Status of the CKM matrix

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Matter comes in 3 generations

$SM = SU(3) \times SU(2)_L \times U(1)$ gauge theory describes electroweak and strong interactions

3 generations of matter spin $\frac{1}{2}$ fields (quarks and leptons)

mechanism of mass generation is still unknown (Higgs?)

Variety of masses and mixing (Yukawa sector) is a mystery

Quark flavor mixing and CP violation are described in the SM by the CKM mechanism
The CKM paradigm

Cabibbo 1963
Kobayashi & Maskawa 1973
The CKM matrix

describes Flavor Violation (mixing between generations of quarks) in the SM

\[
L_W = -\frac{g}{2\sqrt{2}} V_{ij} \bar{u}_i \gamma^\mu W_\mu^+(1 - \gamma^5) d_j + \text{h.c.}
\]

Wolfenstein parameterization

\[
V_{\text{CKM}} = \begin{pmatrix}
V_{ud} & V_{us} & V_{ub} \\
V_{cd} & V_{cs} & V_{cb} \\
V_{td} & V_{ts} & V_{tb}
\end{pmatrix}
= \begin{pmatrix}
0.9740 \pm 0.0010 & 0.2196 \pm 0.0023 & 0.0040^{+0.0006}_{-0.0007} \\
0.224 \pm 0.016 & 0.91 \pm 0.16 & 0.0402 \pm 0.0019 \\
< 0.010 & \simeq 0.0400 & 0.99 \pm 0.29
\end{pmatrix}
\]

3 angles and 1 phase with strong hierarchy:
\(\lambda \sim 0.22 \text{ sine of Cabibbo angle, } \lambda, \rho, \eta = O(1)\)

The CKM phase is the only source of CP violation in the SM
The CKM matrix

Describes Flavor Violation in the SM

\[
L_W = -\frac{g}{2\sqrt{2}} V_{ij} \bar{u}_i \gamma^\mu W_\mu^+ (1 - \gamma^5) d_j + h.c.
\]

Wolfenstein parameterization

\[
V_{CKM} = \begin{pmatrix}
V_{ud} & V_{us} & V_{ub} \\
V_{cd} & V_{cs} & V_{cb} \\
V_{td} & V_{ts} & V_{tb}
\end{pmatrix} = \begin{pmatrix}
1 - \frac{1}{2} \lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\
-\lambda & 1 - \frac{1}{2} \lambda^2 & A\lambda^2 \\
A\lambda^3(1 - \rho - i\eta) & -A\lambda^3 & 1
\end{pmatrix} + O(\lambda^4)
\]

3 angles and 1 phase with strong hierarchy:
\[\lambda \sim 0.22 \text{ sine of Cabibbo angle, } A, \rho, \eta = O(1)\]

At present accuracy, Wolfenstein par must be improved

2 approaches: measure individual elements and test unitarity (PDG)
OR use unitarity and test as many observables as possible
On the way to precision physics
Why precision CKM studies?

We are able to describe the observed flavor violation very well
But we have no theory of flavor.
The SM does not address flavor, but rather accommodates it
Similarly, CP violation is (accidentally) accounted for in the CKM

Most models of new physics include new CP and Flavor violation
but measurements are surprisingly close to SM prediction
scale $\Lambda_{\text{NP}} \gg \text{TeV} \ \Rightarrow \ \text{the flavor & CP problems}$

Need precision studies to uncover new dynamics
and/or degrees of freedom, testing the CKM paradigm.

Strong interactions make CKM studies hard. Learning slowly
but steadily at crossroad of many different fields.
Theory errors dominate almost everywhere.
The Cabibbo angle

\[ V_{CKM} = \begin{pmatrix} 1 - \frac{1}{2} \lambda^2 & \lambda & V_{ub} \\ -\lambda & 1 - \frac{1}{2} \lambda^2 & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \]

Historically, universality of charged currents \( \leftrightarrow \) CKM unitarity

Comparison between \( V_{ud}, V_{us} \) determinations of tests unitarity of the first line of \( V_{CKM} \)

\[ |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 \]

\( \lambda \) could also be measured from 2nd line, \( V_{cd} \) (DIS) at 10%, \( W \) decays at LEP constrains \( \Sigma_{ij} |V_{ij}|^2 \) at 1.3% \( \leftrightarrow \) \( V_{cs} \) at 1.3%
Superallowed Fermi transitions \((0^+ \rightarrow 0^+ \beta \text{ decay})\)
extremely precise, 9 expts, \(\delta V_{ud} \sim 0.0005\) dominated by RC and nuclear structure

\[
\langle p_f; 0^+ | \bar{u} \gamma_\mu d | p_i; 0^+ \rangle = \sqrt{2} (p_i + p_f)_\mu + \text{isospin violation}
\]
Superallowed magic

| Nucleus | $|V_{ud}|$  |
|---------|-----------|
| $^{10}$C | 0.97388(76) |
| $^{14}$O | 0.97445(41) |
| $^{26}$Al | 0.97416(35) |
| $^{34}$Cl | 0.97431(40) |
| $^{38}$K | 0.97424(43) |
| $^{42}$Sc | 0.97351(38) |
| $^{46}$V | 0.97372(43) |
| $^{50}$Mn | 0.97396(44) |
| $^{54}$Co | 0.97409(43) |

Towner & Hardy

$|V_{ud}| = 0.9740(1)(3)(4)$

largest uncertainty due to e.m. rad. corrections

$|V_{ud}| = 0.9739(2)(2)$

Marciano Sirlin 2005

$|V_{ud}| = 0.9740(3)$ theory

Paolo Gambino  Napoli 10/6/2005
Superallowed Fermi transitions \((0^+\rightarrow 0^+\beta\text{ decay})\) are extremely precise, with 9 experiments. \(\delta V_{ud} \approx 0.0005\) is dominated by relativistic corrections (RC) and nuclear structure.

\[
\langle p_f; 0^+ | \bar{u} \gamma_\mu d | p_i; 0^+ \rangle = \sqrt{2}(p_i + p_f)\mu
\]

The neutron \(\beta\) decay is not pure vector, requiring \(g_A/g_V\) but no nuclear structure. \(\delta V_{ud} \approx 0.0015\) will be improved at PERKEO, Heidelberg.

A new measurement of \(n\) lifetime (many \(\sigma\) away) is a serious problem!

\(\pi^+\) decay to \(\pi^0\) \(\nu\) is the cleanest, promising in the long term but with \(BR \approx 10^{-8}\). PIBETA at PSI already suggests \(\delta V_{ud} \approx 0.003\).

\[\text{PDG : } V_{ud} = 0.9738 \pm 0.0005\]
\[\text{Marciano-Sirlin: } 0.9739 \pm 0.0003 \text{ (NEW)}\]
\[\lambda = 0.2274 \pm 0.0021 \Rightarrow 0.2269 \pm 0.0013\]
Discarding old results and using Leutwyler & Roos: $|V_{us}|_{K_{l3}} = 0.2262 \pm 0.0023$
@ CKM WS (march 2005)

Exp error below 0.5%
\[
\Gamma(K_{\ell 3}) = \frac{G_{\mu}^2}{192\pi^3} M_K^5 |V_{us}|^2 C_K^2 |f_+(0)|^2 I(f_+, f_-) + \text{R.C.}...
\]

**AG theorem:** easily calculable

\[
f_+(0) = 1 + f_2 + f_4 + \ldots \quad \text{in the chiral expansion}
\]

New lattice result: Becirevic et al.
at 1% (confirmed by MILC & JLQCD, unquenched)

**Next frontier:** LQCD & measure slopes for \( K_{\mu 3} \) (Dalitz plot) to constrain \( \chi PT \)

Space for theory improvement \( \Rightarrow 0.5\% \)?
**New ideas**

### τ decay ($V_{us}$)  
Jamin et al.  
$m_s$ from sum rules or LQCD as input, may become competitive with B-factory results. At present  
$\delta V_{us} \sim 0.0034$, low values  

### Hyperon decays ($V_{us}$)  
Cabibbo et al. have revisited the subject focusing on vector form fact.  
$\delta V_{us} \sim 0.0027$ (exp) but $O(1\%)$ or more  
SU(3) breaking effects NOT included, lattice calculations under way  

### $\lambda$ using $f_\pi/f_K$ from lattice  
Marciano (2004):  

$$
\frac{\Gamma(K \rightarrow \mu \bar{\nu}_\mu(\gamma))}{\Gamma(\pi \rightarrow \mu \bar{\nu}_\mu(\gamma))} = \frac{|V_{us}|^2 f_K^2 m_K \left(1 - \frac{m_\mu^2}{m_K^2}\right)^2}{|V_{ud}|^2 f_\pi^2 m_\pi \left(1 - \frac{m_\mu^2}{m_\pi^2}\right)^2} \approx 0.9930(35) 
$$

Use LQCD for $f_\pi/f_K$. Present MILC result 1.204(4)(14)  
Staggered fermions, partially unquenched. From there we get  

$$
\lambda = 0.2234 \pm 0.0003(\text{exp}) \pm 0.0004(\text{rc}) \pm 0.0021(\text{lattice})
$$

Compatible with other determinations. MILC error debated.  
**Good potential for improvement**
Summary of $V_{us}$ from $K$ decays:

Preliminary numbers [not to be used yet]:

\[
V_{us} = 0.2262 \pm 0.0012 \cdot \frac{0.961}{f_\pi(0)} \Rightarrow 0.2262 \pm 0.0023
\]

\[
V_{us} = 0.2234 \pm 0.0005 \cdot \frac{1.204 f_\pi}{f_K} \Rightarrow 0.2234 \pm 0.0023
\]

$V_{us}^{\text{unit}} = 0.2269\pm0.0013$
Summary Cabibbo angle

- Lots going on in theory and exp
- First row unitarity problem resolved
- many competing methods
- good prospect of improvement
Determination of $A$

$V_{CKM} = \begin{pmatrix}
1 - \frac{1}{2} \lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\
-\lambda & 1 - \frac{1}{2} \lambda^2 & A\lambda^2 \\
A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1
\end{pmatrix} + O(\lambda^4)$

A can be determined using $|V_{cb}|$ or $|V_{ts}|$

**Two roads to $|V_{cb}|$**

**EXCLUSIVE** | **INCLUSIVE**
At zero recoil, where rate vanishes. Despite extrapolation, exp error ~ 2% 
Main problem is form factor F(1)

The non-pert quantities relevant for excl decays cannot be experimentally determined 
Must be calculated but HQET helps.

\[
F_{B\to D^*(1)} = \eta_A [1 - O(1/m_b, 1/m_c)^2]
\]

Lattice QCD: \( F(1) = 0.91^{+0.03}_{-0.04} \)
Sum rules give consistent results 
Needs unquenching (under way)

\( \delta V_{cb}/V_{cb} \sim 5\% \) and agrees with inclusive det, despite contradictory exps

\( B\to Dl\nu \) gives consistent but less precise results
The advantage of being inclusive

\[ \Lambda_{QCD} \ll m_b : \text{inclusive decays admit systematic expansion in } \frac{\Lambda_{QCD}}{m_b} \]
Non-pert corrections are generally small and can be controlled

Hadronization probability = 1 because we sum over all states
Approximately insensitive to details of meson structure as \( \Lambda_{QCD} \ll m_b \)
(as long as one is far from perturbative singularities)

\[ \frac{d^2\Gamma}{dE dq^2 dq_0} \text{ can be expressed as double series in } \alpha_s \text{ and } \frac{\Lambda_{QCD}}{m_b} \text{ (OPE)} \]
with parton model as leading term No 1/m_b correction!
A double expansion

\[ \frac{d^2 \Gamma}{dE dq^2 dq_0} \]

can be expressed in terms of structure functions related to \( \text{Im} \) of

\[ h_{\mu \nu}(q^2, q_0) = \frac{1}{2MB} \langle B | \int d^4x \ e^{-iqx} iT \left\{ J_\mu(x), J_\nu^\dagger(0) \right\} | B \rangle \]

**OPE (HQE):**

\[ T \ J(x) \ J(0) \approx c_1 \bar{b}b + c_2 \bar{b}D^2 b + c_3 \bar{b}\sigma \cdot Gb + \ldots \]

- The leading term is parton model, \( c_i \) are series in \( \alpha_s \)
- New operators have non-vanishing expectation values in \( B \) and are suppressed by powers of the energy released, \( E_r \sim m_b - m_c \)
- No \( 1/m_b \) correction!

OPE predictions can be compared to exp only after SMEARING and away from endpoints: **they have no LOCAL meaning**
Leptonic and hadronic spectra

Total rate gives CKM elements; global shape parameters tells us about B structure
State of the art

\[ m_b, m_c \quad \mu_G^2, \mu_\pi^2 \quad \lambda_1, \lambda_2 \]

O(1/m_b^2): mean kin. energy of b in B

\[ \mu_\pi^2(\mu) = \frac{1}{2M_B} \langle B \mid \bar{b} (i\bar{D})^2 b \mid B \rangle_\mu \]
State of the art

\[ m_b, m_c \]

\[ \mu_G^2, \mu_\pi^2, \lambda_1, \lambda_2 \]

\[ \rho_D^3, \rho_{LS}^3, \rho_1, \rho_2 \]

Gremm, Kapustin...

\[ O(1/m_b^2): \text{mean kin. energy of b in B} \]

\[ \mu_\pi^2(\mu) = \frac{1}{2M_B} \left\langle B \left| \bar{b} (iD) b \right| B \right\rangle_\mu \]
State of the art

\[ \Gamma_{e\nu} = \frac{G_F^2 m_b^5}{192\pi^3} |V_{cb}|^2 A_{ew} z_0(r) \left( 1 + a_1(r) \frac{\mu^2_G}{m_b^2} + a_2(r) \frac{\mu^2_\pi}{m_b^2} + a_3(r) \frac{\rho^3_D}{m_b^3} + a_4(r) \frac{\rho^3_{LS}}{m_b^3} \right) \]

Recent implementation for moments of lept and hadronic spectra including a cut on the lepton energy

Perturbative Corrections: full \( O(\alpha_s) \) and \( O(\beta_0 \alpha_s^2) \) available

For hadronic moments thanks to NEW calculations

\[ m_b, m_c \quad \mu^2_G, \mu^2_\pi \quad \lambda_1, \lambda_2 \quad \rho^3_D, \rho^3_{LS} \quad \rho_1, \rho_2 \]

Aquila, PG, Ridolfi, Uraltsev...
Implementing the OPE: masses & schemes

\( m_q(\text{pole}) \) is ill-defined, cannot be determined better than \(~100\text{MeV}, \) and induces large uncontrolled higher orders

\[
|V_{cb}| \sim k_0 \left[ 1 - 0.66 \ (m_b - 4.6) + 0.39 \ (m_c - 1.15) + \\
+ 0.01 \ (\mu_\pi^2 - 0.4) + 0.05 \ (\mu_G^2 - 0.35) + 0.09 \ (\rho_D^3 - 0.2) \ldots \right] 
\]

- Need short distance masses: e.g. \( m_b^{\text{kin}}(\mu) \) and \( m_b^{1S} \) and HQ parmts
- Exploit correlations (most moments depend on \(~m_b - 0.7 \ m_c\) like width)
- Avoid unnecessary parameters, avoid \(1/m_c\) expansion
- Define carefully \( \mu_\pi^2 = -\lambda_1 + \ldots \quad \mu_G^2 = 3\lambda_2 + \ldots \)

Traditionally \( m_Q \) reexpressed using

\[
M_{B,D} = m_Q + \bar{\Lambda} + \frac{\mu_\pi^2 - \mu_G^2}{2m_Q} + \frac{\rho_D^3 + \rho_{LS}^3 - \rho_{nl}^3}{4m_Q^2} + O \left( \frac{1}{m_Q^3} \right) 
\]

1/m\( c \) expansion

Non linear ops: \( T_1-4 \)
Using moments to extract HQE parameters

We do know something on HQE par. need to check consistency.

- \(M_{B^*} - M_B\) fix $$\mu_G^2 = 0.35 \pm 0.03$$
- Sum rules: $$\mu_G^2 < \mu_{\pi^2}, \rho_D^3 > -\rho^3_{LS}$$...

Central moments can be VERY sensitive to HQE parameters

$$\left\langle \left( M_X^2 - \langle M_X^2 \rangle \right)^2 \right\rangle \approx \left[ 1.3 + 0.4(m_b - 4.6) - (m_c - 1.2) + 5(\mu_\pi^2 - 0.4) - 6(\rho_D^3 - 0.1) + \ldots \right] \text{GeV}^4$$

Variance of mass distribution

BUT: OPE accuracy deteriorates for higher moments (getting sensitive to local effects)

Provided cut is not too severe (~1.3 GeV) the cut moments give additional info
Global fit to $|V_{cb}|$, BR$_{sl}$, HQE parms

Pioneer work by CLEO & Delphi employed less precise/complete data, some external constraints, and CLEO a different scheme

Not all points included
No external constraint

LEPTONIC MOMENTS

Preliminary, O. Buchmuller

BABAR  o  (NOT FIT)
BELLE  □  (NOT FIT)
CDF  ▼
CLEO  ▲  (NOT FIT)
DELPHI  ★ (NOT FIT)
HFAG  ★
Global fit to $|V_{cb}|$, $\text{BR}_{sl}$, HQE parmts

Preliminary, O. Buchmuller

Excellent agreement within exp and TH errors

Very similar results in a different approach/scheme, Bauer et al
Combined fit in kinetic scheme

Benson, Bigi, Gambino, Mannel, Uraltsev

\[ |V_{cb}| = 41.38 +/− 0.45 \times 10^{-3} \]
\[ m_b = 4.61 +/− 0.06 \text{ GeV} \]
\[ m_c = 1.17 +/− 0.08 \text{ GeV} \]
\[ \mu_{\pi}^2 = 0.40 +/− 0.04 \text{ GeV}^2 \]
\[ \mu_G = 0.29 +/− 0.05 \text{ GeV}^2 \]
\[ \rho_D = 0.16 +/− 0.06 \text{ GeV}^3 \]
\[ \rho_{LS} = -0.18 +/− 0.09 \text{ GeV}^3 \]
\[ BR(B \rightarrow X_{1\nu}) = 10.64 +/− 0.14 \% \]

- Stat., syst. and theo. (HQE, \alpha_s) errors included.
- Error from uncertainty in \Gamma_{SL} (intrinsic charm) not included!
- \[ |V_{cb}| \] error of \( \approx 1\% \)

→ Substantial improvement from combination!

Could also be done in alternative schemes

H.Flaecher, CKM 2005
Comparison with other Determinations

Measurements and Predictions of the $b$-Quark Mass (MS scheme)

**PDG2003**

\[ \bar{m}_b(\bar{m}_b) = 4.22 \pm 0.06 \text{ GeV} \]

Measurements and Predictions of the $c$-Quark Mass (MS scheme)

**PDG2003**

\[ \bar{m}_c(\bar{m}_c) = 1.33 \pm 0.10 \text{ GeV} \]

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**Conversion from kinetic mass scheme to MS scheme with hep-ph/9708372, hep-ph/0302262**

See also report from CKM WS hep-ph/0304132
Theoretical uncertainties are crucial for the fits

- Missing higher power corrections
- Intrinsic charm
- Missing perturbative effects in the Wilson coefficients: $O(\alpha_s^2)$, $O(\alpha_s/m_b^2)$ etc
- Duality violations

How can we estimate all this?
Different recipes, results for $|V_{cb}|$ unchanged
Testing parton-hadron duality

✓ What is it? For all practical purposes:
   No OPE, no duality

✓ Do we expect violations? Yes, because OPE must be continued analytically. There are effects that cannot be described by the OPE, like hadronic thresholds: decays

✓ Can we constrain them effectively? In a self-consistent way: just check the OPE predictions. E.g. leptonic vs hadronic moments. Models may also give hints of how it works

✓ Caveats? HQE depends on many parameters and we know only a few terms of the double expansion in $\alpha_s$ and $\Lambda/m_b$. 
It is not just $V_{cb}$ ...

**HQE parameters describe universal properties of the B meson and of the quarks**

- c and b masses can be determined with competitive accuracy (likely better than 70 and 50 MeV)
- $m_b - m_c$ is already measured to better than 30 MeV: a benchmark for lattice QCD etc?
- most $V_{ub}$ incl. determinations are sensitive to a shape function, whose moments are related to $\mu_{\pi^2}$ etc, moments in $B \rightarrow X_u l\nu$ to constrain WA and to validate MC (Ossola, Uraltsev,PG)
- Bounds on $\rho$, the slope of IW function ($B \rightarrow D^*$ form factor)
- Need precision measurements to probe limits of HQE & test our th. framework
Universality: spectrum of $B \rightarrow X_s \gamma$

Motion of $b$ quark inside $B$ and gluon radiation smear the spike at $m_b/2$

The photon spectrum is very insensitive to new physics, can be used to study the $B$ meson structure

$$<E_{\gamma}> = m_b/2 + ... \quad \text{var}<E_{\gamma}> = \mu^2/12 + ...$$

Importance of extending to $E_{\gamma}^{\text{min}} \sim 1.8$ GeV or less for the determination of both the BR AND the HQE parameters

Bigi Uraltsev

Belle NEW: lower cut at 1.8 GeV

Info from radiative spectrum compatible with semileptonic moments ➔ ➔
**BaBar: Fit to new b → s gamma spectrum**

- **BABAR**: Preliminary

**Kinetic Scheme**

- $\Lambda_K = 0.59^{+0.05}_{-0.04}$ GeV
- $\mu_{\pi K}^2 = 0.30^{+0.07}_{-0.05}$ GeV$^2$

$BF_K = 3.34 \pm 0.18^{+0.12}_{-0.10}$

**Shape Function Scheme**

- $\Lambda_{SF} = 0.63^{+0.04}_{-0.04}$ GeV
- $\mu_{\pi SF}^2 = 0.19^{+0.06}_{-0.05}$ GeV$^2$

$BF_{SF} = 3.42 \pm 0.19^{+0.02}_{-0.03}$

**Benson-Bigi-Uraltsev**

**Neubert**

CKM 2005, Mar. 15-18, 2005
The unitarity triangle

\[ V_{ij} V_{jk}^* = \delta_{ik} \]

Unitarity determines several triangles in complex plane

\[ V_{ud} V_{ub}^* + V_{cd} V_{cb}^* + V_{td} V_{tb}^* = 0 \]

\[ 1 + \frac{V_{ud} V_{ub}^*}{V_{cd} V_{cb}^*} + \frac{V_{td} V_{tb}^*}{V_{cd} V_{cb}^*} = 0 \]

\[ \Lambda = (\rho, \eta) \]

Area = measure of CPV

\[ V_{td} \text{ cannot be accessed directly: we resort to loop transitions} \]

FCNC sensitive to new physics

\[ |V_{ub}/V_{cb}| \text{ describes a circle in the } (\rho, \eta) \text{ plane} \]

\[ V_{td} \text{ cannot be accessed directly: we resort to loop transitions} \]

FCNC sensitive to new physics

\[ O(\lambda^3) \]
$|V_{ub}|$ (not so much) inclusive

$|V_{ub}|$ from total BR($b \to ul\nu$) almost exactly like incl $|V_{cb}|$ but we need kinematic cuts to avoid the ~100x larger $b \to cl\nu$ background:

$$m_X < M_D \quad E_1 > (M_B^2 - M_D^2)/2M_B \quad q^2 > (M_B - M_D)^2$$

or combined $(m_X, q^2)$ cuts

The cuts destroy convergence of the OPE, supposed to work only away from pert singularities

Rate becomes sensitive to “local” $b$-quark wave function properties (like Fermi motion $\Rightarrow$ at leading in $1/m_b$ SHAPE function)
Each strategy has pros and cons

<table>
<thead>
<tr>
<th>cut</th>
<th>% of rate</th>
<th>good</th>
<th>bad</th>
</tr>
</thead>
<tbody>
<tr>
<td>( E_t &gt; \frac{m^2_H - m^2_D}{2m_H} )</td>
<td>( \sim 10% )</td>
<td>don't need neutrino</td>
<td>- depends on ( f(k^+) ) (and subleading corrections)</td>
</tr>
<tr>
<td>( \phi_H &lt; m^2_D )</td>
<td>( \sim 80% )</td>
<td>lots of rate</td>
<td>- WA corrections may be substantial</td>
</tr>
<tr>
<td>( q^2 &gt; (m_B - m_D)^2 )</td>
<td>( \sim 20% )</td>
<td>insensitive to ( f(k^+) )</td>
<td>- reduced phase space - duality issues?</td>
</tr>
<tr>
<td>“Optimized cut”</td>
<td>( \sim 45% )</td>
<td>- insensitive to ( f(k^+) ) lots of rate</td>
<td>- sensitive to ( m_B ) (need +/- 30 MeV for 5% error)</td>
</tr>
</tbody>
</table>

Luke, CKM workshop 2003
**V_{ub} incl. and exclusive**

**Intense theoretical activity:**
- ✓ subleading shape functions
- ✓ optimization of cuts (P_+, P_- etc)
- ✓ weak annihilation contribs.
- ✓ Resum. pert. effects
- ✓ relation to b→sγ spectrum
- ✓ SCET insight

A lot can be learned from exp
(on WA, better constraints on s.f., subleading effects from cut dependence, b→sγ...)

**REQUIRES MANY COMPLEMENTARY MEASUREMENTS** (affected by different uncert.)

Exclusive modes: NO HQET ff normalization

LCSR and LQCD complement each other, but ~20% error. Waiting for unquenching...

**WE ARE ALREADY AT 10%**

New BRECO analyses; new results soon...
Other constraints on the UT

Looking for $V_{td}$ (and $V_{ts}$) through loop processes
To use $\varepsilon_K$ and $\Delta m_{B_{d,s}}$ to extract CKM parameters, we need 3 quantities from lattice: $B_K$, $B_{Bd}F_{Bd}^2$ and $\Delta M_q = \frac{G_F^2}{6\pi^2}\eta_B M_{B_q}(\hat{B}_{B_q}F_{B_q}^2) M_{W}^2 S_0(x_t) |V_{tq}|^2$. Typical errors for quenched results: 10-17%, less for $\xi$.
Progress in LQCD

Despite folklore, there has been progress.

B physics simulations are multiscale: present lattices can resolve neither b (too heavy) nor light q (too light).

3 main sources of systematics:
- Discretization (different complementary approach)
- Chiral extrapolation (needs lighter quarks)
- Quenching (getting there: many new unquenched results)

NEW $B_K = 0.79(4)(9)$ instead of $0.86(6)(14)$

Example of difficulties: $\xi$ parameter

Chiral extrapolation done using ChPT but at NLO large logs appear (+10-20%)

can we trust ChPT in regime of simulations?

(chiral logs are not observed in that range)

Waiting for lower $m_q$, a 10% effect maybe safe

$\xi = 1.18(4)(^{+12}_{-0})$ Lellouch  
$\xi = 1.21(5)(1)$ Becirevic

Paolo Gambino  Napoli  10/6/2005
New unquenched simulations

HPQCD[MILC] & JLQCD

\[ n_f = 2 + 1 \]
\[ n_f = 2 \]

some evidence for chiral logs (Kronfeld, ckm2005)
Two alternative routes to $|V_{td}|$

- A good measurement of $BR(K^+ \rightarrow \pi^+ \nu \nu)$, $O(10^{-10})$, will provide an excellent clean determination of $|V_{td}|$.
- $BR(K_L \rightarrow \pi^0 \nu \nu) \sim 3 \times 10^{-11}$, determines $\eta$
- Both very useful, but theory must be improved, exp is still far and prospects at NA48, CKM, JHF, KOPIO unclear

$BR(K^+ \rightarrow \pi^+ \nu \nu) = (14.7_{-8.9}^{+13.0}) \times 10^{-11}$

$BR(K_L \rightarrow \pi^0 \nu \nu) < 5.9 \times 10^{-7}$

$B \rightarrow \rho \gamma / B \rightarrow K^* \gamma$

New Belle result (first observation of $b \rightarrow d$):

$BR(B \rightarrow (\rho, \omega) \gamma) = (1.8 \pm 0.6 \pm 0.1) \times 10^{-6}$

$R(B \rightarrow \rho \gamma / B \rightarrow K^* \gamma) = (4.2 \pm 1.3)\%$

Ali et al. extract from this $0.16 < |V_{td} / V_{ts}| < 0.29$ at $1\sigma$, in agreement with fits, but less precise. Form factors from LC sum-rules. Exploratory calculations on the lattice confirm LCSR; their improvement is essential.
NEW: the UT from radiative decays

Beneke et al. (dec 2004) use QCD factorization in various exclusive radiative decays (BR, CP and isospin asymmetries) to constrain UT

Bound on $\text{BR}(B \rightarrow \rho^0\gamma)/\text{BR}(B \rightarrow K^*\gamma)$ gives $|V_{td}/V_{ts}| < 0.21$, cutting into the area selected by present standard fits...

VERY PROMISING
But beware of theor. errors!

Standard UT fit
measurement of $\gamma$

Various strategies:
the most effective is Dalitz plot analysis of $D \to 3$ body final states
Strictly tree level
Global fit results

\[ \rho = 0.210 \pm 0.035 \]

\[ \eta = 0.339 \pm 0.021 \]

http://www.utfit.org
Global fit results (II)

\[ \bar{\rho} = 0.207 \pm 0.040 \]

\[ \bar{\eta} = 0.339 \pm 0.024 \]

slightly different inputs

Fitting methods: a matter of taste

Differ in treatment of theory error. Two main groups:

**Bayesian (UTfit)**
Non gaussian errors (th & exp) are assigned a flat pdf, to be convoluted with gaussian pdfs
Pro: conceptually clean, easy for $\Delta m_s$.
Con: does not provide a $\chi^2$ test

**Rfit (CKMfitter)**
Non gaussian parameters have flat likelihood, not pdf
Pro: more conservative (beware of theorists guessing errors!)
Con: CL is at least x%

<table>
<thead>
<tr>
<th>Parameters</th>
<th>68% range</th>
<th>95% range</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B_K$ Rfit (Bayes) [ratio R/B]</td>
<td>0.68-1.06 (0.76-0.98) [1.70]</td>
<td>0.62-1.12 (0.67-1.06) [1.25]</td>
</tr>
</tbody>
</table>

Difference important especially when theory (non-gaussian) error dominates. The 99% CL ranges of global fit are quite similar with SAME INPUTS.

see CKM Yellow book

DO NOT TAKE 1σ RANGES TOO SERIOUSLY
**CP violation in the B and K sectors**

Using only the sides of the UT (CP conserving)

\[
\sin^2 \beta_{\text{UT}} = 0.725 \pm 0.043 \quad \text{(without direct meas.)}
\]

\[
\sin^2 \beta_{J/\psi K_s} = 0.726 \pm 0.037
\]
Prediction of $\Delta m_s$

$\Delta m_s = 21.2 \pm 3.2 \text{ ps}^{-1}$
($\Delta m_s$ not used)

$\Delta m_s = 18.9 \pm 1.7 \text{ ps}^{-1}$
(with all constraints)

DIRECT MEASUREMENT:

$\Delta m_s > 14.5 @ 95 \% \text{ C.L.}$

$\Delta m_s > 30 \Rightarrow \text{ new physics at } 3\sigma$

In the absence of new physics Tevatron should measure it soon
Prediction of $\gamma$ and $\alpha$

$\gamma = (58.1 \pm 5.0) ^\circ$

At 95%CL [48.6-68.6]

$\sin 2\alpha = -0.29 \pm 0.17$

$\gamma = (57.5 \pm 8.7) ^\circ$

$\sin 2\alpha = -0.29 \pm 0.46$

direct determinations

$\sin 2\alpha$
ONLY ANGLES

ALL (reliable) DATA angle measurements start being noticed
The near (unquenched) future?

\[ B_K = 0.79 \pm 0.06 \pm 0.09 \]
\[ \xi = 1.24 \pm 0.04 \pm 0.06 \]
\[ f_{\text{Bs}} \sqrt{B_{\text{Bs}}} = 276 \pm 38 \text{ MeV} \]
\[ \sin 2\beta = 0.734 \pm 0.054 \]

A direct measurement of \( \Delta m_s \) and improvement on \( V_{ub} \) will also have a significant effect.

\[ \Delta \rho = 24\% \rightarrow 15\% \quad \Delta \eta = 7\% \rightarrow 4.6\% \]
Summary

- CKM describes well a host of data. Present errors are dominantly theoretical: LQCD best hope, but theory control can be improved by exploiting new data at B-Factories, Cleo-c, Tevatron.

- First row universality problem resolved by new $K_{l3}$ data

- $|V_{cb}|$ inclusive/momnts analyses: duality verified at % level, $V_{cb}$ at 1.5%, better determination of non-pert B parameters

- Progress in LQCD is slow but sure: learning to unquench etc

- Excellent agreement so far with direct angle measurement (pending scrutiny of $B \rightarrow \Phi K_S$)

...nevertheless, still room for new physics (we have tested only a few FCNC)
A dramatic step forward...

A real advance: non-pert parameters are everywhere in B physics
Fitting non-pert parameters

<table>
<thead>
<tr>
<th></th>
<th>LATTICE QCD</th>
<th>UT FIT</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_B \sqrt{B_B}$</td>
<td>$223 \pm 33 \pm 12$ MeV</td>
<td>$257 \pm 15$ MeV</td>
</tr>
<tr>
<td>$B_K$</td>
<td>$0.86 \pm 0.06 \pm 0.14$</td>
<td>$0.68 \pm 0.10$</td>
</tr>
</tbody>
</table>
Fit Results

\begin{align*}
|V_{cb}| &= (41.4 \pm 0.4_{\text{exp}} \pm 0.4_{\text{HQQ}} \pm 0.2_{\alpha_s} \pm 0.6_{\text{SM}}) \times 10^{-3} \\
\text{Br}(B \to X_c e \nu) &= (10.61 \pm 0.16_{\text{exp}} \pm 0.06_{\text{HQQ}}) \% \\
m_b(1 \text{ GeV}) &= (4.61 \pm 0.05_{\text{exp}} \pm 0.04_{\text{HQQ}} \pm 0.02_{\alpha_s}) \text{GeV} \\
m_c(1 \text{ GeV}) &= (1.18 \pm 0.07_{\text{exp}} \pm 0.06_{\text{HQQ}} \pm 0.02_{\alpha_s}) \text{GeV}
\end{align*}

Kinetic scheme:
- Small pert corrections
- Minimal set of parmts
- No $1/m_c$ expansion

Strong correlation between $m_b$ and $m_c$:

\[ m_b(1 \text{ GeV}) - m_c(1 \text{ GeV}) = (3.44 \pm 0.03_{\text{exp}} \pm 0.02_{\text{HQQ}} \pm 0.01_{a_s}) \text{ GeV} \]

2D projections of the fit result:

\[ \Delta \chi^2 = 1 \] ellipses

No sign of deterioration for higher cuts

Moriond QCD 30. March 04

Henning Flächer (RHUL)