Towards measuring Casimir Energy by a superconducting cavity

G. Bimonte
Napoli 4-2-2005
Summary

- The Casimir Effect (C.E.) in Physics.
- Experiments on the C.E., MEMS.
- The Casimir effect in a superconducting cavity
- The experiment ALADIN
- Conclusions
The Casimir effect is a manifestation of retarded Van der Waals forces between neutral macroscopic bodies, that arise from zero-point quantum fluctuations of the e.m. field.

It is one of the rare quantum phenomena that can be seen on a macroscopic scale.

For two perfectly conducting parallel plates, at $T=0$, Casimir (1948) obtained an attractive force between the plates, of magnitude:

$$F_C = \frac{\pi^2 \hbar c}{240 L^4} A$$

$A$ = area of the plates
$L$ = distance between the plates

The proportionality of $F_C$ to $\hbar$ reveals the quantum origin of the effect.

For $A = 1 \text{ cm}^2$ and $L = 1 \text{ \mu m}$, $F_C = 1.3 \times 10^{-7} \text{ N}$
The scope of the Casimir effect is much broader than this, as it applies to all Quantum Fields under the influence of external boundary conditions.

- **Bag model of hadrons in QCD**: the C.E. Of quarks and gluons make important contributions to the nucleon energy.

- **In Kaluza-Klein models** C.E. Provides an effective mechanism of spontaneous compatification.

- **In Gravitation and Cosmology** (in space-times with nontrivial topology) the vacuum energy resulting from the C.E. Can drive inflation.

- **In Atomic Physics**, the long range Casimir interaction modifies the energy levels of Rydberg states.
In Naples research on Casimir effect goes on since several years. Recently we investigated the possibility of testing experimentally if the Equivalence Principle of General Relativity holds also for vacuum fluctuations: “Do virtual photons have a weight?”

As the Casimir energy of a p.p. cavity is **negative**, it should contribute a **negative weight**!
Some experiments on the Casimir effect

- **Sparnaay (1958)** Spring balance measurement of $F_c$ in plane-parallel geometry. Not conclusive
- **Lamoreaux (1999)** Torsion pendulum measurement of $F_c$ in sphere-plane geometry. 5% error.
- **Mohideen and Roy (1998)** AFM measurement of $F_c$ in sphere-plane geometry. 1% error
- **Bressi et al. (2002)** Plane-parallel geometry. 15% accuracy

The Mohideen-Roy experiment

![Diagram of the experimental setup](image)

**FIG. 1.** Schematic diagram of the experimental setup. Application of voltage to the piezo results in the movement of the plate towards the sphere. The experiments were done at a pressure of 50 mTorr and at room temperature.

**FIG. 4.** The measured average Casimir force as a function of plate-sphere separation for 26 scans is shown as square dots. The error bars show the range of experimental data at representative points. The theoretical Casimir force from Eq. (4) with all corrections is shown as a solid line. The rms deviation between the experiment and theory is 1.6 pN. The dash-dotted line is the Casimir force without the finite conductivity, roughness, or temperature correction [Eq. (1)] which results in a rms deviation of 6.3 pN. The dashed line includes only the finite conductivity correction [Eq. (2)] which results in a rms deviation of 5.5 pN. The dotted line includes only the roughness correction leading to a rms deviation of 48 pN.
Casimir forces are the dominant forces in nanomechanical devices

Recently the first actuator based on the Casimir force was developed by the researchers at Bell labs.


Quantum Mechanical Actuation of Microelectromechanical Systems by the Casimir Force

H. B. Chan, V. A. Aksyuk, R. N. Kleiman, D. J. Bishop, Federico Capasso*

The Casimir force is the attraction between uncharged metallic surfaces as a result of quantum mechanical vacuum fluctuations of the electromagnetic field. We demonstrate the Casimir effect in microelectromechanical systems using a micromachined torsional device. Attraction between a polysilicon plate and a spherical metallic surface results in a torque that rotates the plate about two thin torsional rods. The dependence of the rotation angle on the separation between the surfaces is in agreement with calculations of the Casimir force. Our results show that quantum electrodynamical effects play a significant role in such microelectromechanical systems when the separation between components is in the nanometer range.

Fig. 29. Scanning electron micrographs of (A) the nanofabricated torsional device and (B) a close-up of one of the torsional rods anchored to the substrate. Courtesy of Federico Capasso, Bell Labs, Lucent Technologies.
However, a simple way to obtain the Casimir force between ideal conductors is to consider the associated (free) energy $E_c$, such that:

$$F_c = -\frac{\partial E_c}{\partial L}$$

$E_c$ can be obtained as the zero point energy of the quantized e.m. field within the cavity:

$$E_c = \lim_{\Lambda \to \infty} \left( \sum_i \frac{1}{2} \hbar \omega_i - E_0 \right)$$

where $\omega_i$ are the eigenfrequencies of the e.m. field, $E_0$ is the vacuum energy that would be present in an empty volume equal to the volume of the cavity, and $L$ is a smooth ultraviolet cutoff. The use of a cutoff is physically sensible because at high frequencies all material eventually become transparent.

The computation is easy for a plane-parallel geometry, and gives

$$E_c = -\frac{\pi^2 \hbar c}{720 L^3} A$$

Note the - sign
Though conceptually easy, the computation of $E_c$ for an arbitrary disposition of the perfect conductors is generally very hard. An important feature of the Casimir energy $E_c$, due to the long range nature of the Van der Waals forces, is that it is not an extensive quantity, and so is not proportional to the volume of the system.

$E_c$ depends on the geometry of the cavity!

The dependence on the geometry is clear already in the plane parallel case, as $E_c$ depends separately on $A$ and $L$.

In fact, the dependence on the shape reaches to the point that the Casimir force can be even repulsive, in certain cases. A classic example of this is provided by a perfectly conducting spherical shell (Boyer, 1950). Another nice example is that of a parallelepiped. Here the sign of $F_c$ depends on the ratios among the sides. There is no simple intuitive explanation of the sign of the Casimir energy, as $E_c$ results by taking a difference between two positive infinite numbers.

Whether the Casimir force between real materials can ever be repulsive is in fact controversial (Barton 2000).
Is it possible to measure the Casimir energy, rather than the Casimir force?

Is it possible to observe in a laboratory the influence of the Casimir energy on a phase transition?

Is is possible to study the Casimir effect with rigid cavities, of arbitrary shape?

We think that this is possible by using a superconducting cavity (G.B., E. Calloni, G. Esposito, L. Milano and L.Rosa (2004)).
We consider a rigid cavity containing superconducting plates. For example, a plane-parallel cavity formed by a thin film of a type I superconductor (S), placed between two non superconducting (N) plates.

The idea is simple: Since the Casimir energy depends on the reflective power of the plates, there should be a change $\Delta E_c$ of Casimir energy as soon as the film becomes superconducting, because of the sharp change in the reflective power (in the IR region) of the film.

$\Delta E_c$ should be very small, because the largest contribution to $E_c$ is due to photons of energy $\hbar c/L \approx 20$ eV, while the transition to superconductivity changes the reflective power in the IR region, at the scale $kT_c \approx 10^{-4}$ eV.

Is there a way to measure $\Delta E_c$?
For any $T < T_c$, the difference $\Delta F$ among the free energies of the cavity, when the film is N or S, is the sum of the condensation energy $E_{\text{cond}}$ of the film plus the variation of Casimir energy $\Delta E_c$.

$$\Delta F = E_{\text{cond}} + \Delta E_c$$

$$E_{\text{cond}} \equiv V \left( f_n(T) - f_s(T) \right)$$

$f_{n/s}(T)$ Helmoltz free energy in the n/s state

$V = A \times D$ volume of the film

($E_{\text{cond}}$ and $\Delta E_c$ are both proportional to the area $A$)

A simple direct way to measure $\Delta F$ is by measuring the critical magnetic field $H_c$, that destroys the superconductivity of the film.
Magnetic properties of superconductors

- Meissner effect: s.c. show perfect diamagnets.

- Superconductivity is destroyed by a critical magnetic field $H_c$.

$H_c$ is related to the free-energy difference between the N and the S states:

$$\frac{H_c^2(T)}{8\pi} = f_n(T) - f_s(T)$$

(bulk sample)

$H_c(T)$ follows an approximate Parabolic law

$$H_c(T) = H_c(0) \left[1 - \left(\frac{T}{T_c}\right)^2\right]$$
For our thin film, in parallel field:

\[
\frac{1}{8\pi} \left( \frac{H_c(T)}{\rho} \right)^2 V = E_{\text{cond}} + \Delta E_c
\]

\[\rho = \sqrt{\frac{12\pi}{D}} \left( 1 + \frac{9D^2}{\pi^6 \xi^2} \right)\]

\(\xi = \text{correlation length}, \ \lambda = \text{penetration depth}\)

\(\Delta E_c\) causes a shift of \(H_c\) of the order of:

\[
\frac{\Delta H_c}{H_c} \approx \frac{1}{2} \frac{\Delta E_c}{E_{\text{cond}}}
\]

We can get high sensitivities because \(E_{\text{cond}}\) is very small, and \(E_c\) is large.

For a Be film with \(A=1 \text{ cm}^2\) \(D=5 \text{ nm}\) \(T/T_c=0.97\)

\[E_{\text{cond}} = 3.5 \times 10^{-8} \text{ erg} \quad \left( \frac{T}{T_c} = 0.97 \right)\]

For a cavity with \(A=1 \text{ cm}^2\) \(L=10 \text{ nm}\)

\[E_c = 0.43 \text{ erg}\]

\(E_c\) is 10 million times larger than \(E_{\text{cond}}\)!

So even a tiny fractional change of \(E_c\) can be large compared with \(E_{\text{cond}}\),
and cause a measurable shift of \(H_c\).
Computation of $\Delta E_c$: the Casimir effect in real materials.

The theory for a dispersive cavity was developed by Lifshitz (1956). He computed the Casimir energy by evaluating the v.e.v. of the e.m. stress-energy tensor in the empty space between the bodies. Alternative methods exist today (summation on evanescent modes). The basic assumption of the theory is that one can describe the propagation of e.m. waves inside the plates by a complex permittivity $\varepsilon(\omega, q)$, depending on the frequency $\omega$ and on the wave-vector $q$.

1) Despite the fact that typical cavity modes have short wavelengths ($L=10$ nm) for which space dispersion is important, the computation of $\Delta E_c$ only involves long-wavelength modes of energy $kT_c$, because the reflective power of a superconducting film are indistinguishable from those of a normal film for photon energies larger than a few times $kT_c$ (Glover and Tinkham, 1958). Then, the relevant range of frequencies is the far infrared.

2) In the far IR, space non-local effects are negligible in thin films ($D \ll \xi, \lambda$).
For a film of thickness $D$, placed between two plates, $\Delta E_c$ can be written as:

$$
\Delta E_c = \frac{\hbar A}{4\pi^2 c^2} \int_1^\infty dp \int_0^{\Lambda kT_c} d\zeta \zeta^2 \left( \log \frac{Q_{n}^{TE}}{Q_{s}^{TE}} + \log \frac{Q_{n}^{TM}}{Q_{s}^{TM}} \right)
$$

The expression of the coefficients $Q_{n/s}^{TE/TM}$ for our multilayer system is

$$
Q_{I}^{TE/TM}(\zeta, p) = \frac{1 - \Delta_{11}^{TE/TM} \Delta_{12}^{TE/TM} e^{-2\zeta p L/c} - (\Delta_{11}^{TE/TM} \Delta_{12}^{TE/TM} e^{-2\zeta p L/c})^2 e^{-2\zeta K_T D/c}}{1 - (\Delta_{11}^{TE/TM})^2 e^{-2\zeta K_T D/c}},
$$

$$
\Delta_{I I}^{TE} = \frac{K_j - K_I}{K_j + K_I}; \quad \Delta_{I I}^{TM} = \frac{K_j \epsilon_j(i\zeta) - K_I \epsilon_j(i\zeta)}{K_j \epsilon_j(i\zeta) + K_I \epsilon_j(i\zeta)}, \quad K_j = \sqrt{\epsilon_j(i\zeta) - 1 + p^2}, \quad I = n, s; \quad j, l = 1, 2, n, s.
$$

Note that all permittivities $\epsilon_j(i\zeta)$ are evaluated at imaginary frequencies $i\zeta$.

$\Delta E_c$ has been evaluated numerically.

For the normal plates and for the film in the n state, we used the Drude formula

$$
\epsilon(\omega) = 1 - \frac{\Omega_p^2}{\omega(\omega + i\gamma)}
$$

$\gamma = 1/\tau$ \quad $\tau$=relaxation time

$\Omega_p$ = plasma frequency
For $\varepsilon_s^{\text{film}}(\omega)$ we used the Mattis-Bardeen complex permittivity of BCS theory

$$\varepsilon''(\omega) = \frac{\hbar \Omega_n^2}{2\omega^2 \tau_n} \left[ \int_{\Delta}^{\infty} dE J_T + \theta(\omega - 2\Delta) \int_{\Delta - \omega}^{-\Delta} dE J_D \right]$$

$$J_T := \left[ \tanh \frac{E + \hbar \omega}{2kT} - \tanh \frac{E}{2kT} \right] g(\omega, \tau_n, E)$$

$$J_D := - \tanh \left( \frac{E}{2kT} \right) g(\omega, \tau_n, E) .$$

$$g := \left[ \frac{1 + \frac{E(E + \hbar \omega) + \Delta^2}{P_1 P_2}}{\left( P_1 - P_2 \right)^2 + (\hbar/\tau_n)^2} \right] \left[ \frac{1}{\left( P_1 - P_2 \right)^2 + (\hbar/\tau_n)^2} \right]$$

$P_1 := \sqrt{(E + \hbar \omega)^2 - \Delta^2}$ and $P_2 := \sqrt{E^2 - \Delta^2}$

$\Delta$ is the temperature dependent gap

Using dispersion relations $\varepsilon(i\zeta)$ can be written in terms of $\text{Im}(\varepsilon(\omega))$

$$\varepsilon(i\zeta) = \frac{2}{\pi} \int_0^\infty d\omega \frac{\omega \text{Im}(\varepsilon(\omega))}{\zeta^2 + \omega^2}$$

FIG. 1: Plots of $\omega \varepsilon''(\omega)/(\Omega_n^2 \tau_n)$, for $T/T_c = 0.3$ (solid line), $T/T_c = 0.9$ (dashed line) and $T = T_c$ (point-dashed line). On the abscissa, the frequency $\omega$ is in reduced units $\omega_0 = \hbar \omega/(2\Delta(0))$
$\Delta E_c$ is positive.

This is an intuitive result, because one would think that a superconducting mirror is better than a normal one, leading to a stronger Casimir effect, i.e. to a more negative Casimir energy.

As a result we predict a shift of critical field towards larger values

The effect should be larger for small $L$ and close to $T_c$, because

$$\Delta E_c \propto \frac{1}{L^{0.6}} T_c \times \left(1 - \frac{T}{T_c}\right)$$

while

$$E_{\text{cond}} \propto H_c^2(T) \propto \left(1 - \frac{T}{T_c}\right)^2$$

TABLE I: Values of $\Delta E^{(C)}$ (in erg) for $T/T_c = 0.9$, 0.95, 0.99, and for three values of $\tau_n$ (displayed in the first column).

<table>
<thead>
<tr>
<th>$\tau_n$ (sec)</th>
<th>0.9 $T_c$</th>
<th>0.95 $T_c$</th>
<th>0.99 $T_c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^{-13}$</td>
<td>$1.0 \times 10^{-8}$</td>
<td>$5.6 \times 10^{-9}$</td>
<td>$1.2 \times 10^{-9}$</td>
</tr>
<tr>
<td>$5 \times 10^{-13}$</td>
<td>$1.9 \times 10^{-8}$</td>
<td>$1.0 \times 10^{-8}$</td>
<td>$2.2 \times 10^{-9}$</td>
</tr>
<tr>
<td>$10^{-12}$</td>
<td>$2.15 \times 10^{-8}$</td>
<td>$1.2 \times 10^{-8}$</td>
<td>$2.5 \times 10^{-9}$</td>
</tr>
</tbody>
</table>

$$E_{\text{cond}} = 3.5 \times 10^{-8} \text{ erg} \quad \left(\frac{T}{T_c} = 0.97\right)$$

FIG. 2: Comparison between the parallel critical fields of a Be film in a Casimir cavity (solid curve) and a single Be film of same thickness (dashed line), for $0.93 \leq T/T_c \leq 0.995$. The magnetic field is expressed in Oe. $D = 5 \text{ nm}$, $L = 10 \text{ nm}$, $\tau_n = 5 \times 10^{-13} \text{ sec.}$
The experiment ALADIN aiming at verification of this effect has been recently sponsored by INFN (Gruppo V).

- G. Bimonte
- E. Calloni
- A. Cassinese
- F. Chiarella
- G. Esposito
- L. Rosa
- F. Tafuri
- R. Vaglio
Conclusions

- We propose to use superconducting cavities to measure variations of Casimir energy across phase transitions. This is a novel approach, since all experiments so far are force measurements.
- Use of rigid cavities allows realization of many different geometries.
- Possibility of checking controversial statements about the sign of the Casimir energy.
- Possibility of an experimental verification of the effects of a form of dark energy on a phase transition.