Tomographic approach to Quantum Cosmology

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Papers

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1. "Radon Transform of the Wheeler-De Witt equation and tomography of quantum states of the universe" Gen. Relativ. Gravit. (2005) 37: 99–114
2. "Cosmological dynamics in tomographic probability representation" (gr-qc/0412091) submitted to GRG (see references in this paper for extensive treatment of the tomographic approach)

Motivations for this work

- Why Quantum Gravity?
- Why Quantum Cosmology?
- The initial conditions problem vs the dynamical theories
- Why Tomographic approach to Quantum Cosmology?
- Cosmological tomograms vs cosmological wave functions
- Cosmologies as “ harmonic oscillators”
- Towards a phenomenological approach to Quantum Cosmology
- Perspectives and conclusions

General Relativity

- Gravitation described by the space-time geometry
- Fundamental objects: space-time metrics and curvature

$$ds^2 = g_{ij} dx^i dx^j$$

$$dx^i \quad \text{''infinitesimal'' distance}$$

Cosmological metric

- Homogeneous and isotropic metric

$$ds^2 = -c^2 dt^2 + \frac{a^2(t)}{1 - kr^2}[dx^2 + dy^2 + dz^2]$$

$$k = +1$$
$$k = 0$$
$$k = -1$$

- In conformal time $\quad d\eta = \frac{dt}{a(t)}$

$$ds^2 = a^2(\eta)\left[-c^2 d\eta^2 + \frac{dx^2 + dy^2 + dz^2}{1 - kr^2} \right]$$

Classical cosmology

- Friedmann equations

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$$P = (\gamma - 1)\rho$$

$$\frac{\dot{a}^2}{a^2} + \frac{k}{a^2} = \frac{8\pi G}{3}\rho$$

$$\rho = \rho_0 \frac{a^\alpha{}_0}{a^\alpha}$$

$$\ddot{a} = -\frac{4}{3}\pi G(\rho + 3P)$$

$$\alpha = \begin{cases} 3 : \gamma = 1 \\ 4 : \gamma = 4/3 \end{cases}$$

From General Relativity to Quantum Gravity

- Singularity problem in G.R. and in Relativistic Cosmology
- Quantum Gravity: search for a theory
- Quantum Cosmology, a *simplified* version of Quantum Gravity

Why Quantum Gravity?

- Approaching a singularity that at times (after the Big Bang) of the order of the Planck time

$$t_P \equiv \left(G\hbar / c^5\right)^{1/2} = 5.39 \times 10^{-44}\, s$$

and at distances of the order of

$$\ell_P \equiv (G\hbar / c^3)^{1/2} = 1.62 \times 10^{-33}\, cm$$

Why Quantum Gravity? (2)

- Energies are of the order

$$E_P \equiv (\hbar c^5 / G)^{1/2} = 1.22 \times 10^{19}\, GeV$$

- energy densities are of the order of

$$\rho_P \equiv c^5 / \hbar G^2 = 5.16 \times 10^{93}\, g / cm^3$$

It is a common opinion that at these scales of distance and energy density General Relativity in its classical form is not reliable and has to be substituted by a Quantum theory of Gravitation

Different approaches to Quantum Gravity

1. Perturbative approaches:
    - Superstring theory
    - Super gravity
2. Non perturbative canonical quantum gravity:
    - 3+1 Hamiltonian
    - Ashtekar variables
3. Alternative theories of spacetime

Quantum Mechanics

- Uncertainty principle $\Delta p \Delta q \geq \hbar / 2$
- Schrödinger Equation

$$\hat{H}\psi(\mathrm{x}, \mathrm{t}) = i\hbar \frac{\partial \psi(\mathrm{x}, \mathrm{t})}{\partial t}$$

- Observables and measurements
- Physical interpretation

Wheeler-de Witt equation in Quantum Gravity

- canonical approach
- equation in the space of three dimensional metrics

$x \longrightarrow h_{ij}$ metrica spaziale

$$\left(-G_{ijkl}\frac{\delta^2}{\delta h_{ij}\delta h_{kl}} - {}^3R(h)h^{1/2} + 2\Lambda h^{1/2} \right)\psi\left(h_{ij}\right) = 0$$

G_{ijkl} metric the space of three geometries (superspace)

Why Quantum Cosmology?

1. Minisuperspace: considering only homogeneous metrics
2. cosmological models as a point particles
3. working with a finite number of degrees of freedom
4. violates the uncertainty principle fixing contemporarily a zero infinite variables and their momenta
5. Not Copenhagen interpretation of this quantum theory

Wheeler-de Witt equation in Quantum Cosmology

Here is an example of a Wheeler-deWitt equation in the space of homogeneous and isotropic metrics

$$\frac{1}{2}\left\{\frac{1}{a^p}\frac{d}{da}a^p\frac{d}{da} - a^2 + \Lambda a^4\right\}\psi(\mathrm{a}) = 0$$

a model with cosmological constant and no matter fields is considered, the exponent ''p'' reflects the ambiguity of the theory in fixing the order of operators.

For large values of the expansion factor a the solution is

$$\psi(\mathrm{a}) \approx \cos\frac{\mathrm{Ha}^3}{3}$$

Boundary Conditions

- Wick rotation Euclidean 4 space
- It is important to note that the cosmological evolution is determined by the (initial) boundary conditions

Two proposals:

- Hartle and Hawking, no boundary conditions
- Vilenkin, the universe ‘’tunnels into existence from nothing’’

Need or a fundamental law of the initial condition (Hartle) or to derive it from the phenomenology

Motivation for a Tomographic approach

- quantum mechanics without wave function and density matrix
- new formulation of Q. M. based on the ”probability representation” of quantum states
- we deal with the evolution of a measurable quantity
- its evolution is classical or quantum depending on the initial conditions, that can be classical or quantum

Relation between tomograms and wave function

Tomograms contain the same information of wave functions

$$X = \mu q + \nu p$$

is a ''coordinate'' transformation on phase space

$$W(X,\mu,\nu) = \frac{1}{2\pi|\nu|}\left|\int \psi(y)\exp\left(\frac{i\mu}{2\nu}y^2 - \frac{iX}{\nu}y\right)dy\right|^2$$

$$\mu = \cos\varphi \qquad \nu = \sin\varphi$$

Tomographic equation for a harmonic oscillator

$$\frac{\partial W}{\partial t} - \mu \frac{\partial W}{\partial \nu} + \omega^2 \nu \frac{\partial W}{\partial \mu} = 0$$

The initial condition problem

- Quantum cosmology can be considered as the theory of the initial conditions of the universe
- Differently from the wave function approach we can deal classical and quantum cosmology with the *same* variable.
- The difference is just in the initial conditions
- Do we have to postulate these conditions?

Cosmological models as harmonic oscillators

- Let us go back to the homogeneous and isotropic model in conformal time
- Make the change of variables

$$z = a^{\chi} \qquad \chi = \frac{3}{2}\gamma - 1$$

- The evolution cosmological equation takes the form

$$z'' + k\,\chi^2 z = 0$$

$$k = +1$$
$$k = 0$$
$$k = -1$$

Propagators in the tomographic approach

- The evolution of a tomogram can be described by the transition probability

$$\Pi(X, \mu, \nu, t, X', \mu', \nu', t_0)$$

with the equation

$$W(X,\mu,\nu,t) = \int \Pi(X,\mu,\nu,t,X',\mu',\nu',t_0)\, W(X',\mu',\nu',t_0)\, dX'\, d\mu'\, d\nu'$$

Evolution of a tomogram of the universe

- The transition probility Π satisfies the following equation

$$\frac{\partial \Pi}{\partial t} - \mu \frac{\partial \Pi}{\partial \nu} + \omega^2 \nu \frac{\partial \Pi}{\partial \mu} = \delta(\mu - \mu')\delta(\nu - \nu')\delta(X - X')\delta(t - t_0)$$

Solutions for the transition probabilities

In the minisuperspace considered in this talk, the transition probabilities are

$$\Pi^{osc}(X, \mu, \nu, t, X', \mu', \nu', t_0) = \delta(X - X')\,\delta(\mu' - \mu\cos\omega(t - t_0) + \nu\sin\omega(t - t_0))$$
$$\times\,\delta\left(\nu' - \nu\cos\omega(t - t_0) - \frac{\mu}{\omega}\sin\omega(t - t_0)\right)$$

$$\Pi^{free}(X, \mu, \nu, t, X', \mu', \nu', t_0) = \delta(X - X')\,\delta(\mu' - \mu)\,\delta(\nu' - \nu - \mu(t - t_0))$$

$$\Pi^{rep}(X, \mu, \nu, t, X', \mu', \nu', t_0) = \delta(X - X')\,\delta(\mu' - \mu\cosh\omega(t - t_0) - \nu\sinh\omega(t - t_0))$$
$$\times\,\delta\left(\nu' - \nu\cosh\omega(t - t_0) - \frac{\mu}{\omega}\sinh\omega(t - t_0)\right)$$

Phenomenological Quantum Cosmology

- Our approach appears to be promising, because tomograms are in principle measurable
- In the particular case discussed before the classical and quantum equations are the same
- In future work we shall need to define the measurement of a cosmological tomogram
- This will enable us to study the initial conditions problem from a phenomenological point of view
- Moreover we hope to be able to distinguish the quantum evolution from the classical one.

What we can know from observations?

We can expect that observations put some constraints on the present tomogram (and consequently to the initial conditions)

- Entropy
- Cosmic background radiation fluctuations
- Approximate homogeneity and isotropy
- Formation of structures

Conclusions and perspectives

Conclusions

- We have proposed a novel way to deal with Quantum Cosmology
- We saw that there are models that have a simple description
- This result seems promising to develop a phenomenological study of the initial conditions problem

Perspectives

- Determine how to measure a cosmological tomogram
- Analyze quantum decoherence from our point of view

Moreover

- Formulate a classical theory of fluctuations in G.R.
- Apply our approach to Quantum Gravity