Universita' "Federico II" di Napoli

Italy

Kondo effect in Quantum Dot

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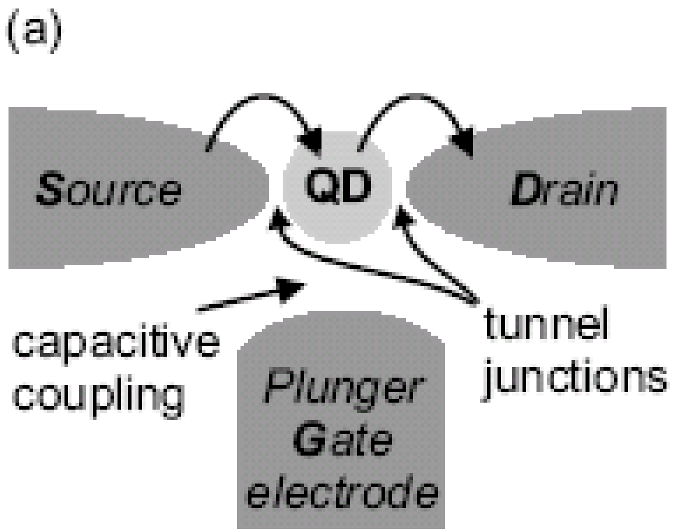
Napoli 2/05

Quantum Dots:

Artificial atoms



e-e correlations:

Charge quantization

Spin properties

it is nice to scale atomic energies down to meV !

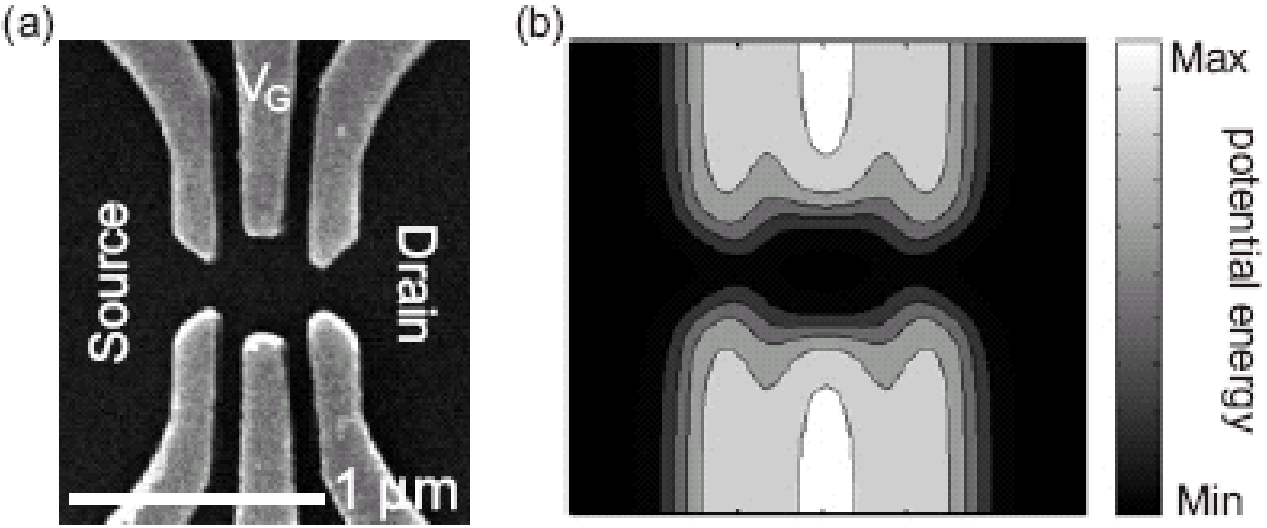
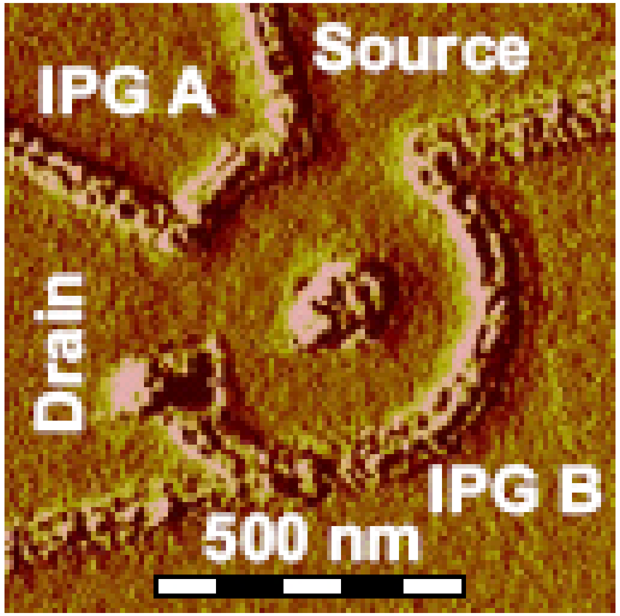


Figure 3.2: (a) SEM micrograph of the central gate



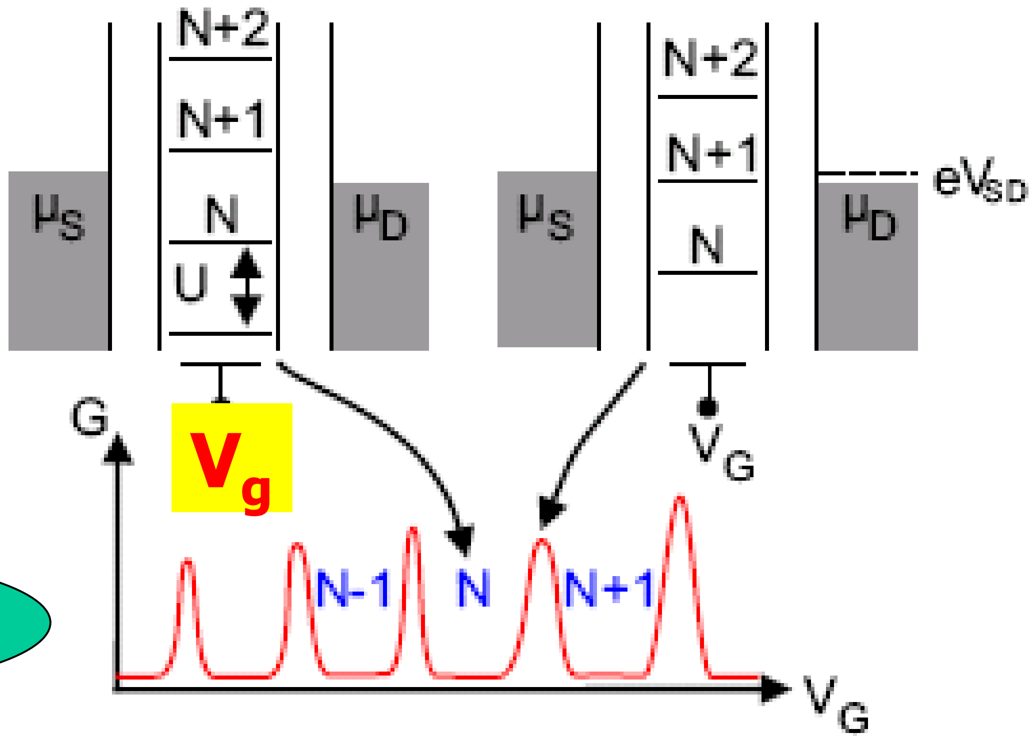
Nanodevices in Hannover

QUANTUM NUMBERS for the ISOLATED DOT in presence of e-e interactions:

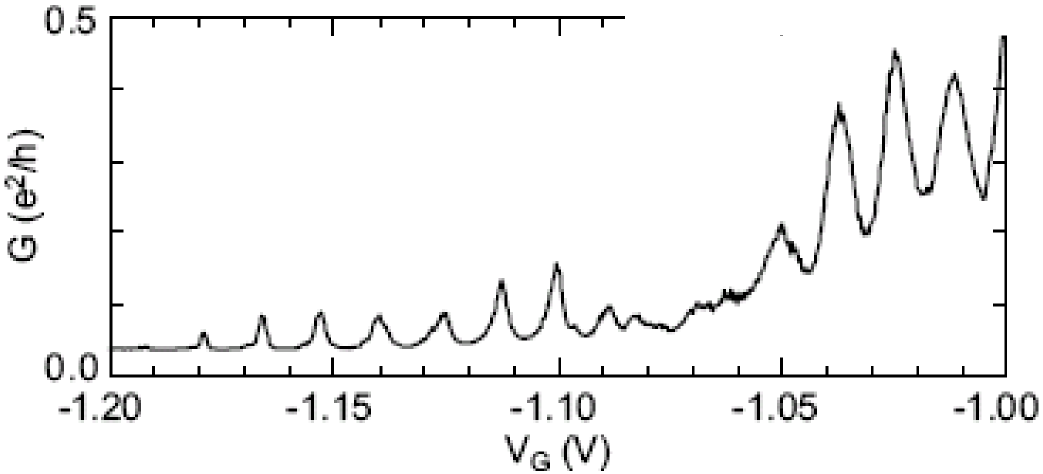
N electron number (1-10.000)

- S total spin
- S_z spin projection
- M total angular momentum ⊥ to the dot plane

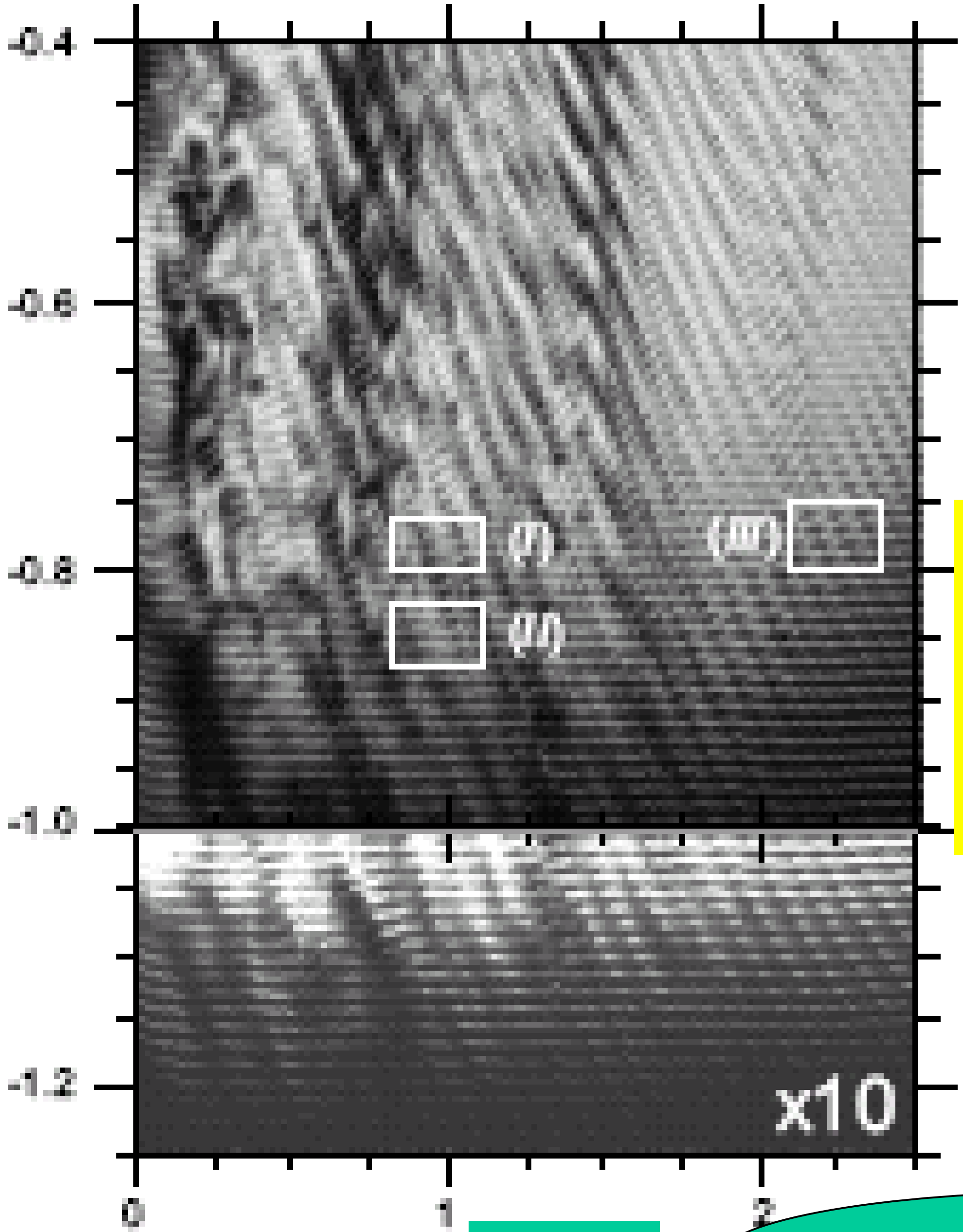
Coulomb blockade



conductance



vs **gate voltage**



V_g

Anche con centinaia di elettroni !

B

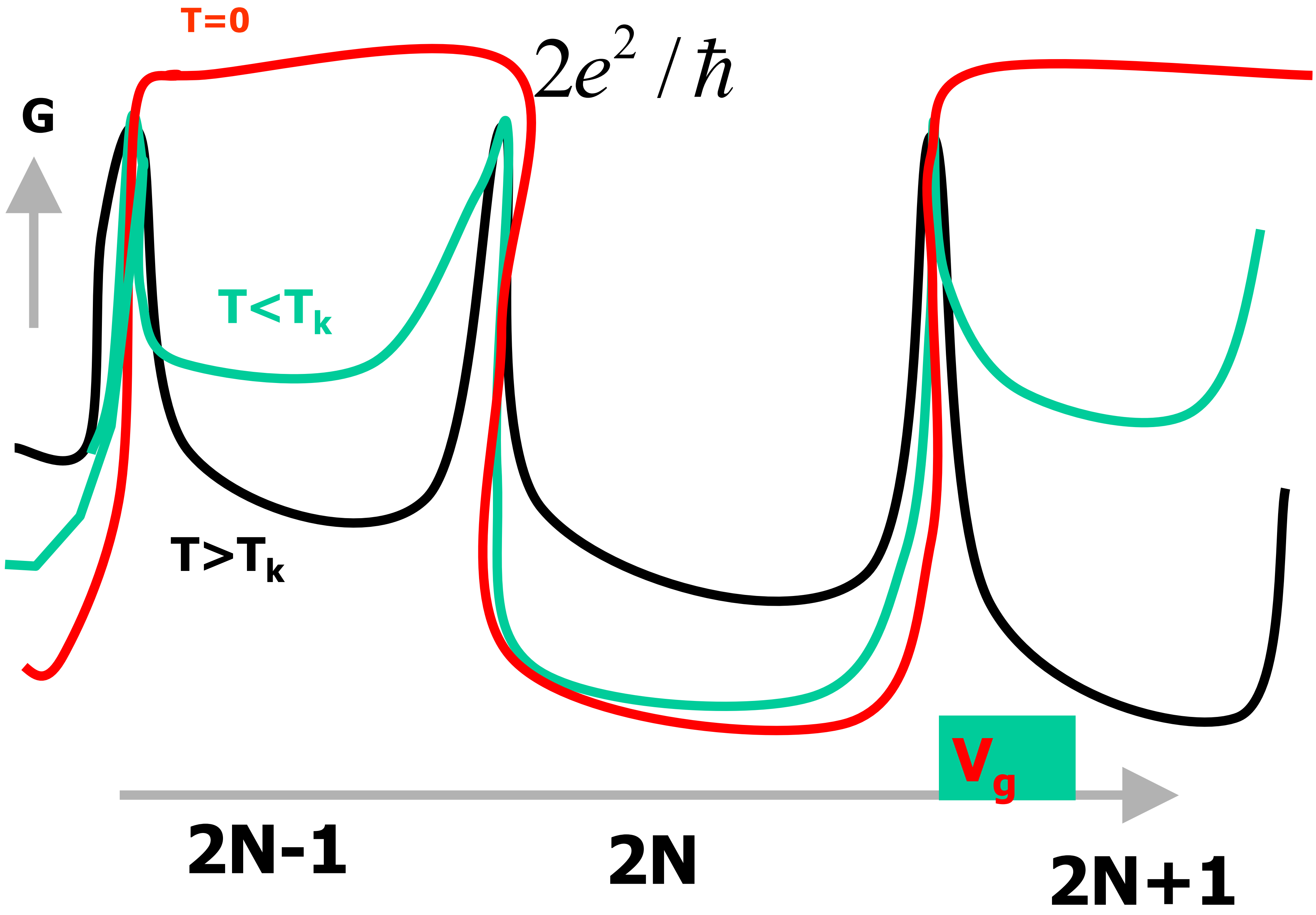
campo magnetico

Massimo possibile di conduttanza:

$$2\ e^2/h$$

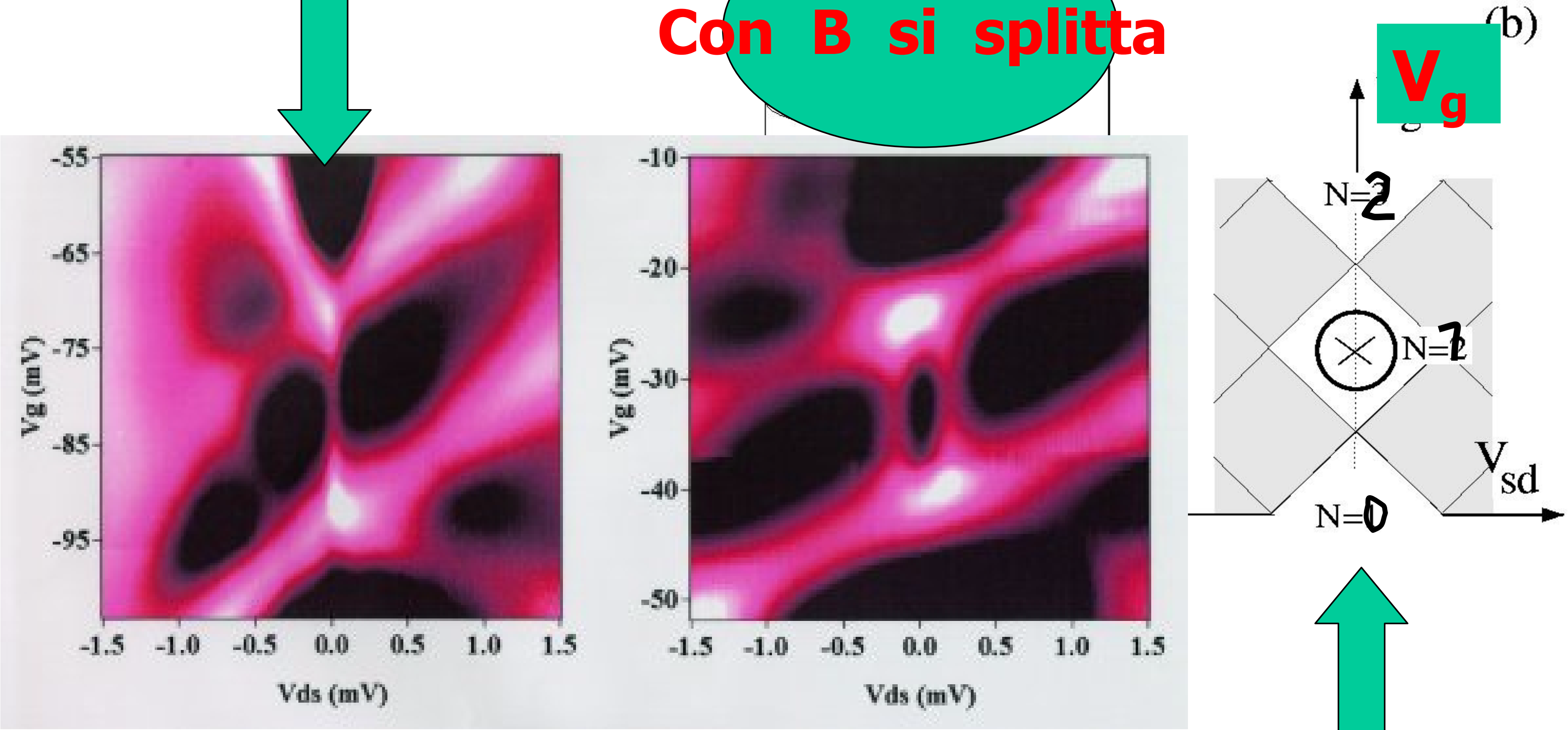
Diminuendo la temperatura sotto T_K cosa avviene nelle valli di conduttanza?

tunnel risonante Kondo



T=0
$2e^2/\hbar$
G
T<T_k
T>T_k
V_g
2N-1
2N
2N+1

Con B si splitta

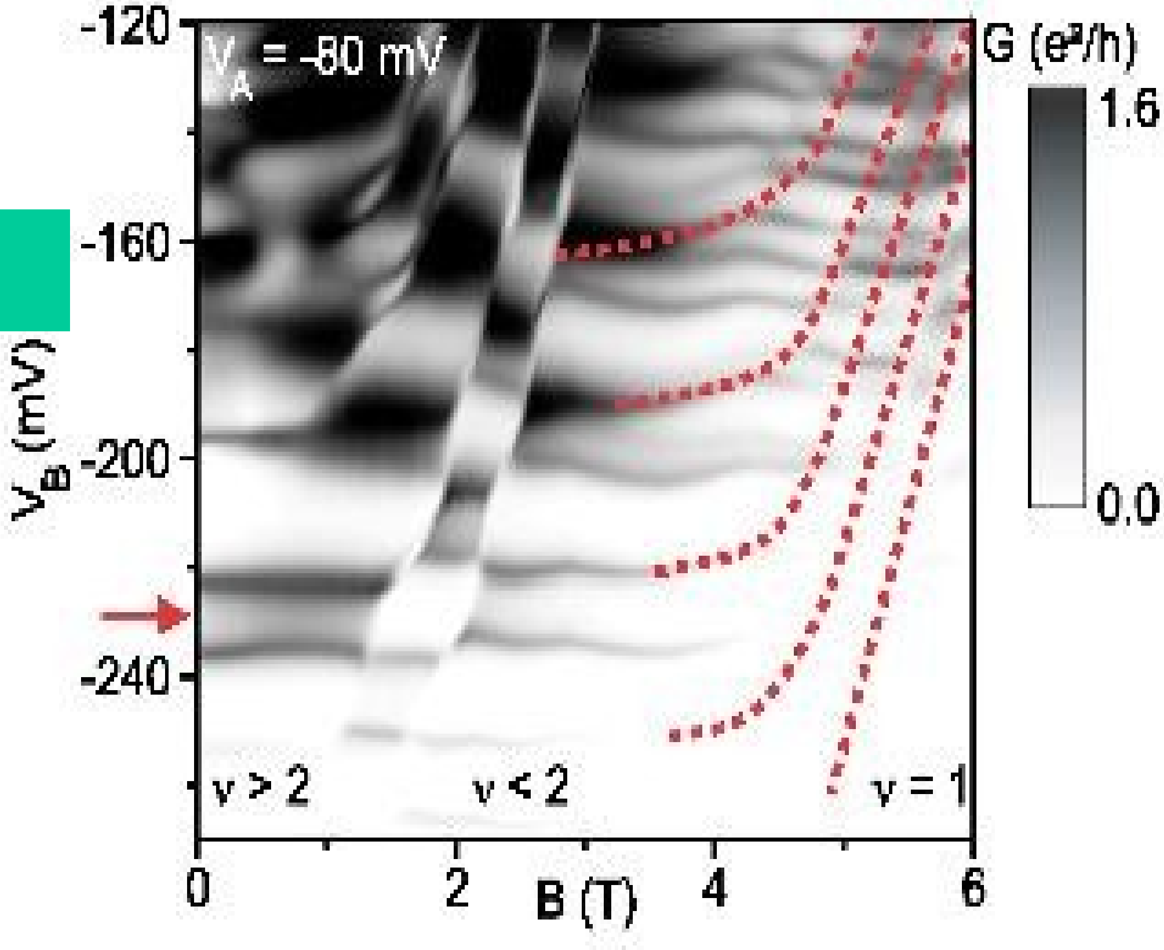


V_{sd}

Tensione applicata

courtesy of M.Kastner

V_g



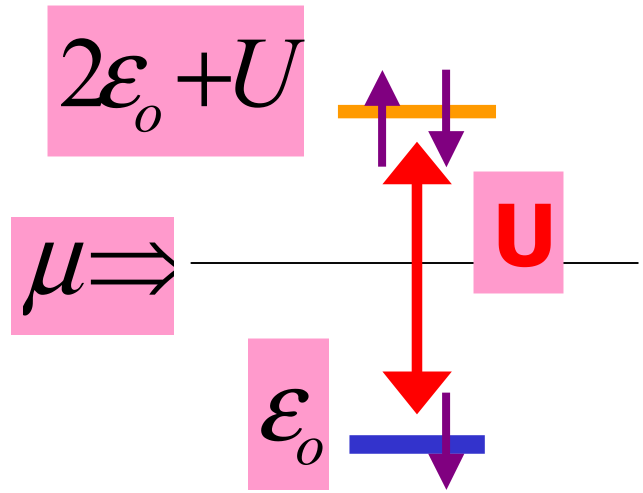
B

Anderson paradigm to include the contact electrons on the dot in the limit $U \rightarrow \infty$

$$H = \sum_{\sigma} \varepsilon_d n_\sigma + U n_\uparrow n_\downarrow + \sum_{k\sigma} \varepsilon_{k\sigma} c^{+}_{k\sigma} c_{k\sigma} + \Gamma \cdot \sum_{k\sigma} d^{+}_{\sigma} c_{k\sigma} + h.c.$$

Single level dot

Impurity level is singly occupied:



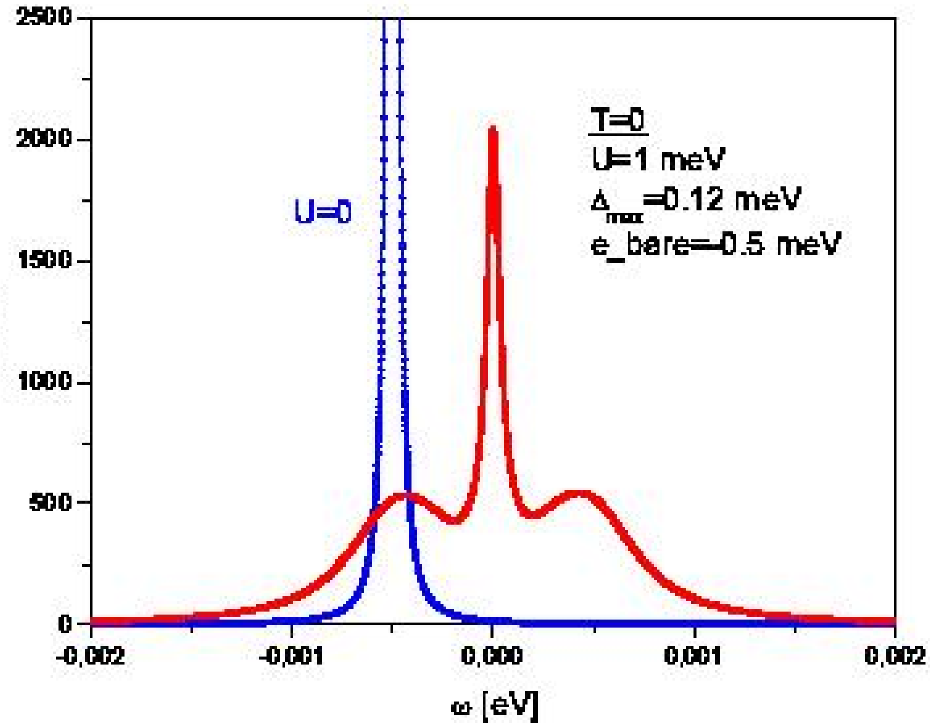
Spin on the level can be ↑ or ↓

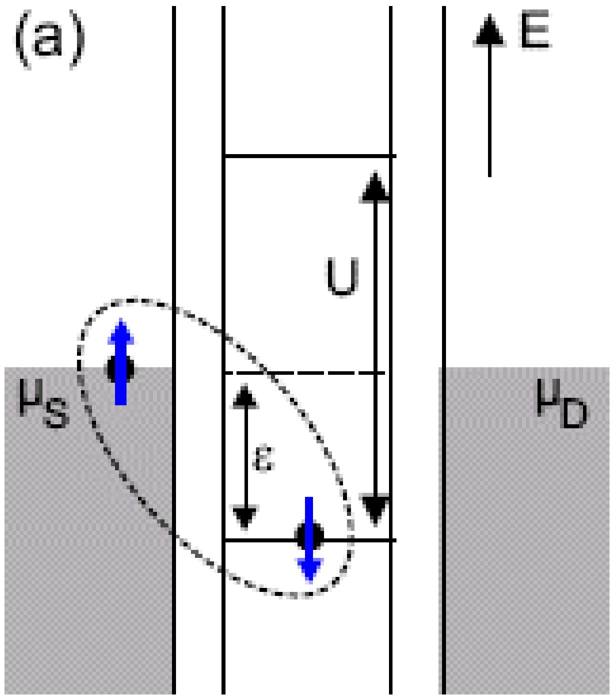
→ Impurity acts just as a spin **S_d = 1/2**

Quantum dot can sustain

Kondo conductance

at low temperatures





Σ_K *calculated via IPS* (*Horvatic*['87])

Schrieffer-Wolff projection onto the singly occupied subspace:

yelds the AF Kondo hamiltonian:

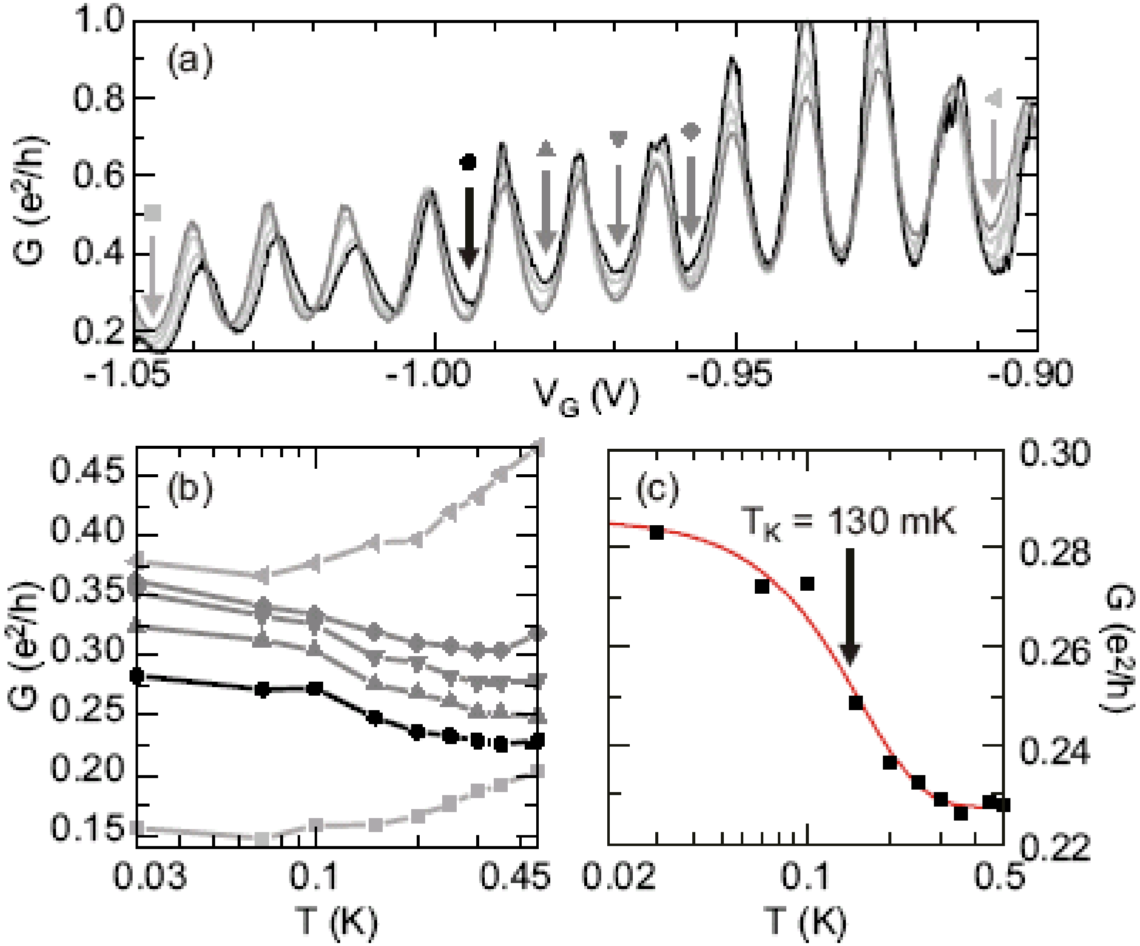
$$H=\sum_{k\sigma}\varepsilon_{k\sigma}c^{+}_{k\sigma}c_{k\sigma}+J\,\vec{S}_d\cdot\sum_{kk'\sigma}c^{+}_{k'\alpha}\vec{\sigma}_{\alpha\beta}\,c_{k\beta}$$

$4t^2/U$

Strong coupling fixed point: $J\rightarrow\infty$

Impurity spin is compensated :

$$S_d=0$$



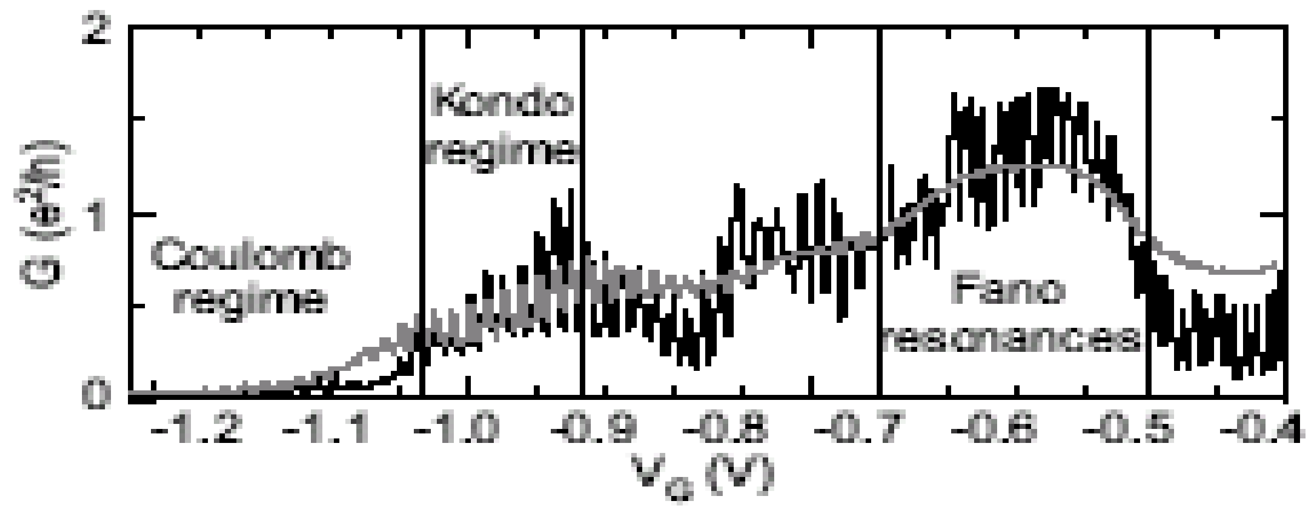
$$T_K \propto \sqrt{U\Gamma}\; e^{-U/\Gamma}$$

Liquido di Fermi !

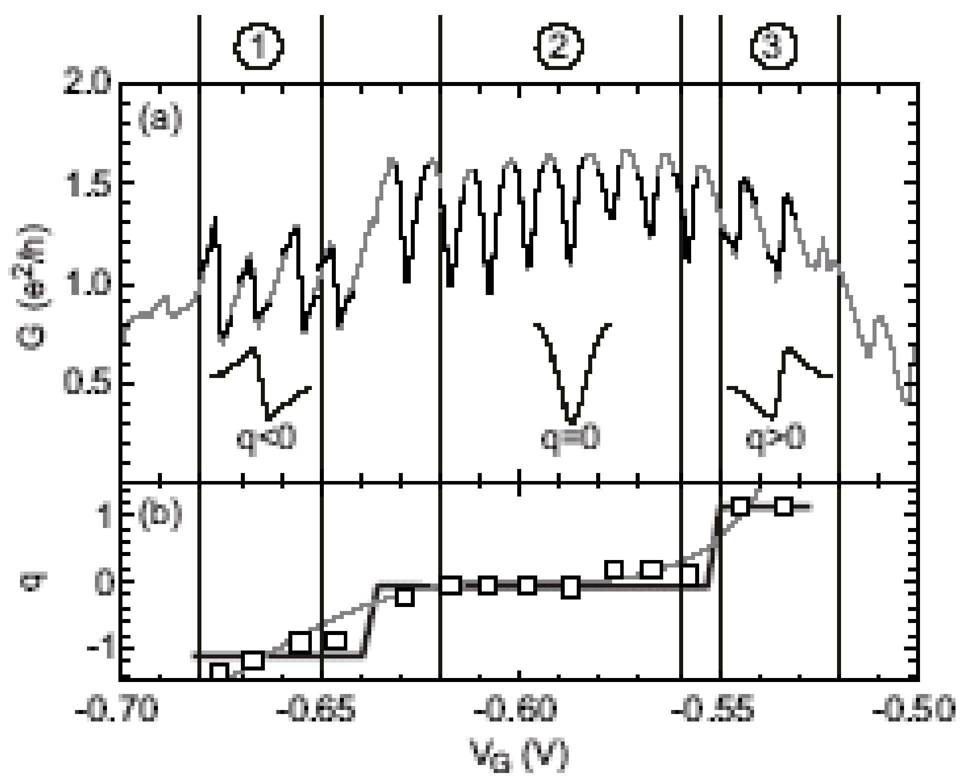
$$G(T) = G(0)\left[1 - c\left(T/T_K\right)^2\right]$$

Il dot a molti elettroni possono avere anche risonanze nella conduzione di tipo Fano

Pensiamo possa esservi la struttura Fano sovrapposta alla risonanza Kondo

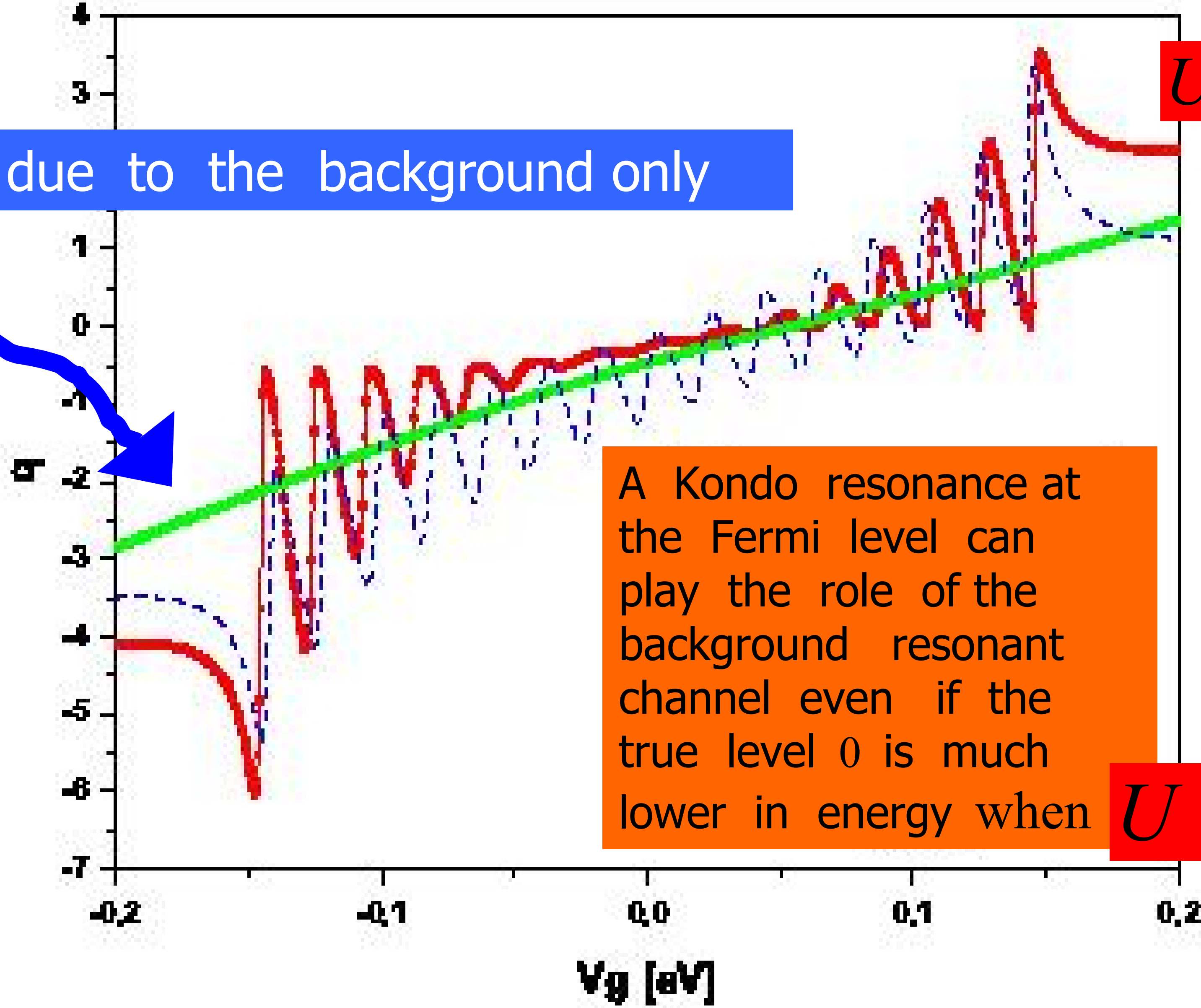


G (e²/h)
V_G (V)
Coulomb regime
Kondo regime
Fano resonances



1
2
3
(a)
(b)
G (e²/h)
q
q<0
q=0
q>0
V_G (V)

Concentrate now on the red curve:

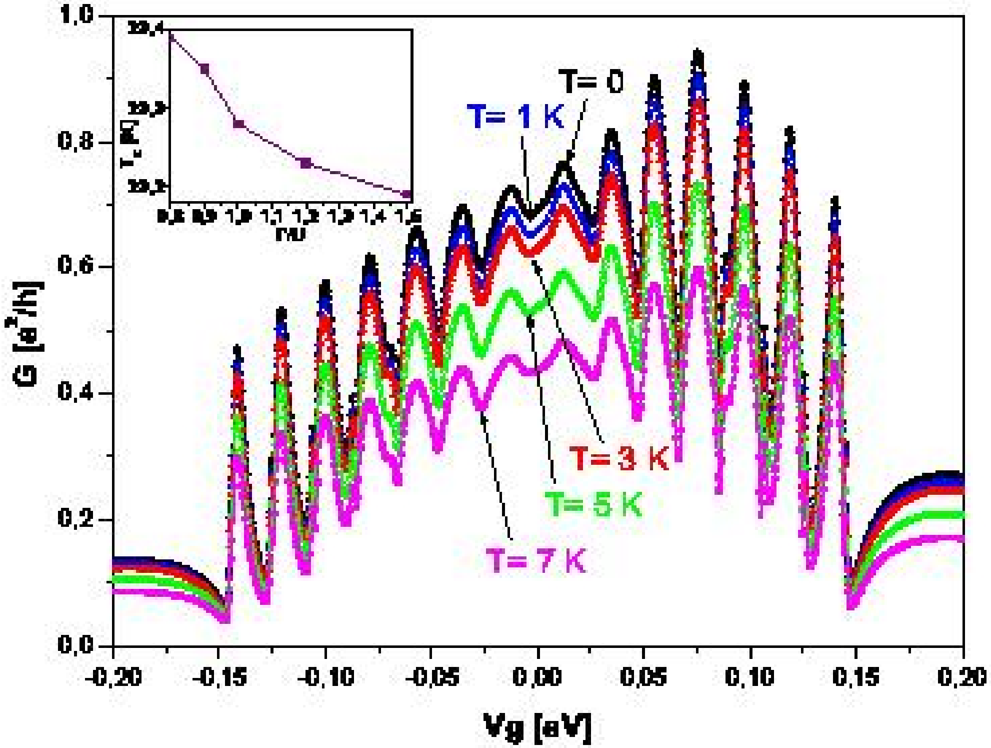


$U \neq 0$

q due to the background only

A Kondo resonance at the Fermi level can play the role of the background resonant channel even if the true level 0 is much lower in energy when $U \neq 0$

Temperature dependence:



P.Stefanski, A.T., B.Bulka , *PRL 93,186805(2004)*
P.Stefanski, A.T., B.Bulka, *cond-mat/05011385*

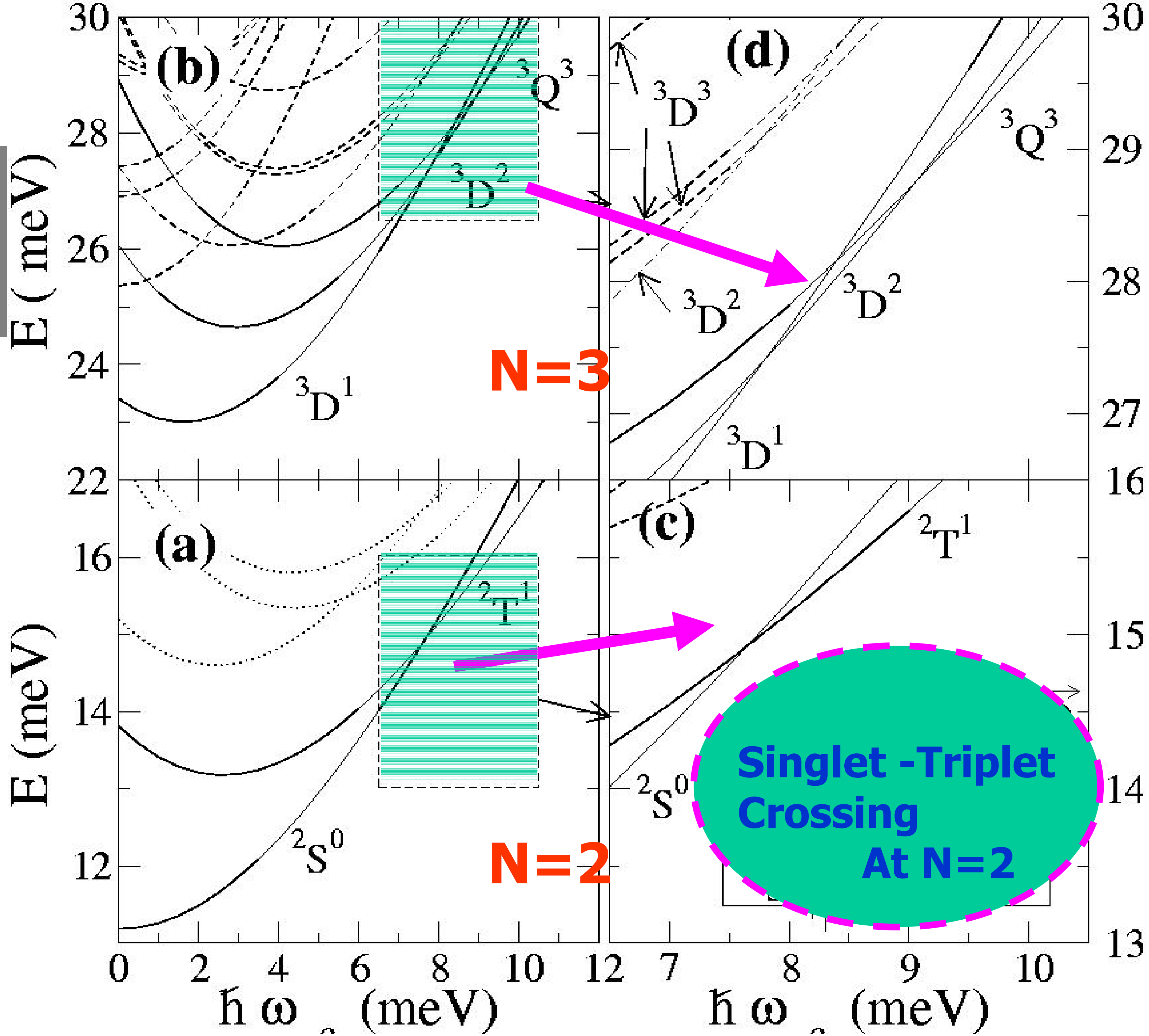
In a vertical structure with magnetic field

$B_{\perp}$ increases both M and S of the GS

Level crossing

to a higher spin state !

Prototype example :



Exotic Kondo coupling in strong $B_{\perp}$

Orbital Kondo

But it involves also exchange of angular momentum

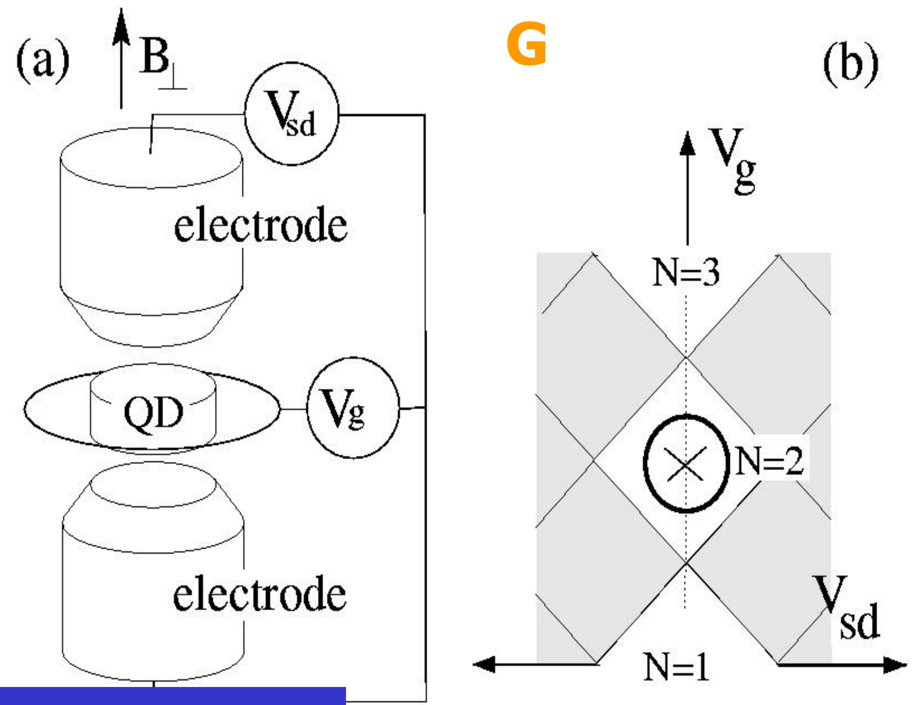
N=2

is enforced by

$U \to \infty$

At Kondo coupling $J = 0$:

4 spin states:



$${}^2S^0_0,\ {}^2T^1_{-1},\ {}^2T^1_0,\ {}^2T^1_1$$

Exchange of angular momentum **m** and spin **σ** with the contacts allows for Kondo fluctuations between and

At strong coupling :

Only an effective spin ½ survives:

$$\otimes,\ {}^{2}T^{1}_{-1},\ {}^{2}T^{1}_{0},\ \otimes$$

- **N = 2**
- **S =1/2**

Spin fractionalization !

Partial screening of the spin in Kondo strong coupling:

- D.Giuliano, A.T. PRL 84, 4677(2000)
- D.Giuliano,B.Jouault,A.T, PRB63(2001)

Multichannel Kondo allows for studying the crossover from the *underscreened* to the *overscreened* coupling

No convincing experimental proof of spin ½ -two channels Kondo

a) U –heavy fermion materials

b) cuprate superconductors

c) electron assisted tunneling in metallic glasses

Multichannel Kondo requires:

Our suggestion

-- multicontacts

--- one contact with more subbands of propagating states

D.Giuliano,B.Jouault,A.T., Europhys.Lett.65,401(2002)

Single electron states in the leads in cylindrical coordinates:

ρ, θ, z

$$\psi_{\varepsilon\approx\varepsilon_F,q,m,\sigma} \equiv e^{i(k_F+q)z} \; \varphi_{\approx 0,m}(\rho) \; e^{im\theta} \; \chi_\sigma$$

incoming plane wave in the z direction

wave in the cross- section of the contact

spin function

Two channels are involved:

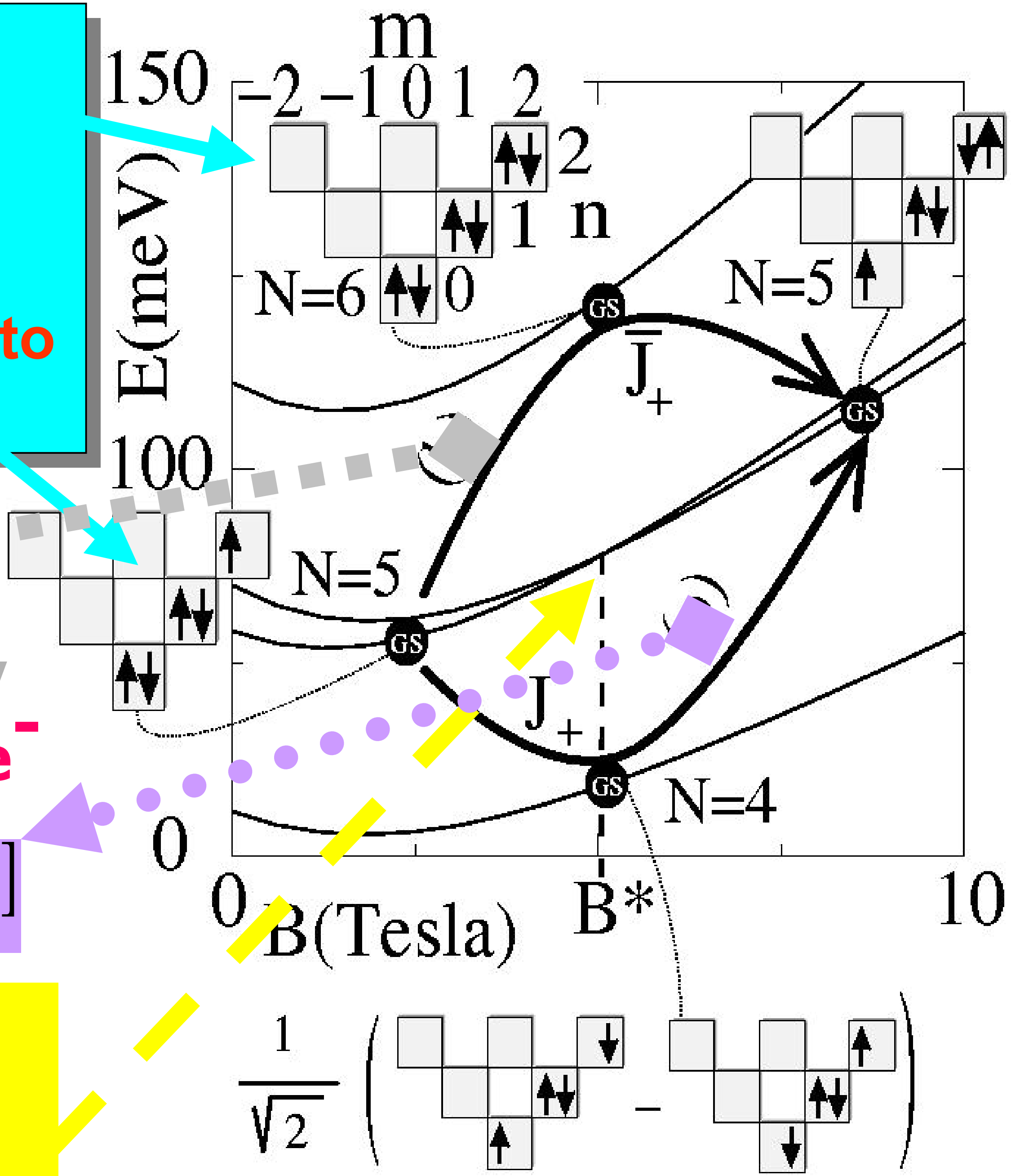
$[m=0,\sigma]$ and $[m=2,\sigma]$

Largest Slater determinant contributing to each state

$$\pm[m=2,\downarrow] \quad \mp[m=0,\downarrow]$$

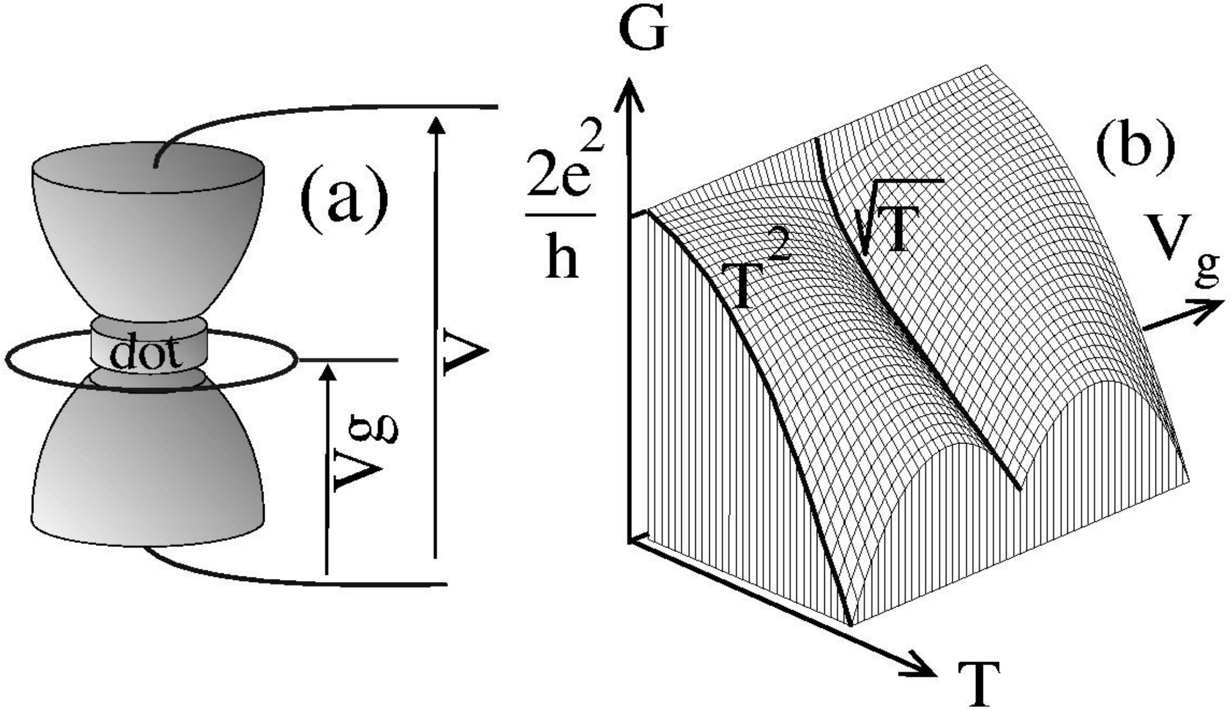
All hops are obtained by addition / removal of e^-

$$\pm[m=0,\uparrow] \quad \mp[m=2,\uparrow]$$



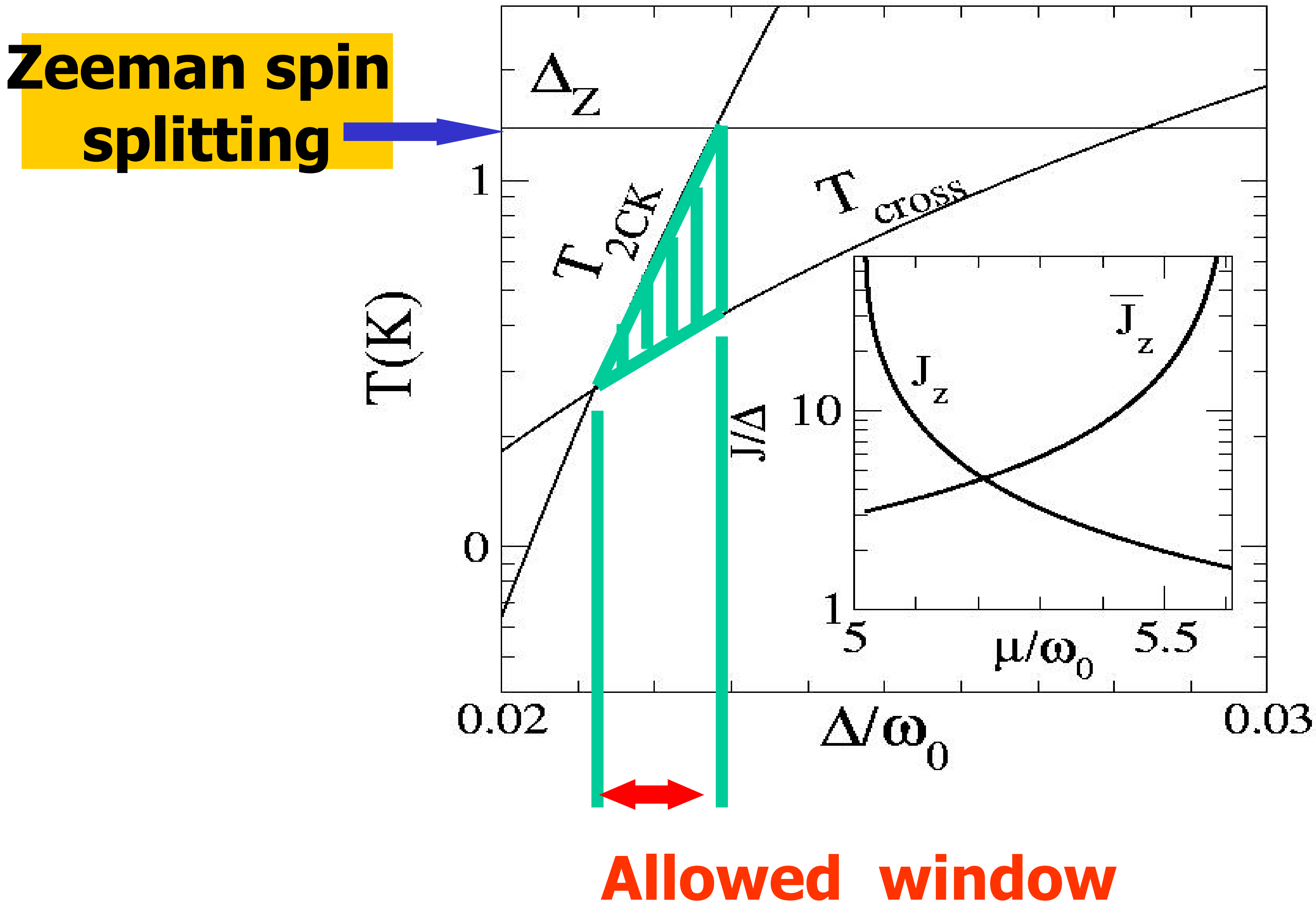
U and B give a crossing at N=5

crossover from the *underscreened* to the *overscreened* Kondo coupling



Non Fermi liquid behavior is expected !

The overscreend two-channel S=1/2 Kondo Effect



Scattering at the origin only involves the even parity channel

Degrees of freedom are charge,

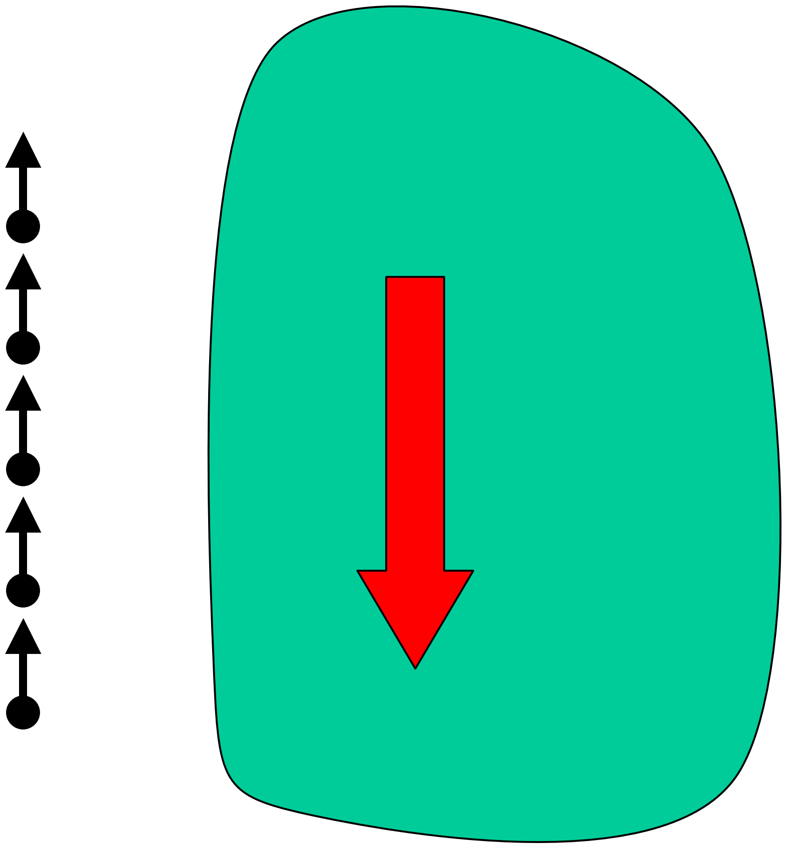
flavor (i.e. channel : $\alpha=1,2$), spin

$$H^{2ck} = \sum_{k\alpha\sigma} \varepsilon_{k\alpha\sigma} c^{+}_{k\alpha\sigma} c_{k\alpha\sigma} + J\, \vec{\mathbf{S}}_{\mathbf{d}} \bullet [\, \vec{\sigma}_1(0) + \vec{\sigma}_2(0)]$$

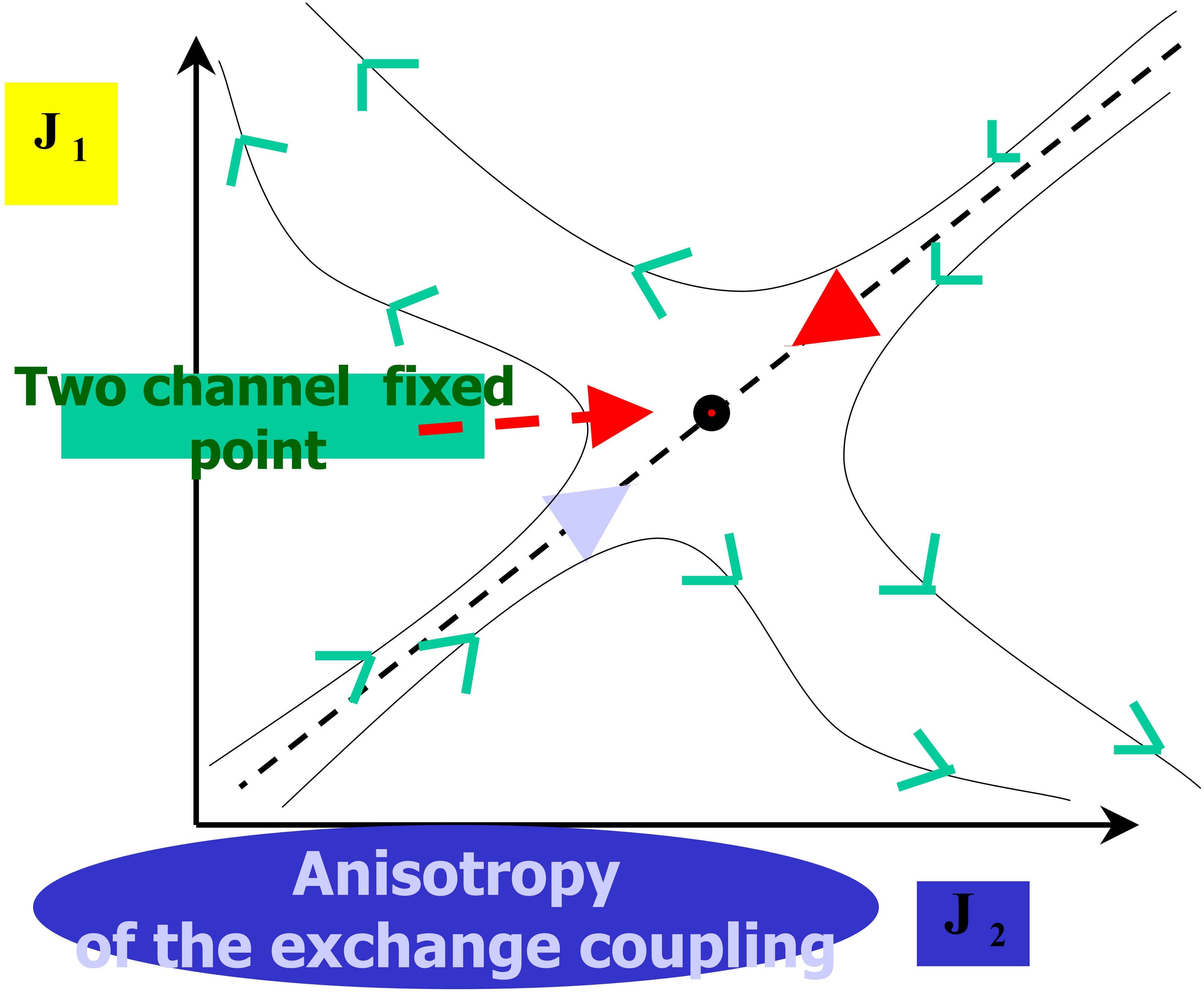
fixed point implies screening of the spin ½ impurity

how to get rid of overscreening?

Overscreening of **S= ½ impurity**



and again


J 1
Two channel fixed
point
Anisotropy
of the exchange coupling
J 2

TWO GOALS:

1) SEPARATE RELEVANT DEGREES OF FREEDOM

2) AVOID OVERSCREENING

To achieve this:

bosonize

an extra degree required :

the **spin-flavour** index : **sf**

ψ 's are Bose fields for each degree of freedom :

Charge, spin , flavour , and spin –flavour:

Scattering theory for two channel Kondo

$$\psi_{kl}^{out}(r) = S^{l}(k)\ \psi_{kl}^{in}(r)$$

Labels of single electron states: k ,l,σ

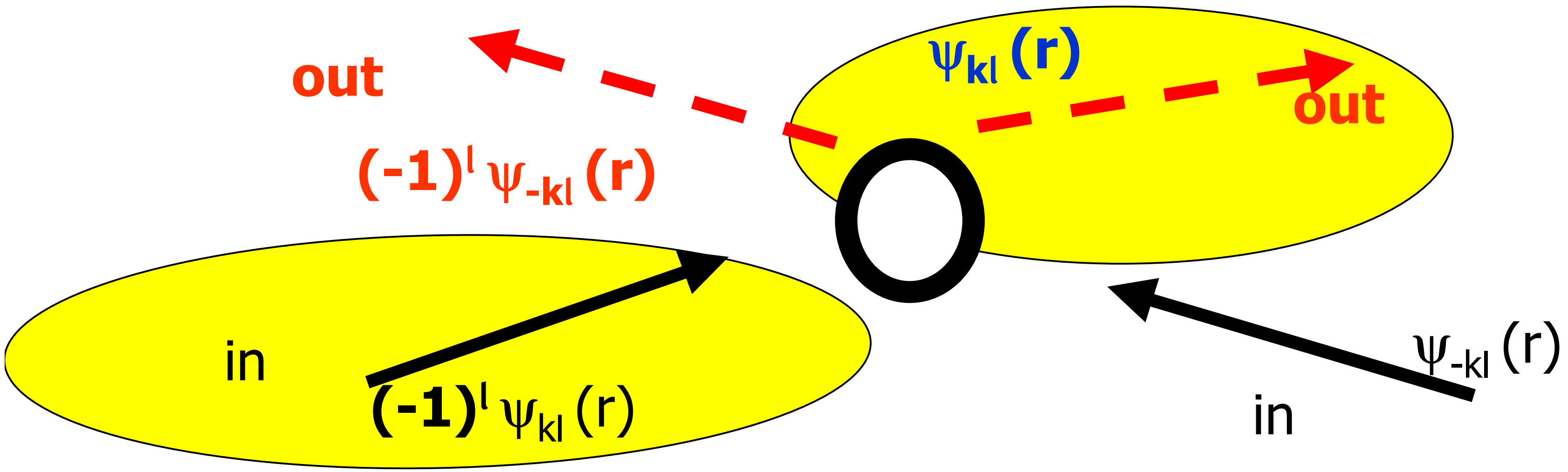
k : chirality: $\pm k - v\,t$

l : parity : **even or odd :**

Odd parity gives trivially $S^{odd} = -1$

r= | x |

σ : spin



Charge, spin, flavour, and
spin –flavour:

Two possible representations of fermions:

even

Fermion fields

$$\phi^{e}_{\alpha\sigma;\,I}(r) = :e^{-i/2\,[\psi_c + \sigma\psi_s + \alpha\psi_f \oplus \alpha\sigma\psi_{sf}](r)}:$$

Representation label : I or II

$$\phi^{e}_{\alpha\sigma;\,II}(r) = :e^{-i/2\,[\psi_c + \sigma\psi_s + \alpha\psi_f \ominus \alpha\sigma\psi_{sf}](r)}:$$

ψ 's are Bose fields for each degree of freedom

$\alpha = 1,2$: the channel (flavour) index

Bosonized Kondo coupling does not contain

charge and flavor

$$H_K = \tfrac{1}{2} J \left[S_d^+ : e^{i\Psi_{sp}(0)} :: \cos(\Psi_{sf}(0)) : + S_d^- : e^{-i\Psi_{sp}(0)} :: \cos(\Psi_{sf}(0)) : + S_d^z \frac{1}{\pi} \frac{d\Psi_{sf}(0)}{dx} \right]$$

Can be rewritten as two SU(2) spin currents

$$\Sigma^z{}_{A,B} = \frac{1}{2\pi} \frac{d}{dx} \left[\Psi_{sp}(0) \pm \Psi_{sf}(0) \right]$$

$$\Sigma^{\pm}{}_{A,B} = \frac{1}{\sqrt{2}} : e^{\pm i \left[\Psi_{sp}(0) \pm \Psi_{sf}(0) \right]} :$$

$$H^{2ck} = J\, \vec{\mathbf{S}}_{\mathbf{d}} \bullet [\, \vec{\Sigma}_A(0) + \vec{\Sigma}_B(0)]$$

$$H^{2ck} = J\, \vec{S}_d \bullet [\, \vec{\Sigma}_A(0) + \vec{\Sigma}_B(0)]$$

Define ($\sigma = \downarrow\uparrow$)

$$\left|\sigma; A/B\right\rangle =: e^{\sigma \frac{i}{2}[\Psi_{sp}(0) \pm \Psi_{sf}(0)]} : \left|bvac\right\rangle$$

bosonic vacuum

This is a powerful representation because singlets of type A or B are allowed at the origin ,

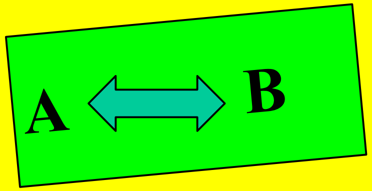
$$\left|Sin\ , A/B\right\rangle = \frac{1}{\sqrt{2}}\left\{\left|\Uparrow\right\rangle \otimes \left|\downarrow A/B\right\rangle - \left|\Downarrow\right\rangle \otimes \left|\uparrow A/B\right\rangle\right\}$$

but not triplet ! :

$$\vec{\Sigma}_B\left|\sigma; A\right\rangle = \vec{\Sigma}_A\left|\sigma; B\right\rangle = 0$$

fixed point is now moved to $J \longrightarrow \infty$

Scattering of delocalized electrons flip the composite object at the origin



First perturbation is hopping onto the impurity site:

$$h_T = -t \sum_{X=sp,sf} c_X^+(0)\left[c_X(a) + c_X(-a)\right] + h.c.$$

Matrix elements of scattering potential are

$$\left\langle Sin\,;B\middle| h_T \middle| Sin\,;A\right\rangle = -\frac{t}{2}\left[c_{sf}(a) + c_{sf}(-a) - c_{sf}^+(a) - c_{sf}^+(-a)\right]$$

$$\left\langle Sin\,;A\middle| h_T \middle| Sin\,;A\right\rangle = \left\langle Sin\,;B\middle| h_T \middle| Sin\,;B\right\rangle = 0$$

can be diagonalized and gives two degenerate states

$$\varsigma = \pm$$

with a superposition on the impurity :

$$\left| Sin, \varsigma \right\rangle = \frac{1}{\sqrt{2}} \left[\left| Sin; A \right\rangle + i^{\varsigma} \left| Sin; B \right\rangle \right]$$

Nozieres picture: incoming electrons should not produce extra type A and B spin density in the same combination

$$P_{\varsigma} = \sum_{\sigma} \left(\left| \sigma; A \right\rangle \left\langle \sigma; A \right| + i^{\varsigma} \left| \sigma; B \right\rangle \left\langle \sigma; B \right| \right)$$

Attach the charge and flavor degree of freedom and refermionize this projector :

$$P_\varsigma = \sum_{\sigma\alpha} \left[\phi^{I+}_{\sigma\alpha}(0)\phi^{I}_{\sigma\alpha}(0) + \phi^{II+}_{\sigma\alpha}(0)\phi^{II}_{\sigma\alpha}(0)\right] + i^\varsigma \left[\phi^{I+}_{\uparrow 1}(0)\phi^{II}_{\uparrow 1}(0) + \phi^{II+}_{\uparrow 2}(0)\phi^{I}_{\uparrow 2}(0)\right]$$

$$+ i^\varsigma \left[\phi^{II+}_{\downarrow 1}(0)\phi^{I}_{\downarrow 1}(0) + \phi^{I+}_{\downarrow 2}(0)\phi^{II}_{\downarrow 2}(0)\right] + h.c.$$

Mixes the two representations !

Final result: scattering hamiltonian include a pointlike interaction:

$$V(x) = \lambda P_\varsigma \quad \delta(x)$$

with $\lambda \rightarrow \infty$

Back to the Fermion representation:

Hamiltonian with pointlike scattering between representations

$$H_\lambda \approx -iv_F \int dx$$

$$\sum_\sigma \left(\phi^*_{\alpha\sigma,I}(x) \quad \phi^*_{\alpha\sigma,II}(x) \right) \left[\frac{d}{dx} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \lambda\delta(x) \cdot \begin{pmatrix} 1 & \pm i \\ \mp i & 1 \end{pmatrix} \right] \begin{pmatrix} \phi_{\alpha\sigma,I}(x) \\ \phi_{\alpha\sigma,II}(x) \end{pmatrix}$$

$$\alpha \rightarrow \overline{\alpha} \quad : \qquad \pm \rightarrow \mp$$

Motion eq.s for Green's functions:

$$\left(\frac{\partial}{\partial\tau}+iv_F\frac{\partial}{\partial x}\right)G^{I,I}(x,\tau;x',\tau')=\delta(x-x')\delta(\tau-\tau')-\lambda\delta(x)G^{I,I}(x,\tau;x',\tau')$$
$$-i\lambda\delta(x)G^{II,I}(x,\tau;x',\tau')$$

$$\left(\frac{\partial}{\partial\tau}+iv_F\frac{\partial}{\partial x}\right)G^{II,I}(x,\tau;x',\tau')=\delta(x-x')\delta(\tau-\tau')-\lambda\delta(x)G^{II,I}(x,\tau;x',\tau')$$
$$+i\lambda\delta(x)G^{I,II}(x,\tau;x',\tau')$$

The solution allows for the recognition of the **S** matrix

In the space of both representations:

$$S = \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix}$$

$$G = G_0 + G_0 T G_0$$

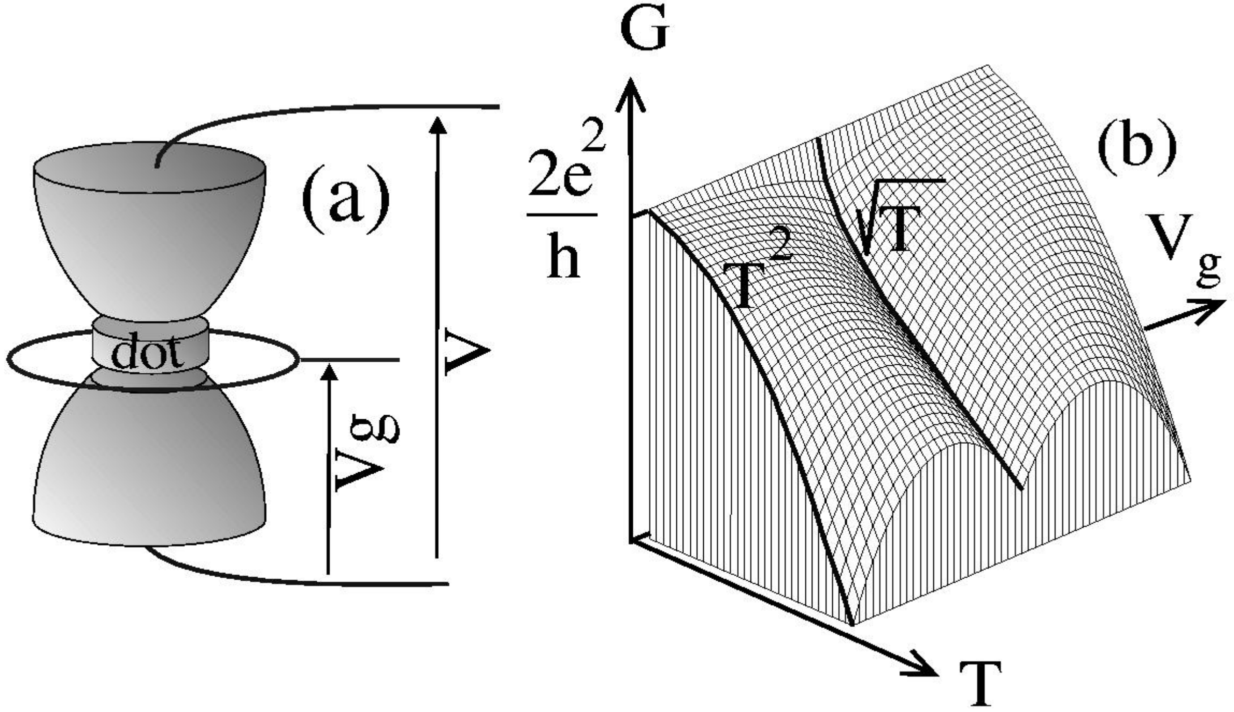
no unitarity paradox!

The conductance at the unitary limit: $\mathcal{G}$

$$\mathcal{G} = \frac{e^2}{h} trT = \frac{e^2}{h} tr\left[\frac{1}{2}\sum_l S^l\right]^+ \left[\frac{1}{2}\sum_l S^l\right] = \frac{2e^2}{h}$$

D.Giuliano, A.T. , J. Phys. C:Condens. Matt. 16,2846 (2004)
D.Giuliano, A.Naddeo, A.T., J. Phys. C : Condens. Matt. 16 S1453-S1483 (2004)

Non Fermi liquid behavior is expected !



crossover from the *underscreened*
to the *overscreened* Kondo coupling

Conclusions:

- The dot can act as an impurity spin
- The orbital and spin state can be tuned by an orthogonal magnetic field
- The spin can be maximized in the GS.

- various multichannel Kondo couplings can be studied: in particular the overscreened case