From hadronic B decays to the angles of the unitarity triangle

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Outline:

- Introduction
- Theoretical tools: Heavy quark expansion for exclusive processes – QCD factorization, soft-collinear effective theory
- CP violation in hadronic 2-body decays: results and puzzles

Introduction

Flavour and CP violation in the Standard Model, Status of the unitarity triangle, Theoretical challenges

Flavour and CP violation in the SM

Fundamentally related to the scalar sector

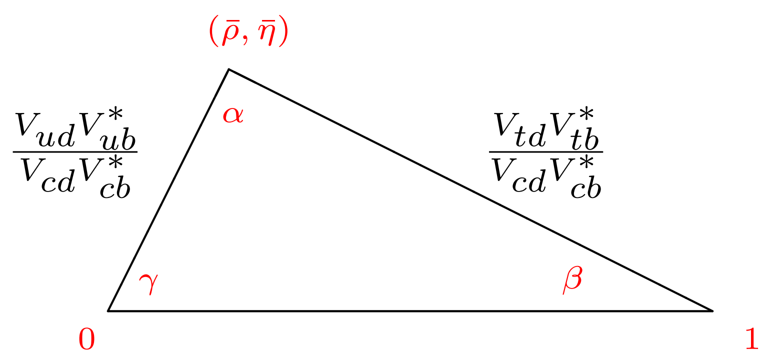
$$\mathcal{L} = -\lambda^D_{ij}\bar{Q}_i\phi D_j - \lambda^U_{ij}\bar{Q}_i\tilde{\phi}U_j + \text{h.c.}$$

At low energies (use mass basis): flavour-changing W-interactions
Observables: 6 diagonal elements (quark masses)

$V_{\rm CKM} \equiv U_{U_L}U^\dagger_{D_L}$ (unitary, three angles, one CP-violating phase)

$$V^{\rm CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

$$V_{ud}V^*_{ub} + V_{cd}V^*_{cb} + V_{td}V^*_{tb} = 0$$



Fundamental goals of flavour physics

- determine constants of Nature
  B physics: $|V_{cb}|$, $|V_{ub}|$, $\delta_{\rm CKM}$, m_b

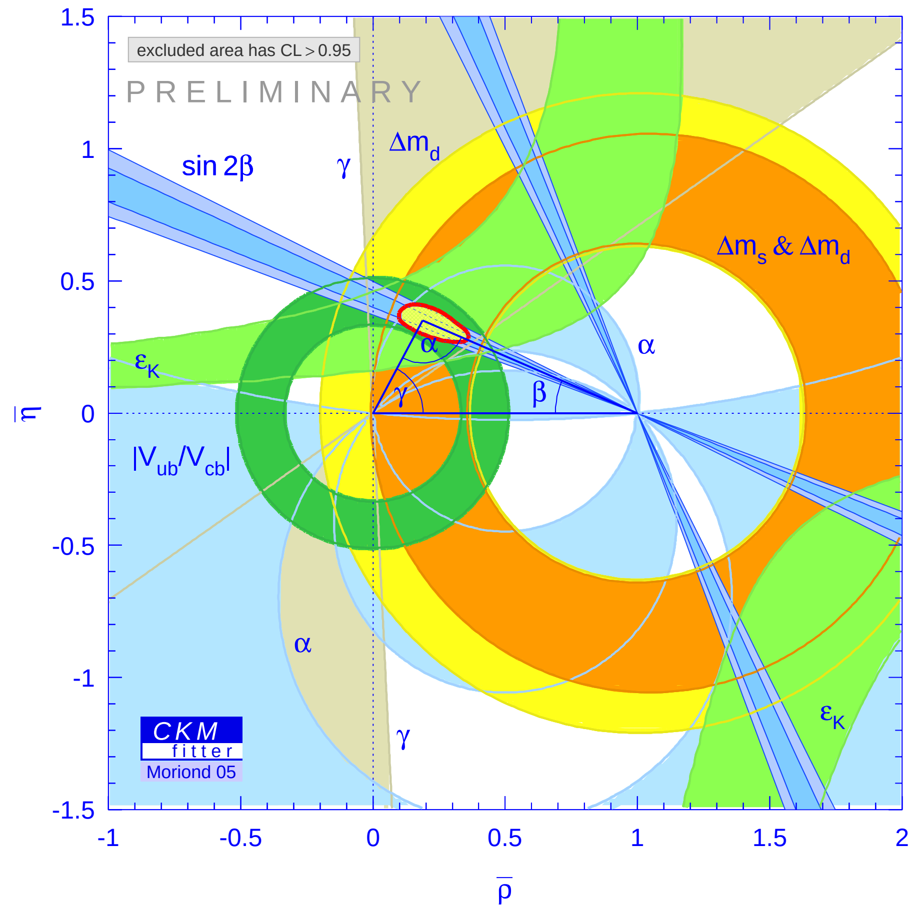
- confirm (or falsify) CKM mechanism of CP violation:
  Kaons (ϵ, ϵ'/ϵ, future: $K \to \pi\nu\bar{\nu}$)
  B mesons ($\sin(2\beta)$, many other observables)

- probe new interactions at the TeV scale
  high sensitivity, because flavour-changing interactions in the SM are suppressed by small CKM couplings

$$\lambda_{\rm CKM} G_F \ll \frac{g^2}{1\,\text{TeV}^2}$$

Status of the unitarity triangle

- Very consistent picture. KM mechanism has passed a decisive test with the measurement of $\sin(2\beta)$.
- Knowledge about CKM phase comes from $B\bar{B}$ and $K\bar{K}$ meson mixing ($\Delta F = 2$ processes) – the top sector (V_{td}, β)
- Establish further consistency through B decays ($\Delta B = 1$ processes) – the bottom sector (V_{ub}, γ) – and search for anomalous effects in specific flavour transitions.



Heavy Flavour Physics in the B factory era

[after the $\sin(2\beta)$ measurement from $B \to J/\psi K_S$]

- High statistics accesses branching fractions in the 10^{-6} range, "rare decays".
  Many exclusive decays to light hadrons:
  - $B \to \pi\pi, \pi K, \pi\rho, \ldots$ – about 40 different final states from the lightest pseudoscalar or vector meson nonet observed to date,
  - $B \to K^*\gamma$, $B \to K^{(*)}l^+l^-$, $B \to \rho\gamma$
  - Time-resolved studies of mixing and decay

- First precise explorations of $b \to d$ FCNCs as well as $b \to s$ hadronic and electroweak FCNCs.
  Measurements of γ (α)
  Opportunities to detect new flavour-changing or CP-violating interactions (probably very weak) through a larger variety of observables than provided by meson mixing.

- New challenges for theory.

Heavy Flavour Physics in the 1990's

- Spectroscopy
  $B \to D^{(*)} l\nu$ – $|V_{cb}|$
  Inclusive heavy quark decays – $|V_{cb}|$, $|V_{ub}|$, lifetimes
  $B \to X_s\gamma$ – electromagnetic $b \to s$ FCNC

- Theory is based on expansion in Λ/m_b (heavy quark expansion):

  - HQET for $B \to D$ matrix elements
  - Operator product expansion for inclusive decays

- New challenge is exclusive $B \to$ light decays, i.e. detected light particles with large energy (like jets or light hadrons in high-energy collisions)

  – HQET and the OPE cannot be applied, because they assume that the light degrees of freedom are soft, i.e. energy, momentum of order Λ
  – Methods from jet physics cannot be applied because soft physics IS important (the light degrees of freedom in the B meson)

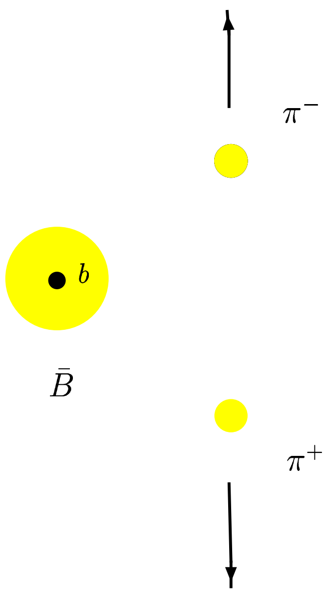
Theoretical tools

Weak effective Hamiltonian, QCD factorization, Soft-collinear effective theory (SCET)

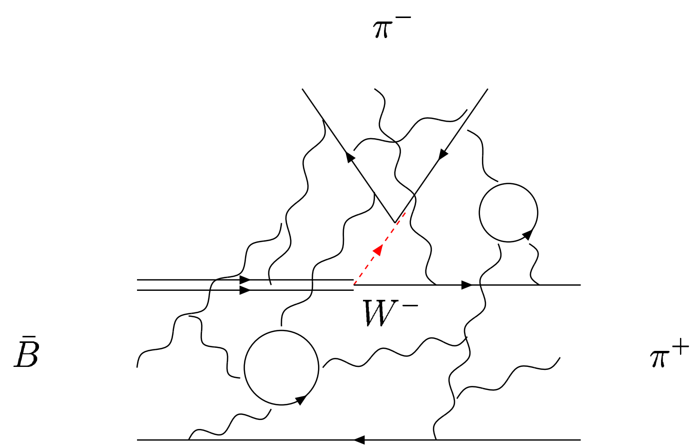
References:
– QCDF (MB, Buchalla, Neubert, Sachrajda)
– SCET (Bauer, Pirjol, Stewart, Fleming and others; MB, Feldmann, Chapovsky, Diehl; Hill, Neubert, Becher, Lange and others; Chay and Kim)

Theory problem: computation of decay amplitudes

- Focus on hadronic 2-body decays
  (such as $B \longrightarrow \pi\pi$, applies also to radiative decays such as $B \longrightarrow \rho\gamma$)
- Basic theoretical strategies:
  - "data-driven": use SU(3) to relate different decays
  - "theory-driven": compute amplitudes in heavy quark expansion (QCD factorization [BBNS])



$$\langle \pi^- \pi^+ | \bar{B} \rangle_{\mathcal{L}_{\rm SM}} \quad =$$



Multiple scales

- Construct heavy quark expansion by integrating out the various large scales – in practice we keep the leading term in Λ/m_b only, plus some corrections

  - weak scale: $p^2 \sim M_W^2$
  - heavy quark (hard) scale: $p^2 \sim m_b^2$
  - intermediate (hard-collinear) scale: $p^2 \sim m_b\Lambda$

    energy $\sim m_b$, $p_\perp \sim \sqrt{m_b\Lambda}$
  - QCD scale: $p^2 \sim \Lambda^2$

    collinear: energy $\sim m_b$, $p_\perp \sim \Lambda$

    soft: $p \sim \Lambda$

  The hard-collinear scale is a consequence of the relevance of soft AND collinear IR physics: $(p_s + p_c)^2 \sim m_b\Lambda$

- Use a sequence of effective theories:

$$\text{SM} \rightarrow \text{QCD+QED} \rightarrow \text{SCET}_{\text{I}} \rightarrow \text{SCET}_{\text{II}}$$

Step I: Integrating out the weak scale

Standard procedure: Remove W, Z, top and highly virtual light fields to obtain the "effective weak Lagrangian"

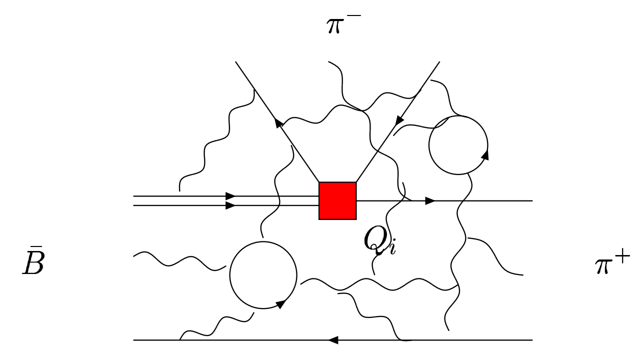
$$\mathcal{H}_{\rm eff} = \frac{G_F}{\sqrt{2}} \sum_{p=u,c} \lambda_p^{(D)} \sum_i C_i(\mu_W)\, Q_i(\mu_W) + \text{h.c.}$$

$$[\lambda_p^{(D)} = V_{pb} V_{pD}^*,\ \ D = d, s]$$

C_i's known to NNLL since 2004 (Bobeth, Misiak, Urban; Gambino, Gorbahn, Haisch)
New flavour violating interactions enter only here: modify the C_i, add more operators

What remains is QCD ($\times$ QED):

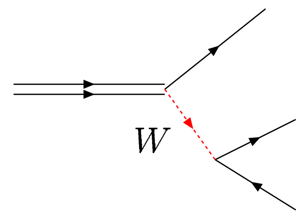
$$\langle \pi^- \pi^+ | Q_i | \bar{B} \rangle_{\rm QCD \times QED} \quad =$$



Weak effective Hamiltonian [Colour indices dropped]

Tree operators

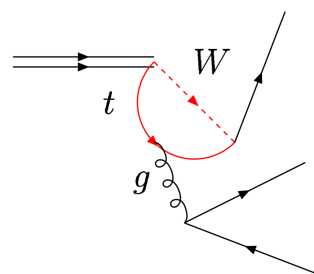
$$Q^p_{1,2} \quad = \quad (\bar{p}b)_{\mathrm{V-A}}(\bar{D}p)_{\mathrm{V-A}}$$



QCD penguin operators

$$Q_{3-6} \quad = \quad (\bar{D}b)_{\mathrm{V-A}} \sum\nolimits_q (\bar{q}q)_{\mathrm{V\mp A}}$$

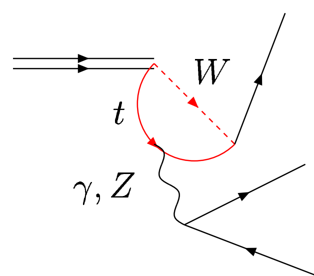
$$Q_{8g} \quad = \quad -\frac{g_s}{8\pi^2}\, m_b\, \bar{D}\sigma_{\mu\nu}(1+\gamma_5)G^{\mu\nu}b$$



EW penguin operators

$$Q_{7-10} \quad = \quad (\bar{D}b)_{\mathrm{V-A}} \sum\nolimits_q \tfrac{3}{2}e_q(\bar{q}q)_{\mathrm{V\pm A}}$$

$$Q_{7\gamma} \quad = \quad -\frac{e}{8\pi^2}\, m_b\, \bar{D}\sigma_{\mu\nu}(1+\gamma_5)F^{\mu\nu}b$$



Step II: Integrating out the heavy quark scale: QCD $\rightarrow$ SCET$_{\mathrm{I}}$

- Integrate out fluctuations (modes) with virtuality $m_b^2 \gg \Lambda^2$
  Can be done perturbatively

- Identify long-distance degrees of freedom
  hard-collinear ($p \sim (1, \sqrt{\Lambda}, \Lambda)$), collinear ($p \sim (1, \Lambda, \Lambda^2)$) and soft ($p \sim (\Lambda, \Lambda, \Lambda)$)

- Find the operators that parameterize the long-distance part of the decay process.

- Can be done with diagrammatic methods (as in jet physics) or – more elegeantly – with soft collinear effective theory (SCET)
  - Fields have defined power counting in Λ.
  - Write down the possible interactions.
  - Match to QCD.

$$[p \sim (n_+p, p_\perp, n_-p), n_\mp^2 = 0, n_-n_+ = 2]$$

SCET$_\mathrm{I}$ Lagrangian [pure Yang-Mills terms not shown]

$\mathcal{L} = \mathcal{L}^{(0)} + \mathcal{L}^{(1)} + \ldots.$ [expansion in $\sqrt{\Lambda/m_b}$]

$$\begin{aligned}\mathcal{L}^{(0)} &= \bar{\xi}_c \left(i n_- D + i \not{D}_{\perp c} \frac{1}{i n_+ D_c} i \not{D}_{\perp c} \right) \frac{\not{n}_+}{2} \, \xi_c \; + \bar{q} i \not{D}_s q + \bar{h}_v i v D_s h_v \\ \mathcal{L}^{(1)} &= \bar{\xi}_c (x_\perp^\mu n_-^\nu W_c g_s F^s_{\mu\nu} W_c^\dagger) \frac{\not{n}_+}{2} \, \xi_{hc} \; + \bar{q} \, W_c^\dagger i \not{D}_{\perp c} \xi_c - \bar{\xi}_c \, i \overleftarrow{\not{D}}_{\perp c} W_c q\end{aligned}$$

$$W_c \equiv P \exp \left(i g_s \int_{-\infty}^0 ds \, n_+ A_c(x + s n_+) \right)$$ "collinear Wilson line"

$\not{n}_- \xi_c = 0$ "collinear quark field"

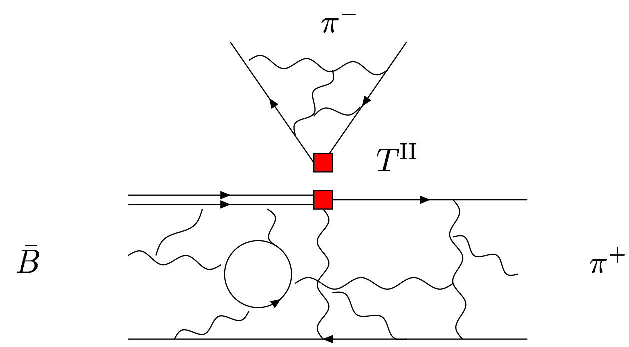
SCET is a non-local EFT, because $n_+ p_{(h)c} \sim m_b$ is large $\longrightarrow$ factorization in convolutions (rather than products) just as in DIS (+ many further technicalities such as light-front multipole expansion of fields ...)

Hard factorization [omit Dirac and colour structures]

Basic result of the second step:

$$\langle \pi^-\pi^+|(\bar{u}b)(\bar{d}u)|\bar{B}\rangle_{\rm QCD} = F^{B\pi} \times T^{\rm I} \star \Phi_\pi + \Xi^{B\pi} \star T^{\rm II} \star \Phi_\pi$$

- $T^{\rm I,II}$ involve virtualities m_b^2 (perturbative)
- Below the scale m_b the second pion has factorized (see figure).
  Strong (rescattering phases) appear only in $T^{\rm I,II}$ – perturbative!
- Φ_π is a light-cone distribution amplitude: $\langle \pi|\bar{\xi}_c(sn_+)\xi_c(0)|0\rangle_{\rm SCET}$



- $F^{B\pi}(q^2 = 0)$ is a standard heavy-to-light form factor (from QCD sum rules or lattice extrapolations). In SCET$_{\rm I}$ this is related to $\langle \pi|\bar{\xi}_c(0)\Gamma h_v(0)|\bar{B}\rangle_{\rm SCET}$
- $\Xi^{B\pi}(q^2 = 0, \tau)$ is a complicated non-local form factor. In SCET$_{\rm I}$ this is related to $\langle \pi|\bar{\xi}_c(0)\Gamma A_c(rn_+)h_v(0)|\bar{B}\rangle_{\rm SCET}$

Step III: Integrating out the hard-collinear scale: $\text{SCET}_{\text{I}} \longrightarrow \text{SCET}_{\text{II}}$

To remove the unknown non-local form factor, integrate out hard-collinear modes (virtualities $m_b\Lambda \longrightarrow$ perturbative, though at comparatively lower scale).

Basic result of the third step:

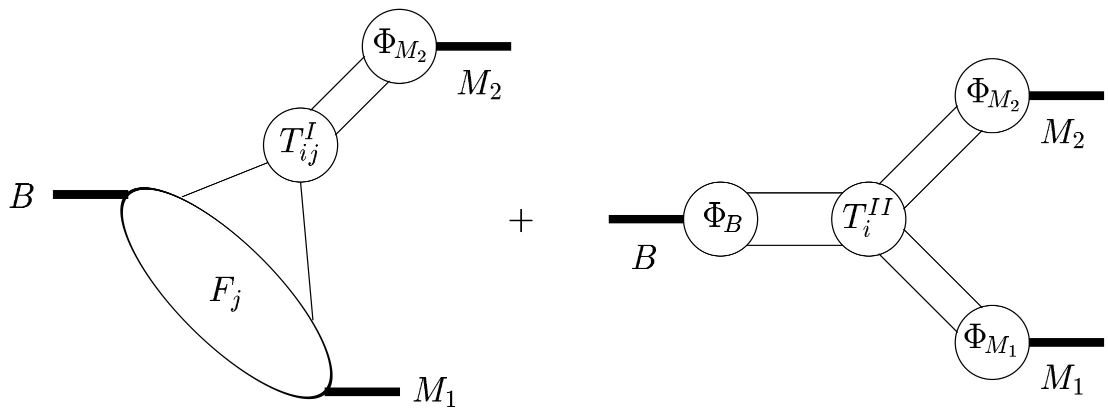
$$\Xi^{B\pi}(q^2 = 0, \tau) \sim \text{FT}\left[\langle \pi | \bar{\xi}_c(0)\Gamma A_c(rn_+)h_v(0)|\bar{B}\rangle_{\text{SCET}}\right] = \Phi_B \star J^{\text{II}} \star \Phi_\pi,$$

where J^{II} is a perturbative hard-collinear function and Φ_B the B meson light-cone distribution amplitude related to $\langle 0|\bar{q}_s(tn_-)h_v(0)|\bar{B}\rangle_{\text{SCET}}$ – essentially only one new non-perturbative parameter

Note:

Contrary to $\Xi^{B\pi}(q^2 = 0, \tau)$ the standard form factor $F^{B\pi}(q^2 = 0)$ does not factorize into $\Phi_B \star J^{\text{I}} \star \Phi_\pi$, because the matrix element is dominated by a non-factorizable soft overlap contribution [MB, Feldmann; Lange, Neubert].
Hence keep the standard QCD form factor as an input parameter.

Final QCD factorization formula



$$A(B \to M_1 M_2) \quad = \quad \text{factor} \times \sum_{\text{terms}} C(\mu_h) \times \Big\{ F^{BM_1} \times T^{\mathrm{I}}(\mu_h, \mu_s) \star f_{M_2} \Phi_{M_2}(\mu_s)$$

$$+ f_B \Phi_B(\mu_s) \star \Big[T^{\mathrm{II}}(\mu_h, \mu_I) \star J^{\mathrm{II}}(\mu_I, \mu_s) \Big] \star f_{M_1} \Phi_{M_1}(\mu_s) \star f_{M_2} \Phi_{M_2}(\mu_s) \Big\}$$

$+$ first term with $M_1 \leftrightarrow M_2$ if allowed

$+ \Lambda/m_b$ corrections

Phenomenological implementation

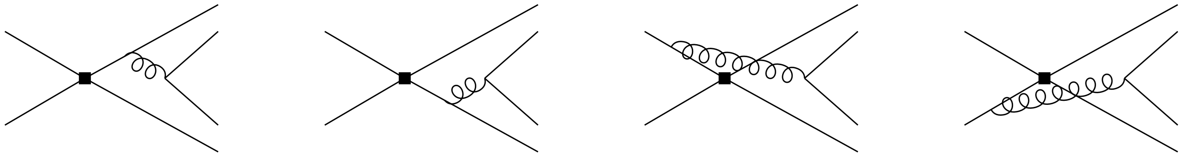
- Currently NLO, i.e.

$$T^{\mathrm{I}} \sim 1 + \alpha_s \qquad T^{\mathrm{II}} \sim 1 \qquad J^{\mathrm{II}} \sim \alpha_s$$

- This means LO for strong phases of the amplitudes. [NLO calculation of phases is in progress, important for direct CP asymmetries.]

- (Unfortunately) power corrections cannot be entirely ignored:
  - Add calculable Λ/m_b correction to the form factor term (from "scalar penguins")
  - Phenomenological model for power correction from weak annihilation used to estimate errors from power corrections.

  No systematic treatment of all power corrections is known.



Applications: Results and Puzzles

Selective:

$B \to \pi K^{(*)}$, $B \to \pi\pi, \pi\rho$, ...

Determinations of γ

References:

– (MB, Buchalla, Neubert, Sachrajda)

– (MB, Neubert, hep-ph/0308039)

Observables

Decay amplitude can always be decomposed into a CP-even and CP-odd term. In the SM,

$$\begin{aligned} \mathcal{A}_f = A(B \to f) &= A_1 + A_2 e^{i(\delta+\gamma)} \qquad (A_{1,2} > 0) \\ \bar{\mathcal{A}}_{\bar f} = A(\bar B \to \bar f) &= A_1 + A_2 e^{i(\delta-\gamma)} \end{aligned}$$

- CP-averaged branching fractions

$$\frac{1}{2}(\overline{\mathrm{Br}}(\bar f) + \mathrm{Br}(f)) = A_1^2 + A_2^2 + 2A_1A_2\cos\delta\cos\gamma$$

Sensitive to magnitudes of amplitudes. For $\gamma \sim 70^\circ$ and small strong phases δ large sensitivity to γ if A_1 and A_2 are comparable.

- (Direct) CP-asymmetries

$$A_{\mathrm{CP}}(\bar f) = \frac{\overline{\mathrm{Br}}(\bar f) - \mathrm{Br}(f)}{\overline{\mathrm{Br}}(\bar f) + \mathrm{Br}(f)} \propto 2A_1A_2\sin\delta\sin\gamma$$

Proportional to CP phases. For $\gamma \sim 70^\circ$ and small strong phases δ smaller sensitivity to γ.

- Mixing-induced CP asymmetries, if f is a common final state of B and $\bar{B}$.
  Interference of $e^{i\Phi_{B_d}}\mathcal{A}_f$ and $\bar{\mathcal{A}}_{\bar{f}}$. Theoretically clean, if one of $A_{1,2}$ is negligible (cf. $B \to J/\psi K$).

- Decay angle distributions in $B \to VV$
  Three independent helicity amplitudes with

$$A_0 \gg A_- \gg A_+ \quad \text{(large energy, left-handed weak interactions)}$$

Sensitive to handedness of new flavour-violating interactions.

Flavour amplitudes

$$
\begin{aligned}
\mathcal{A}_{B^-\to\pi^-\bar{K}^0} &= A_{\pi\bar{K}}\left[\delta_{pu}\,\beta_2 + \alpha_4^p - \tfrac{1}{2}\alpha_{4,\mathrm{EW}}^p + \beta_3^p + \beta_{3,\mathrm{EW}}^p\right], \\
\sqrt{2}\,\mathcal{A}_{B^-\to\pi^0K^-} &= A_{\pi\bar{K}}\left[\delta_{pu}\,(\alpha_1 + \beta_2) + \alpha_4^p + \alpha_{4,\mathrm{EW}}^p + \beta_3^p + \beta_{3,\mathrm{EW}}^p\right] \\
&+ A_{\bar{K}\pi}\left[\delta_{pu}\,\alpha_2 + \tfrac{3}{2}\alpha_{3,\mathrm{EW}}^p\right], \\
\mathcal{A}_{\bar{B}^0\to\pi^+K^-} &= A_{\pi\bar{K}}\left[\delta_{pu}\,\alpha_1 + \alpha_4^p + \alpha_{4,\mathrm{EW}}^p + \beta_3^p - \tfrac{1}{2}\beta_{3,\mathrm{EW}}^p\right], \\
\sqrt{2}\,\mathcal{A}_{\bar{B}^0\to\pi^0\bar{K}^0} &= A_{\pi\bar{K}}\left[-\alpha_4^p + \tfrac{1}{2}\alpha_{4,\mathrm{EW}}^p - \beta_3^p + \tfrac{1}{2}\beta_{3,\mathrm{EW}}^p\right] \\
&+ A_{\bar{K}\pi}\left[\delta_{pu}\,\alpha_2 + \tfrac{3}{2}\alpha_{3,\mathrm{EW}}^p\right].
\end{aligned}
$$

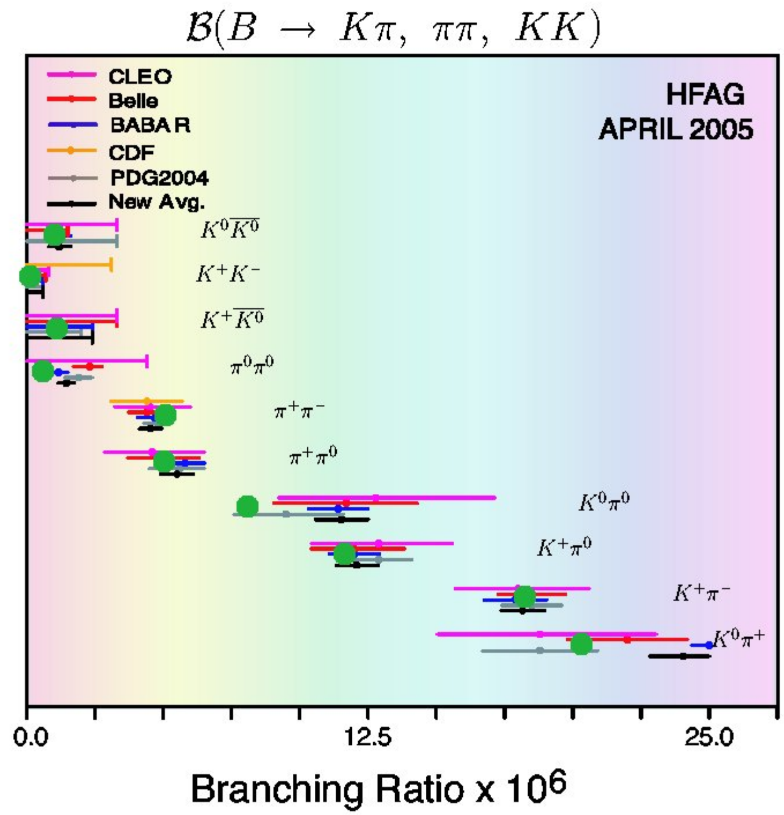
$$A_{M_1M_2} = i\frac{G_F}{\sqrt{2}}m_B^2F_0^{B\to M_1}(0)f_{M_2}V_{pb}V_{ps}^*$$

Sum over $p = u, c$. For $\Delta S = 1$ decays $p = u$ is CKM-suppressed. Generically, these decays are therefore penguin-dominated.

$\alpha_{1,2}$ tree, $\alpha_{3,4}$ QCD penguin, $\alpha_{3,4,\mathrm{EW}}$ electroweak penguin, β_i^p weak annihilation.

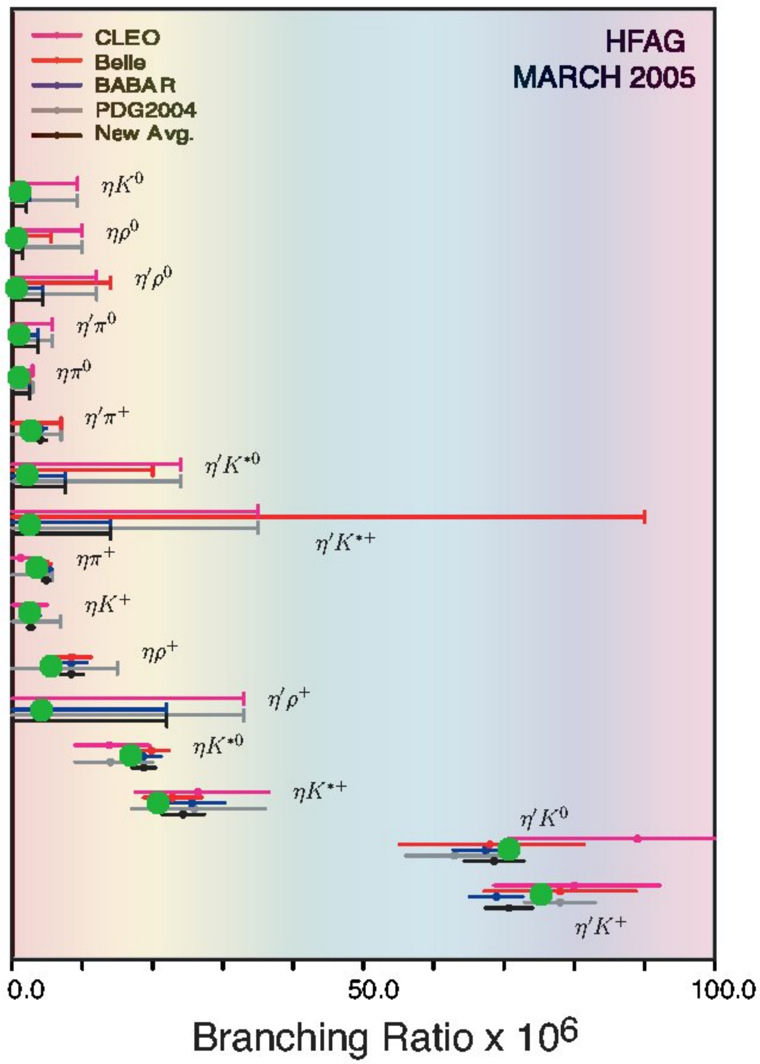
Same coefficients for SU(2) isospin related decays, but different for πK vs $\pi\pi$ or πK^*.

Overall comparison

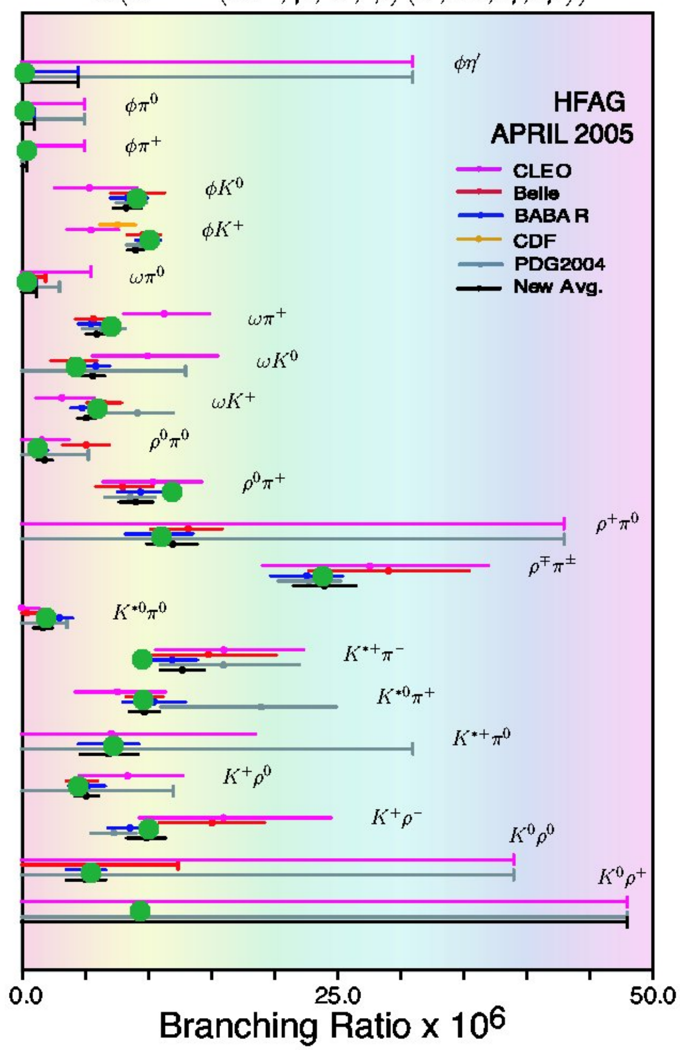


Branching fractions are correctly predicted for modes whose branching fractions vary over three orders of magnitude.

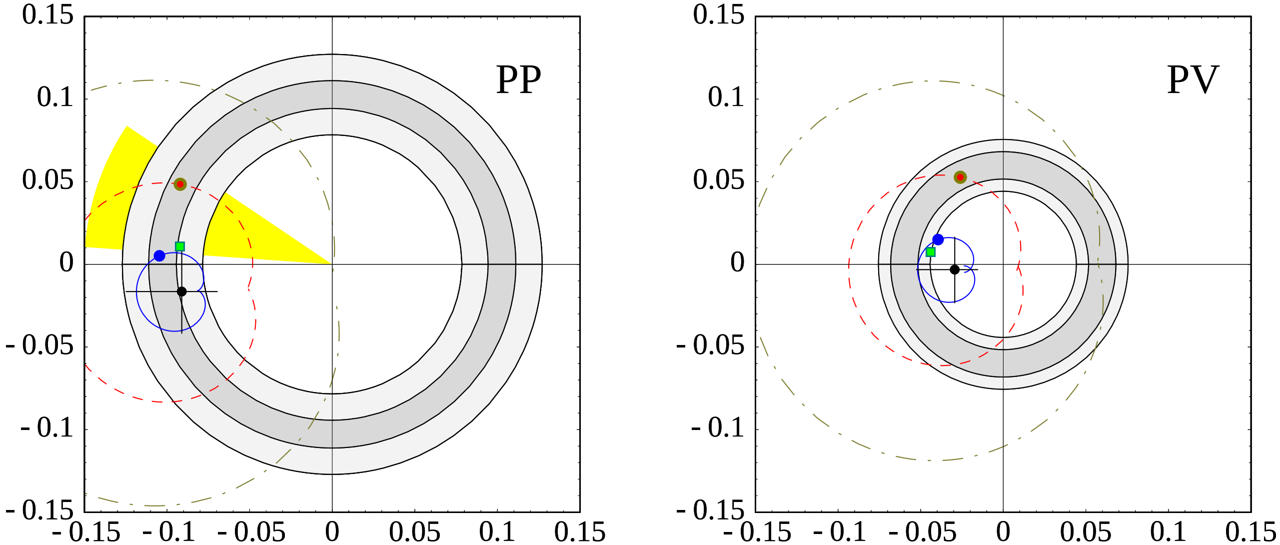
$\mathcal{B}(B \to (\eta,\ \eta')\ (K^{(*)},\ \pi,\ \rho))$



$\mathcal{B}(B \to (K^*, \rho, \omega, \phi)(\pi, K, \eta, \eta'))$



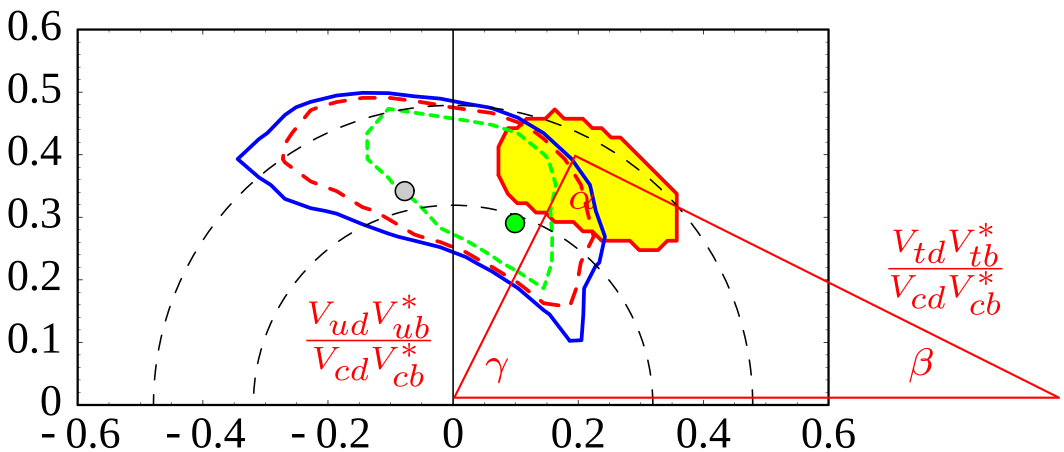
Penguin-dominated decays $B \to \pi K^{(*)}$



- Good agreement of the calculated PP QCD penguin amplitude $\longrightarrow$ account for magnitude of πK BR's contrary to LO prediction
- Predicted phase is small in agreement with data (from $A_{\rm CP}(\pi^+ \bar{K}^-)$) but preferentially the wrong sign.
- Suppression of PV penguin amplitude is predicted but magnitude falls short of data by about (20-50)% (but data maybe controversial).

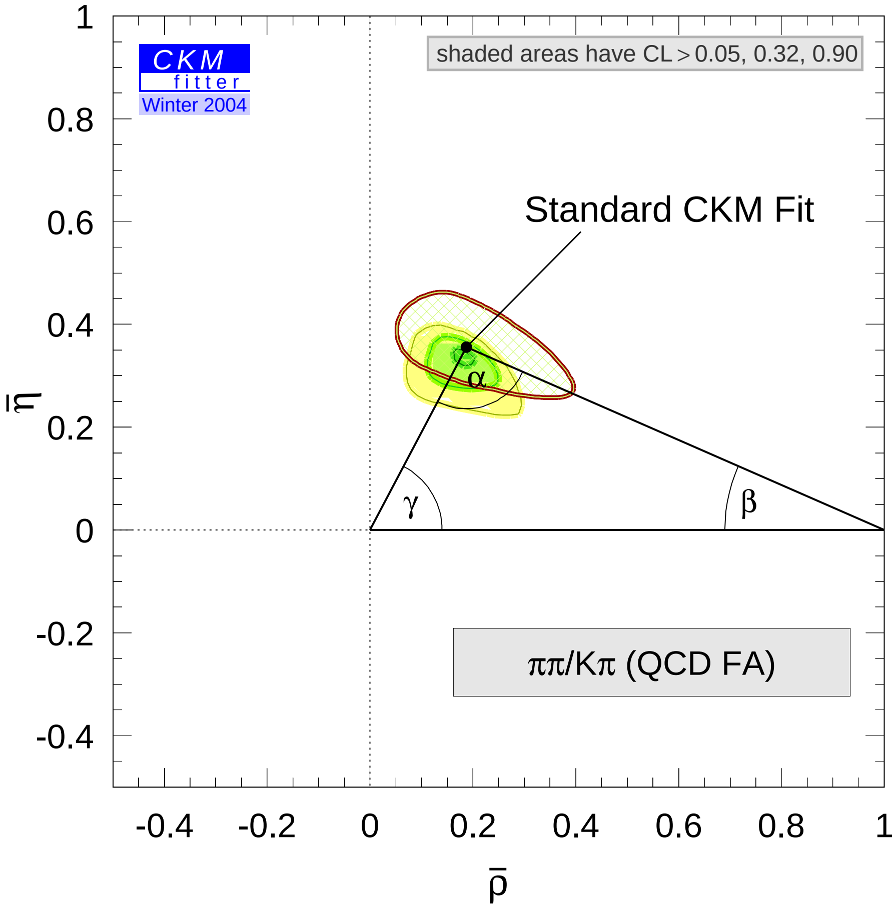
$(\bar{\rho}, \bar{\eta})$ fit from $B \to \pi K, \pi\pi$

Global fit to six CP-averaged branching fractions (using the same fit procedure as in the standard fit by Höcker et al.)



(BBNS, 2001; update from Neubert, 2003)

- Consistent with "standard" fit. Favours slightly larger γ. (See also CKMfitter next slide.)
- Establishes CP-violation in the bottom sector (phase of V_{ub}) – but maybe it is more difficult to quantify the theoretical uncertainty than in the standard fit ...



Updated QCDF fit from CKMfitter
Charles et al. [hep-ph/0406184]

$$\gamma = (62^{+6}_{-9})^\circ \qquad \text{(charmless)}$$

versus

$$\gamma = (62^{+10}_{-12})^\circ \qquad \text{(standard fit)}$$

A $B \to \pi K$ puzzle?

Construct ratios with little dependence on γ, but sensitive to electroweak penguins.

$$R_{00} = \frac{2\Gamma(\bar{B}^0 \to \pi^0 \bar{K}^0)}{\Gamma(B^- \to \pi^- \bar{K}^0)} = |1 - r_{\rm EW}|^2 + 2\cos\gamma\, \text{Re}\, r_C + \ldots$$

$$R_L = \frac{2\Gamma(\bar{B}^0 \to \pi^0 \bar{K}^0) + 2\Gamma(B^- \to \pi^0 K^-)}{\Gamma(B^- \to \pi^- \bar{K}^0) + \Gamma(\bar{B}^0 \to \pi^+ K^-)} = 1 + |r_{\rm EW}|^2 - \cos\gamma\, \text{Re}(r_T\, r_{\rm EW}^*) + \ldots$$

$$r_{\rm EW} = \frac{3}{2} R_{\pi K} \frac{\alpha^c_{3,\rm EW}(\pi \bar{K})}{\hat{\alpha}^c_4(\pi \bar{K})} \approx 0.12 - 0.01i$$

$$r_C \approx 0.03 - 0.02i, \qquad r_T \approx 0.18 - 0.02i$$

	theory	data
R_{00}	0.79 ± 0.08	1.04 ± 0.11
R_L	1.01 ± 0.02	1.12 ± 0.07

- To explain data need $r_{\rm EW} \approx 0.3 e^{\pm 90^\circ}$ – a (2-3) fold enhancement of the EW $b \to s$ penguin amplitude with a large CP-violating phase.
- Hints of new physics or a persistent problem with the $B \to \pi^0 K^0$ measurement? [> dozen papers]

Another πK problem:

$$\delta A_{\mathrm{CP}} \equiv A_{\mathrm{CP}}(\pi^0 K^\pm) - A_{\mathrm{CP}}(\pi^\mp K^\pm) = -2\sin\gamma\Big(\mathrm{Im}(r_C) - \mathrm{Im}(r_T\, r_{\mathrm{EW}})\Big) + \ldots$$

where the second term is negligible in the SM.

	theory	data
δA_{CP}	0.03 ± 0.03	0.15 ± 0.04

$B \to \pi\pi, \pi\rho$ (tree-dominated)

Mode	Theory Br (top) and ACP (bottom)	S1	S2	S3	S4	Experiment
$B^- \to \pi^-\pi^0$	$6.0^{+3.0+2.1+1.0+0.4}_{-2.4-1.8-0.5-0.4}$	5.8	5.5	6.0	5.1	5.5 ± 0.6
$\bar{B}^0 \to \pi^+\pi^-$	$8.9^{+4.0+3.6+0.6+1.2}_{-3.4-3.0-1.0-0.8}$	6.0	4.6	9.5	5.2	4.5 ± 0.4
$\bar{B}^0 \to \pi^0\pi^0$	$0.3^{+0.2+0.2+0.3+0.2}_{-0.2-0.1-0.1-0.1}$	0.7	0.9	0.4	0.7	1.5 ± 0.3
$B^- \to \pi^-\pi^0$	$-0.02^{+0.01+0.05+0.00+0.01}_{-0.01-0.05-0.00-0.01}$	-0.02	-0.02	-0.02	-0.02	-2 ± 7
$\bar{B}^0 \to \pi^+\pi^-$	$-6.5^{+2.1+3.0+0.1+13.2}_{-2.1-2.8-0.3-12.8}$	-9.6	-9.1	5.6	10.3	37 ± 28
$\bar{B}^0 \to \pi^0\pi^0$	$45.1^{+18.4+15.1+\ 4.3+46.5}_{-12.8-13.8-14.1-61.6}$	23.0	21.7	5.6	-19.0	28 ± 40

- Results for $\pi\pi$ counter to naive expectations.
  In particular Br$(\pi^+\pi^-)$/Br$(\pi^-\pi^0)$ too small, Br$(\pi^0\pi^0)$ and $A_{\rm CP}(\pi^+\pi^-)$ too large.
  For $\pi\rho$ reasonable agreement, but still large experimental errors.
- Discrepancies can be blamed on hadronic physics: Large colour-suppressed tree (α_2), smaller $B \to \pi$ form factor or large strong phase of penguin amplitude?
  Not completely understood, $\Gamma(B \to \pi l \nu)$ at $q^2 = 0$ should help.
- Focus on quantities insensitive to $F^{B\to\pi}$, α_2 and depending only on $\cos\delta_P$.

Time-dependent CP asymmetries

$$|\mathcal{A}_f(t)|^2 \equiv |\langle f|B(t)\rangle|^2 = \frac{e^{-\Gamma t}}{2}\left(|\mathcal{A}_f|^2 + |\bar{\mathcal{A}}_f|^2\right)\left\{1 + C_f\cos(\Delta m_B\, t) - S_f\sin(\Delta m_B\, t)\right\},$$

$$\rho_f = \frac{\bar{\mathcal{A}}_f}{\mathcal{A}_f}, \qquad C_f = \frac{1-|\rho_f|^2}{1+|\rho_f|^2}, \qquad S_f = -2\,\frac{\mathrm{Im}\,(e^{-2i\beta}\rho_f)}{1+|\rho_f|^2}$$

Focus on $S_{\pi\pi}$ and $S \equiv \frac{1}{2}(S_{\pi^-\rho^+} + S_{\pi^+\rho^-})$, because it is very sensitive to γ and less sensitive to hadronic uncertainties. To see this, expand in the penguin-to-tree ratio

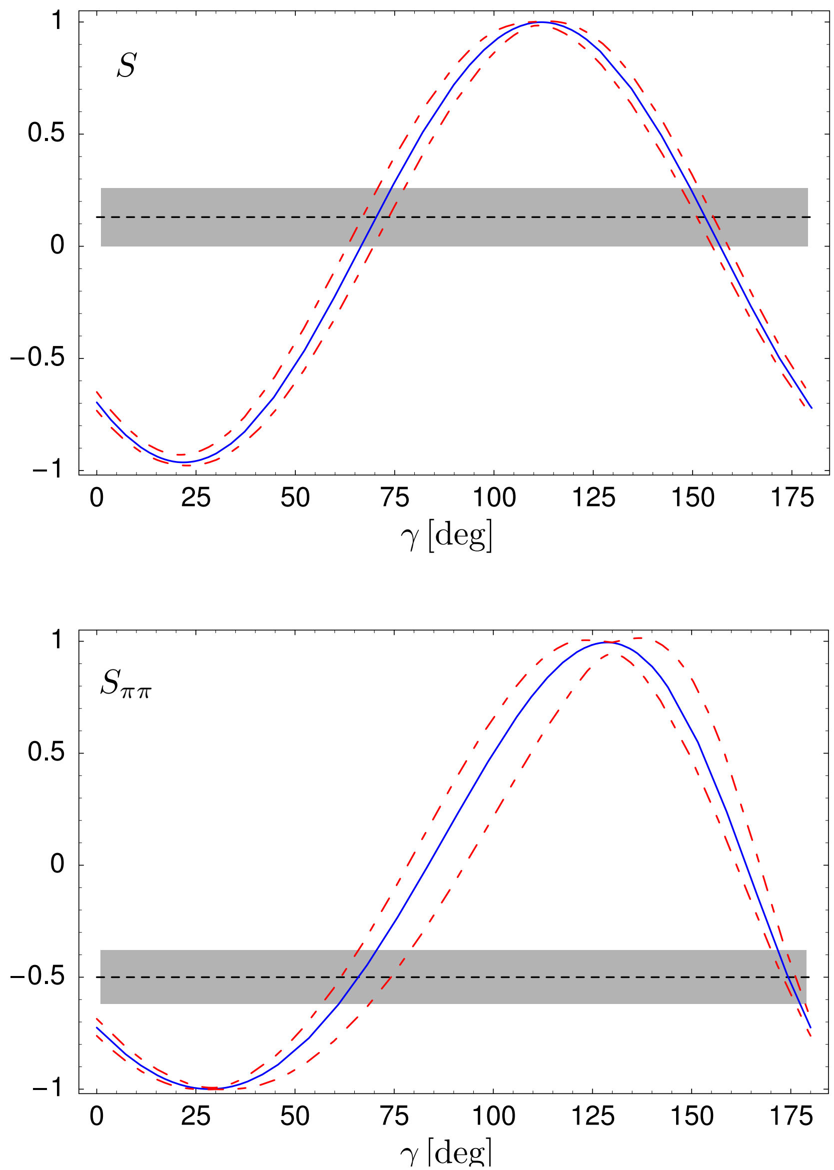
$$S = \frac{2R}{1+R^2}\sin 2\alpha - \frac{2R}{1+R^2}\left\{a\cos\delta_a\left(\frac{2\sin 2\alpha}{1+R^2}\cos\gamma + \sin(2\beta+\gamma)\right)\right.$$

$$\left. - b\cos\delta_b\left(\frac{2R^2\sin 2\alpha}{1+R^2}\cos\gamma + \sin(2\beta+\gamma)\right)\right\} + \dots \qquad (\alpha = \pi - \beta - \gamma)$$

$$A_{\rho\pi}T_{\rho\pi}/(A_{\pi\rho}T_{\pi\rho}) = R\,e^{i\delta_T} \quad R = 0.91\,^{+0.26}_{-0.21},\ \delta_T \approx 0$$

$$P_{\pi\rho}/T_{\pi\rho} = a\,e^{i\delta_a},\ P_{\rho\pi}/T_{\rho\pi} = -b\,e^{i\delta_b}, \quad a \approx b \approx 0.1,\ \cos\delta_{a,b} \approx 1$$

For $S_{\pi\pi}$ put $R = 1$, $\delta_T = 0$, $a = -b \approx 0.3$, $\delta_a = \delta_b$.



from $S = -0.13 \pm 0.13$:

$\gamma = (70\,^{+8}_{-8})^\circ$ or $\gamma = (153\,^{+6}_{-6})^\circ$

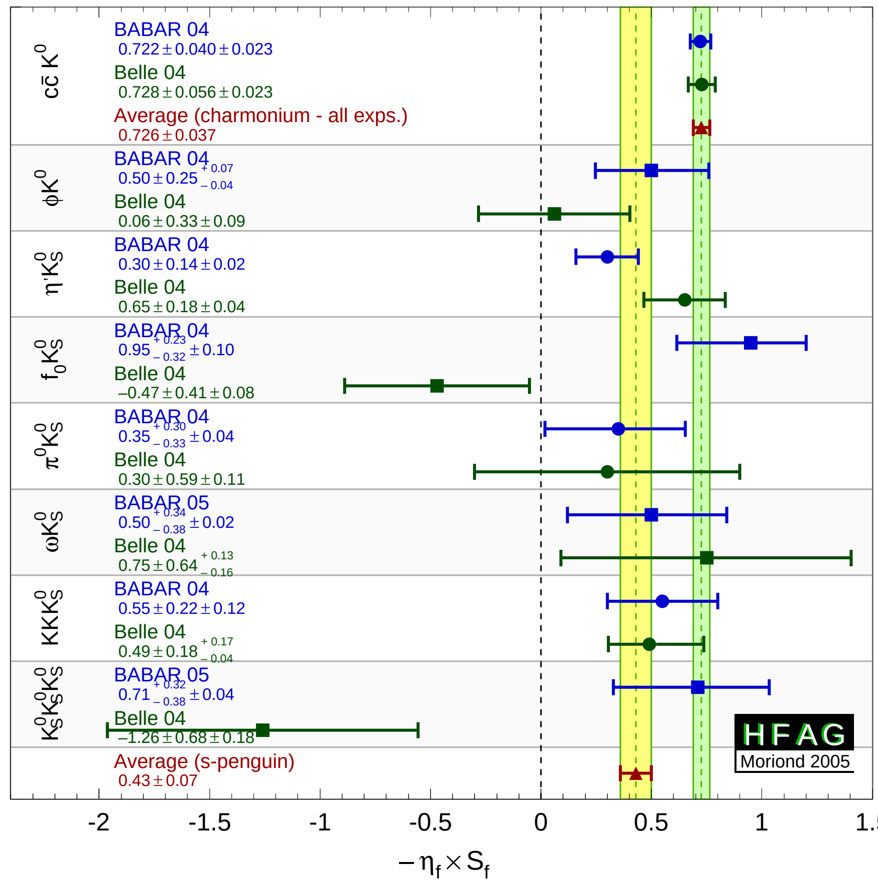
from $S_{\pi\pi} = -0.50 \pm 0.12$:

$\gamma = (66\,^{+13}_{-12})^\circ$ or $\gamma = (174\,^{+5}_{-5})^\circ$

The first ranges are mutually consistent and consistent as well with the global fit to the BR's and the standard mixing-based fit.

$\sin(2\beta)$ from $b \to s$ transitions

Mixing-induced (time-dependent) CP asymmetry S_f in $B \to J/\psi K_S$ and $B \to \Phi K_S$ should both be nearly the same, $\sin(2\beta) \approx 0.7$, as $b \to c\bar{c}s$ and $b \to s\bar{s}s$ have (nearly) the same weak phase.



Theoretical results $(\sin(2\beta) = 0.726 \pm 0.037)$

Mode	Theory $\delta\sin(2\beta)_f$	[Range]*	Experiment
πK_S	$0.07^{+0.05}_{-0.04}$	$[+0.02, 0.13]$	$-0.39^{+0.27}_{-0.29}$
ρK_S	$-0.08^{+0.08}_{-0.12}$	$[-0.24, 0.02]$	—
$\eta' K_S$	$0.01^{+0.01}_{-0.01}$	$[-0.01, 0.03]$	-0.32 ± 0.11
ηK_S	$0.10^{+0.11}_{-0.07}$	$[-1.45, 0.27]$	—
ϕK_S	$0.02^{+0.01}_{-0.01}$	$[+0.01, 0.04]$	-0.39 ± 0.20
ωK_S	$0.13^{+0.08}_{-0.08}$	$[+0.03, 0.19]$	$0.02 \pm 0.64^{+0.13}_{-0.16}$

* Range from a random scan of 10^4 input parameter sets and requiring that experimental branching fractions are reproduced within $\pm 3\sigma$.

- $\delta\sin(2\beta)_f$ is positive except for ρK_S and ηK_S.
- The large range for ηK_S disappears once a lower limit on Br(ηK) is imposed.
- Smallest deviations and uncertainties for $\eta' K_S$ and ϕK_S.

$\Rightarrow$ Many speculations on anomalous CP violation in $b \longrightarrow s\bar{s}s$.

Used-to-be puzzles

- $B \to \eta^{(\prime)} K^{(*)}$ (MB, Neubert; 2002)

  Interesting pattern Br$(\eta' K) \approx 20\,$Br(ηK) but Br$(\eta' K^*) <$ Br(ηK^*)
  QCD factorization explains this as an interference of QCD $b \to s$ penguin amplitudes which are different for PP and PV final states and can have different signs for η and η'.
  QCD factorization accounts for the large $B \to \eta' K$ branching fraction because penguin amplitudes are enhanced by short-distance radiative corrections.
  No need to invoke anomalous $b \to sg$ interactions

- Polarization in $B \to VV$ (Kagan 2004; MB, Rohrer, Yang; others)

  For $m_b \to \infty$ both vector mesons are longitudinally polarized. Expect
  $A_0 \gg A_- \, [1/m_b] \gg A_+ \, [1/m_b^2]$, hence

$$f_L = |A_0|^2 / \sum_{0,\pm} |A_i|^2 = 1 + \mathcal{O}(1/m_b^2) \qquad \text{but ...}$$

f_L	data	theory
$\rho^+\rho^-$	$0.99^{+0.05}_{-0.04}$	
$\rho^-\rho^0$	$0.97^{+0.05}_{-0.07}$	
ΦK^{*-}	0.50 ± 0.07	
ΦK^{*0}	0.48 ± 0.04	

- Agreement with expectations for tree-dominated decays ($\rho\rho$).
- For $b \longrightarrow s$ penguin dominated modes (ϕK^*), however

$$A_0 \sim A_- \qquad \text{(no } 1/m_b \text{ suppression!)}$$

On the other hand, as predicted

$$A_- \sim A_+ \qquad \text{(no evidence for anomalous right-handed interactions)}$$

$\Rightarrow$ “Polarization puzzle” (2003)

f_L	data	theory
$\rho^+\rho^-$	$0.99^{+0.05}_{-0.04}$	$0.95^{+0.02}_{-0.03}$
$\rho^-\rho^0$	$0.97^{+0.05}_{-0.07}$	$0.96^{+0.02}_{-0.03}$
ΦK^{*-}	0.50 ± 0.07	$0.81^{+0.23}_{-0.44}$
ΦK^{*0}	0.48 ± 0.04	$0.81^{+0.23}_{-0.45}$

- Agreement with expectations for tree-dominated decays ($\rho\rho$).
- For $b \longrightarrow s$ penguin dominated modes (ϕK^*), however

$$A_0 \sim A_- \qquad \text{(no } 1/m_b \text{ suppression!)}$$

On the other hand, as predicted

$$A_- \sim A_+ \qquad \text{(no evidence for anomalous right-handed interactions)}$$

- Theoretical calculation: The VV penguin amplitude may receive a large contribution from weak annihilation, which precludes a reliable prediction of f_L. No contradiction (but also no prediction).

Conclusion

- Over the past five years the theoretical description of hadronic exclusive B decays has advanced from models to theory based on the heavy quark expansion, QCD factorization and soft-collinear effective theory.

- Overall the comparison with data is successful and seems to imply

$$\gamma \sim (60 - 70)^\circ$$

  based on B decays in very good agreement with the standard unitarity triangle fit based mainly on meson mixing. After $\sin(2\beta)$ this is the second major accomplishment of the B factory experiments (and theory).

- There are persistent intriguing anomalies mostly related to hadronic $b \longrightarrow s$ FCNSs. Individually neither significant nor conclusive.
  Nevertheless the source of many theoretical speculations.