GAUGE/GRAVITY CORRESPONDENCE
and
OPEN/CLOSED STRING DUALITY

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PLAN

INTRODUCTION

ASPECTS OF GAUGE/GRAVITY CORRESPONDENCE

WHY DOES IT WORK FOR SUPERSYMMETRIC THEORIES?
(Perturbative Aspects)

STRING CONSTRUCTION OF ORIENTIFOLD FIELD THEORIES
AND GAUGE/GRAVITY CORRESPONDENCE FOR THEM

CONCLUSION

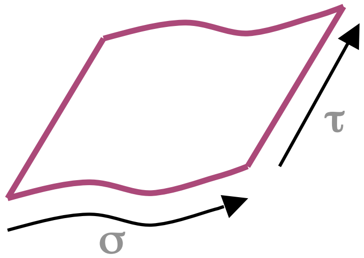
INTRODUCTION AND MOTIVATIONS

String Theory is an excellent candidate for a unified theory for all forces in nature. These are unified in a deep and significant way: *all the particles (bosons and fermions) are unified, being identified as vibrational modes of an elementary miscroscopic STRING.*

The theory is characterized by the presence of only one adjustable and dimensional parameter α'

A string sweeps out a two-dimensional world-surface (world-sheet)

$$S = -\frac{1}{2\pi\alpha'}\int dA$$



$X^{\mu}(\tau,\sigma)$

$\sigma \in [0,\pi]$ $\tau \in [-\infty,+\infty]$

α' has to involve the fundamental constants c, $\hbar, G_N$

Lunghezza e massa di Planck:

$$l_P = \sqrt{\alpha'} = \left(\frac{\hbar G_N}{c^3}\right)^{3/2} \approx 1.6 \times 10^{-33}\, cm$$

$$m_p = \left(\frac{\hbar c}{G_N}\right)^{1/2} \approx 1.2 \times 10^{19}\, GeV/c^3$$

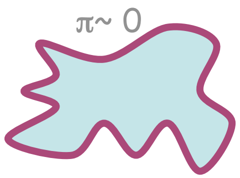
Equations of motion + boundary conditions

$$(\partial_\sigma^2 - \partial_\tau^2) X^\mu = 0 \qquad , \qquad \partial_\sigma X_\mu \delta X^\mu|_{\sigma=\pi} - \partial_\sigma X_\mu \delta X^\mu|_{\sigma=0} = 0$$

$$X^\mu(\tau,\sigma) = X_L^\mu(\tau+\sigma) + \overline{X}_R^\mu(\tau-\sigma)$$

Open

$$\partial_\sigma X_\mu \delta X^\mu|_{0,\pi} = 0 \Rightarrow \begin{cases} \partial_\sigma X_\mu|_{0,\pi} = 0 \rightarrow \text{Neumann boundary conditions} \\ \delta X^\mu|_{0,\pi} = 0 \rightarrow \text{Dirichlet boundary conditions} \end{cases} .$$



Closed

$$X^\mu(\tau,\sigma) = X^\mu(\tau,\sigma+\pi)$$

Supersymmetry is embodied in a very natural way.

World-sheet supersymmetry extension $X^{\mu}(\tau,\sigma) \leftrightarrow \psi^{\mu}(\tau,\sigma)$

The quantization of the theory fixes the dimensionality of the space-time: D=26 for the bosonic string and D=10 for the superstring.

5 different and consistent perturbative 10-dimensional susperstring models:

TYPE IIA
TYPE IIB
HETEROTIC E8xE8
HETEROTIC SO(32)
TYPE I

CLOSED STRINGS

OPEN AND CLOSED STRINGS

OPEN STRING MASSLESS SPECTRUM

NS sector A_μ vector boson (spin 1)

R sector ψ gaugino (spin ½)

CLOSED STRING MASSLESS SPECTRUM

NS-NS sector

$G_{\mu\nu}$ graviton (spin 2)

$B_{\mu\nu}$ Kalb-Ramond (spin 2)

ϕ dilaton (spin 0)

R-R sector (n+1)-form potential

n={1,3} in type IIA
n={0,2,4} in IIB

NS-R R-NS sectors
gravitinos+dilatinos

ST is a quantum theory of gravity since it includes a vibrational mode that can be identified with the graviton.

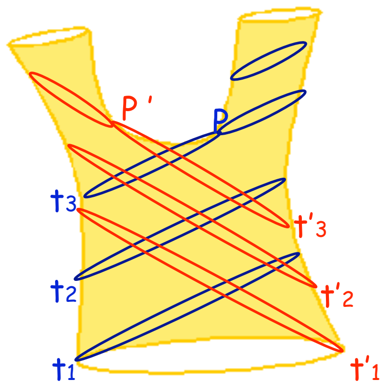
Tachyon projected out by GSO-Projection

ST naturally describes quantum gauge theories

It provides a *finite* perturbative quantum gravity.

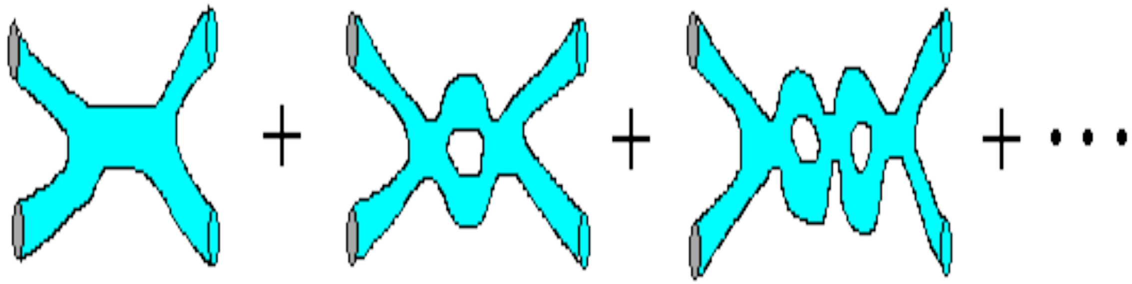
In perturbative QFT contributions to the amplitudes are associated to Feynman diagrams which give all possible configurations of the interacting particles trajectories. In particular interactions correspond to the trajectories junctions.

Interactions arise very elegantly in string theory because they are described by processes in which strings join and split. In these processes, the world-sheets of free strings combine to form a single world-sheet, which represents the interaction. For a free open string the world-sheet is an infinite strip, for a closed string it is an infinite cylinder.



No Lorentz invariant notion of interaction point

String perturbative expansion sums over all the world-sheets (Riemann surfaces) topologies with a give number of external legs and an increasing number of handles (*loops*).

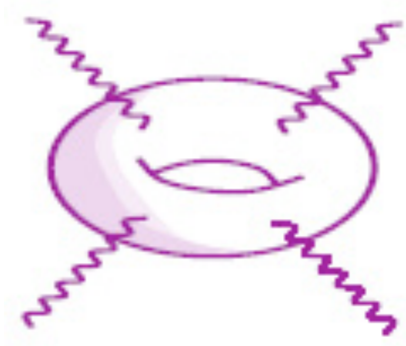


The smoothness of the world-surfaces implies

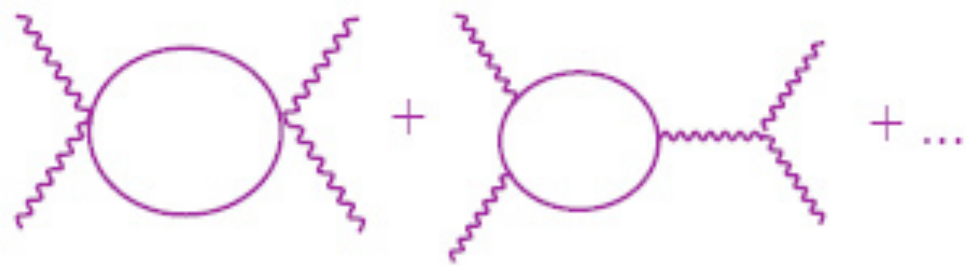
UV finiteness

as a consequence of the presence of the natural cut-off α'.

String theory provides a powerful tool to analyze gauge theories. Indeed a single string scattering amplitude:

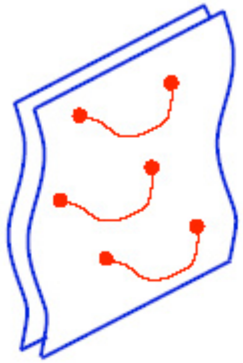


in the field theory limit $(\alpha' \to 0)$ reproduces a sum of different Feynman diagrams:



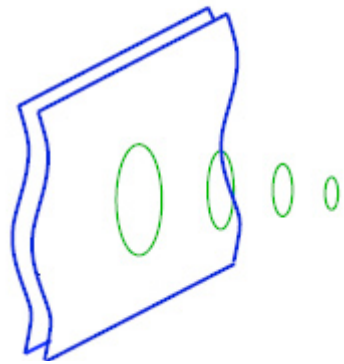
The role played by D-branes in string theory has opened new perspectives in deriving properties of gauge theories.

D*p*-BRANES



A D*p*-brane is a *p*-dim object on whose *p+1*-dim world-volume open strings can attach their endpoints (satisfying Dirichlet b.c.). This implies the existence of a gauge theory on the brane world-volume.

A D*p*-brane is a classical solution of the low-energy effective closed string state (supergravity), hence it is coupled to the closed string fields (graviton,dilaton, RR *p+1*-form, etc.)



They are dynamical and not rigid objects that can fluctuate in shape and on which external fields can live.

$$\mathcal{S} = \frac{1}{2\kappa^2}\int d^{10}x\,\sqrt{-G}\,\Big\{\mathcal{R} - \frac{1}{2}G_{\mu\nu}\,\partial_\mu\Phi\partial_\nu\Phi - \frac{1}{12}\,\mathrm{e}^{-\Phi}H_{\mu\nu\rho}H^{\mu\nu\rho}$$

$$- \sum_p \frac{1}{2(p+2)!}\,\mathrm{e}^{\frac{3-p}{2}\Phi}\,F^2_{(p+2)} + \text{ferm.} + \mathcal{O}(\alpha')\Big\}$$

$$\kappa = 8\pi^{7/2}\alpha'^2 g_s$$

This two-fold nature of a

D-BRANE

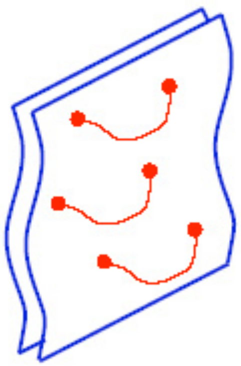
of having a gauge theory living on its world-volume

OPEN STRING DESCRIPTION

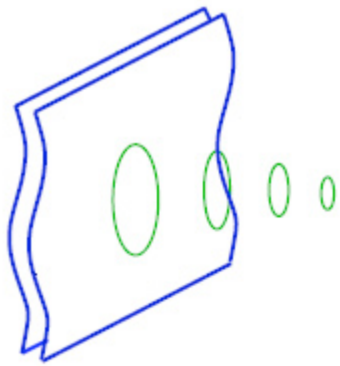
of being sources of classical SUGRA profiles

CLOSED STRING DESCRIPTION

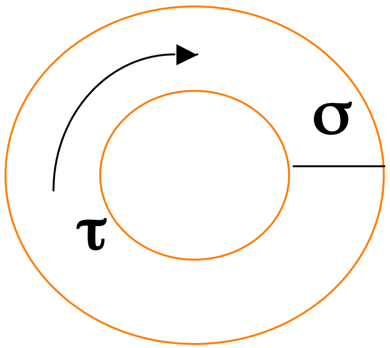
provides a powerful tool for studying the *quantum* properties of the world-volume gauge theory from the *classical* supergravity solution and viceversa



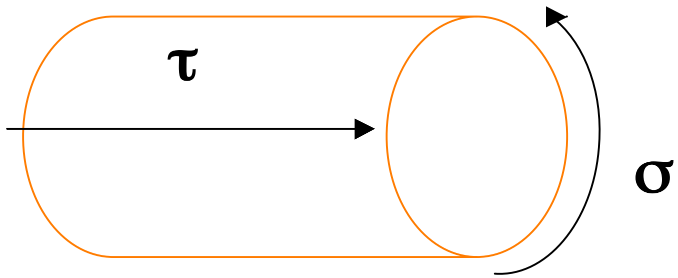
GAUGE/GRAVITY CORRESPONDENCE



OPEN/CLOSED STRING DUALITY



$(\sigma,\tau) \longrightarrow (\tau,\sigma)$

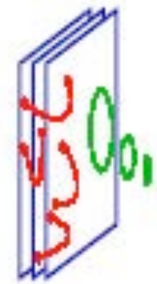


The physical content of these two diagrams is completely different:
the one-loop open string annulus diagram has a quantum content;
the tree-level closed string amplitude is a classical quantity.
They are made equivalent by the *conformal symmetry* of String Theory.

Interaction between two D*p*-branes is given by the one-loop open string annulus diagram that can also be expressed as a tree diagram involving closed string exchange.

The two-fold nature of D-branes has led to the formulation

of the Maldacena conjecture



A stack of N D3-branes (of Type IIB) in d=10 dim flat space

supports an $\mathcal{N}=4$ SYM $SU(N)$ D=4 on its world-volume

produces a non-trivial geometry being sources of closed strings

The ten-dimensional near horizon geometry is $\boxed{AdS_5 \times S^5}$

$X^{\mu}\,(\mu=0,\ldots,3)$: longitudinal directions

$X^{a}\,(a=4,\ldots,9)$: transverse directions

In the decoupling limit $\left(\alpha' \to 0\right.$, g_s fixed, $\dfrac{X^a}{\alpha'}$ fixed $\left.\right)$

$\mathcal{N}=4$ SYM $SU(N)$ $\approx$ Type IIB on $AdS_5 \times S^5$

It is also possible to study properties of less supersymmetric and non-conformal gauge theories (for instance, beta functions and anomalies).

More realistic theories obtained by:

- Reducing the amount of supersymmetry:
  $\mathcal{N}=4 \longrightarrow \mathcal{N}=2 \longrightarrow \mathcal{N}=1 \ (\mathcal{N}=0\ )$
- Breaking conformal invariance

1. To reduce SUSY the *d=10* geometry one has to consider:

$$R^{1,9} \rightarrow R^{1,3} \times X^6$$

X^6 ⇨ Orbifold backgrounds

2. To break conformal invariance one has to introduce

⇨ Fractional branes

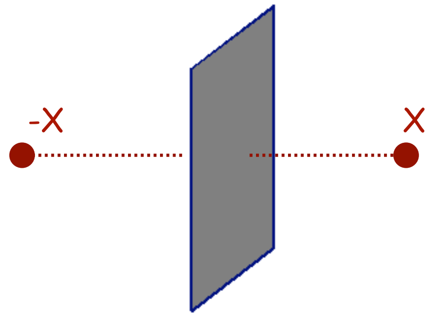
3. To add flavor one has to introduce different kinds of D-branes

⇨ D3 and D7-branes.

ORBIFOLD BACKGROUNDS

Quotient spaces $\mathcal{M}/\mathcal{G}$ $\mathcal{G}$ → discret symmetry of $\mathcal{M}$

$\mathcal{M}$ → space – time
ex. $\mathcal{G}$ → reflection
g: X→-X X=0 → fixed point



The physical spectrum has to be $\mathcal{G}$-invariant

- Some states are projected out
- Some new states appear in the twisted sector

UNTWISTED SECTOR $X^{\mu}(\tau,\sigma) = X^{\mu}(\tau,\sigma+\pi)$

TWISTED SECTOR $X^{\mu}(\tau,\sigma) = -X^{\mu}(\tau,\sigma+\pi)$

Twisted states live at the orbifold fixed point

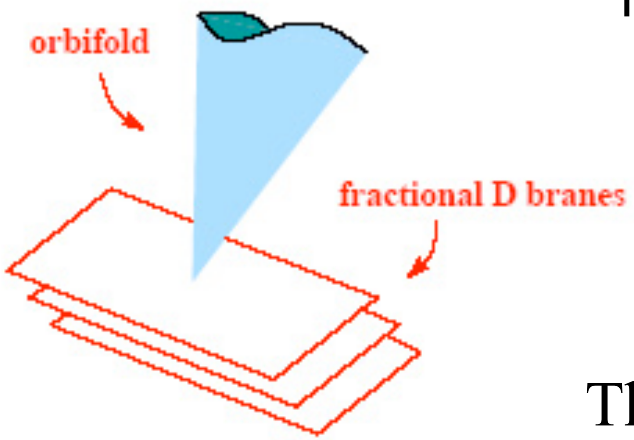
$\mathcal{N}$=2 ORBIFOLD AND FRACTIONAL BRANES

Consider Type IIB String Theory on the orbifold

$$R^{1,3} \otimes R^6 / Z_2 \qquad\qquad X^i \to +X^i \quad \text{for } i=4,5$$

$$Z_2 = (e,h) \qquad h: \quad X^a \to -X^a \quad \text{for } a=6,7,8,9$$

and fractional D3 branes stuck at the orbifold fixed point $\left(X^a = 0\right)$



The massless open string excitations on these fractional branes are:

$$A_\mu, \phi^i \qquad \mu = 0,\ldots,3; i = 4,5$$

$$\lambda_{\alpha A}, \bar{\lambda}^{\dot{\alpha} A} \qquad \alpha, \dot{\alpha} = 1,2; A = 1,2$$

The theory defined on these D3-branes is

$\mathcal{N}$=2 *SYM* in d=4

If we add *M* fractional D7 branes one gets $\mathcal{N}$=2 *SQCD* with M flavours.

$\mathcal{N}=1$ ORBIFOLD

$$z_1 = X^4 + iX^5 \; ; \; z_2 = X^6 + iX^7 \; ; \; z_3 = X^8 + iX^9$$

Consider Type IIB String Theory on the orbifold

$$R^{1,3} \otimes C^3 / Z_2 \times Z_2$$

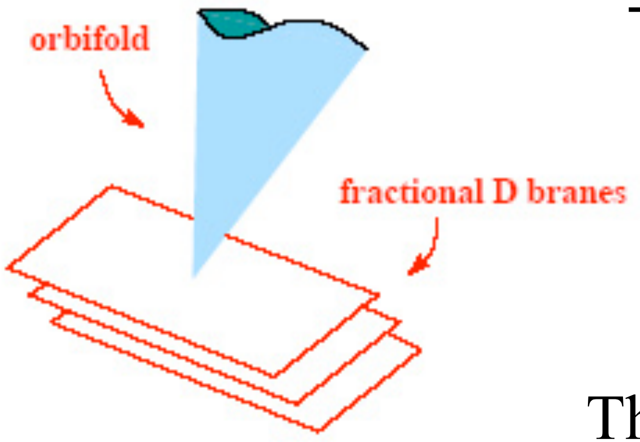
$$Z_2 \times Z_2 = (e, h_1, h_2.h_3)$$

$$h_1 : (z_1, z_2, z_3) \rightarrow (z_1, -z_2. - z_3)$$

$$h_2 : (z_1, z_2, z_3) \rightarrow (-z_1, z_2, -z_3)$$

$$h_3 : (z_1, z_2, z_3) \rightarrow (-z_1, -z_2, z_3)$$

and fractional D3 branes stuck at the orbifold fixed points



The massless open string excitations on these fractional branes are:

$$A_\mu \qquad \mu = 0, \ldots, 3$$

$$\lambda_\alpha, \bar{\lambda}^{\dot{a}} \qquad \alpha, \dot{\alpha} = 1,2$$

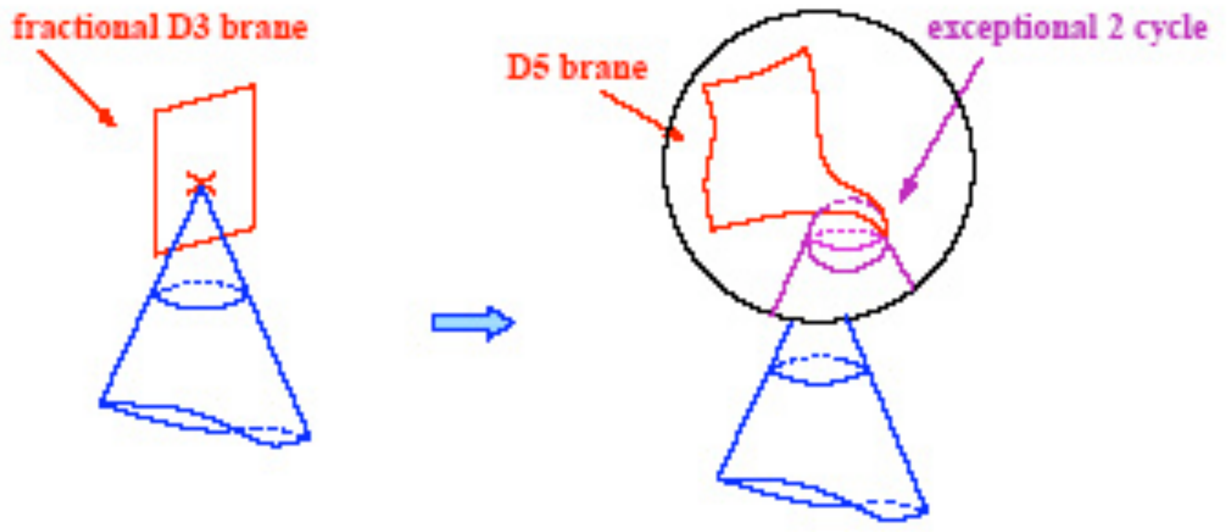
The theory defined on these D3-branes is

$\boxed{\mathcal{N}=1 \textit{ SYM} \textbf{ in d=4}}$

If we add *M* fractional D7 branes one gets $\mathcal{N}=2$ *SQCD* with M flavours.

Fractional D3 branes of $R^{1,3} \times C^2 / Z_2 [R^{1,3} \times C^3 /(Z_2 \times Z_2)]$

can be seen as D5 branes wrapped around the [three] singular exceptional 2-cycle[s] with vanishing volume at the orbifold fixed point



This explains why the fractional D3 branes are stuck at the origin of the orbifold and cannot move.

Closed String Description

Fractional branes emit the following massless bosonic closed strings:

- untwisted fields:

$G_{\mu\nu}\,,\ B_{\mu\nu}\,,\ \phi$

NS-NS sector

Propagating in the bulk

$C_0\,, C_4$

R-R sector

N=2 case

- twisted fields:

$B_2=\omega_2 b, C_2=\omega_2 c$

NS-NS twisted sector

Propagating in two dimensions

two-form dual to the vanishing two-cycle located at the orbifold fixed point.

The twisted states play an important role in deriving the so-called

HOLOGRAPHIC RELATIONS

HOLOGRAPHIC IDENTIFICATIONS

RELATIONS BETWEEN THE GAUGE THEORY PARAMATERS AND THE SUPERGRAVITY FIELDS.

$$\boxed{\frac{1}{g_{YM}^2} = f(\text{sugra fields})} \qquad \boxed{\theta_{YM} = g(\text{sugra fields})}$$

MAKING THE GAUGE/GRAVITY CORRESPONDENCE TO WORK:

- ❑ Find the SUGRA solution corresponding to the stack of branes;
- ❑ Take the world-volume action of a fractional D3 brane with F;
- ❑ Take the field theory limit to select only massless states fluctuations;
- ❑ Expand this latter up to 2n order in F obtaining identifications between the gauge theory parameter and the SUGRA fields;
- ❑ Plug the classical solution into those holographic relations;
- ❑ In so doing the correct running of the coupling constant and chiral anomaly are obtained!!

Classical geometry of fractional *N* D3 branes and *M* D7 branes

- **Metric**

$$ds^2 = H^{-1/2}\eta_{\alpha\beta}dx^\alpha dx^\beta + H^{-1/2}\left(\delta_{lm}dx^l dx^m + e^{-\phi}\delta_{ij}dx^i dx^j\right)$$

- **Dilaton + NS-NS 2-form**

$$be^{-\varphi} = \frac{\left(2\pi\sqrt{\alpha'}\right)^2}{2}\left[1 + \frac{2N - M}{\pi} g_s \log\left(y/\varepsilon\right)\right]$$

$$z \equiv x^4 + ix^5 = ye^{i\vartheta}$$

- **R-R self-dual 5-form field strength**

$$F_5 = d\left(H^{-1}dx^0 \wedge \ldots \wedge dx^3\right) + {}^*d\left(H^{-1}dx^0 \wedge \ldots \wedge dx^3\right)$$

- **Dilaton+RR 0-form**

$$\tau \equiv C_0 + ie^{-\phi} = i\left(1 - \frac{Mg_s}{2\pi}\log\frac{z}{\varepsilon}\right)$$

- **Dilaton** $e^\phi = g_s$

$$c + C_0 b = -2\pi\alpha'\,\theta g_s(2N - M)$$

The classical solution has a naked singularity (at a short distance y_r from the branes) of the repulson type but if a brane probe moves in the background of the solution, it becomes tensionless at the enhancon radius $y_e > y_r$.

For shorter distance in supergravity corresponding to larger distances in the gauge theory the classical solution does not give any information about the gauge theory.

WORLD-VOLUME ACTION (Born Infeld + Wess-Zumino)

The world-volume action for a fractional D3 brane seen as a wrapped D5 brane with a gauge field *F* turned-on on its worldvolume is:

$$S_{BI} = \tau_5 \left[-\int d^6\xi e^{-\phi} \sqrt{-\det(G_{ab} + B_{ab} + 2\pi\alpha' F_{ab})} + \right.$$

$$\left. + \int_{V_6} \sum_n C_n \wedge e^{2\pi\alpha' F + B} \right] \quad , \quad \tau_5 = [g_s \sqrt{\alpha'}(2\pi\sqrt{\alpha'})^5]^{-1}$$

$a, b \equiv (\alpha, \beta; A, B)$ **with** $\alpha, \beta = 0, 1, 2, 3$ **are flat, while** $A, B \in S^2$ **are curved.**

- Assume that supergravity fields are independent on the (α,β) directions

$+$ determinant factorizes into the product of two determinants, a four-dimensional and a two-dimensional one ⇨

$$-\tau_5 \frac{(2\pi\alpha')^2}{8} \int d^6\xi e^{-\phi} \sqrt{-\det G_{\alpha\beta}} G^{\alpha\gamma} G^{\beta\delta} F^c_{\alpha\beta} F^c_{\gamma\delta} \sqrt{\det(G_{AB} + B_{AB})}$$

In the field theory limit, keeping fixed the scalar field $\boxed{\phi = \frac{z}{2\pi\sqrt{2\alpha'}}}$

of the gauge multiplet of N=2 SYM

$$S_{YM} = -\frac{1}{g_{YM}^2}\int d^4x \left\{ \frac{1}{4} F^a_{\alpha\beta} F_a^{\alpha\beta} + \frac{1}{2}\partial_\alpha \bar{\phi} \partial^\alpha \phi \right\} + \frac{\theta_{YM}}{32\pi^2}\int d^4x F^a_{\alpha\beta}\tilde{F}_a^{\alpha\beta}$$

with

$$\boxed{\frac{4\pi}{g_{YM}^2} = \frac{1}{g_s(2\pi\sqrt{\alpha'})^2}\int d^2\xi e^{-\phi}\sqrt{\det\left(G_{AB} + B_{AB}\right)}}$$

$$\boxed{\theta_{YM} = \tau_5(2\pi\alpha')^2(2\pi)^2\int_{\mathcal{C}_2}(C_2 + C_0 B_2) = \frac{1}{2\pi\alpha' g_s}\int_{\mathcal{C}_2}(C_2 + C_0 B_2)}$$

• By inserting the explicit supergravity solution described above, one gets parameters of the gauge theory expressed in terms of supergravity fields:

$$\frac{1}{g_{\rm YM}^2} = \tau_5 \frac{(2\pi\alpha')^2}{2} \int_{C_2} e^{-\phi} B_2 = \frac{1}{8\pi g_s} + \frac{2N-M}{(4\pi)^2} \log \frac{y^2}{\epsilon^2}$$

$$\theta_{\rm YM} = \tau_5 (2\pi\alpha')^2 (2\pi)^2 \int_{C_2} (C_2 + C_0 B_2) = -(2N-M)\theta$$

• The scale and chiral transformations are realized on the supergravity coordinate $z = \frac{x_4 + i x_5}{\sqrt{2}} \equiv y e^{i\theta}$ as: $\phi \to \mu e^{2i\alpha} \Leftrightarrow z \to \mu e^{2i\alpha} z$

This implies:

$$\frac{1}{g_{\rm YM}^2} \to \frac{1}{g_{\rm YM}^2} + \frac{2N-M}{16\pi^2} \log \mu^2 \quad \text{and} \quad \theta_{\rm YM} \to \theta_{\rm YM} - 2(2N-M)\,\alpha$$

These equations reproduce the beta-function and the chiral anomaly of SU(N) $\mathcal{N}=2$ SYM

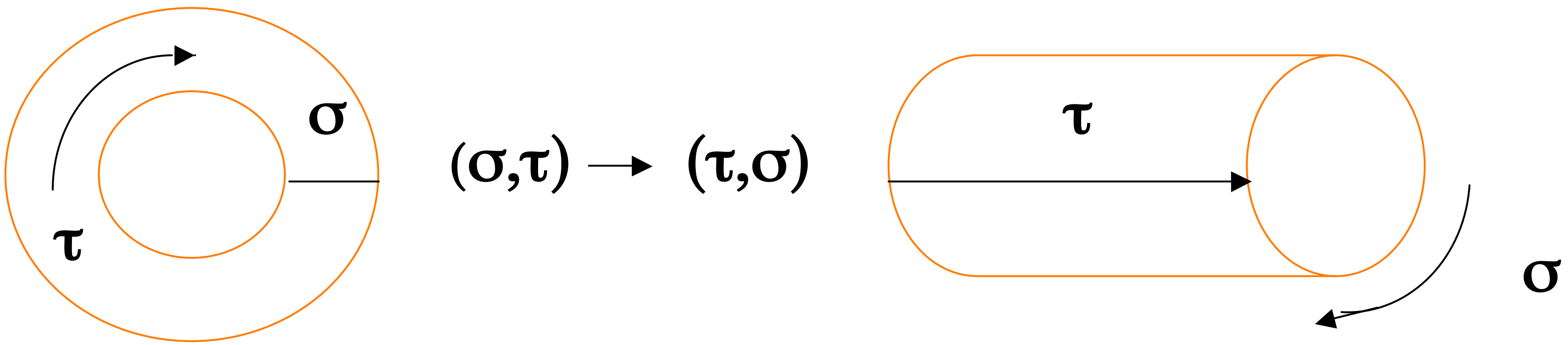
The beta-function and the chiral anomaly of *SU(N)* $\mathcal{N}=2$ *SQCD* are reproduced:

$$\beta(g_{\rm YM}) = -\frac{2N-M}{16\pi^2}\, g_{\rm YM}^3$$

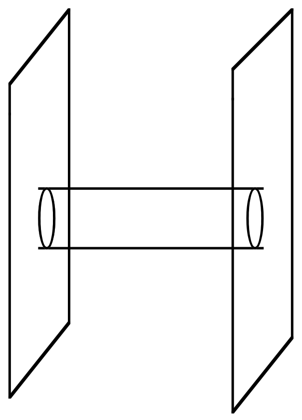
The field theory 1-loop results for the scale and chiral anomalies are obtained from the supergravity classical solution.

In this framework we have shown that gauge/gravity correspondence is a direct consequence of

OPEN/CLOSED STRING DUALITY



The interaction between two Dp-branes can be described in two ways



1-loop vacuum energy of the open string stretched between the two branes

Tree-level exchange of the closed string propagator between the two branes

$$\tau \rightarrow t = \frac{1}{\tau}$$

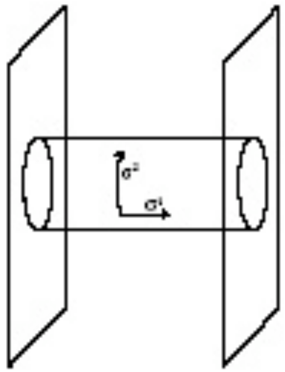
We have computed the annulus diagram describing the interaction between a set of fractional N D3 branes and M D7 branes and a fractional D3 brane having an external gage field in its world-volume and located at a distance y from the other branes.

$$Z^0 \equiv Z^0{}_e + Z^0{}_h$$

$$N\int_0^\infty \frac{d\tau}{\tau}\, \mathrm{Tr}_{\text{NS-R}}\left[\left(\frac{e+h}{2}\right) \mathrm{P}_{GSO}\; e^{-2\pi\tau L_0}\right]$$

- One can extract from the annulus diagram the contribution to the gauge coupling constant and to the θ -angle.
- The same computation can be performed in the closed string channel

$$Z^c = Z^c{}_e + Z^c_h \equiv \langle B_p \,|\, D \,| B^F_p \rangle$$



In the open channel:

$$Z_h^o(F) \rightarrow \left[-\frac{1}{4}\int d^4xF^2\right]\left\{-\frac{N}{8\pi^2}\int_{1/\alpha'\Lambda^2}^{\infty}\frac{d\tau}{\tau}e^{-\frac{y^2\tau}{2\pi\alpha'}}\right\}+iN\left[\frac{1}{32\pi^2}\int d^4xF\cdot\tilde{F}\right]\int_{1/\alpha'\Lambda^2}^{\infty}\frac{d\tau}{\tau}e^{-\frac{y^2\tau}{2\pi\alpha'}}$$

In the closed channel:

$$Z_h^c(F) \rightarrow \left[-\frac{1}{4}\int d^4xF^2\right]\left\{-\frac{N}{8\pi^2}\int_0^{\alpha'\Lambda^2}\frac{dt}{t}e^{-\frac{y^2}{2\pi\alpha' t}}\right\}-iN\left[\frac{1}{32\pi^2}\int d^4x F\cdot\tilde{F}\right]\int_0^{\infty}\frac{dt}{t}e^{-\frac{y^2}{2\pi\alpha' t}}$$

- Threshold corrections (due to the massive states) are vanishing.

- The interaction is logarithmically divergent at the string level in the UV (open-string channel) or in IR (closed string channel).

- The distance between the two systems of branes y makes the integral convergent in the IR (open-string channel) or in the UV (closed-string channel).

- The UV divergence in the open string channel, due to the massless open string states circulating in the loop, is transformed under open/closed string duality, in IR divergence in the closed string channel due to the massless closed string states exchanged among the branes.

$$\Downarrow$$

This provides the reason why the one-loop running of the coupling constant can be derived from supergravity.

From the open string channel:

$$\frac{1}{g_{YM}^2} = \frac{1}{g_{YM}^2(\Lambda)} - \frac{N}{8\pi^2}\int_{1/\Lambda^2}^{\infty}\frac{d\sigma}{\sigma}e^{-\frac{y^2\sigma}{2\pi(\alpha')^2}} \qquad (\sigma \equiv \alpha'\tau)$$

$$\theta_{YM} = -iN\int_{1/\Lambda^2}^{\infty}\frac{d\sigma}{\sigma}e^{-\frac{y^2\sigma}{2\pi(\alpha')^2}}$$

From the closed string channel: $(s = \alpha' t)$

$$\frac{1}{g_{YM}^2} = \frac{1}{g_{YM}^2(\Lambda)} - \frac{N}{8\pi^2}\int_{0}^{(\alpha'\Lambda)^2}\frac{ds}{s}e^{-\frac{y^2}{2\pi s}}$$

$$\theta_{YM} = -iN\int_{0}^{(\alpha'\Lambda)^2}\frac{ds}{s}e^{-\frac{y^2}{2\pi s}}$$

Introducing the quantity:

$$I(z) \equiv \int_{1/(e^{-i\theta}\Lambda)^2}^{\infty} \frac{d\sigma}{\sigma} e^{-\frac{y^2\sigma}{2\pi(\alpha')^2}} \simeq \log \frac{2\pi(\alpha')^2\Lambda^2}{y^2 e^{2i\theta}} \quad ; \quad z \equiv y e^{i\theta}$$

one gets:

$$\frac{1}{g_{YM}^2} = \frac{1}{g_{YM}^2(\Lambda)} - \frac{N}{8\pi^2} \cdot \frac{1}{2}\left[I(z) + I(\bar{z})\right] = \frac{1}{g_{YM}^2(\Lambda)} + \frac{N}{8\pi^2} \log \frac{y^2}{2\pi(\alpha')^2\Lambda^2}$$

and

$$\theta_{YM} = -iN \cdot \frac{1}{2}\left[I(z) - I(\bar{z})\right] = -2N\theta$$

precisely as from the supergravity solution!!!

This result provides a quantitative evidence of why the 1-loop β–function and the chiral anomaly could be derived from the classical sugra solution.

- **Is it possible to extend these results to non supersymmetric gauge theories?**

Problem analyzed in the case of consistent non-supersymmetric gauge theories that reduce to the supersymmetric ones in the large N limit

ORIENTIFOLD GAUGE THEORIES

TYPE 0B STRING THEORY

Non supersymmetric closed string model obtained by applying the following projectors on the NS-R model:

$$P_{NS-NS} = \frac{1+(-1)^{F_{NS}+\overline{F}_{NS}}}{2} \qquad P_{R-R} = \frac{1+(-1)^{F_R+\overline{F}_R}}{2}$$

$$F_{NS} = \sum_{t=1/2}^{\infty} \psi_{-t}.\psi_t - 1 \qquad (-1)^{F_R} = \psi_{11}(-1)^{\sum_{n=1}^{\infty}\psi_{-n}\psi_n}$$

Physical spectrum invariant under these projectors

$\Rightarrow$ **No space-time fermions**

$$(NS-,NS-)\oplus(NS+,NS+)\oplus(R+,R+)\oplus(R-,R-)$$

$\rightarrow$ **Tachyon + Graviton, Dilaton, Kalb-Ramond**

Doubling of the Dp-branes spectrum

World-sheet parity $\Omega : \sigma \rightarrow -\sigma$

Alternative definition: $\boxed{\Omega' = \Omega(-1)^{F_L}}$

ORIENTIFOLD THEORY: Type 0' $\equiv \dfrac{0B}{\Omega'} \Leftrightarrow$ Type I $\equiv \dfrac{IIB}{\Omega}$

$\Downarrow$

No tachyon + same spectrum of RR states as in IIB +same branes

There is a gauge theory living on N D3 branes consisting of 1 gluon + 6 adjoint scalars + 2 Dirac fermions in the two-index symmetric + 2 Dirac fermions in the two-index antisymmetric representations of SU(N).

It is a conformal theory (at least at one-loop level)

In order to get an orientifold gauge theory one has to consider:

$$\frac{0\mathrm{B}}{\Omega' I_6 (-1)^{F_L}}$$

I_6: Inversion operator of 6 space-time coordinates (transverse to the branes)

$(-1)^{F_L}$: space-time fermion number in the left sector

- Construct consistent non-supersymmetric string theories based on this orientifold projector having D3 branes that on their world-volume support non-supersymmetric *SU(N)* gauge theories.

- These gauge theories have the property of being equivalent to *SU(N)* $\mathcal{N}=1,2,4$ Super Yang-Mills for large *N* and are of the same type as the ones recently discussed by Armoni, Shifman and Veneziano.

• Consider a stack of N D3 branes in this orientifold theory. The gauge theory of living on them has an *SU(N)* gauge group and consists of:

1 gluon + 6 adjoint scalars + 4 Dirac fermions in the two-index (anti)symmetric representation of *SU(N)*.

• Number of degrees of freedom: $8N^2$ $\quad 4\times 4\times \frac{N(N\pm 1)}{2}$

$$\beta(g_{YM}) = \frac{g_{YM}^3}{(4\pi)^2}\left[-\frac{11}{3}N + 6\cdot\frac{N}{6} + 4\cdot\frac{4}{3}\frac{N\pm 2}{2}\right] = \frac{g_{YM}^3}{(4\pi)^2}\left[0\cdot N \pm \frac{16}{3}\right]$$

• It reduces to that of $\mathcal{N}=4$ super Yang-Mills for large N.

• For large *N* also Bose-Fermi degeneracy.

Consider N fractional D3 branes in the orbifold C^2/Z_2 of the previous orientifold.

The gauge theory living on them has an *SU(N)* gauge group with:

- 1 gluon + 2 adjoint scalars + 2 Dirac fermions in the two-index (anti)symmetric representation of *SU(N)* ;
- a number of degrees of freedom: $\underbrace{4N^2}_{\text{bosonic}}$ $\underbrace{2\times 4\times\frac{N(N\pm 1)}{2}}_{\text{fermionic}}$

- Its one-loop beta-function is given by

$$\beta(g_{YM}) = \frac{g_{YM}^3}{(4\pi)^2}\left[-\frac{11}{3}N + 2\cdot\frac{N}{6} + 2\cdot\frac{4}{3}\frac{N\pm 2}{2}\right] = \frac{g_{YM}^3}{(4\pi)^2}\left[2N \pm \frac{8}{3}\right]$$

reducing to that of $\mathcal{N}=2$ SYM for large N !!

- Again Bose-Fermi degeneracy for $N \rightarrow\infty$

Consider N fractional D3 branes in the orbifold $C^3/(Z_2 \times Z_2)$ of the previous orientifold.

The gauge theory living on them has an *SU(N)* gauge group with:

- 1 gluon + 1 Dirac fermion in the two-index (anti)symmetric representation of *SU(N)* ;
- a number of degrees of freedom: $\underbrace{2N^2}_{\text{bosonic}}$ $\underbrace{4 \times \frac{N(N \pm 1)}{2}}_{\text{fermionic}}$

One-loop beta-function

$$\beta(g_{YM}) = \frac{g_{YM}^3}{(4\pi)^2}\left[-\frac{11}{3}N + \frac{4}{3}\frac{N \pm 2}{2}\right] = \frac{g_{YM}^3}{(4\pi)^2}\left[3N \pm \frac{4}{3}\right]$$

reducing to that of $\mathcal{N}=2$ SYM for large N.

$\boxed{\text{QCD FOR } N=3}$

- Again degeneracy in the limit of infinite N.

GAUGE/GRAVITY CORRESPONDENCE FOR THE ORIENTIFOLD FIELD THEORIES

Interaction between a stack of *N* D3 branes and a dressed D3-brane:

$$Z^o \equiv Z^o_e + Z^o_{\Omega' I_6} =$$

$$\int_0^\infty \frac{d\tau}{\tau} Tr_{\mathrm{NS-R}} \left[\frac{e + \Omega' I_6}{2} \frac{1 + (-1)^{F_s}}{2} (-1)^{G_{bc}} \frac{(-1)^{G_{\beta\gamma}} + (-1)^F}{2} e^{-2\pi\tau L_0} \right]$$

The term corresponding to *e* is the annulus diagram, while the other one corresponds to the Moebius diagram.

The annulus diagram does not give any contribution to the term *FF* as in $\mathcal{N}$=4 SYM.

The Moebius diagram is not vanishing

Performing the field-theory limit ($\alpha' \to 0$ and $\sigma = 2\pi\alpha'\tau$ fixed) $\Longrightarrow$
UV divergences and the cut-off μ has to be introduced. One gets:

$$\frac{1}{g^2_{\rm YM}(\mu)} = \left[\frac{1}{g^2_{\rm YM}(\Lambda)}\right] \pm \frac{1}{3\pi^2}\int_{\mu^2/\Lambda^2}^{\infty} \frac{d\sigma}{\sigma} e^{-\sigma} = \left[\frac{1}{g^2_{\rm YM}(\Lambda)}\right] \mp \frac{1}{3\pi^2}\log\frac{\mu^2}{\Lambda^2},$$

leading to the right β-function.

The field theory limit in the closed string channel

$$t \to \infty, \alpha' \to 0, s = 2\pi\alpha' t$$

gives a vanishing contribution, not reproducing the correct β-function.

The gauge/gravity correspondence works only in the large N limit where the contribution of the Moebius strip is suppressed and the gauge theory recovers the Bose-Einstein degeneracy in the spectrum.

ORBIFOLD C^2 / Z_2

$$Z = \int_0^\infty \frac{d\tau}{\tau} Tr_{\text{NS-R}} \left[(\frac{1+h}{2}) \left(\frac{e+\Omega' I_6}{2} \right) \left(\frac{1+(-1)^{F_s}}{2} \right) \right.$$
$$\left. \times (-1)^{G_{bc}} \left(\frac{(-1)^{G_{\beta\gamma}} + (-1)^F}{2} \right) e^{-2\pi\tau L_0} \right] \equiv Z^o_e + Z^o_{\Omega' I_6} + Z^o_{he} + Z^o_{h\Omega' I_6}$$

By extracting the quadratic term in the gauge field (in the field theory limit performed in the open channel):

$$\frac{1}{g^2_{YM}} = \frac{1}{16\pi^2} \left[2N \mp \frac{8}{3} \right] \log \frac{\mu^2}{\Lambda^2}$$

from the twisted annulus

from the untwisted Moebius strip

Extracting the running coupling constant from the closed string channel one gets again only the leading term in N.

The twisted annulus and the twisted Moebius produce also a term proportional to the topolical charge of the gauge theory:

$$-\frac{i(N\pm 2)}{32\pi^2}\int d^4x\, F^a_{\alpha\beta}\tilde{F}^{a\,\alpha\beta}\int_0^\infty \frac{d\tau}{\tau}e^{-\frac{\rho^2\tau}{2\pi\alpha'}}$$

$$\theta_{YM} = -i(N\pm 2)\int_0^\infty \frac{d\tau}{\tau}e^{-\frac{\rho^2\tau}{2\pi\alpha'}} \rightarrow -2\theta(N\pm 2)$$

It gets contribution only from the massless open string states that in this case is transformed, under open/closed string duality, into the massless closed string states contribution, hence it can be equivalently obtained from either channels!

CONCLUSION

It has been shown that the gauge/gravity correspondence can be regarded as a consequence of the open/closed string duality for supersymmetric theories, due to the absence of threshold corrections.

It has been constructed a stringy description of the

Orientifol Field Theories

The gauge/gravity correspondence works for these theories only in the large N limit.