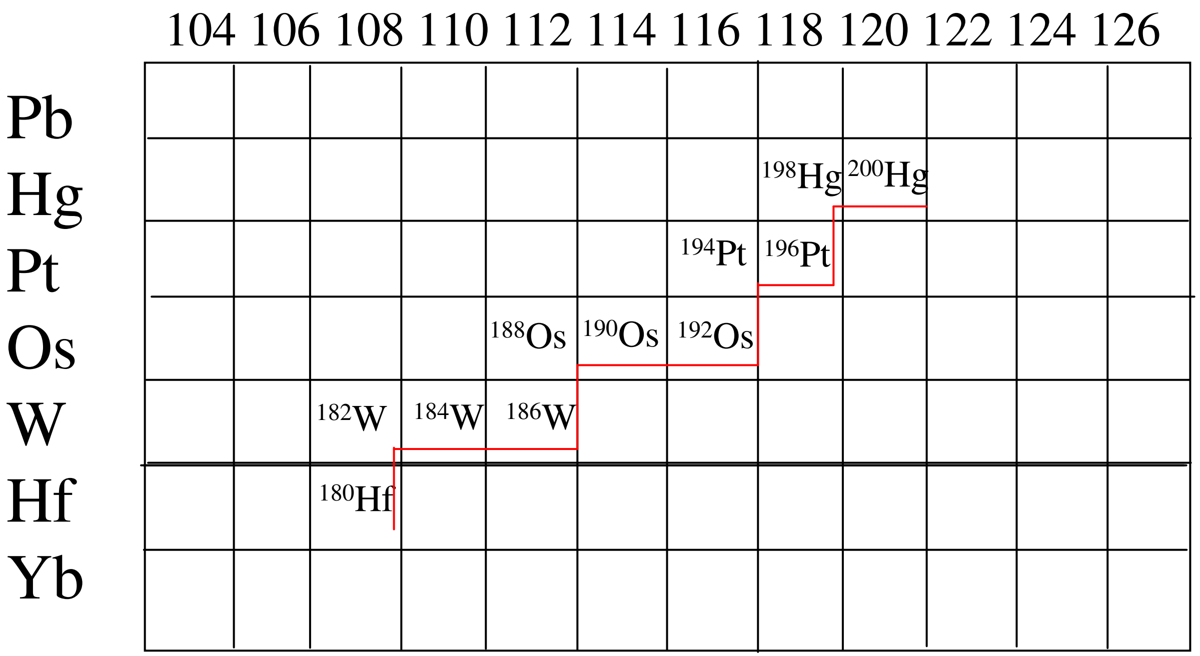
$$C_P = -T\frac{\partial^2 \phi(P,T;\xi_0)}{\partial T^2} \longrightarrow C_\chi = -\frac{\partial^2 E(N,\eta,\chi;\beta_0)}{\partial \eta^2}$$

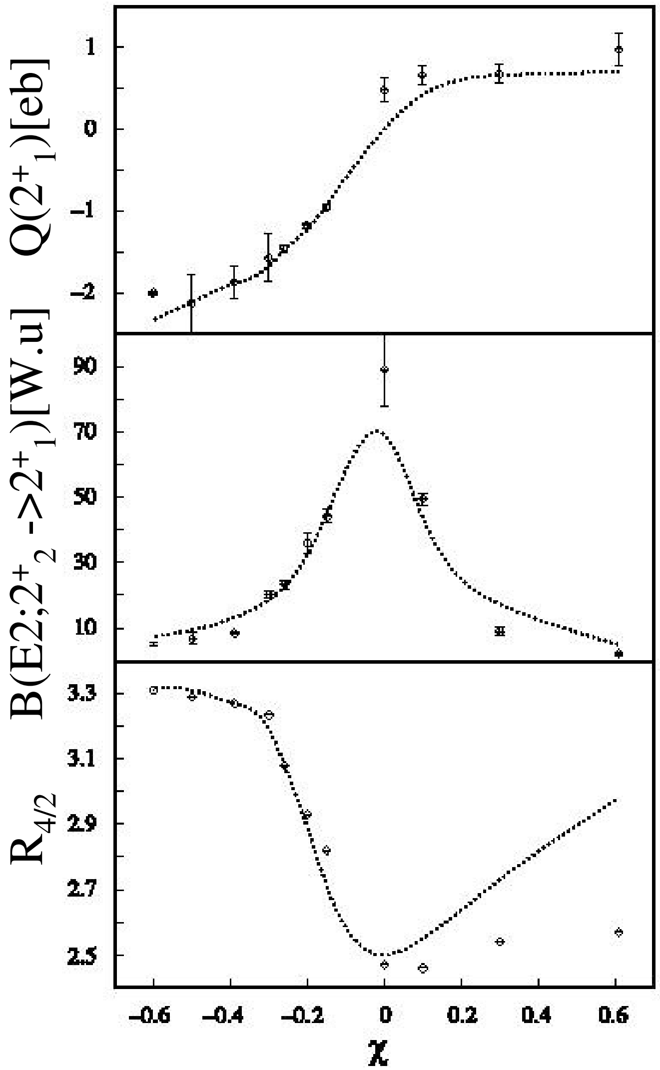


P. Cejnar, S. Heinze, J. Jolie, Phys. Rev. C68 (2003) 034326.

Experimental examples for the prolate-oblate phase transition







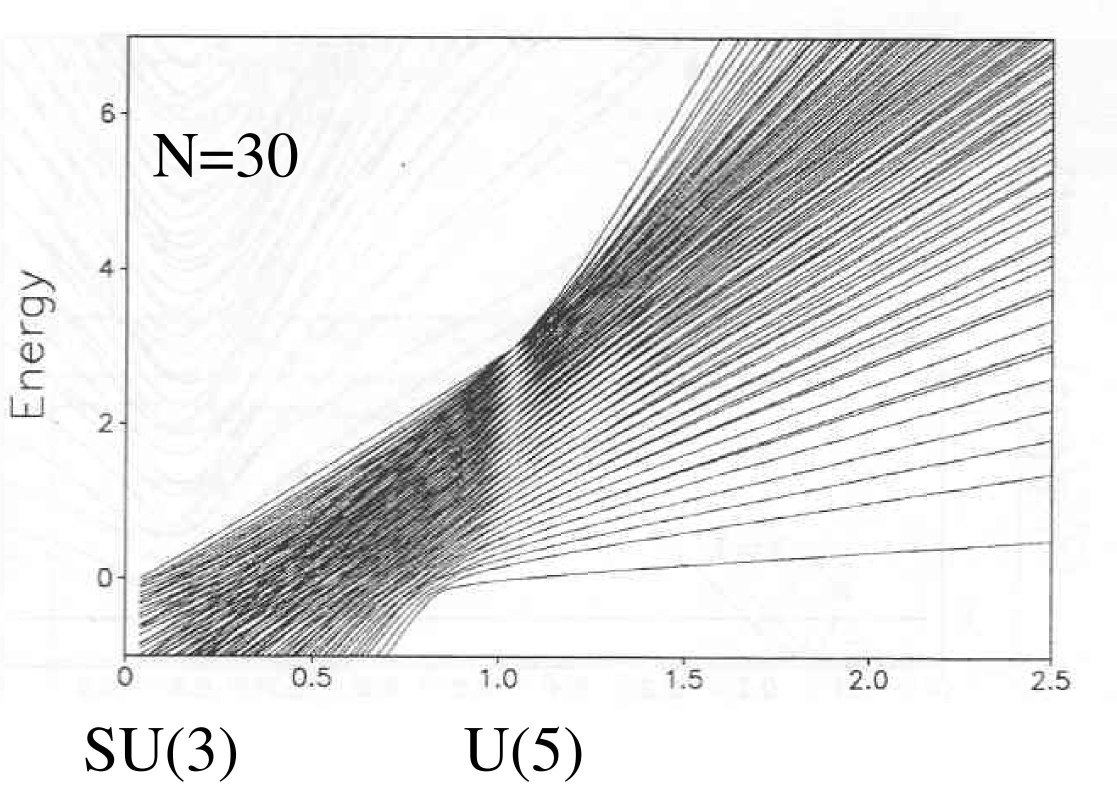
J.Jolie, A. Linnemann
Phys. Rev. C 68 (2003) 031301.

3. Level dynamics and phase transitions

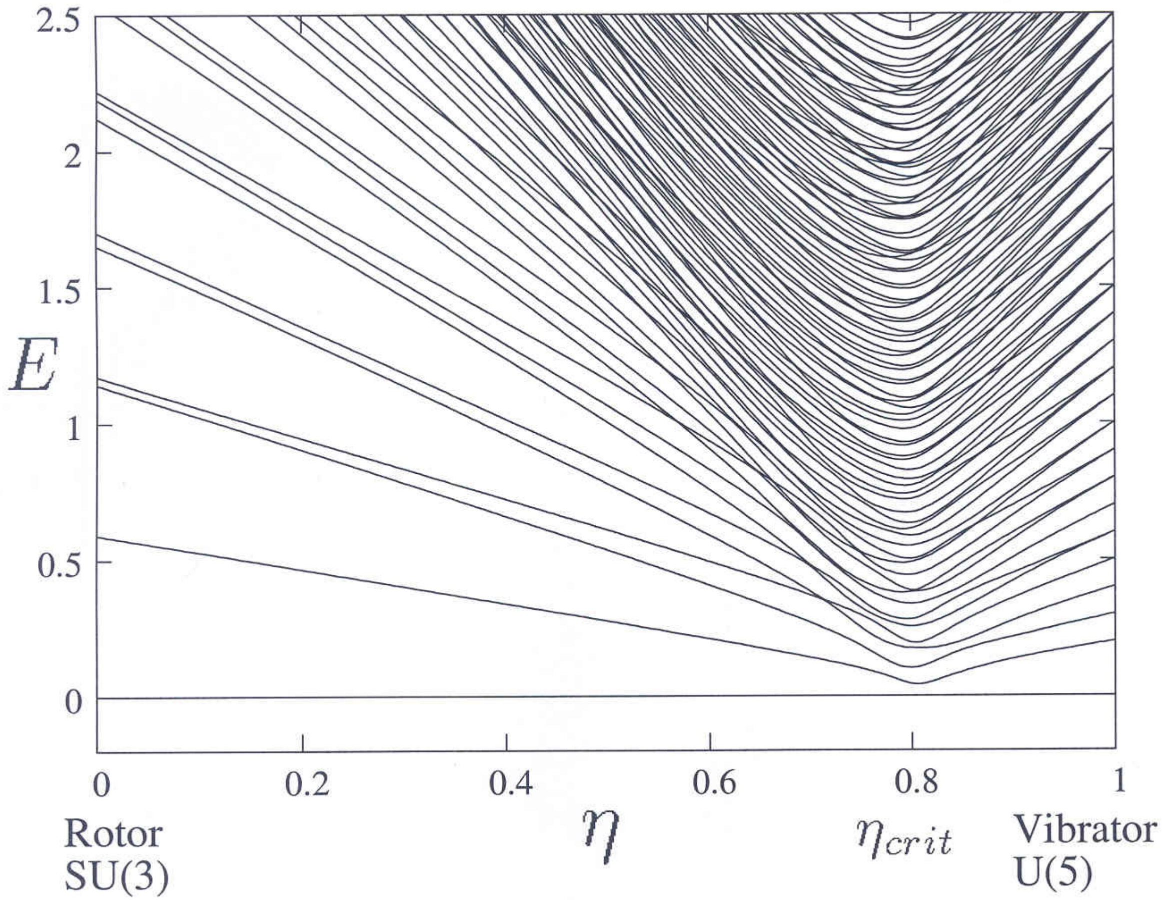
Up to now we concentrated only on the lowest states, what happens with higher excited states and the level density?

U(5)-SU(3) first order shape phase transition

Energies of 0^+ states



Energy of 0^+ states up to 2.5 MeV as a function of η in the spherical-deformed transition for N = 30.



From P. Cejnar and J. Jolie, Phys. Rev. E (2000) 6237

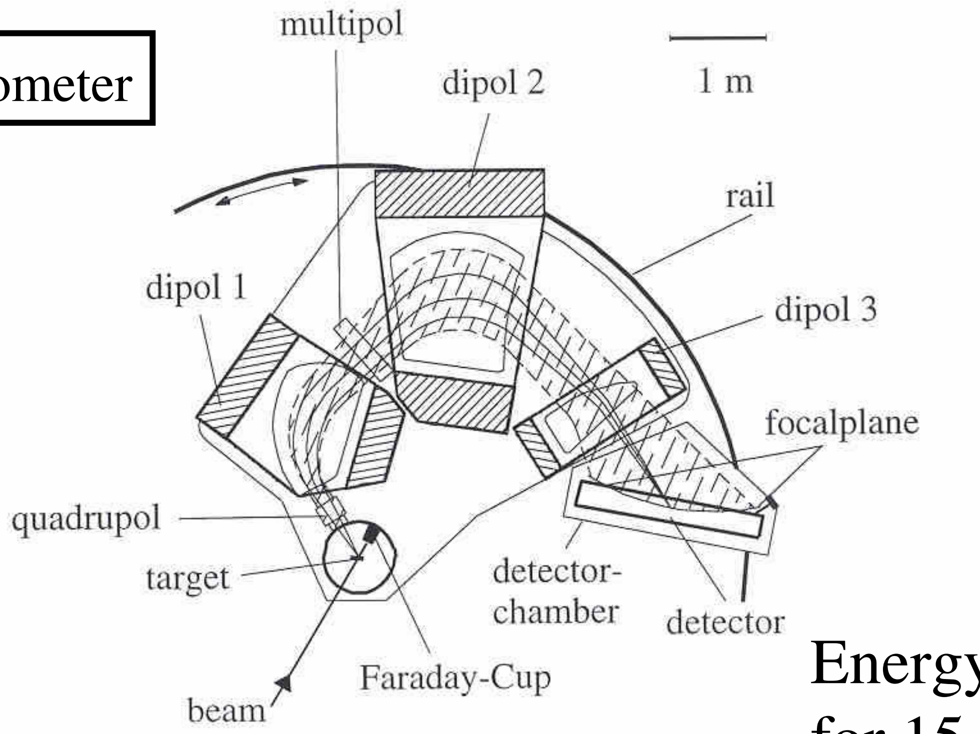
Can this be experimentally observed?

To excite the 0^+ states the ideal and very complete way is using the (p,t) transfer reaction at the high resolution Q3D spectrometer (Garching).

A recent (p,t) study of ^{158}Gd lead to the observation of 13 0^+ states below 3 MeV of which seven new.

S. R. Lesher et al., Phys. Rev. C66, 051305(R) (2002)

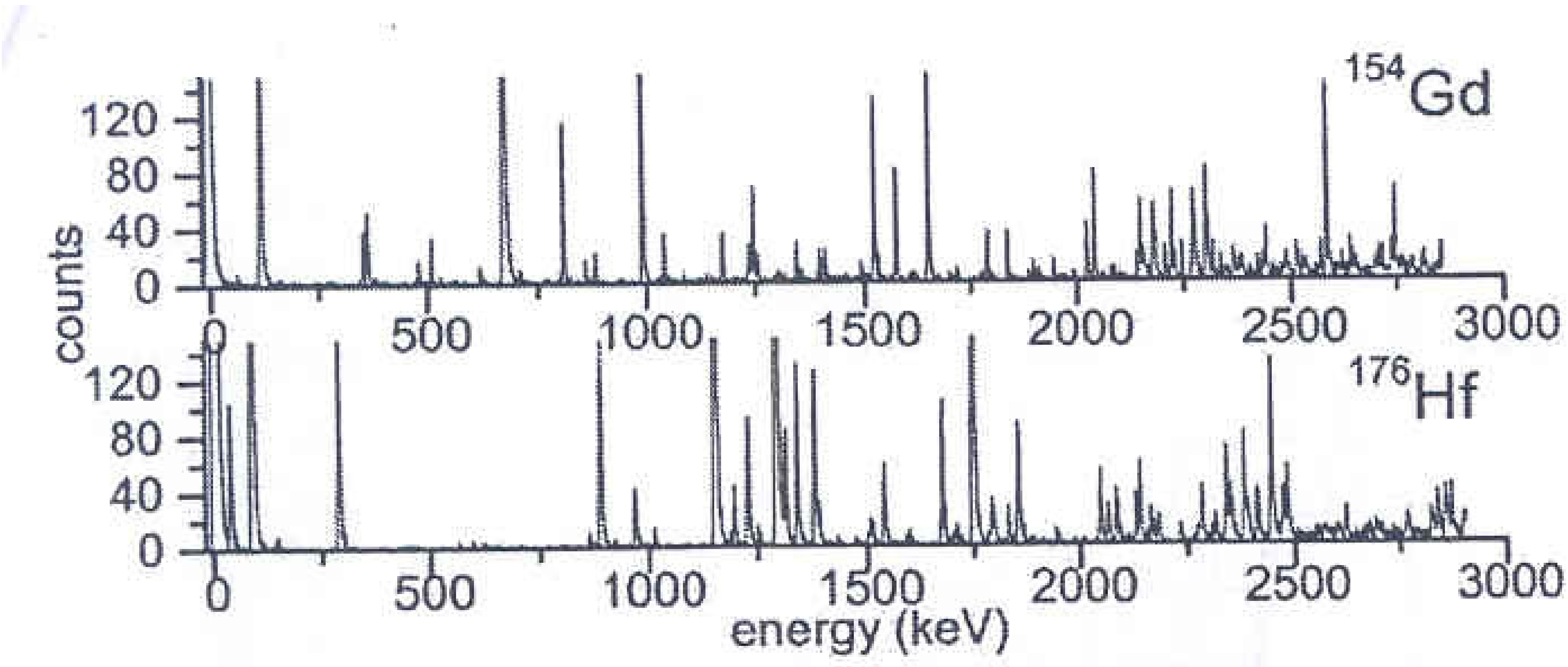
Q3D Spectrometer

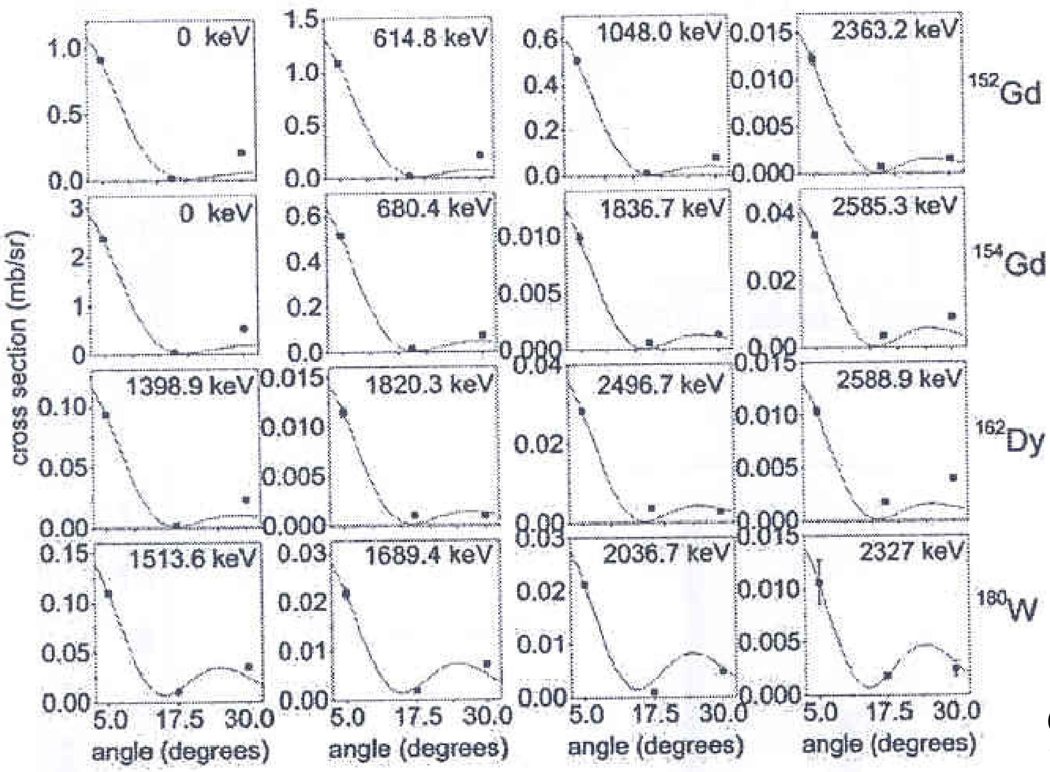


Energy resolution: ~4 keV for 15-20 MeV tritons.

This motivated us to study 15 additional rare-earth nuclei using this reaction (Yale/Köln/Bukarest/ Surrey/LMU-TU Munchen collaboration).

Eight nuclei in the rare earth region were systematically studied up to 3 MeV.





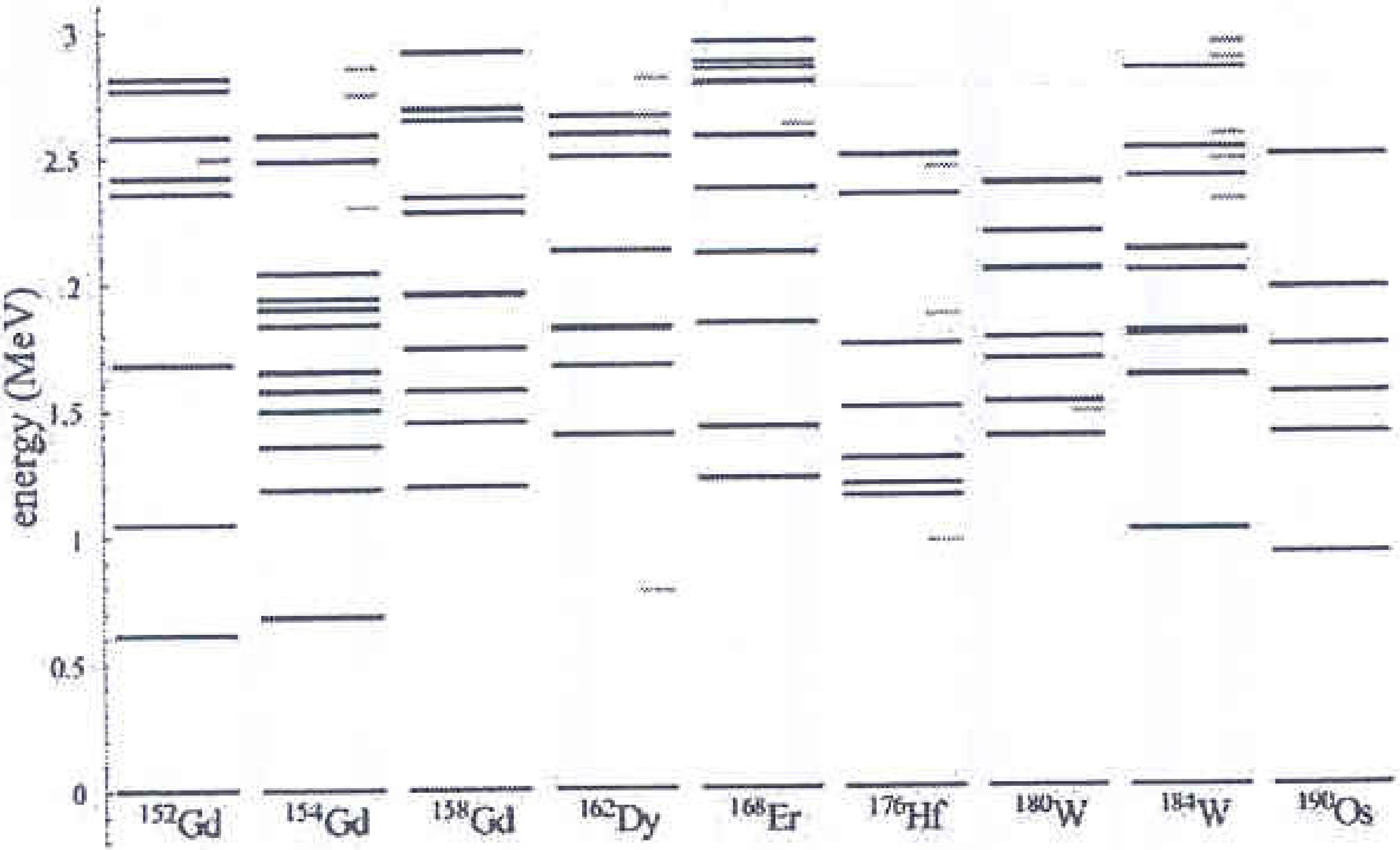
Identification by forward peaked cross section.

94 0^+ states observed
66 new of which 18 tentative

94 18 28

	observed 0^+ states	tentative 0^+ states	previously known 0^+ states
^{152}Gd	10	1	3
^{154}Gd	16	3	6
^{162}Dy	12	3	4
^{168}Er	12	1	4
^{176}Hf	11	3	4
^{180}W	9	1	3
^{184}W	16	6	2
^{190}Os	7	0	3

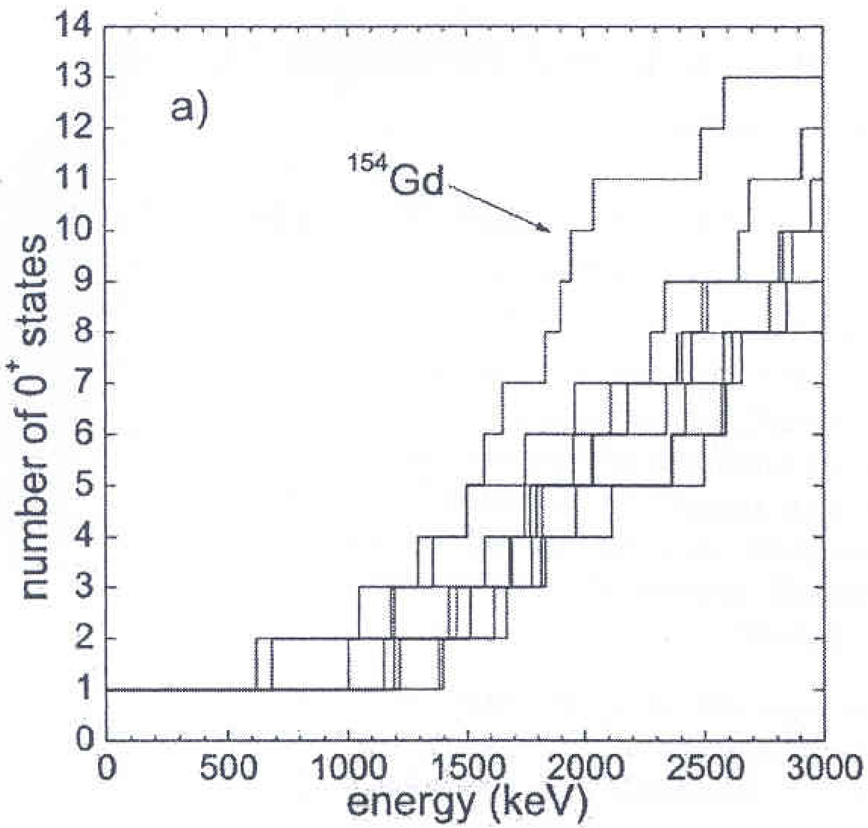
Result:

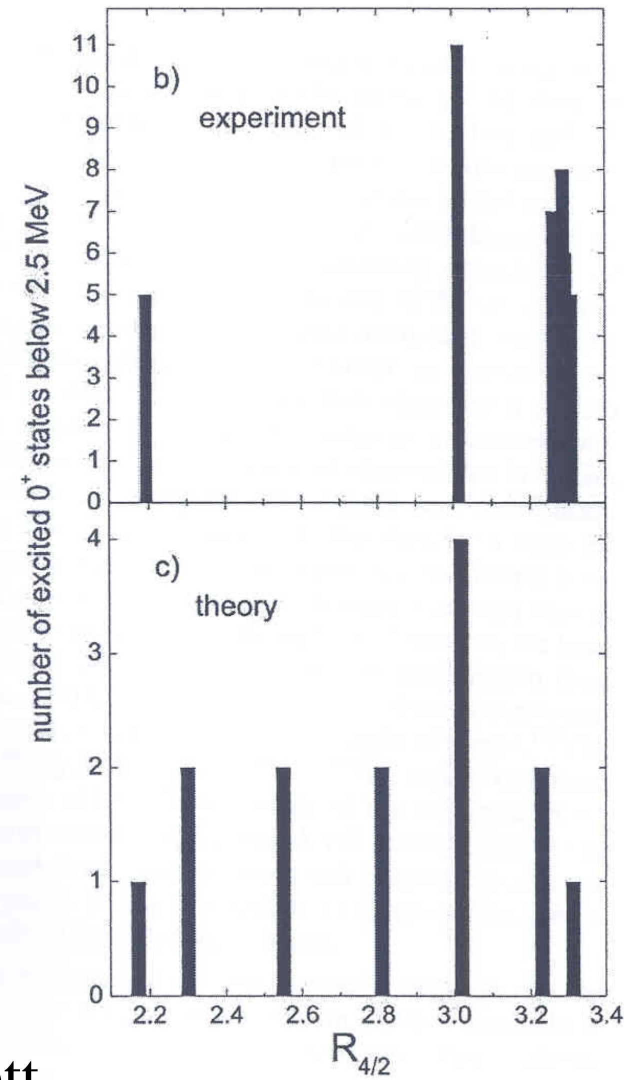


energy (MeV)
3
2.5
2
1.5
1
0.5
0
152Gd
154Gd
158Gd
162Dy
168Er
176Hf
180W
184W
190Os

^{154}Gd shows clearly a higher level density than the other nuclei and is a known phase transitional nucleus

N-dependent calculation confirm the increased density



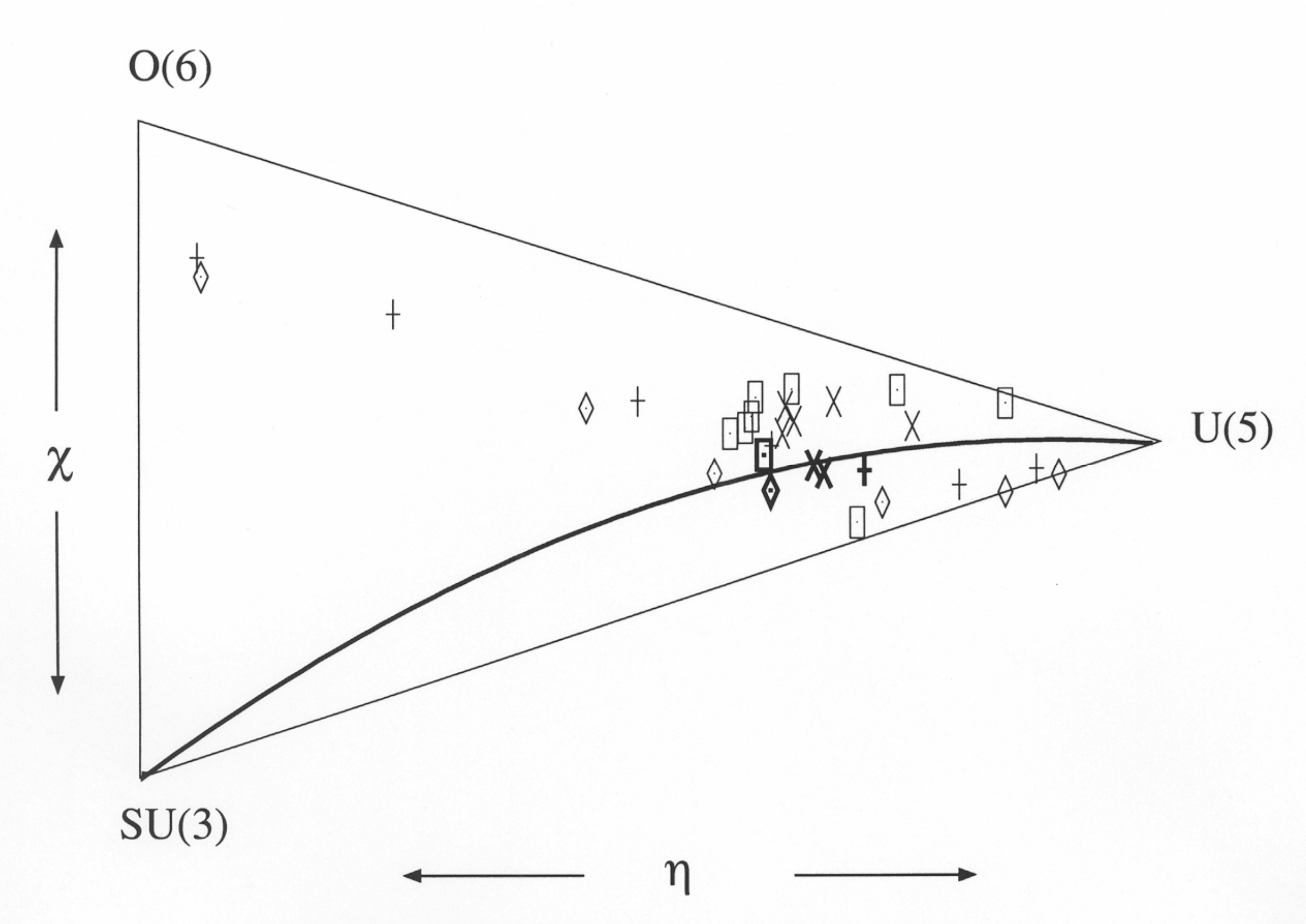


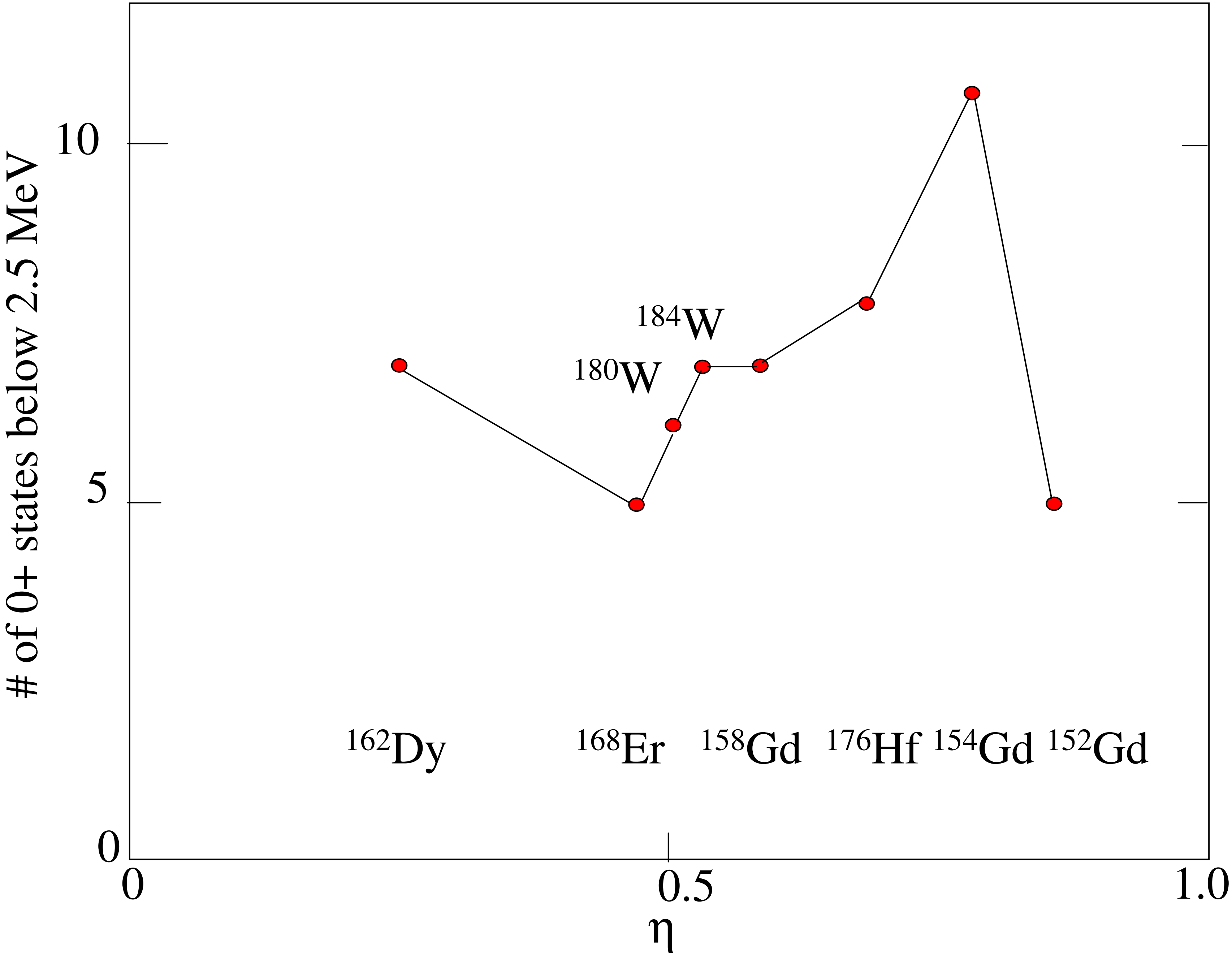
D.A. Meyer et al, subm. to Phys. Rev. Lett.

Can we make this more quantitative?

Recently, E.A. McCutchan, N.V. Zamfir and R.F. Casten fitted all rare earth nuclei.

E.A. McCutchan et al. Phys. Rev. C69, 064306 (2004)





# of 0+ states below 2.5 MeV
10
5
0
0
0.5
1.0
η
184W
180W
162Dy
168Er
158Gd
176Hf
154Gd
152Gd

4. Chaos and regularity in the Casten triangle

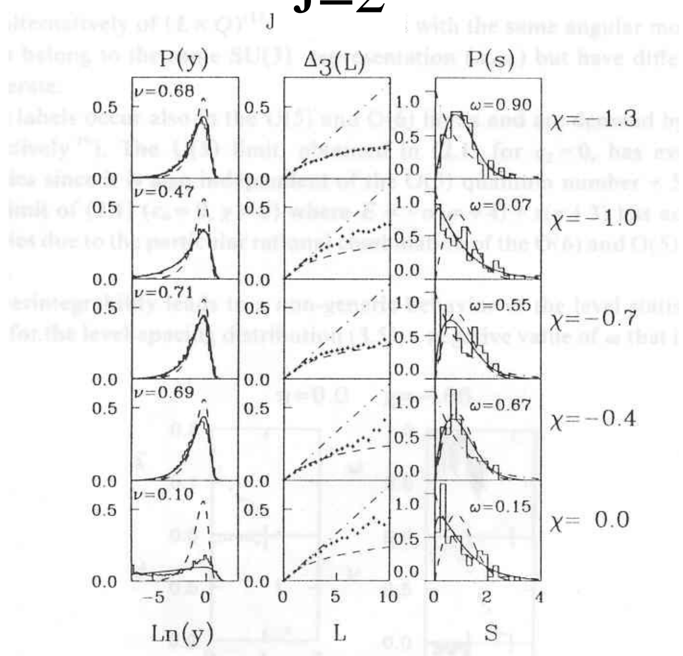
J. Jolie, R.F. Casten, P. Cejnar, S. Heinze, E.A. McCutchan, N.V. Zamfir, Phys. Rev. Lett.93(2004)132501

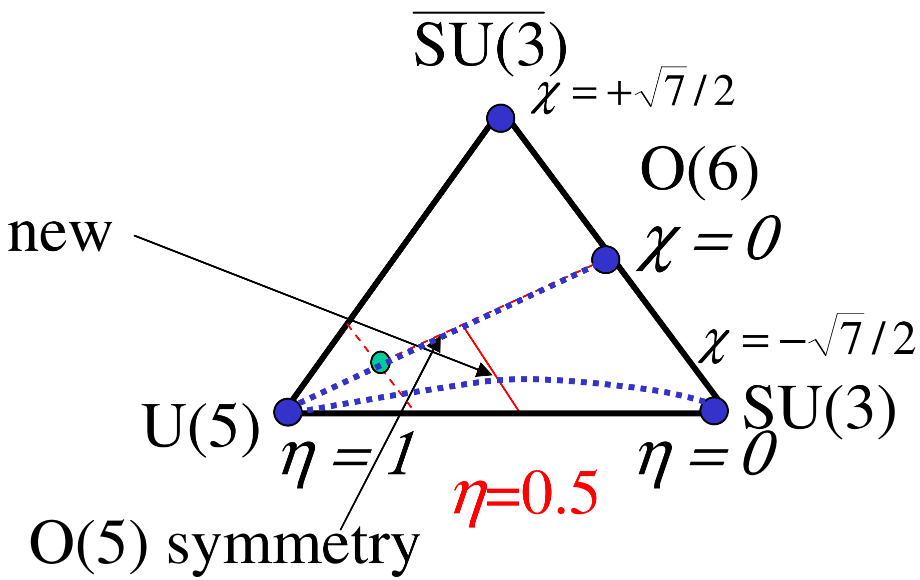
In the early nineties Alhassid and Whelan studied the chaotic properties of the hamiltonian:

$$\hat{H} = a\left[\eta \hat{n}_d - \frac{(1-\eta)}{N}\hat{Q}_\chi \cdot \hat{Q}_\chi\right]$$

They found a region of regularity that is still unexplained:

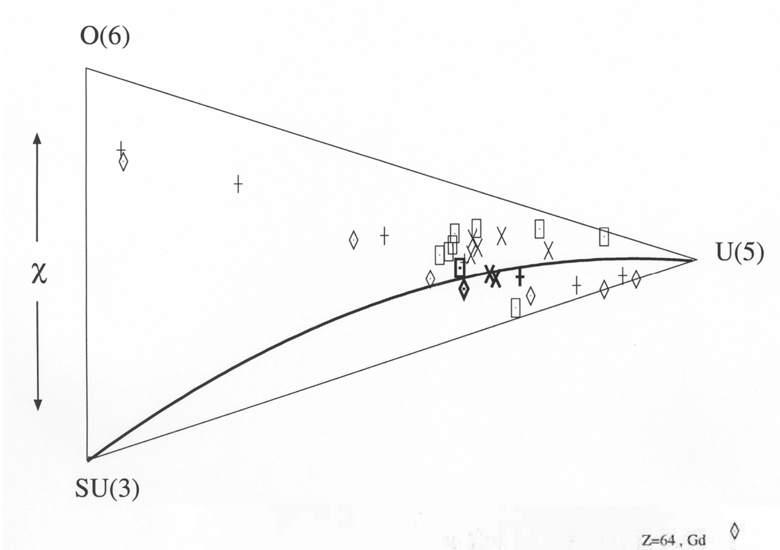
J=2



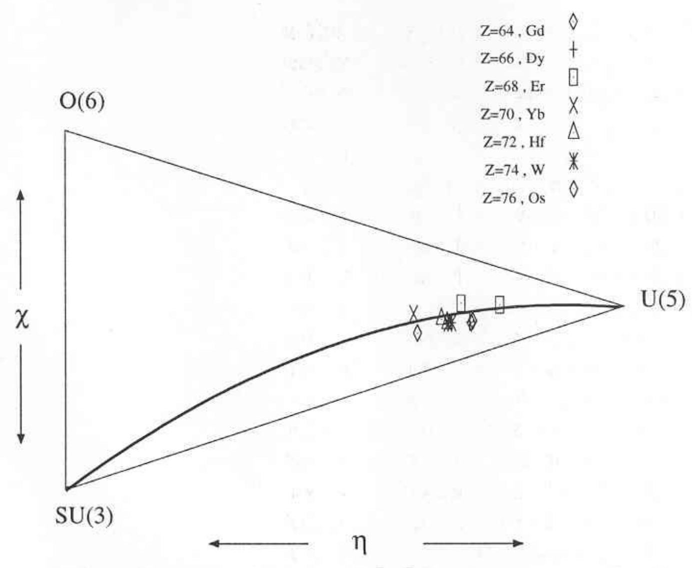


Nuclei in this region fulfil the relation: $$\chi = \frac{\sqrt{7}-1}{2}\eta - \frac{\sqrt{7}}{2}$$

Recently, E.A. McCutchan, N.V. Zamfir and R.F. Casten fitted all rare earth nuclei.

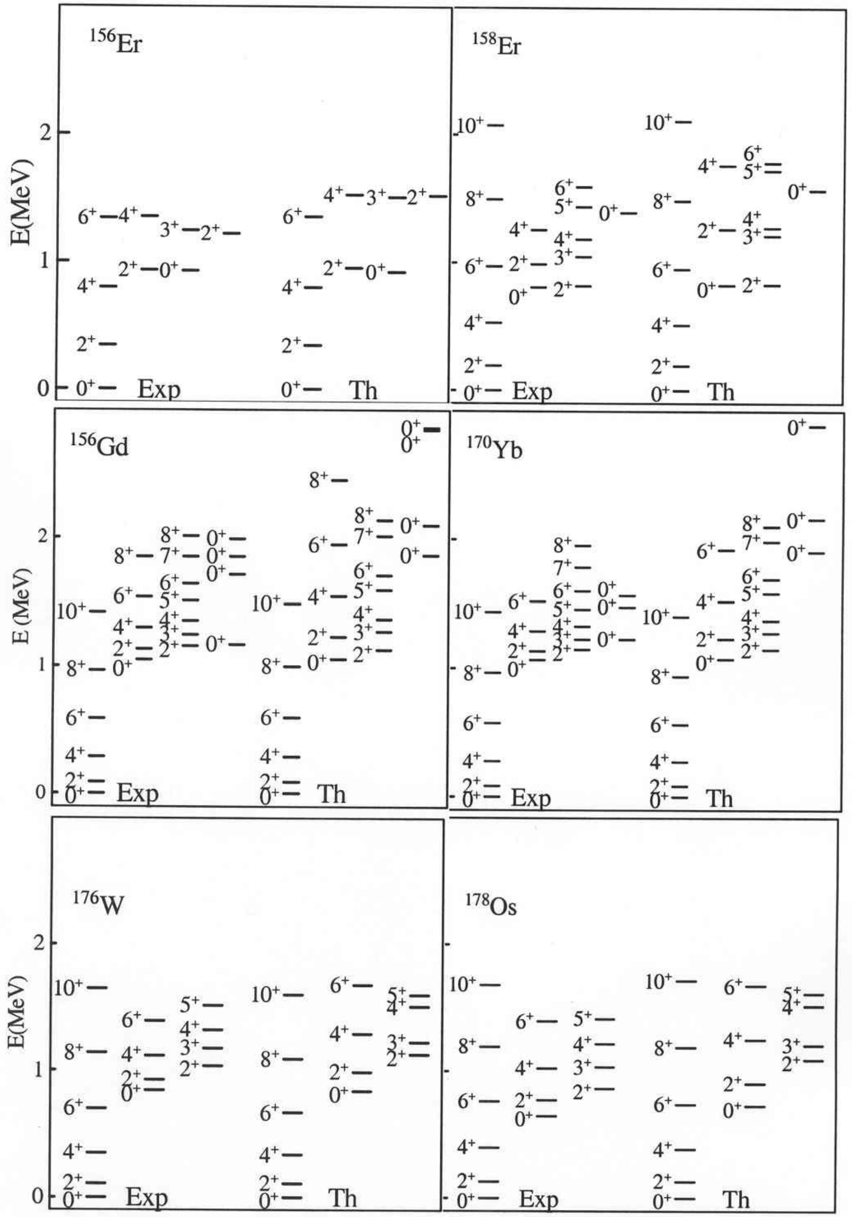


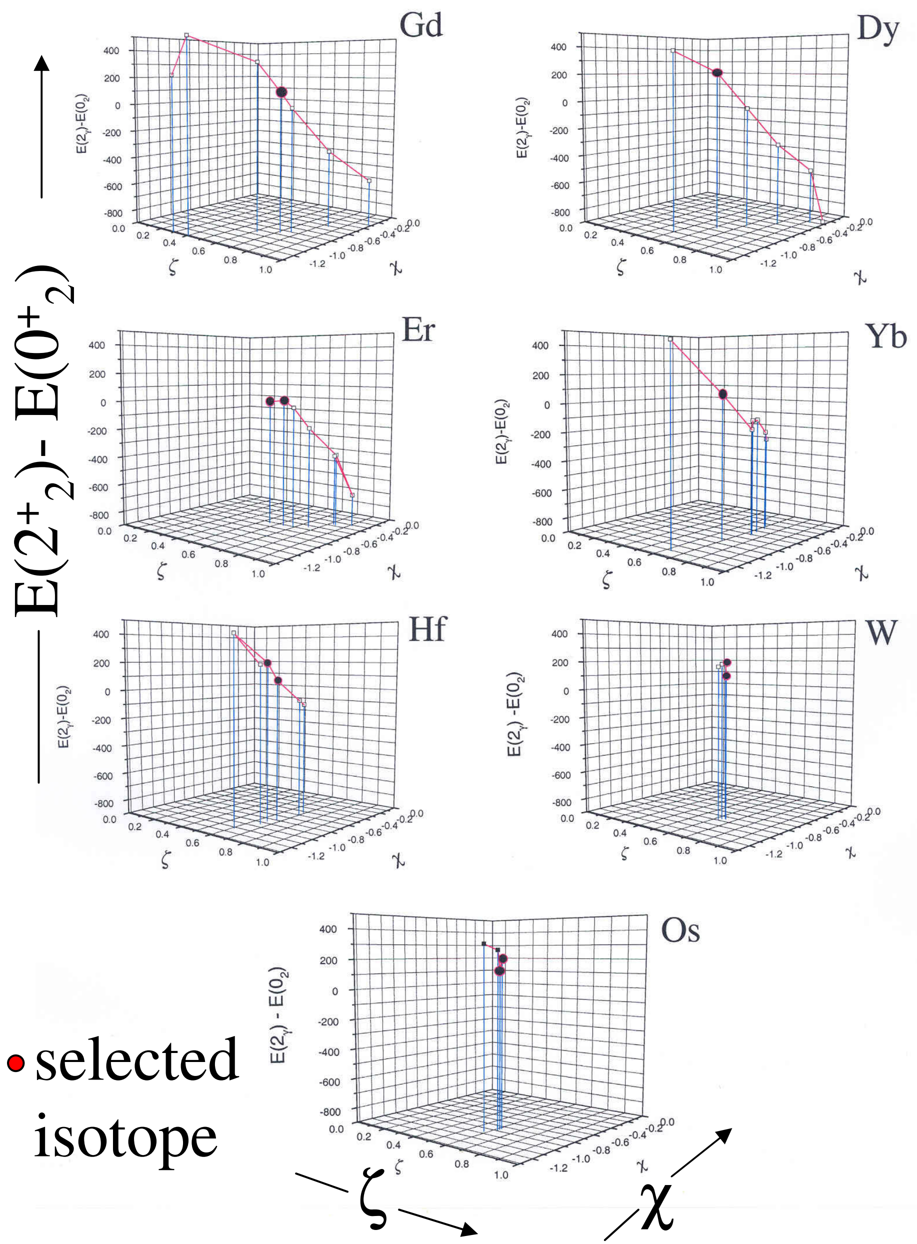
Eleven candidates situated on the arc of regularity could be identified using their data set.



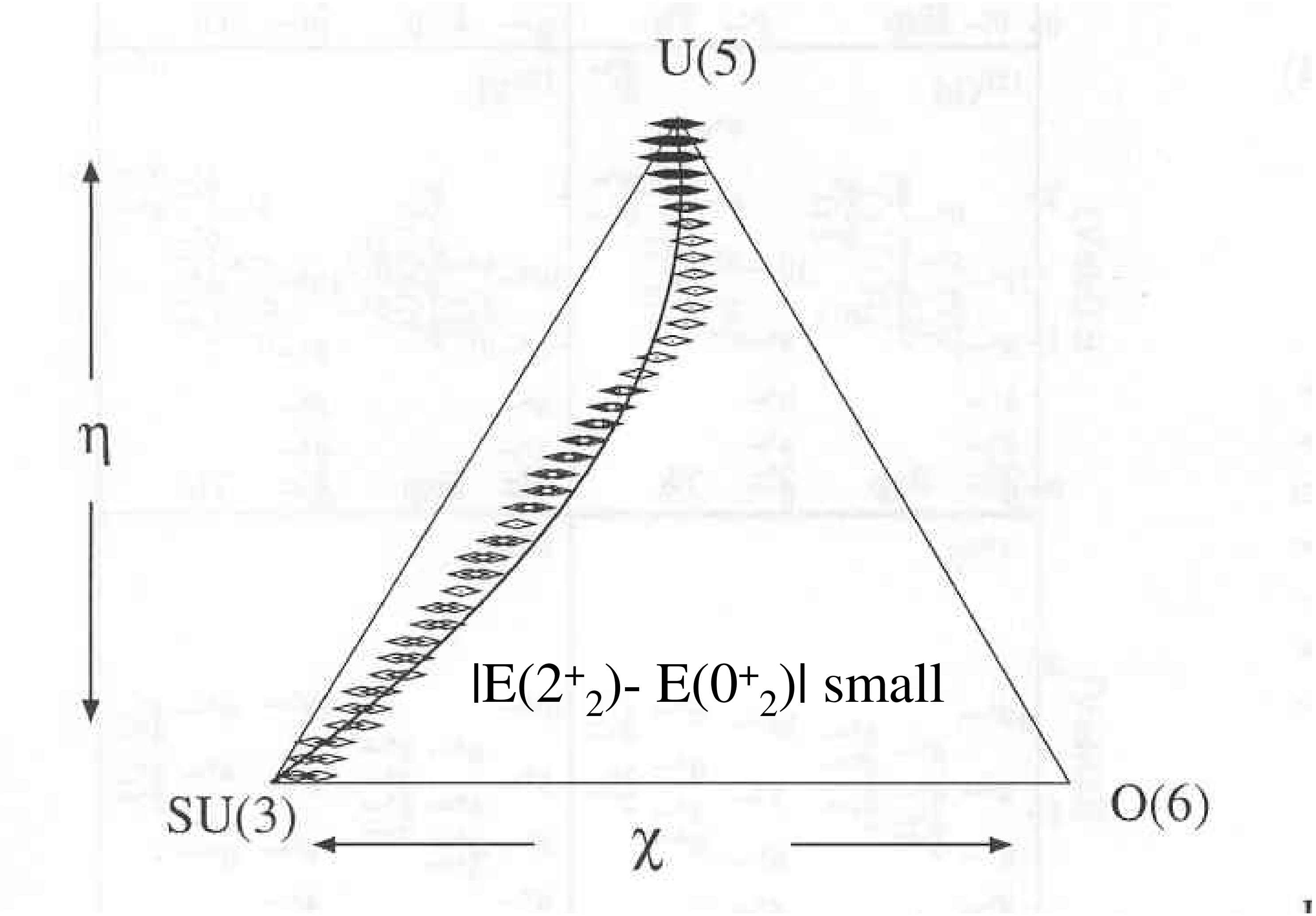
^{156,158}Er, ^{156}Dy, ^{156}Gd, ^{170,172}Hf, ^{170}Yb, ^{176,178}W, ^{178,180}Os,

They look different in their structure so what do they have in common?



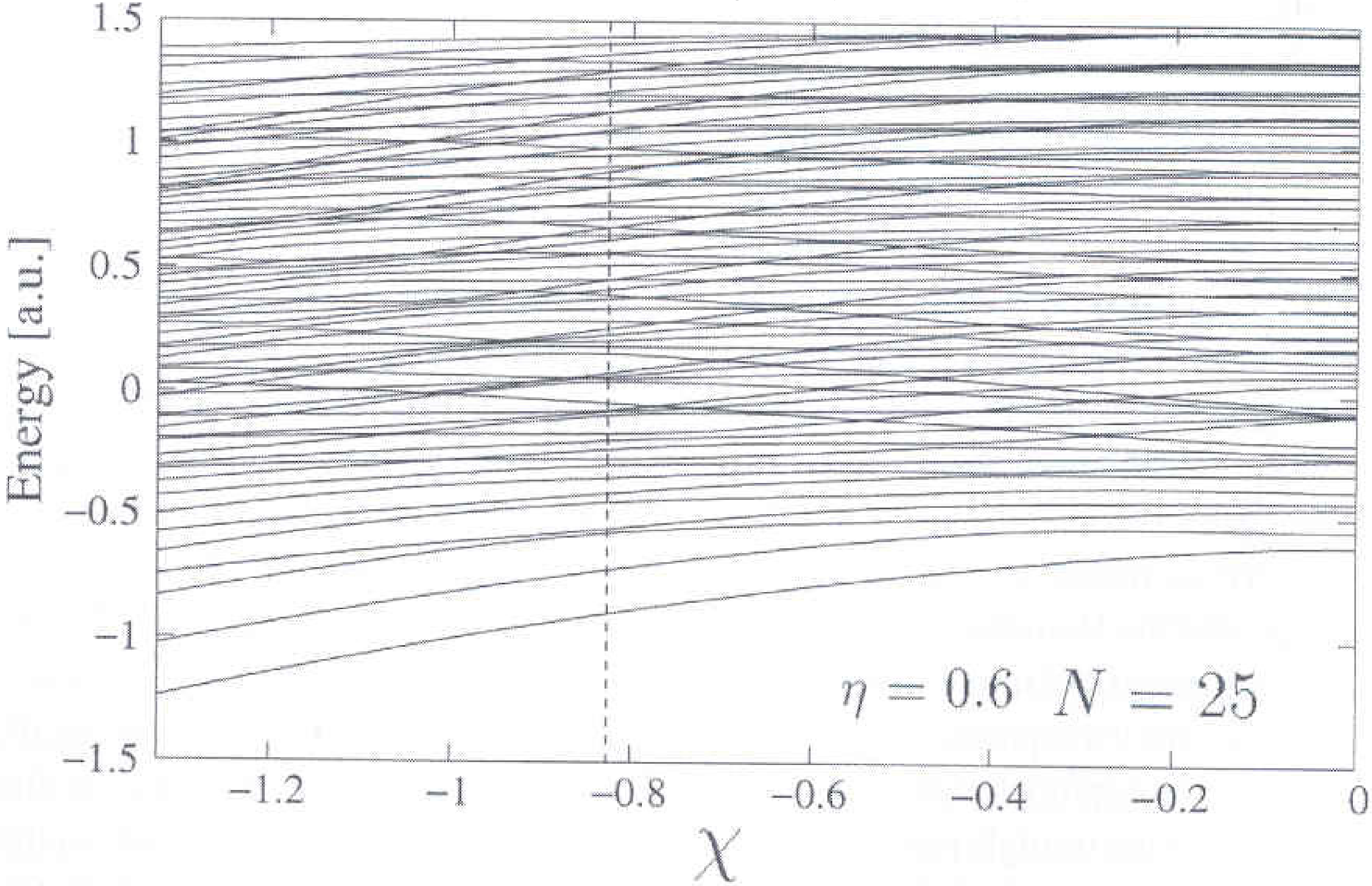


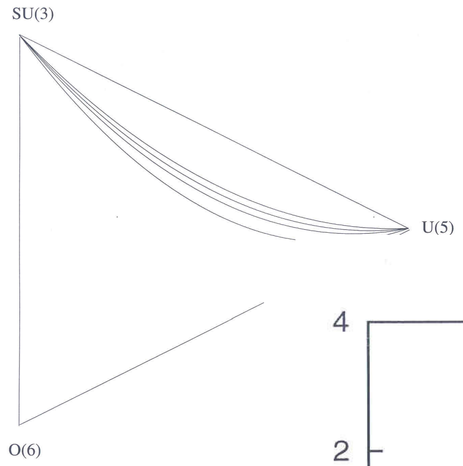
Moreover : $E(2^+_2)$- $E(0^+_2)$ is only ~ 0 in the regular region.



The level dynamics presents quasi crossings at the regular region

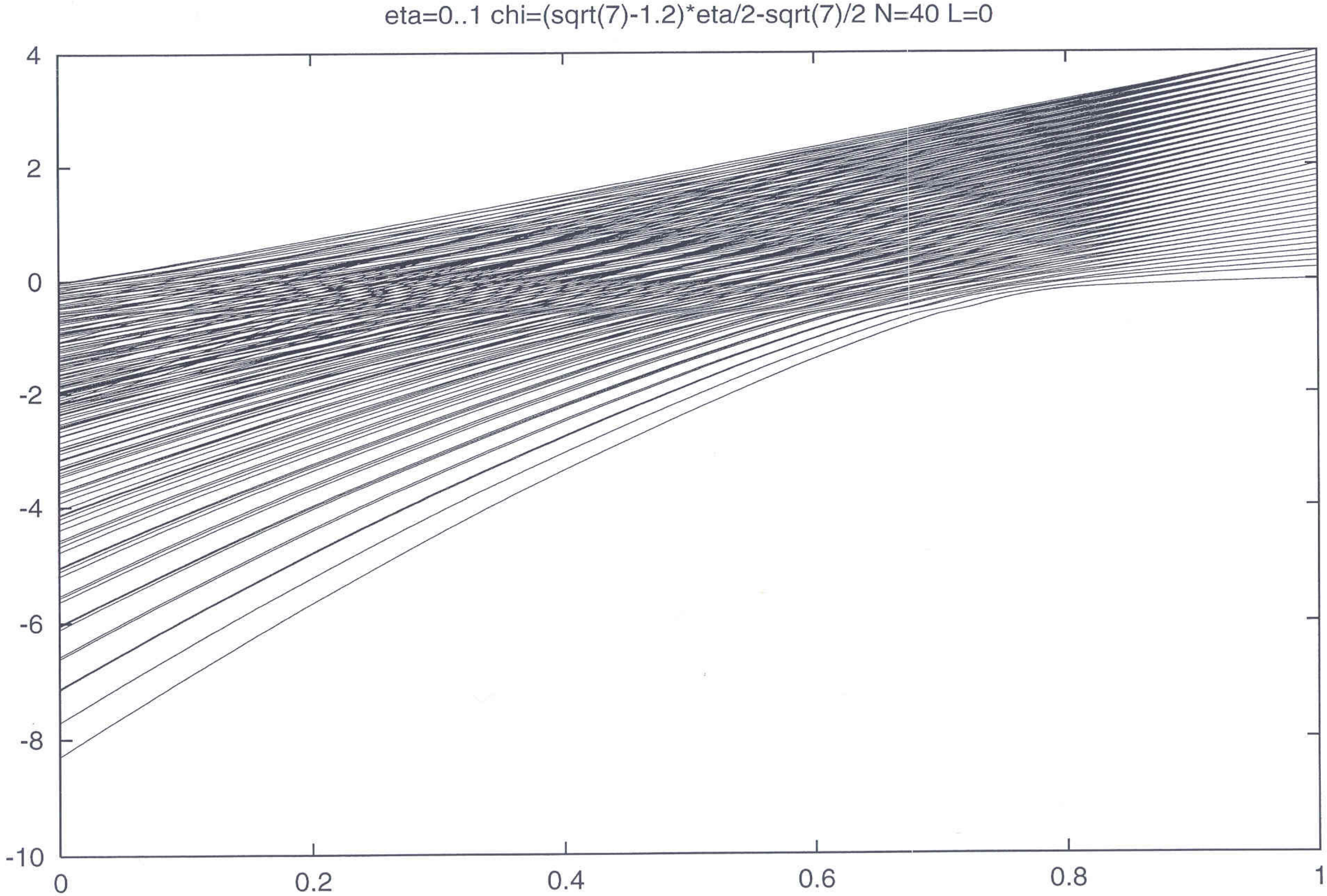
0+ levels





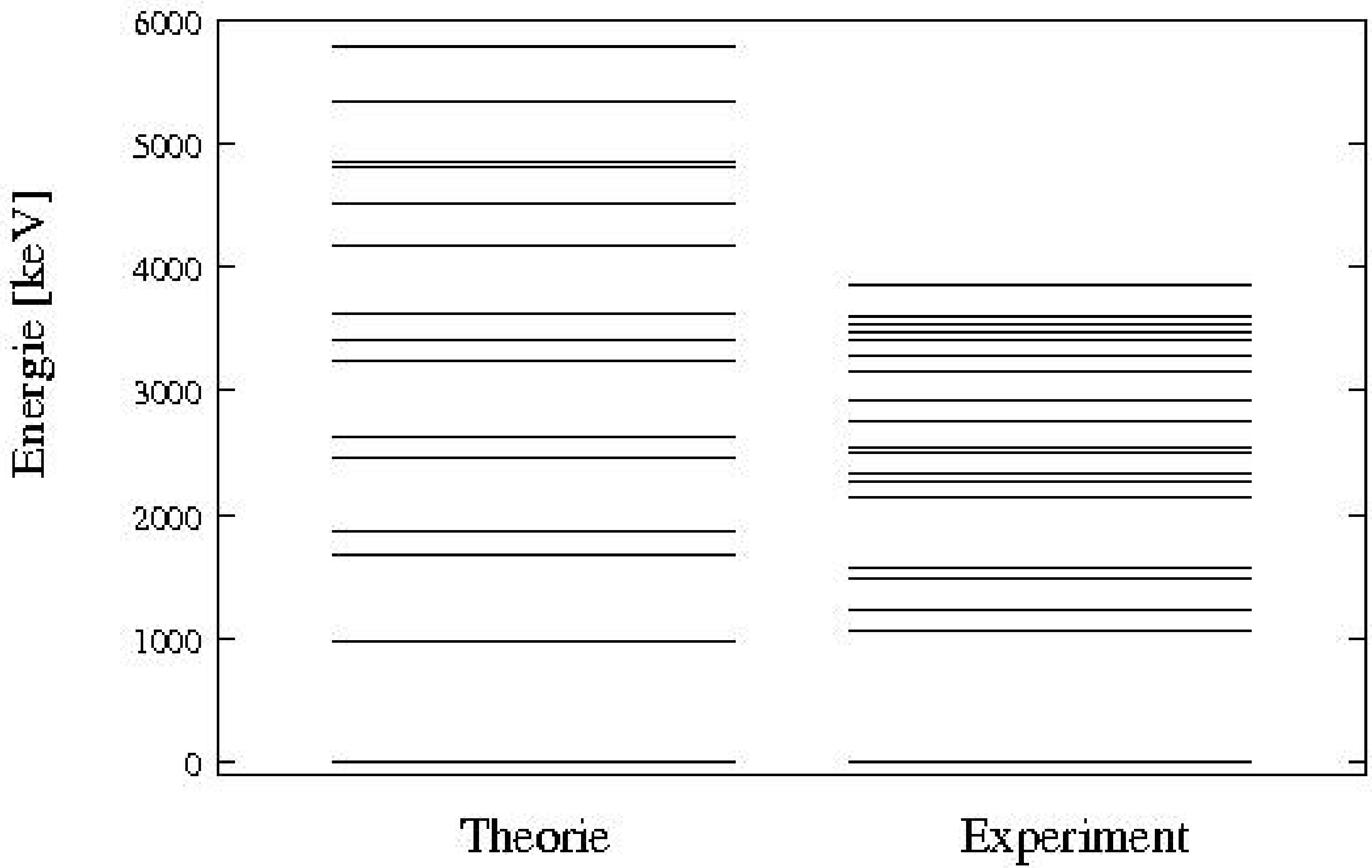
$$\chi = \frac{\sqrt{7} - a}{2}\eta - \frac{\sqrt{7}}{2}$$

a=1.2



Can this be observed?

$^{172}Yb(p,t)^{170}Yb$ reveals nearly all 0^+ states below 4 MeV

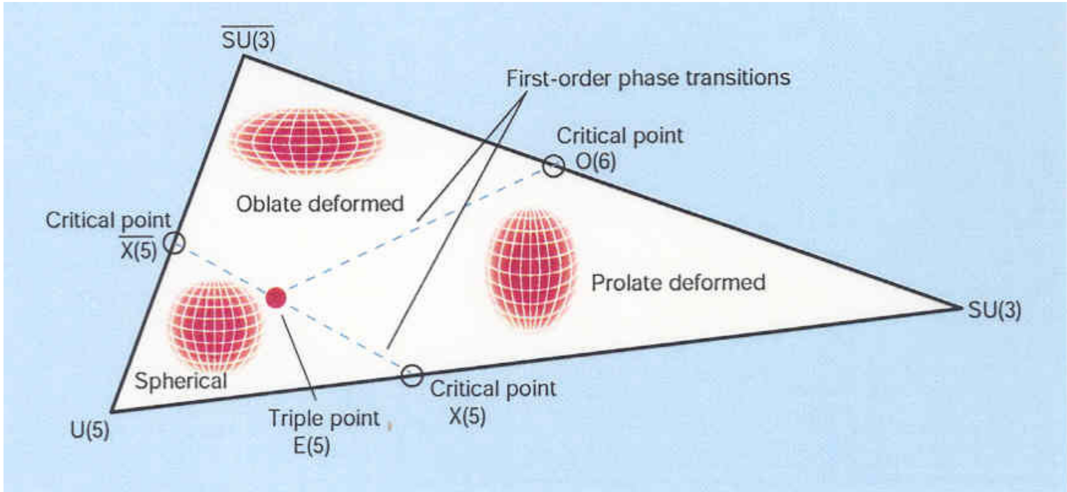


N=15

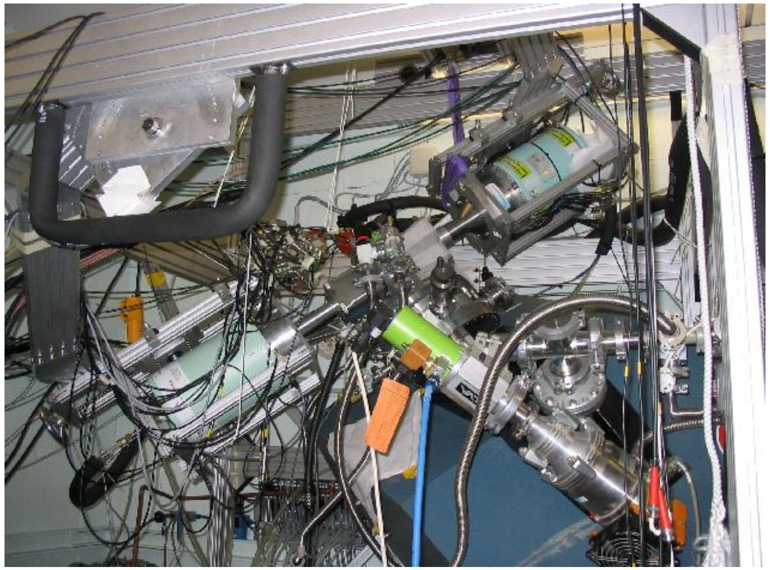
Conclusion: The atomic nucleus is a genereous finite-N mesoscopic quantal system. To study it fully we need

$$\hat{H} = c\left[\eta \hat{n}_d - \frac{(1-\eta)}{N} \hat{Q}_\chi . \hat{Q}_\chi\right]$$

simple but rich models



new concepts



new instrumentation

Thanks to (amongst others):

S. Heinze, A. Linnemann, V. Werner,
University of Cologne,

P. Cejnar,
Charles University Prague;

R.F. Casten, E. A. McCutchan, D. Meyer and co, N.V. Zamfir,
Yale University;

H.F. Wirth, R. Hertenberger, R. Kreucken,
TU and LMU Munchen

D. Bucurescu,
Bucharest

This work is supported by the DFG under grant JO391/2-3.