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Consiglio Nazionale delle Ricerche

Solitoni sine-Gordon in sistemi reali

Un approccio matematico nonperturbativo e nuove prospettive fenomenologiche

1a parte:

C. Nappi

C.N.R. Istituto di Cibernetica "E Caianiello", Pozzuoli, Napoli

http://www.cib.na.cnr.it

Sommario

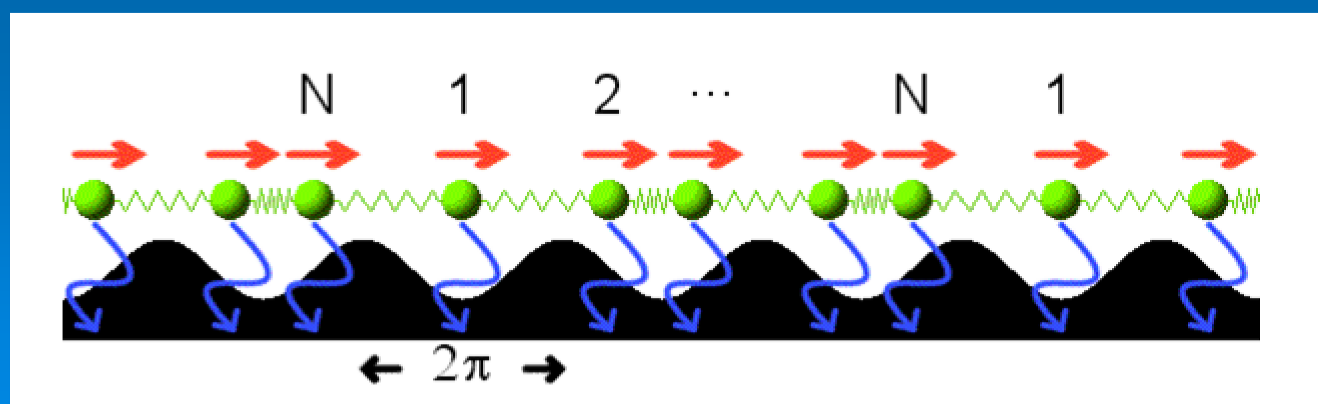
- A Klein-Gordon equation without "Klein"
- Soliton parade
- A famous mechanical analog
- sine-Gordon equation and long Josephson junctions

Frenkel and Kontorova Model (1939)

A “nonlinear Klein-Gordon equation”

$$\frac{\partial^2 \phi}{\partial x^2} - \frac{\partial^2 \phi}{\partial t^2} = \sin\phi$$

Infinite chain of harmonically bound atoms slipping over a periodic potential



Sine-Gordon Equation

$$\frac{\partial^2 \varphi}{\partial x^2} - \frac{\partial^2 \varphi}{\partial t^2} = \sin \varphi$$

Nonlinearity may compensate dispersion:

Permanent profile wave or “solitary” wave solution possibility

A. C. Scott
A nonlinear
Klein-Gordon equation
Am. Journ. Phys. 37, 52 (1969)

A Barone, F Esposito, C J Magee, A C Scott
Theory and Applications of sine-Gordon Equation
Rivista del Nuovo Cimento **1** p. 227-267 (1971)

PROCEEDINGS OF THE IEEE, VOL. 61, NO. 10, OCTOBER 1973 1443

The Soliton: A New Concept in Applied Science

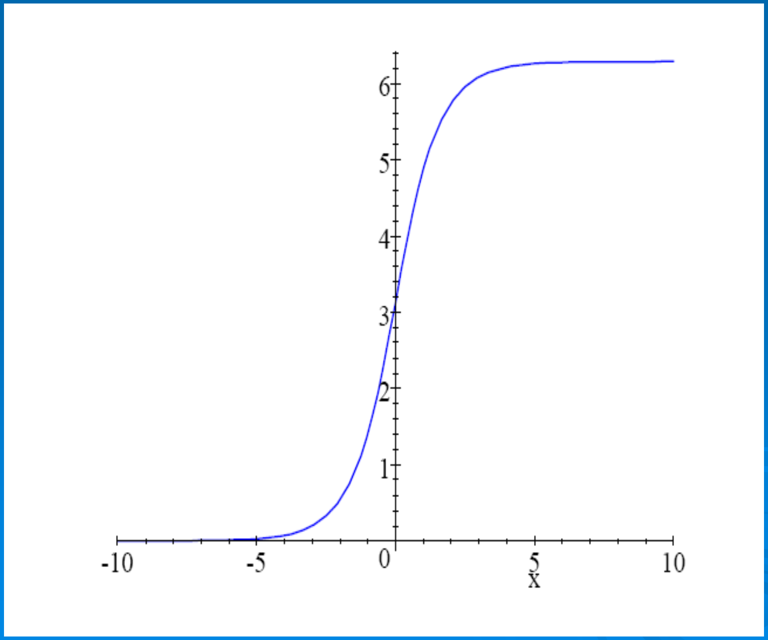
ALWYN C. SCOTT, F. Y. F. CHU, AND DAVID W. McLAUGHLIN

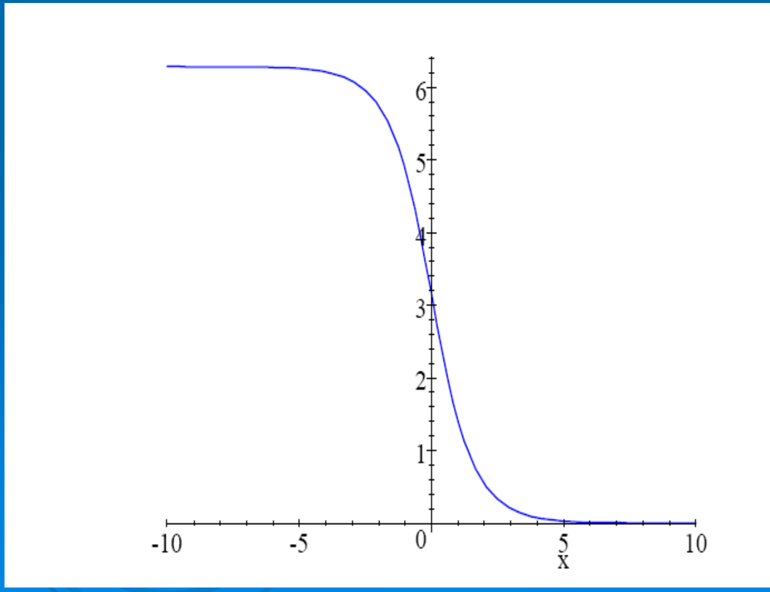
Invited Paper

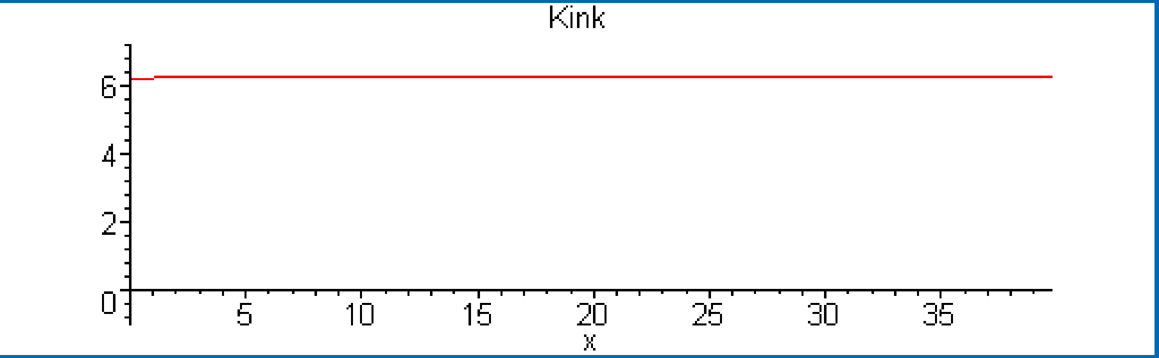
“Kink” solution or a “soliton”

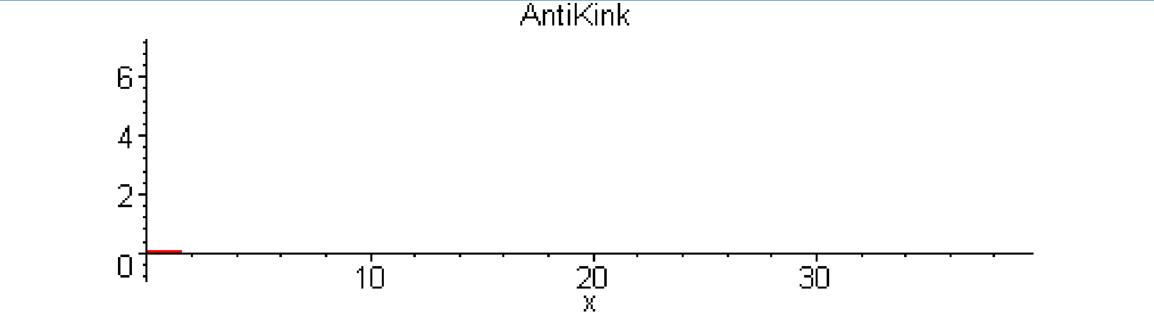
$$\varphi_0(x,t) = 4\arctan[\exp\frac{x - vt - x_0}{\sqrt{1-v^2}}]$$

$$\overline{\varphi_0}(x,t) = 4\arctan[\exp(-\frac{x-vt-x_0}{\sqrt{1-v^2}})]$$

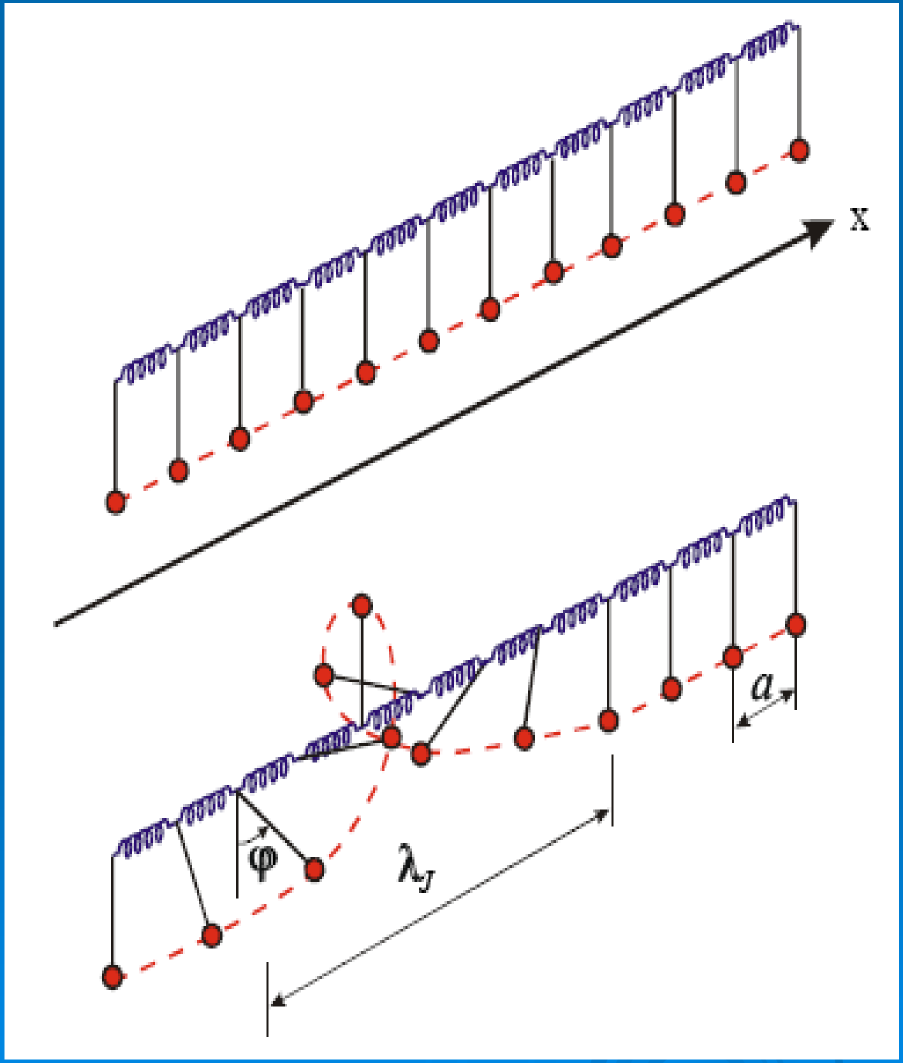





Kink
6
4
2
0
5
10
15
20
25
30
35
x


AntiKink
6
4
2
0
10
20
30
x

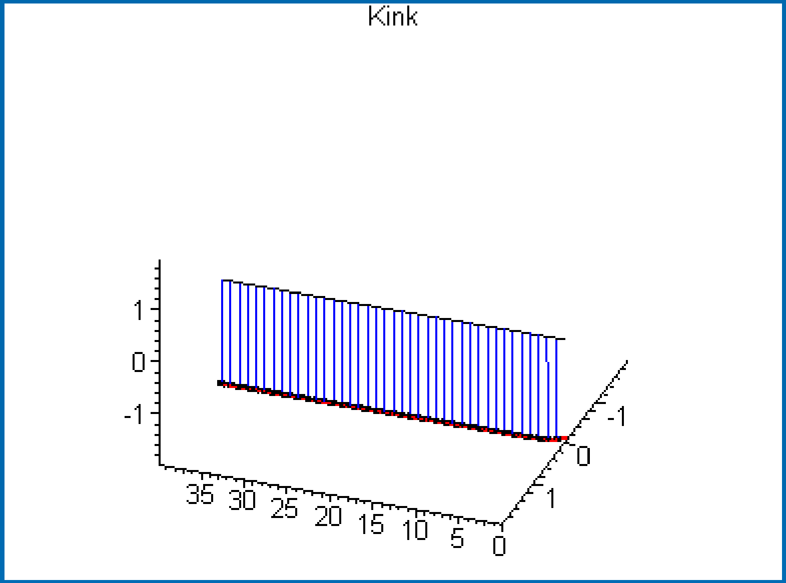
Mechanic analog: chain of pendula



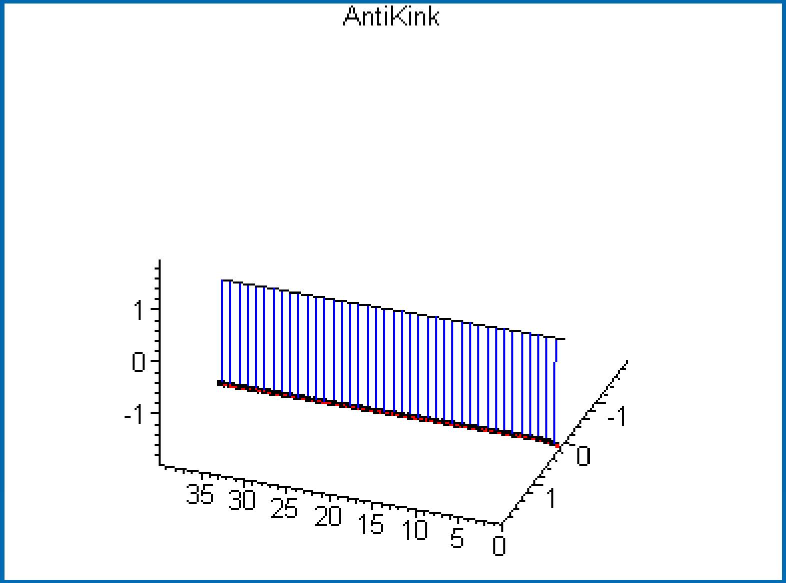
See for example Barone & Paternò
Physics and Applications of the Josephson Effect
N Y Wiley 1982

(In the book a ‘pocket ‘ version is also shown)

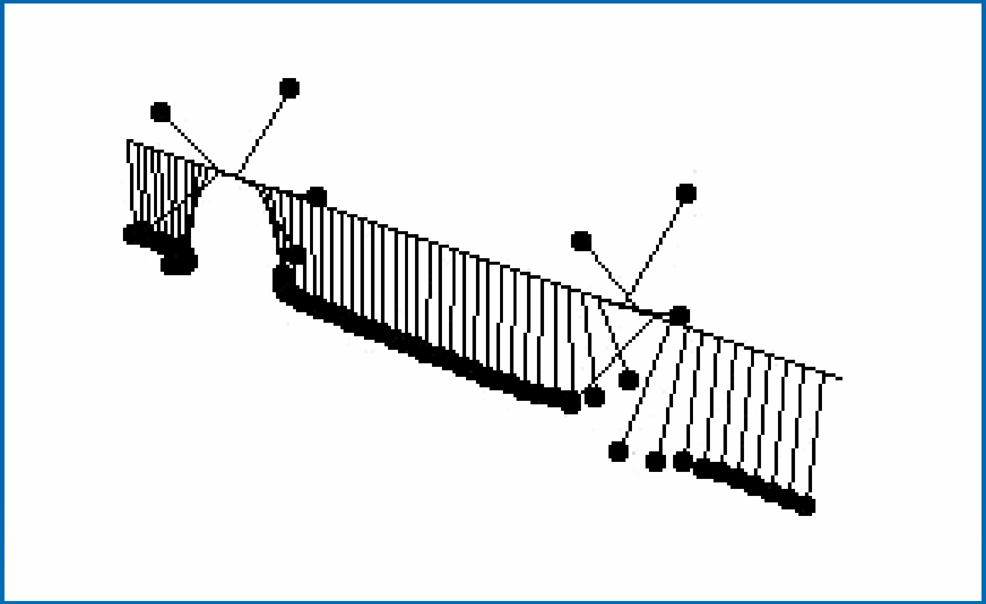
Kink in motion



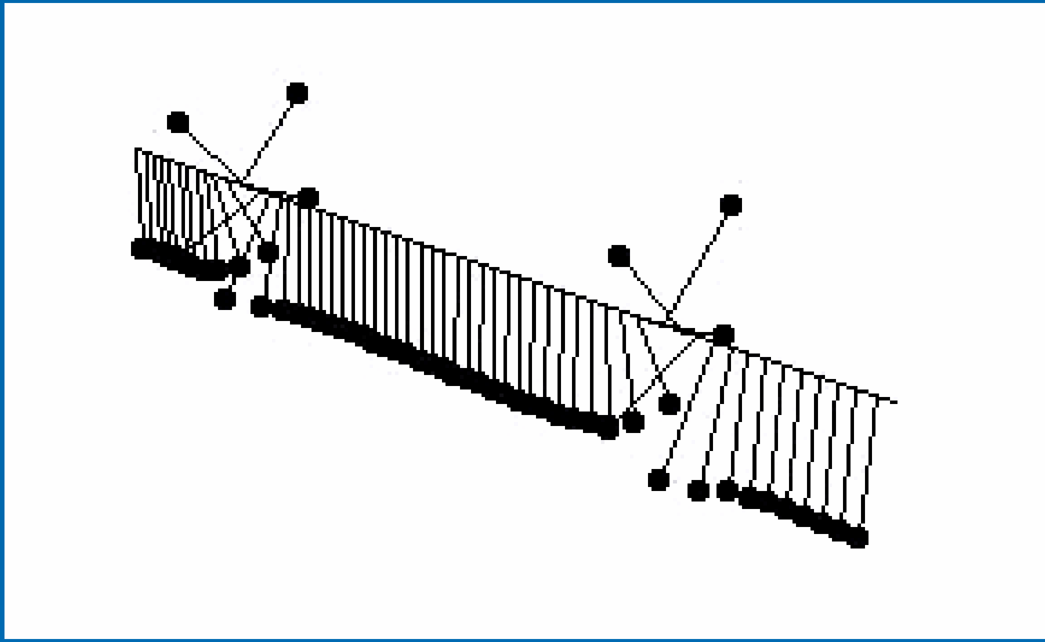
AntiKink in motion



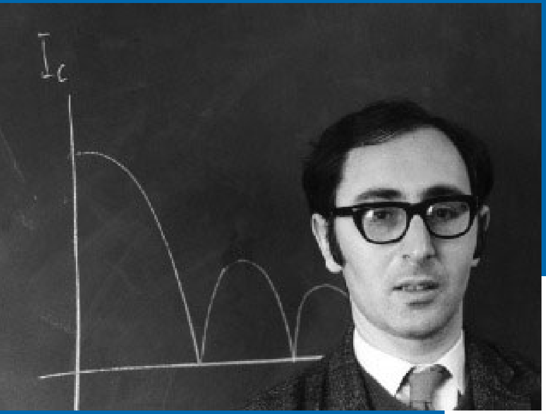
kink anti-kink collision



kink-kink collision

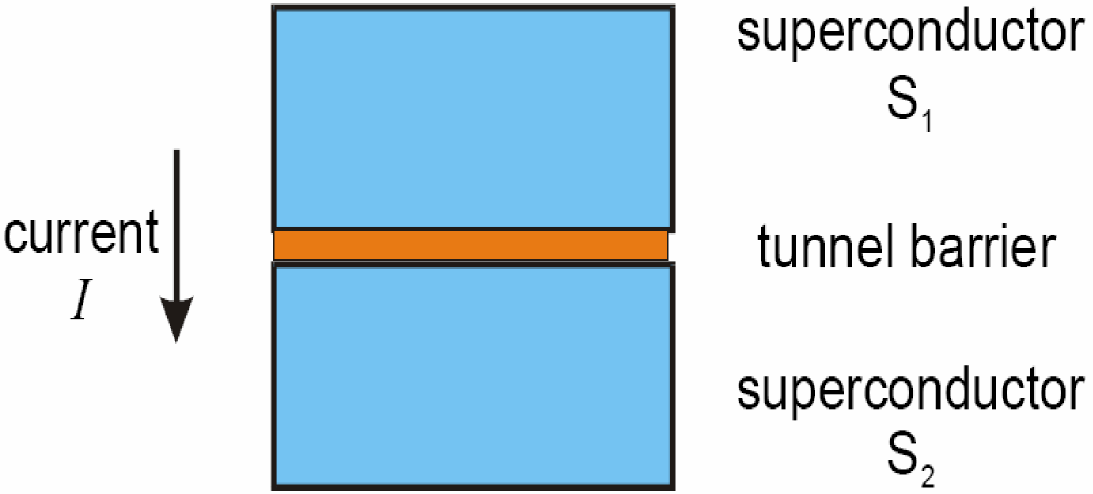


Josephson effect



Nobel Prize in Physics 1973 with
Leo Esaki
Ivar Giaever

Obtained by B. Josephson in 1962



Wavefunctions of superconducting electrodes:

$$\Psi_1 = |\Psi_1| \exp(i\theta_1), \quad \Psi_2 = |\Psi_2| \exp(i\theta_2)$$

Phase difference: $\varphi = \theta_2 - \theta_1$

Josephson equations

dc Josephson effect:

$$I_s(\varphi) = I_c \sin\varphi \quad (1)$$

ac Josephson effect:

$$\frac{d\varphi}{dt} = \frac{2e}{\hbar} V \qquad (2)$$

Josephson junction is a *quantum dc voltage - to - frequency converter*

$1\,\mu\text{V} \leftrightarrow 483.59767\ \text{MHz}$

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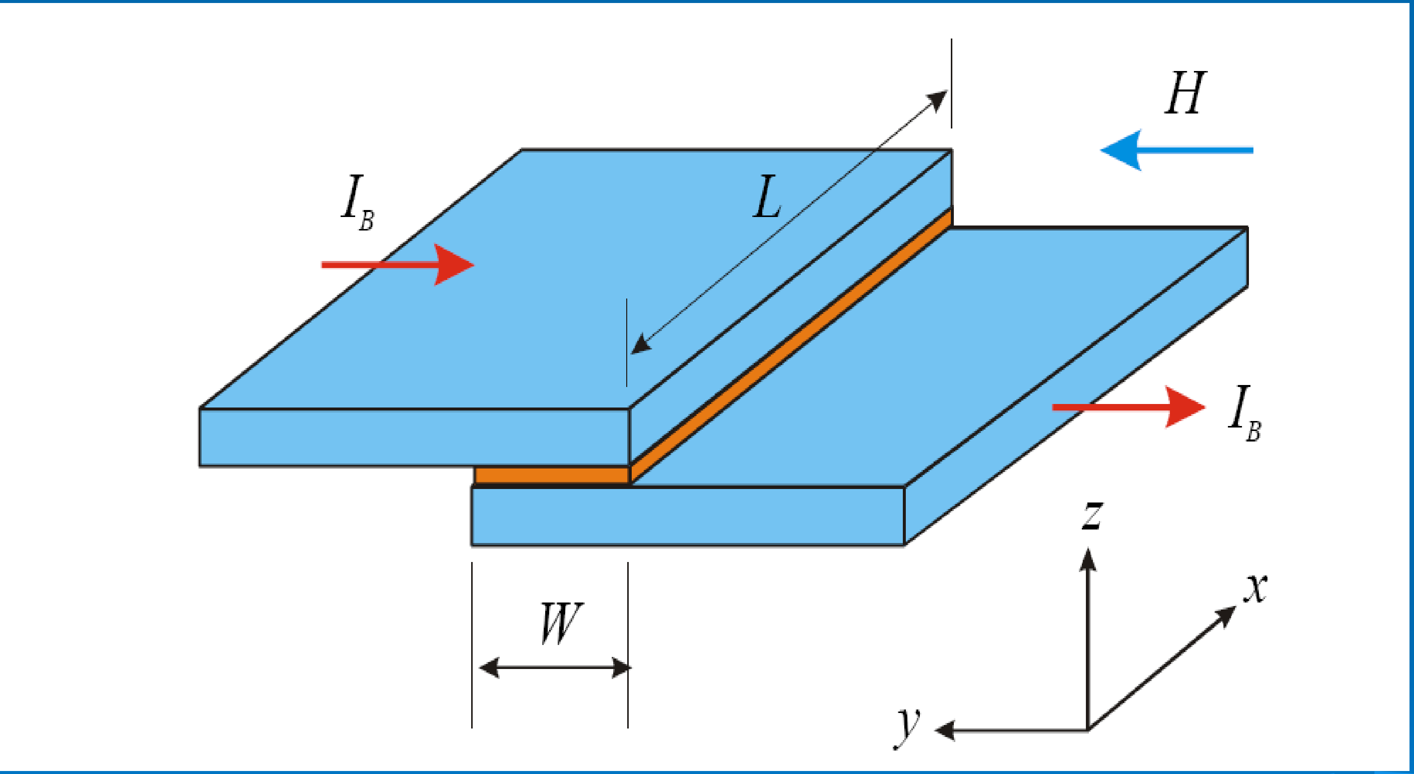
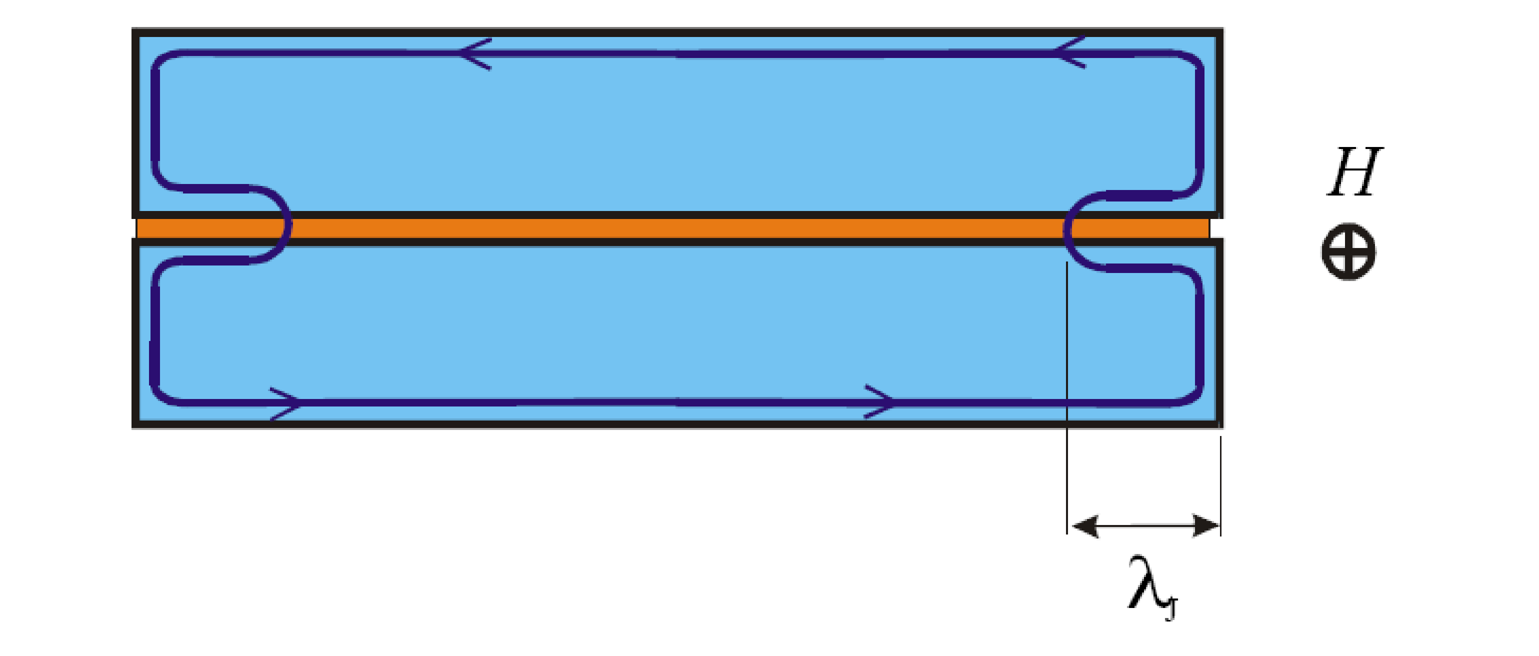
Overlap geometry



Josephson junction in magnetic field

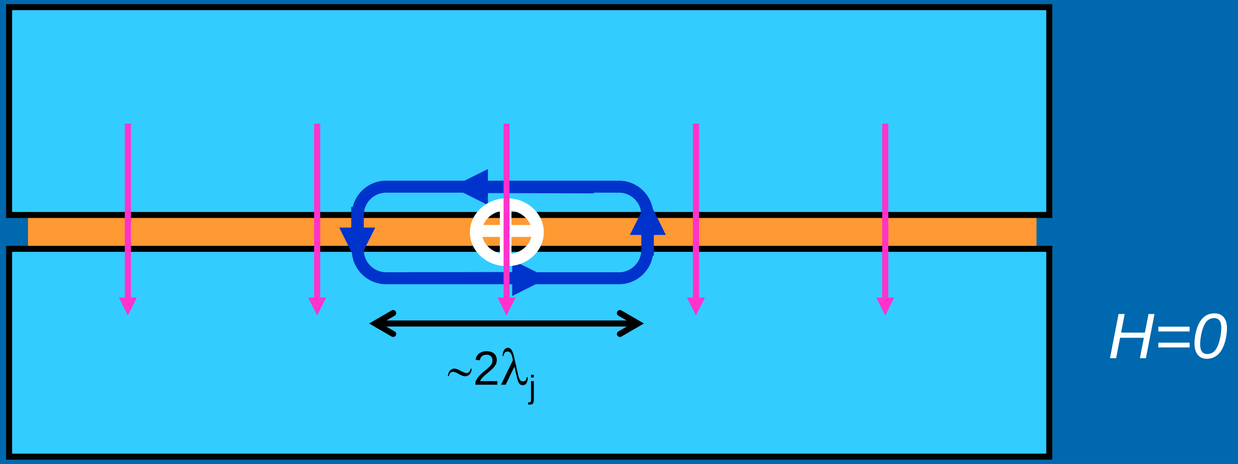


$$\lambda_J = \sqrt{\frac{\Phi_0}{2\pi j_c \mu_0 (2\lambda_L + t_0)}}$$

typical value: 10µm

Josephson vortex

- A stable self-sustained tunneling current structure



- Associated flux is the elementary flux quantum:

$\Phi_0 = 2.07\times10^{-15}$ tesla • m^2

$= 2.07\times10^{-15}$ V • s

magnetic field in the center of static fluxon

$H_F \sim 10^{-4}\,\mathrm{T} = 1\,\mathrm{Oe}$

Perturbed sine-Gordon models

$$\varphi_{xx} - \varphi_{tt} = \sin\varphi + \alpha\varphi_t - \beta\varphi_{xxt} - \gamma$$

Boundary conditions:

$$\left(\varphi_x + \beta\varphi_{xt}\right)\Big|_{x=0} = \eta_1$$

$$\left(\varphi_x + \beta\varphi_{xt}\right)\Big|_{x=\ell} = \eta_2$$

Space and time normalized to:

$$\lambda_J = \sqrt{\frac{\Phi_0}{2\pi j_c \mu_0 (2\lambda_L + t_0)}}$$

$$\omega_p = \sqrt{\frac{2\pi j_c}{\Phi_0 C}}$$

more perturbations

S.Pagano, C. Nappi, R. Cristiano, E. Esposito, L. Frunzio, L. Parlato, G. Peluso, G. Pepe, U. Scotti di Uccio in “Nonlinear Superconducting Devices and High-Tc Materials”, R.D. Parmentier and N.F. Pedersen Eds, World Scientific, Singapore, 1995

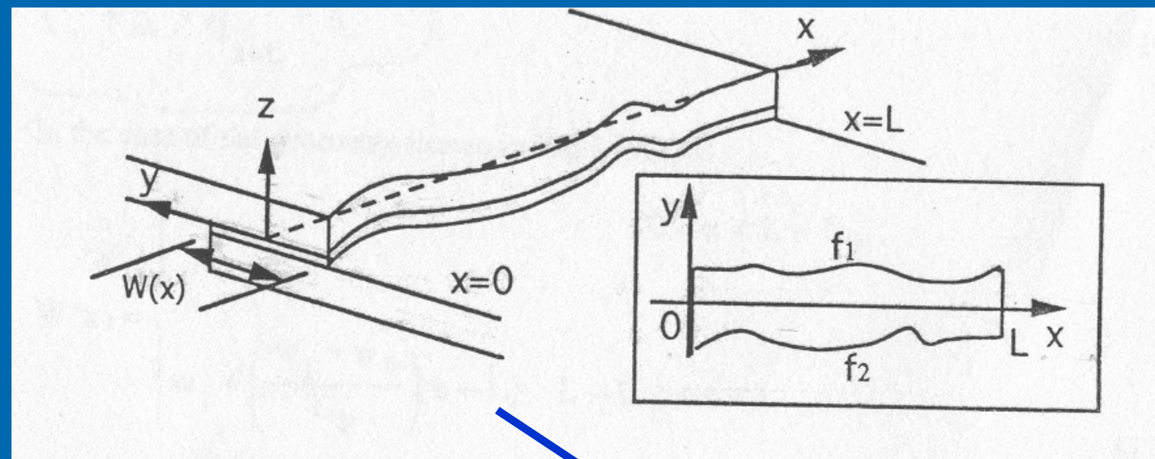


Fig. 4. A schematic view of a long junction with variable width and its boundary C

$$\varphi_{xx} - \varphi_{tt} = \sin\varphi + \alpha\varphi_t - \beta\varphi_{xxt} - \gamma - \delta\varphi_x$$

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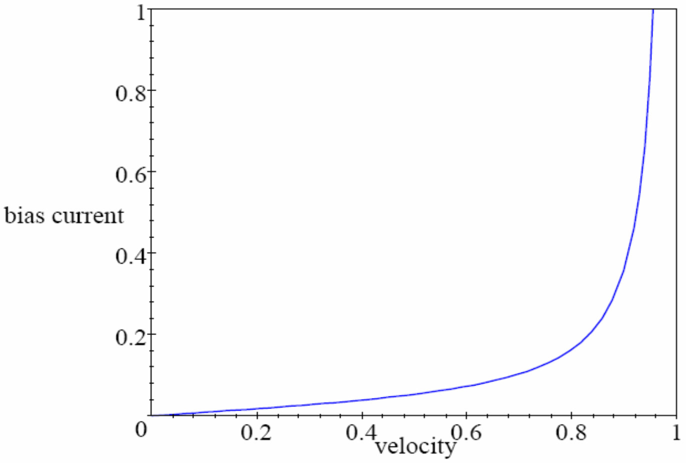
Energy and perturbation analysis for a single fluxon

D.W.McLaughlin and A.C.Scott, Phys.Rev.B **18**, 1652 (1978)

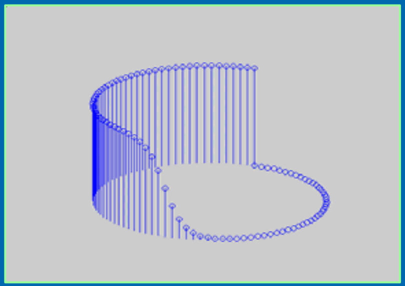
In this approximation fluxon behaves as a relativistic “particle”

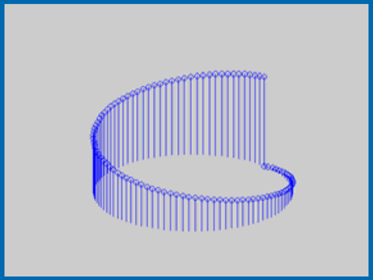
$$E_0 = \frac{8}{\sqrt{1-v^2}}$$

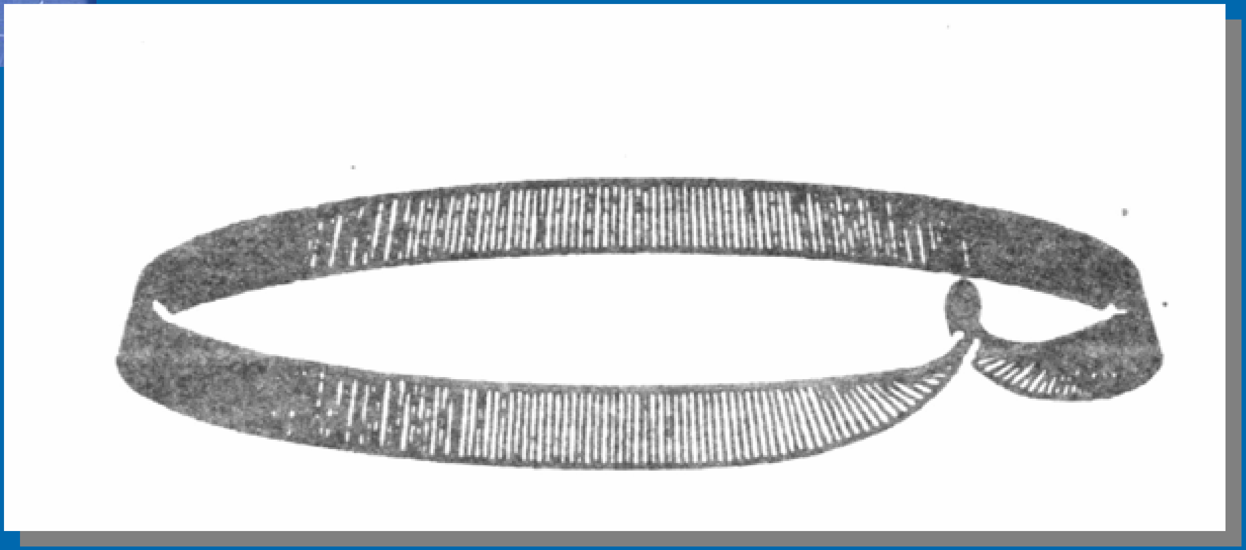


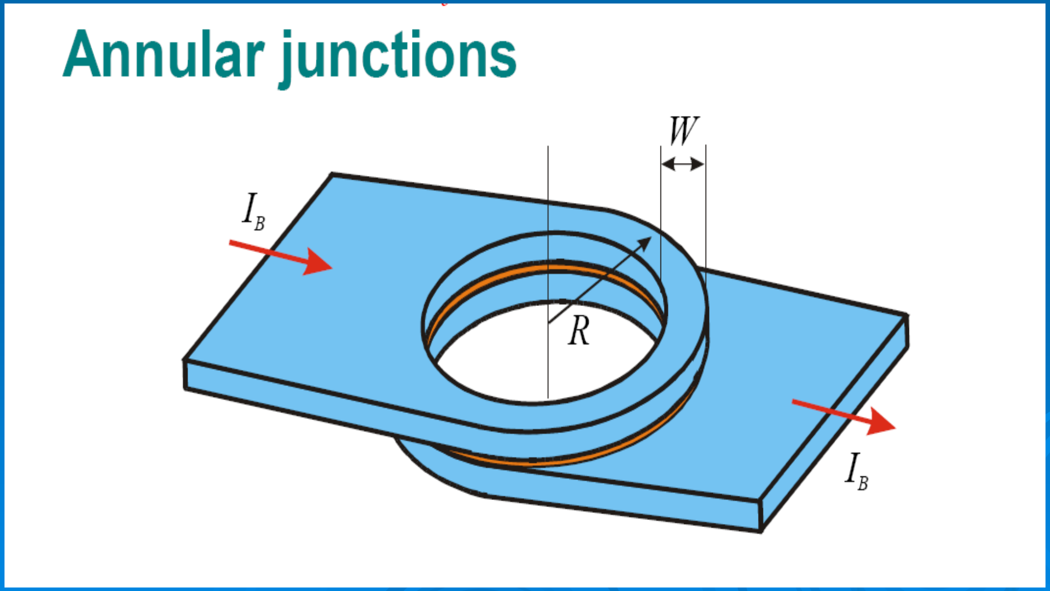
Eq.(12) at $\alpha = \beta = 0.05$

$$\frac{dv}{dt} = -\alpha v(1-v^2) - \frac{1}{3}\beta v - \frac{1}{4}\pi\gamma(1-v^2)^{3/2}$$

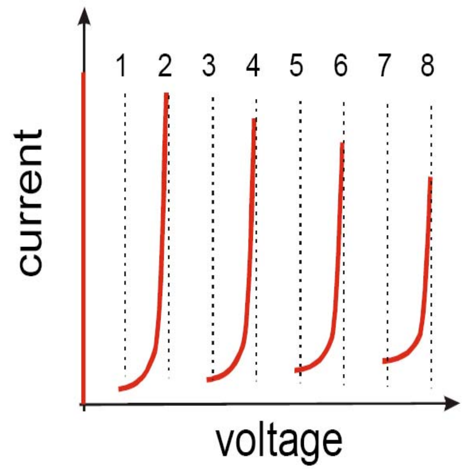




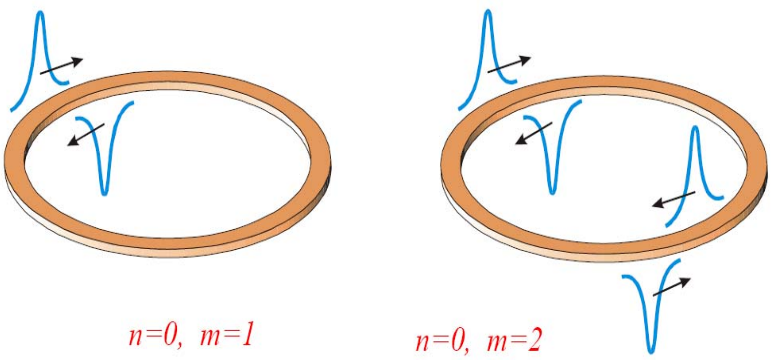



Annular junctions
W
R
I_B
I_B

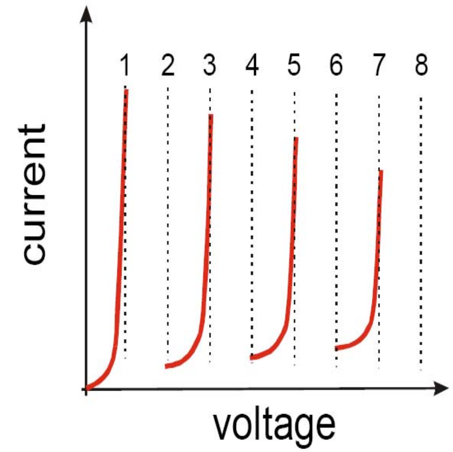
For $n = 0$:



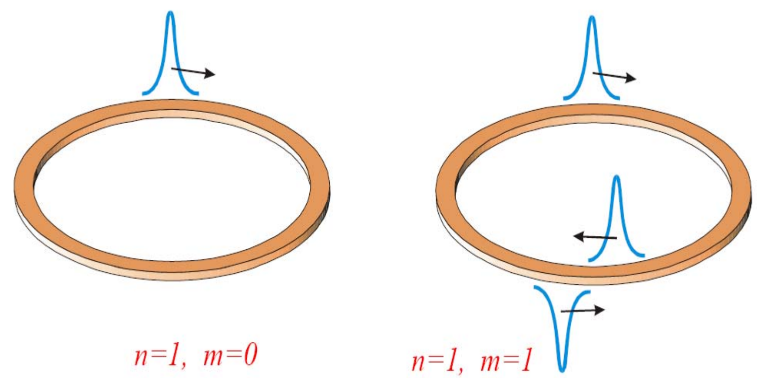
Only even fluxon steps



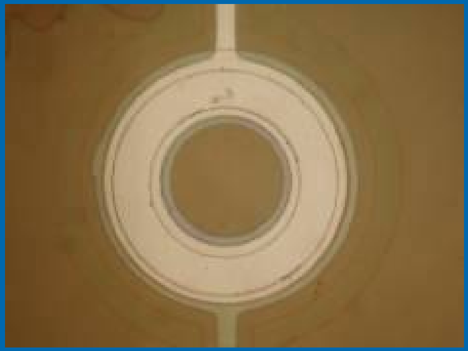
For $n = 1$:

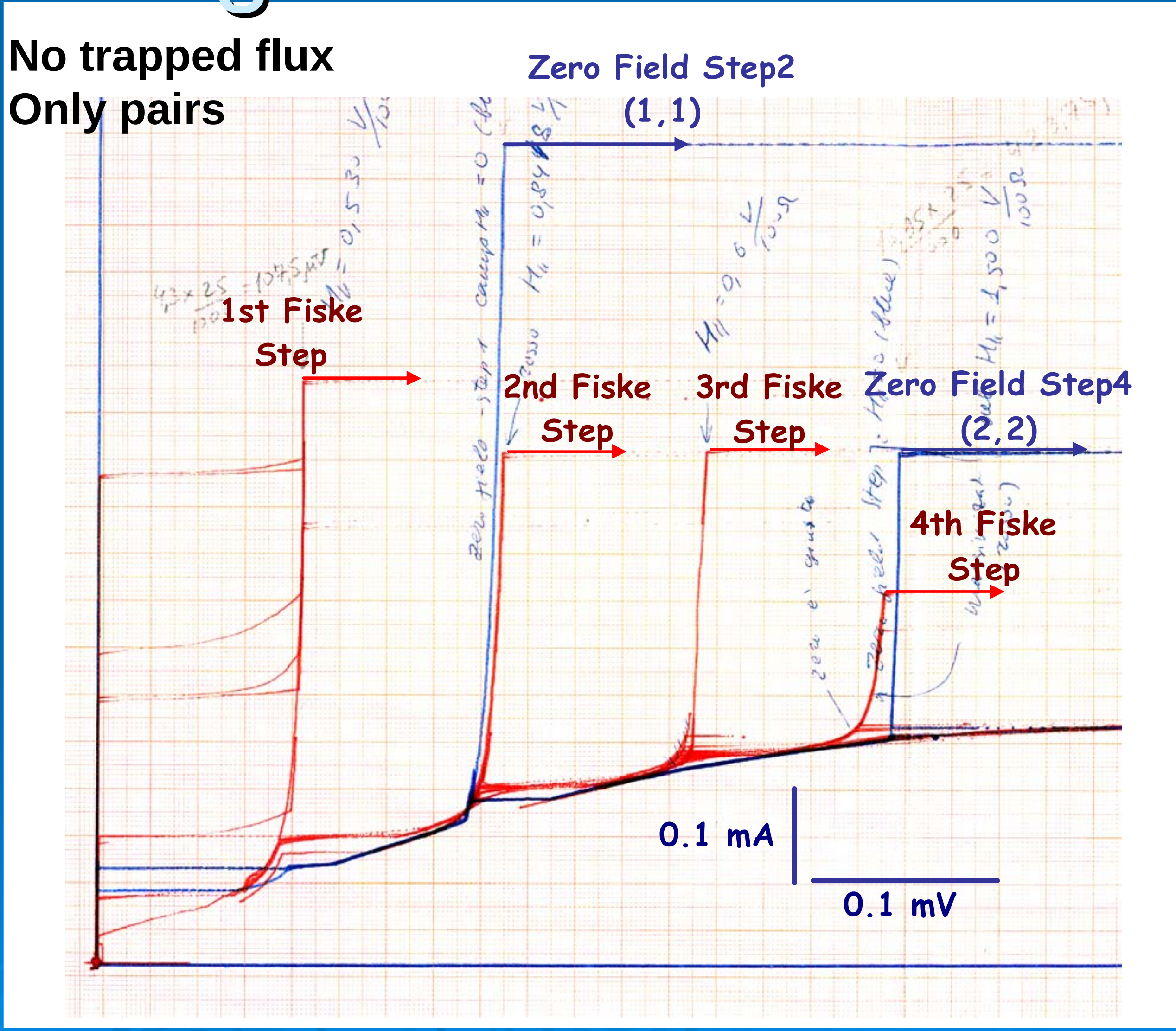


Only odd fluxon steps



Current Voltage characteristic





R=45µm. R_{int}=30µm, λ_j~ 45µm
jc=54 A/cm^2

Nb/Al/Al2O3/Al/Nb

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A long JJ clock based on a rotating fluxon

Supercond. Sci. Technol. **12** (1999) 769–772. Printed in the UK PII: S0953-2048(99)04955-6

Low-jitter on-chip clock for RSFQ circuit applications

Yongming Zhang† and Deepnarayan Gupta‡

† Conductus, Inc., 969 West Maude Avenue, Sunnyvale, CA 94086, USA
‡ HYPRES, 175 Clearbrook Road, Elmsford, NY 10523, USA

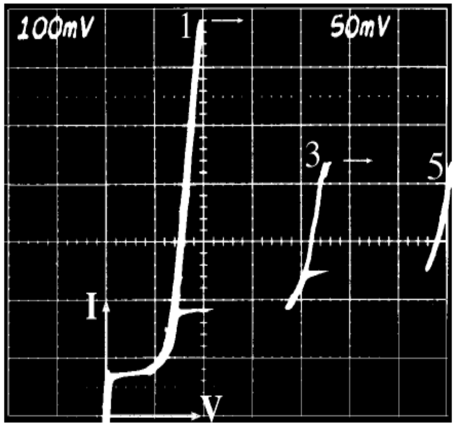


Figure 3. Current against voltage for various numbers of solitons at 4.2 K. There is one trapped soliton. The number next to each curve is the total number of solitons and antisolitons moving to produce that curve.

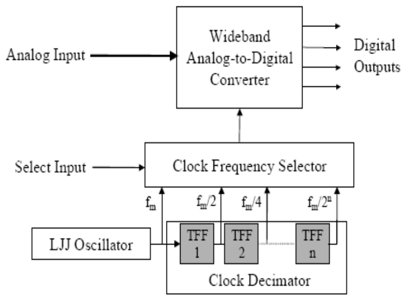


Figure 1. A proposed flash ADC with integrated LJJ clock.

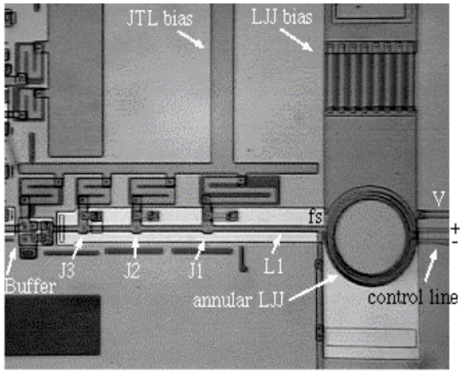
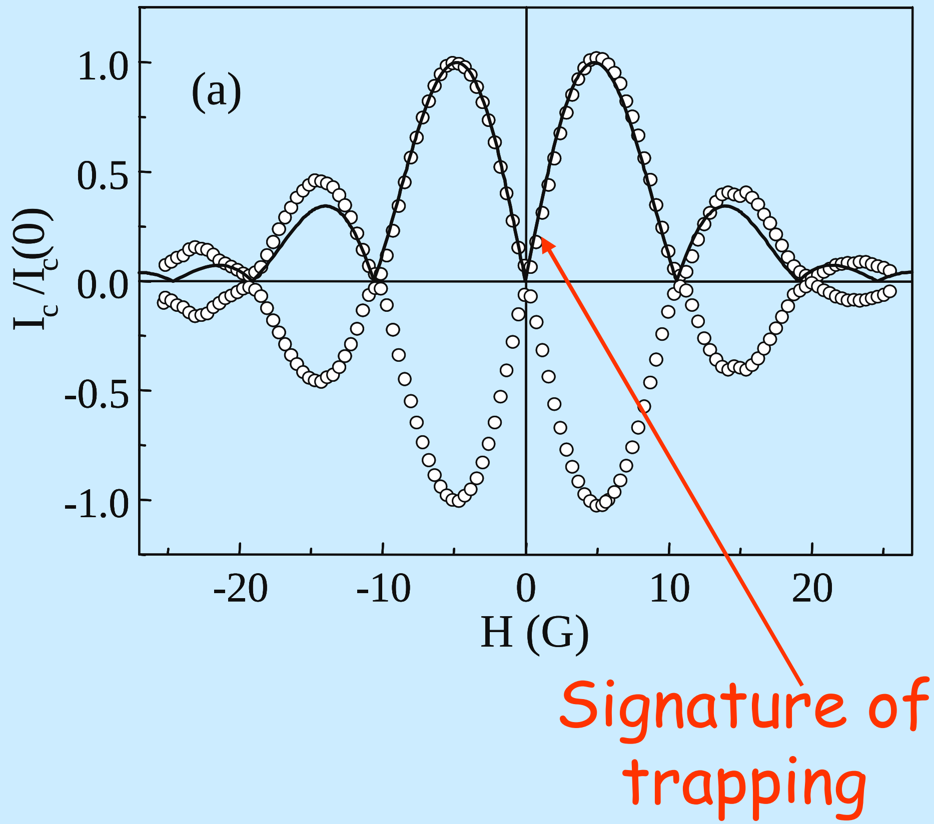


Figure 2. An optical micrograph of a fabricated annular long junction that is coupled to the SFQ circuit. The length of the junction is 230 μm, and the width is 5 μm. The circuit was fabricated using a HYPRES 1 kA cm^{-2} Nb/AlO$_x$/Nb junction fabrication process.

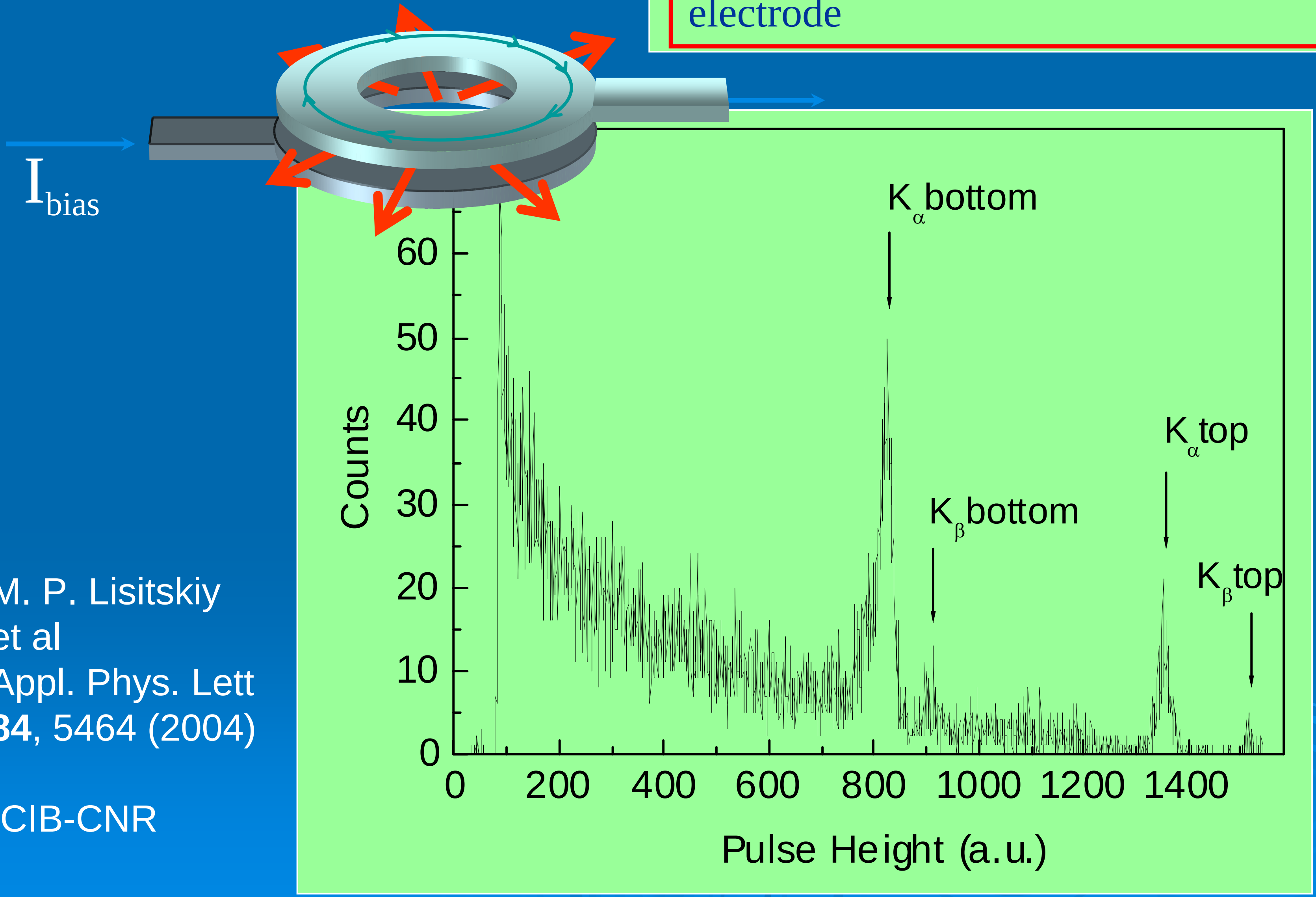
Trapping of one fluxon in a small junction



2Re = 16 μm
2Ri = 7.5 μm
Nb λ_j = 50 μm @4.2K
(ξo=38nm)
ICIB-C.N.R.

R. Cristiano et al. APL **74**, (1999) 3389

An energy resolution of about 100 eV was obtained for the K_α line of the top electrode



M. P. Lisitskiy et al
Appl. Phys. Lett **84**, 5464 (2004)

ICIB-CNR

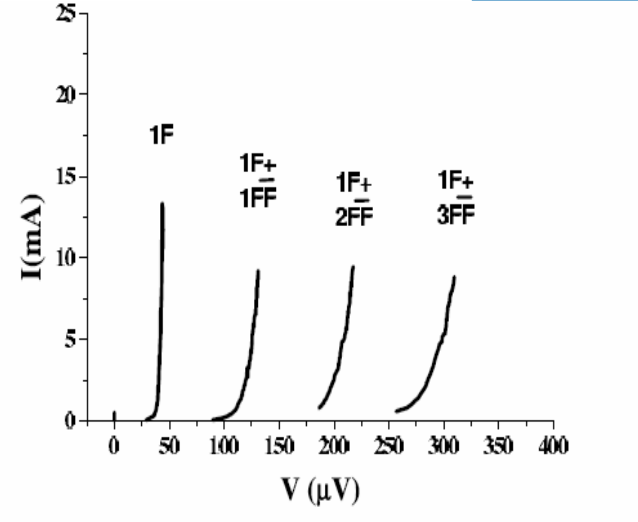
Using Annular Josephson Tunnel Junctions to Monitor Causal Horizons

R. Monaco[a] and R. J. Rivers[b],
a) Istituto di Cibernetica del C.N.R., I-80078, Pozzuoli, Italy
and Unita' INFM-Dipartimento di Fisica,
Universita' di Salerno,
I-84081 Baronissi, Italy
b) Blackett Laboratory, Imperial College London,
London SW7 2AZ, U.K.

(Dated: March 2, 2005)

If systems change as fast as possible as they pass through a phase transition then the initial domain structure is constrained by causality. We shall show how we can trace these causal horizons by measuring the spontaneous production of flux in annular Josephson Tunnel Junctions as a function of the quench time τ_Q into the superconducting phase. A specific test of our analysis is that the probability P_1 to trap a single fluxon at the N-S transition clearly follows an allometric dependence on τ_Q as $P_1 = a\,\tau_Q^{-\sigma}$, with a scaling exponent $\sigma = 0.25$, in agreement with the data.

PACS Numbers : 11.27.+d, 05.70.Fh, 11.10.Wx, 67.40.Vs



Abstract

Jackiw-Teitelboim (JT) theory is a theory of gravity in two dimensions with black hole solutions. The sine-Gordon equation is a nonlinear equation with soliton solutions. Black holes in JT theory can be parametrized in such a way that the JT field equations are the sine-Gordon equation. As a result, we can study JT black holes using the solutions of the sine-Gordon equation. In this thesis, we use the sine-Gordon breather solution to study JT black holes.

VOLUME 88, NUMBER 23 PHYSICAL REVIEW LETTERS 10 JUNE 2002

Mode Locking in Reversed-Field Pinch Experiments

H. K. Ebraheem and J. L. Shohet
University of Wisconsin–Madison, Madison, Wisconsin 53706

A.C. Scott
University of Arizona, Tucson, Arizona 85721
(Received 16 November 2001; revised manuscript received 3 May 2002; published 24 Ma

The MHD mode trajectory in the Madison Symmetric Torus reversed-field pinch has bee
obey the sine-Gordon equation. Corresponding to experiment, a perturbation analysis predict
tions of mode locking to be at the vacuum chamber poloidal and/or toroidal gaps. The mod
dissipates when it locks, as shown by a decaying spiral phase-plane trajectory. Unlocked m
around the torus without an abrupt energy loss. By varying key machine parameters obtained
tical analysis, the probability of locking in accordance with the experimental results can be p

DOI: 10.1103/PhysRevLett.88.235003 PACS number

Accordingly, we sum torques around the magnetic axis to produce the sine-Gordon equation

$$M\frac{\partial^2\phi}{\partial t^2} = K\frac{\partial^2\phi}{\partial x^2} - T\sin\phi\,, \qquad (3)$$

VOLUME 88, NUMBER 23 PHYSICAL REVI

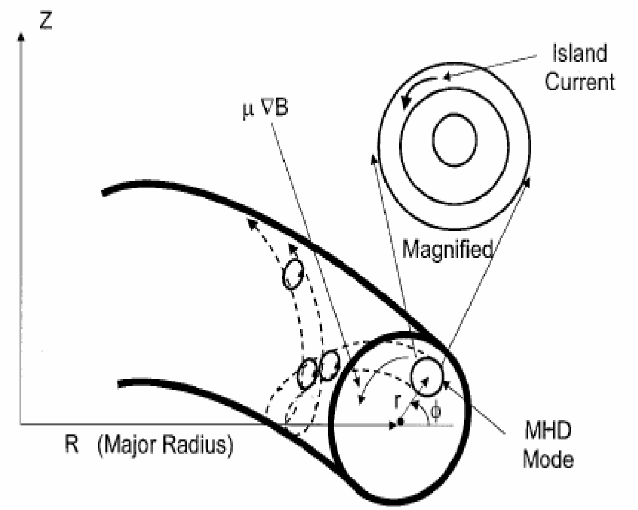


FIG. 1. A conceptual drawing of the MHD kink mode in the MST torus. The mode threads its way along the torus passing both the poloidal and toroidal vacuum chamber gaps.

Part of the material of this seminar has been taken from the web sites:

For the animations:

- http://homepages.tversu.ru/~s000154/collision/solpen_m/SOLPEN1.html
- http://www.math.h.kyoto-u.ac.jp/~takasaki/soliton-lab/gallery/solitons/sg-e.html

Figure and other, A. Ustinov group web site:

- http://fluxon group.physik.uni-erlangen.de/