Towards measuring Casimir Energy by a superconducting cavity

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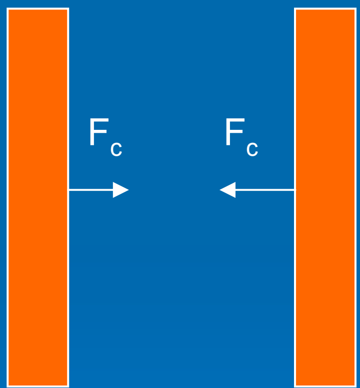
Summary

- The Casimir Effect (C.E.) in Physics.
- Experiments on the C.E., MEMS.
- The Casimir effect in a superconducting cavity
- The experiment ALADIN
- Conclusions

The Casimir effect is a manifestation of retarded Van der Waals forces between neutral macroscopic bodies, that arise from zero-point quantum fluctuations of the e.m. field.

It is one of the rare quantum phoenomena that can be seen on a macroscopic scale.

For two perfectly conducting parallel plates, at T=0, Casimir (1948) obtained an attractive force between the plates, of magnitude:



$$F_C = \frac{\pi^2 \hbar c}{240 L^4} A$$

A = area of the plates
L= distance between the plates

The proportionality of F_c to ħ reveals the quantum origin of the effect

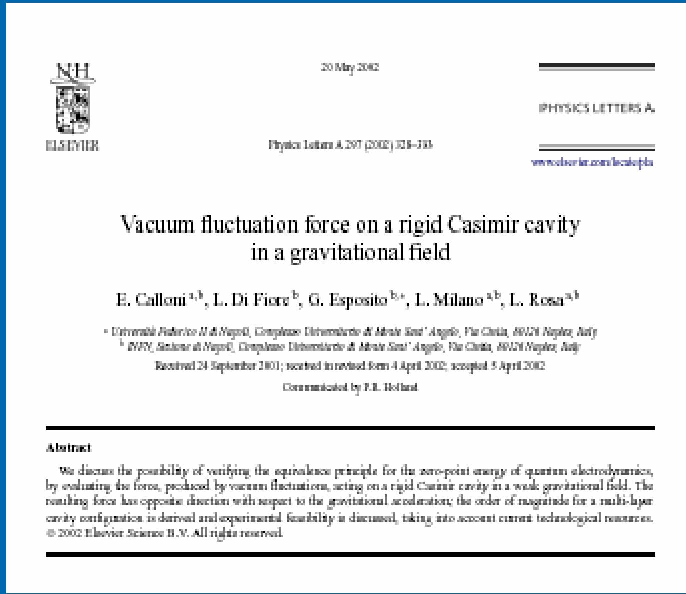
For A= 1 cm^2 and L=1 μm F_c=1.3 10^{-7} N

The scope of the Casimir effect is much broader than this, as it applies to all Quantum Fields under the influence of external boundary conditions.

- Bag model of hadrons in QCD: the C.E. Of quarks and gluons make important contributions to the nucleon energy.
- In Kaluza-Klein models C.E. Provides an effective mechanism of spontaneous compatification.
- In Gravitation and Cosmology (in space-times with nontrivial topology) the vacuum energy resulting from the C.E. Can drive inflation.
- In Atomic Physics, the long range Casimir interaction modifies the energy levels of Rydberg states.

In Naples reserch on Casimir effect goes on since several years.

Recently we investigated the possibility of testing experimentally if the Equivalence Principle of General Relativity holds also for vacuum fluctuations:
“ Do virtual photons have a weight?”



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Vacuum fluctuation force on a rigid Casimir cavity in a gravitational field

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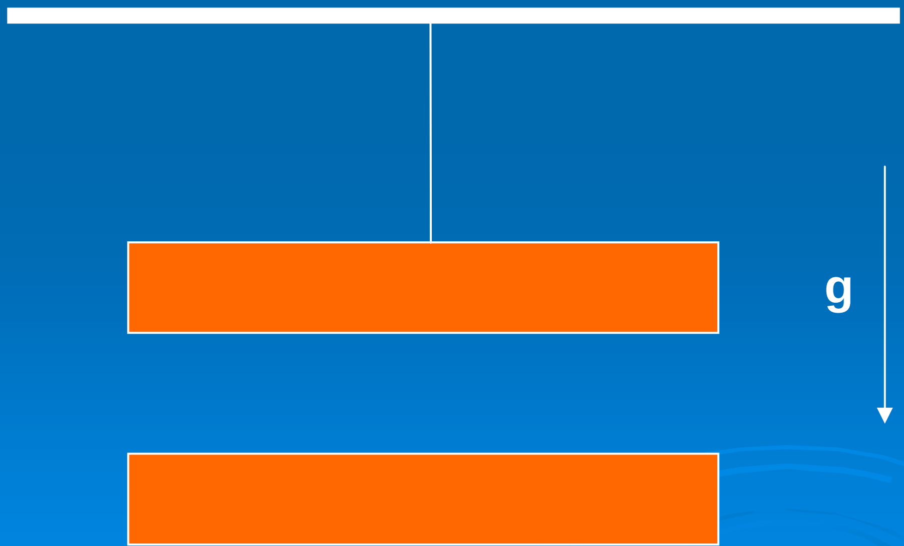
Abstract

We discuss the possibility of verifying the equivalence principle for the zero-point energy of quantum electrodynamics, by evaluating the force, produced by vacuum fluctuations, acting on a rigid Casimir cavity in a weak gravitational field. The resulting force has opposite direction with respect to the gravitational acceleration; the order of magnitude for a multi-layer cavity configuration is derived and experimental feasibility is discussed, taking into account current technological resources.



As the Casimir energy of a p.p. cavity is negative, is should contribute a negative weight!

Some experiments on the Casimir effect

- Sparnaay (1958) Spring balance measurement of F_c in plane-parallel geometry. Not conclusive
- Lamoreaux (1999) Torsion pendulum measurement of F_c in sphere-plane geometry. 5% error.
- Mohideen and Roy (1998) AFM measurement of F_c in sphere-plane geometry. 1% error
- Bressi et al. (2002) Plane-parallel geometry. 15% accuracy

The Mohideen-Roy experiment

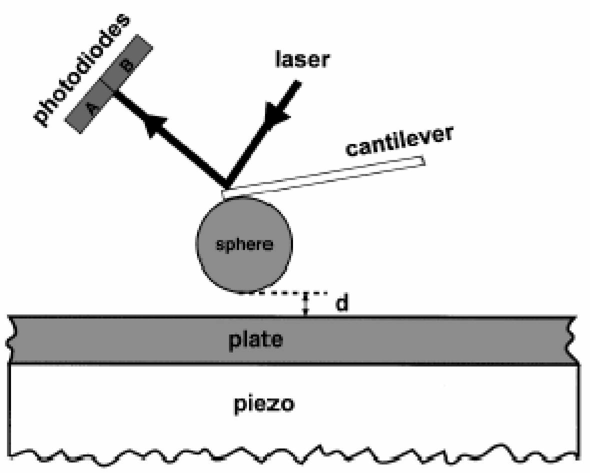


FIG. 1. Schematic diagram of the experimental setup. Application of voltage to the piezo results in the movement of the plate towards the sphere. The experiments were done at a pressure of 50 mTorr and at room temperature.

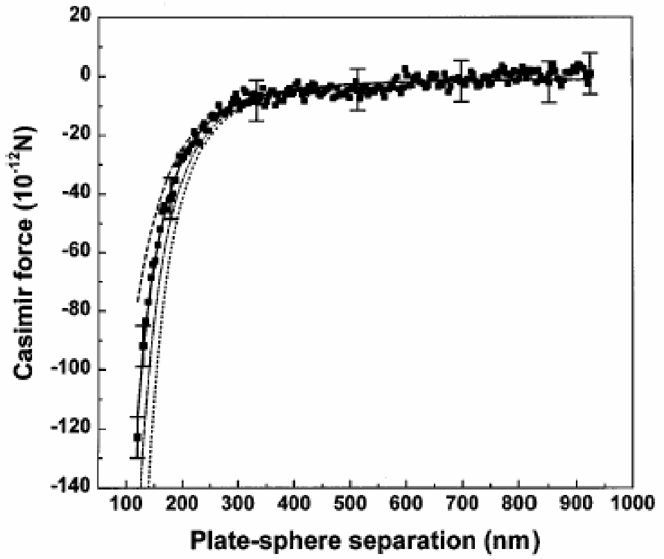


FIG. 4. The measured average Casimir force as a function of plate-sphere separation for 26 scans is shown as square dots. The error bars show the range of experimental data at representative points. The theoretical Casimir force from Eq. (4) with all corrections is shown as a solid line. The rms deviation between the experiment and theory is 1.6 pN. The dash-dotted line is the Casimir force without the finite conductivity, roughness, or temperature correction [Eq. (1)] which results in a rms deviation of 6.3 pN. The dashed line includes only the finite conductivity correction [Eq. (2)] which results in a rms deviation of 5.5 pN. The dotted line includes only the roughness correction leading to a rms deviation of 48 pN.

Casimir forces are the dominant forces in nanomechanical devices

Recently the first actuator based on the Casimir force was developed by the researchers at Bell labs.

H.B. Chan, V.A. Aksyuk, R.N. Kleiman, D.J. Bishop, F. Capasso, Science 291 (2001) 1941.

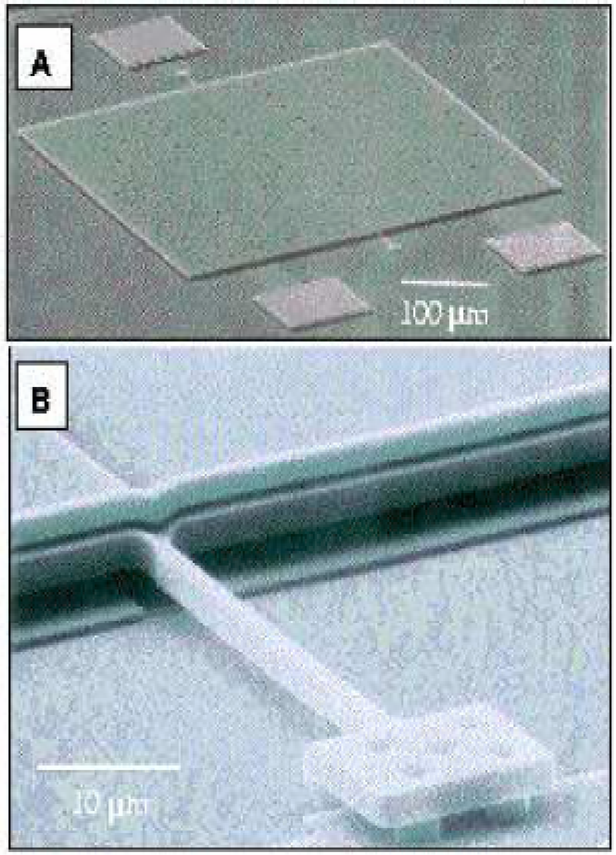
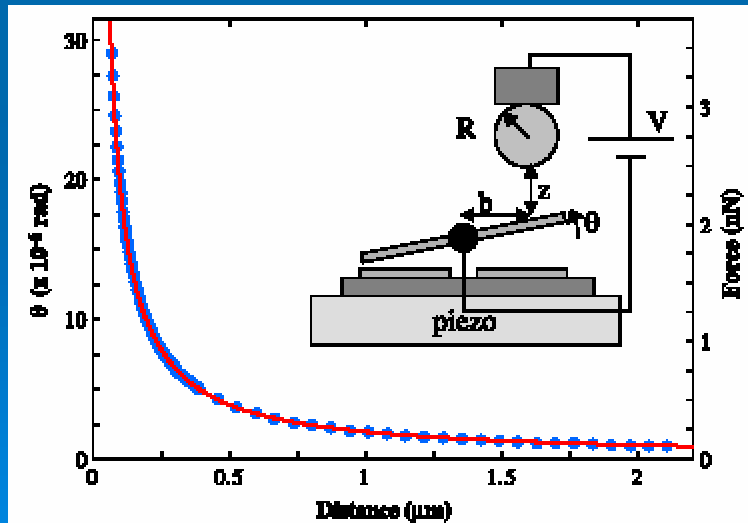


Fig. 29. Scanning electron micrographs of (A) the nanofabricated torsional device and (B) a close-up of one of the torsional rods anchored to the substrate. Courtesy of Federico Capasso, Bell Labs, Lucent Technologies.

Quantum Mechanical Actuation of Microelectromechanical Systems by the Casimir Force

H. B. Chan, V. A. Aksyuk, R. N. Kleiman, D. J. Bishop, Federico Capasso*

The Casimir force is the attraction between uncharged metallic surfaces as a result of quantum mechanical vacuum fluctuations of the electromagnetic field. We demonstrate the Casimir effect in microelectromechanical systems using a micromachined torsional device. Attraction between a polysilicon plate and a spherical metallic surface results in a torque that rotates the plate about two thin torsional rods. The dependence of the rotation angle on the separation between the surfaces is in agreement with calculations of the Casimir force. Our results show that quantum electrodynamical effects play a significant role in such microelectromechanical systems when the separation between components is in the nanometer range.



From a QFT point of view the C.E. is a one-loop radiative correction to the external background provided by some boundary conditions.

However, a simple way to obtain the Casimir force between ideal conductors is to consider the associated (free) energy E_c, such that:

$$F_c = -\frac{\partial E_c}{\partial L}$$

E_c can be obtained as the zero point energy of the quantized e.m. field within the cavity:

$$E_c = \lim_{\Lambda \to \infty} \left(\sum_i \frac{1}{2} \hbar \omega_i - E_0 \right)$$

where ω_i are the eigenfrequencies of the e.m. field, E_0 is the vacuum energy that would be present in an empty volume equal to the volume of the cavity, and L is a smooth ultraviolet cutoff. The use of a cutoff is physically sensible because at high frequencies all material eventually become transparent.

The computation is easy for a plane-parallel geometry, and gives

Note the - sign

$$E_C = -\frac{\pi^2 \hbar c}{720 L^3} A$$

Though conceptually easy, the computation of Ec for an arbitrary disposition of the perfect conductors is generally very hard. An important feature of the Casimir energy E_c, due to the long range nature of the Van der Waals forces, is that it is not an extensive quantity, and so is not proportional to the volume of the system.

Ec depends on the geometry of the cavity!

The dependence on the geometry is clear already in the plane parallel case, as E_c depends separately on A and L

In fact, the dependence on the shape reaches to the point that the Casimir force can be even repulsive, in certain cases. A classic example of this is provided by a perfectly conducting spherical shell (Boyer, 1950). Another nice example is that of a parallelepiped. Here the sign of F_c depends on the ratios among the sides. There is no simple intuitive explanation of the sign of the Casimir energy, as E_c results by taking a difference between two positive infinite numbers.

Whether the Casimir force between real materials can ever be repulsive is in fact controversial (Barton 2000)

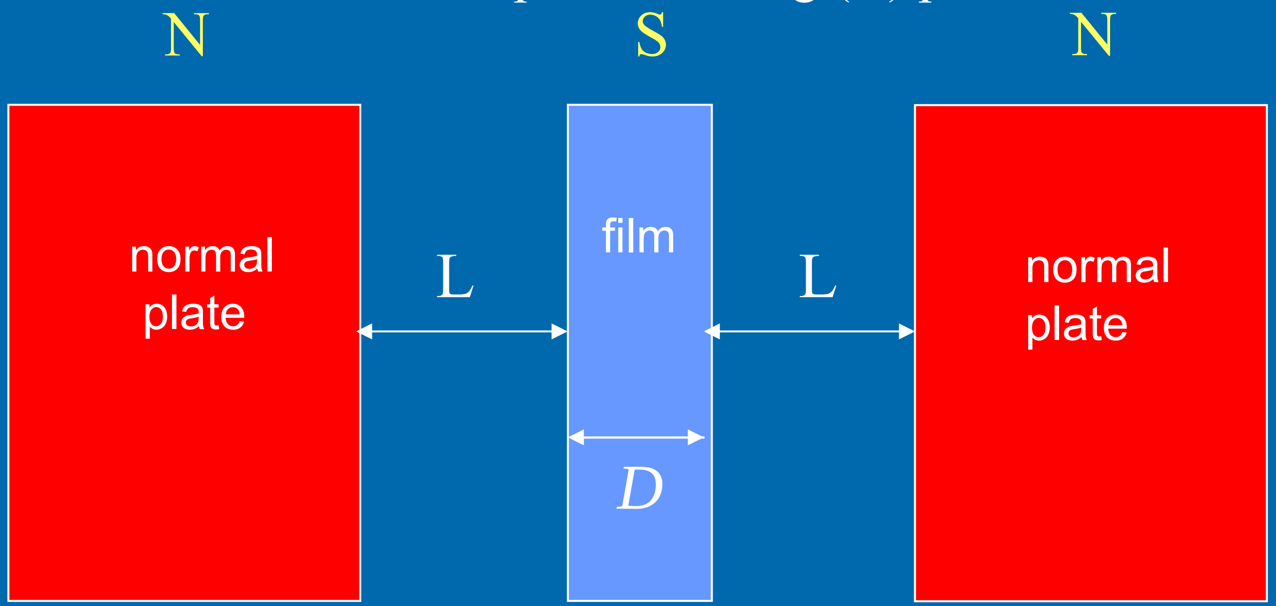
All experiments so far are force measurements, in the plane-parallel or in the sphere-plate geometry.
No experiments yet available for geometries in which F_c is expected to be repulsive.

Questions

- Is it possible to measure the Casimir energy, rather than the Casimir force?
- Is it possible to observe in a laboratory the influence of the Casimir energy on a phase transition?
- Is is possible to study the Casimir effect with rigid cavities, of arbitrary shape?

We think that this is possible by using a superconducting cavity (G.B., E. Calloni, G. Esposito, L. Milano and L.Rosa (2004)).

We consider a rigid cavity containing superconducting plates. For example, a plane-parallel cavity formed by a thin film of a type I superconductor (S), placed between two non superconducting (N) plates.



D=5 nm

L=10 nm

The idea is simple: Since the Casimir energy depends on the reflective power of the plates, there should be a change ΔE_c of Casimir energy as soon as the film becomes superconducting, because of the sharp change in the reflective power (in the IR region) of the film.

ΔE_c should be very small, because the largest contribution to E_c is due to photons of energy $\hbar c/L \approx 20$ eV, while the transition to superconductivity changes the reflective power in the IR region, at the scale $kT_c \approx 10^{-4}$ eV .

Is there a way to measure ΔE_c?

For any $T<T_{c,}$ the difference ΔF among the free energies of the cavity, when the film is N or S, is the sum of the condensation energy E_{cond} of the film plus the variation of Casimir energy ΔE_c

$$\Delta F = \mathrm{E}_{\mathrm{cond}} + \Delta E_C$$

$$\mathrm{E}_{\mathrm{cond}} \equiv V\left(f_n(T) - f_S(T)\right)$$

$f_{n/S}(T)$ Helmoltz free energy in the n/s state

$V = A \times D$ volume of the film

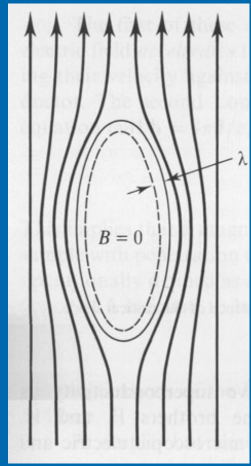
(E_{cond} and ΔE_c are both proportional to the area A)

A simple direct way to measure ΔF is by measuring the critical magnetic field H_c, that destroys the superconductivity of the film.

Magnetic properties of superconductors

- Meissner effect: s.c. show perfect diamagnetsim.

- Superconductivity is destroyed by a critical magnetic field $H_{c.}$



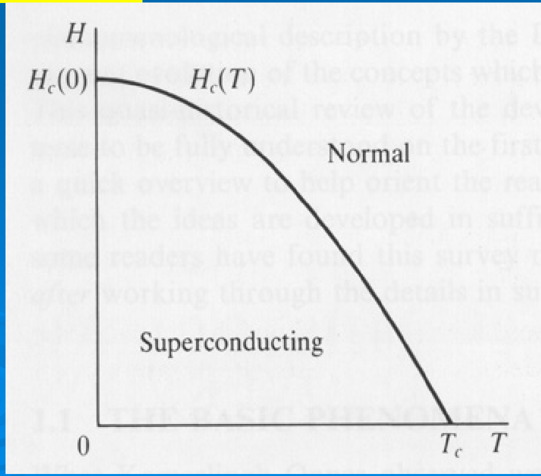
H_c is related to the free-energy difference between the N and the S states:

$$\frac{H_c^2(T)}{8\pi} = f_n(T) - f_s(T)$$

(bulk sample)

$H_c(T)$ follows an approximate Parabolic law

$$H_c(T) = H_c(0)\left[1 - \left(\frac{T}{T_c}\right)^2\right]$$



For our thin film, in parallel field:

$$\frac{1}{8\pi}\left(\frac{H_c(T)}{\rho}\right)^2 V = \mathrm{E}_{\mathrm{cond}} + \Delta E_c$$

$$\rho = \sqrt{24}\frac{\lambda}{D}\left(1+\frac{9D^2}{\pi^6\xi^2}\right)$$

ρ takes account of incomplete field expulsion, and nucleation

ξ=correlation length, λ=penetration depth

ΔE_c causes a shift of H_c of the order of:

$$\frac{\delta H_c}{H_c} \approx \frac{1}{2}\frac{\Delta E_c}{\mathrm{E}_{\mathrm{cond}}}$$

We can get high sensitivities because E_{cond} is very small, and E_c is large.

For a Be film with A=1 cm² D=5 nm T/T_c=0.97

$$\mathrm{E}_{\mathrm{cond}} = 3.5 \times 10^{-8}\ erg \qquad \left(\frac{T}{T_c} = 0.97\right)$$

For a cavity with A=1 cm² L=10 nm

$$E_c = 0.43\ erg$$

E_c is 10 million times larger than E_{cond}!

So even a tiny fractional change of E_c can be large compared with E_{cond}, and cause a measurable shift of H_c

Computation of ΔE_c: the Casimir effect in real materials.

The theory for a dispersive cavity was developed by Lifshitz (1956). He computed the Casimir energy by evaluating the v.e.v. of the e.m. stress-energy tensor in the empty space between the bodies. Alternative methods exist today (summation on evanescent modes). The basic assumption of the theory is that one can describe the propagation of e.m. waves inside the plates by a complex permittivity $\varepsilon(\omega,q)$, depending on the frequency ω and on the wave-vector q.

Space dispersion can be neglected in the computation of ΔE_c because:

1) Despite the fact that typical cavity modes have short wavelengths (L=10 nm) for which space dispersion is important, the computation of ΔE_c only involves long-wavelength modes of energy kT_c, because the reflective power of a supercondcuting film are indistinguishable from those of a normal film for photon energies larger than a few times kT_c (Glover and Tinkkam, 1958).
Then, the relevant range of frequencies is the far infrared.

2) In the far IR, space non-local effects are negligible in thin films ($D << \xi,\lambda$).

For a film of thickness D, placed between two plates, ΔE_c can be written as:

$$\Delta E_C = \frac{\hbar A}{4\pi^2 c^2} \int_1^{\infty} p\,dp \int_0^{\Lambda k T_c} d\zeta\, \zeta^2 \left(Log \frac{Q_n^{TE}}{Q_s^{TE}} + Log \frac{Q_n^{TM}}{Q_s^{TM}} \right)$$

The expression of the coefficients $Q_{n/s}^{TE/TM}$ for our multilayer system is

$$Q_I^{TE/TM}(\zeta,p) = \frac{(1-\Delta_{1I}^{TE/TM}\Delta_{12}^{TE/TM} e^{-2\zeta p L/c})^2 - (\Delta_{1I}^{TE/TM} - \Delta_{12}^{TE/TM} e^{-2\zeta p L/c})^2 e^{-2\zeta K_I D/c}}{1-(\Delta_{1I}^{TE/TM})^2 e^{-2\zeta K_I D/c}},$$

$$\Delta_{jl}^{TE} = \frac{K_j - K_l}{K_j + K_l}, \quad \Delta_{jl}^{TM} = \frac{K_j \epsilon_l(i\zeta) - K_l \epsilon_j(i\zeta)}{K_j \epsilon_l(i\zeta) + K_l \epsilon_j(i\zeta)}, \quad K_j = \sqrt{\epsilon_j(i\zeta) - 1 + p^2}, \quad I = n,s \;;\; j,l = 1,2,n,s.$$

Note that all permittivities $\varepsilon_j(i\zeta)$ are evaluated at imaginary frequencies $i\zeta$

ΔE_c has been evaluated numerically

For the normal plates and for the film in the n state, we used the Drude formula

$$\varepsilon(\omega) = 1 - \frac{\Omega_P^2}{\omega(\omega + i\gamma)}$$

$\gamma = 1/\tau$ τ=relaxation time

Ω_p plasma frequency

For $\varepsilon_s^{film}(\omega)$ we used the Mattis-Bardeen complex permittivity of BCS theory

$$\epsilon_s''(\omega) = \frac{\hbar\Omega_n^2}{2\omega^2\tau_n}\left[\int_\Delta^\infty dE\ J_T + \theta(\omega - 2\Delta)\int_{\Delta-\hbar\omega}^{-\Delta} dE\ J_D\right]$$

$$J_T := \left[\tanh\frac{E+\hbar\omega}{2kT} - \tanh\frac{E}{2kT}\right] g(\omega,\tau_n,E)$$

$$J_D := -\tanh\left(\frac{E}{2kT}\right) g(\omega,\tau_n,E)\ .$$

$$g := \left[1 + \frac{E(E+\hbar\omega)+\Delta^2}{P_1 P_2}\right]\frac{1}{(P_1-P_2)^2+(\hbar/\tau_n)^2} - \left[1 - \frac{E(E+\hbar\omega)+\Delta^2}{P_1 P_2}\right]\frac{1}{(P_1-P_2)^2+(\hbar/\tau_n)^2}$$

$$P_1 := \sqrt{(E+\hbar\omega)^2-\Delta^2} \text{ and } P_2 := \sqrt{E^2-\Delta^2}$$

Δ is the temperature dependent gap

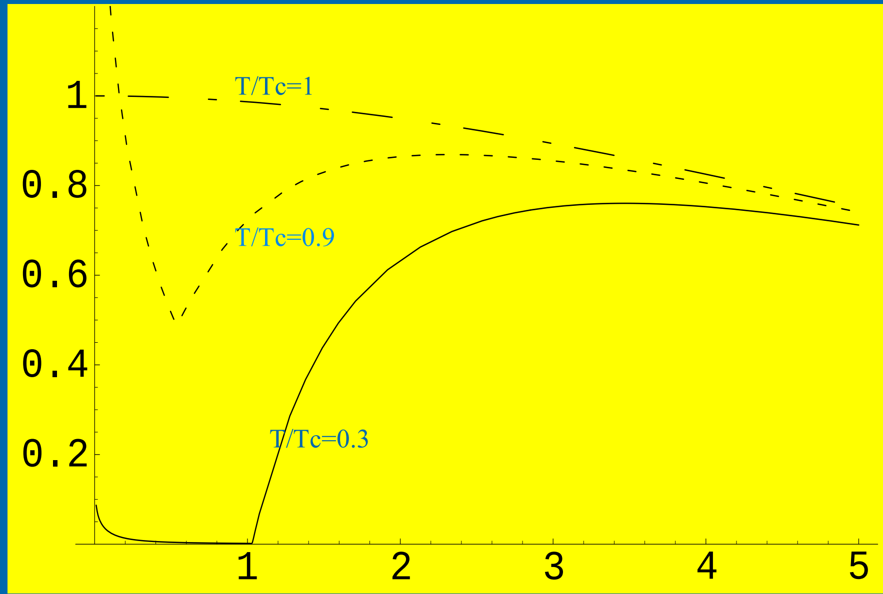


FIG. 1: Plots of $\omega\,\epsilon_s''(\omega)/(\Omega_n^2\tau_n)$, for $T/T_c = 0.3$ (solid line), $T/T_c = 0.9$ (dashed line) and $T = T_c$ (point-dashed line). On the abscissa, the frequency ω is in reduced units $x_0 = \hbar\omega/(2\Delta(0))$

Using dispersion relations $\varepsilon(\imath\zeta)$ can be written in terms of $\mathrm{Im}(\varepsilon(\omega))$

$$\varepsilon(i\zeta) = \frac{2}{\pi}\int_0^\infty d\omega\ \frac{\omega\ \mathrm{Im}(\varepsilon(\omega))}{\zeta^2+\omega^2}$$

ΔE_c is positive.

This is an intuitive result, because one would think that a superconducting mirror is better than a normal one, leading to a stronger Casimir effect, i.e. to a more negative Casimir energy.

TABLE I: Values of $\Delta E^{(C)}$ (in erg) for $T/T_c = 0.9, 0.95, 0.99$, and for three values of τ_n (displayed in the first column). $D = 5$ nm, $L = 10$ nm, $A = 1$ cm^2.

τ_n (sec)	$0.9\ T_c$	$0.95\ T_c$	$0.99\ T_c$
10^{-13}	1.0×10^{-8}	5.6×10^{-9}	1.2×10^{-9}
5×10^{-13}	1.9×10^{-8}	1.0×10^{-8}	2.2×10^{-9}
10^{-12}	2.15×10^{-8}	1.2×10^{-8}	2.5×10^{-9}

As a result we predict a shift of critical field towards larger values

$$E_{cond} = 3.5 \times 10^{-8}\ erg \quad \left(\frac{T}{T_c} = 0.97 \right)$$

The effect should be larger for small L and close to T_c., because

$$\Delta E_c \propto \frac{1}{L^{0.6}} \times T_c \times \left(1 - \frac{T}{T_c} \right)$$

while

$$E_{cond} \propto H_c^2(T) \propto \left(1 - \frac{T}{T_c} \right)^2$$

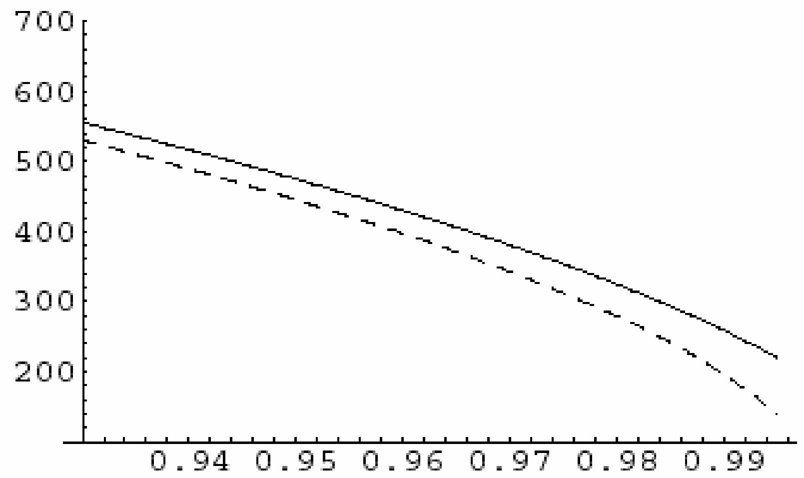


FIG. 2: Comparison between the parallel critical fields of a Be film in a Casimir cavity (solid curve) and a single Be film of same thickness (dashed line), for $0.93 \leq T/T_c \leq 0.995$. The magnetic field is expressed in Oe. $D = 5$ nm, $L = 10$ nm, $\tau_n = 5 \times 10^{-13}$ sec.

Aladin

The experiment ALADIN aiming at at verification of this effect has been recently sponsored by INFN (Gruppo V).

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- L. Rosa
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Conclusions

- We propose to use superconducting cavities to measure variations of Casimir energy across phase transitions. This is a novel approach, since all experiments so far are force measurements.
- Use of rigid cavities allows realization of many different geometries.
- Possibility of checking controversial statements about the sign of the Casimir energy.
- Possibility of an experimental verification of the effects of a form of dark energy on a phase transition