Napoli-17.12.04

A unitary S-Matrix up to the black-hole production threshold

based on hep-th/0410166 & on much old stuff

The problem at hand

(Super)-Planckian-energy collisions of light particles within a consistent quantum theory of gravity (superstring theory) in d = D-1 “large” dimensions

Since superstring theory is essentially an S-matrix theory, the description will be, naturally,

quantum and unitary

Three reasons for going about it

1. Weinberg's: "Because I can!"
2. Theoretical: Information paradox:
   - Is the S-matrix description breaking down in some regime?
   - And, if not, is the final state (close to) thermal?
3. "Phenomenological": finding signatures of string/quantum gravity @ future colliders:
   - In KK models with large extra dimensions
   - In brane-world scenarios

NB. Future colliders will be at best marginal for producing BHs!

Outline

1. General considerations and two complementary approaches
2. The phase diagram of super-Planckian collisions
3. A unitary S-matrix for regions I and III
4. Analysis of the final state, "anti-scaling"
5. Implications for the information paradox
6. Summary

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- With very few exceptions, we do not have much of a handle on string theory in extreme regimes (strong coupling, strong curvature)
- Superplanckian collisions may help as gedanken experiments by providing:
  - A way to find out how QST is able to reproduce expectations from CGR at large distances
  - A way to find out how QST modifies gravity at short distances
- Two complementary approaches (from 1987 on):
  1. Gross-Mende + Mende-Ooguri (1987-1990)
  2. 't-Hooft; Muzinich & Soldate; Amati, Ciafaloni & GV; Verlinde & V.; FPVV… Arcioni, de Haro …(1987-'04)

Gross-Mende-Ooguri (GMO)

- Genus by genus (i.e. loop by loop) calculation (GM, 1987-'88) of elastic scattering at very high energy and fixed sc. angle θ (h = number of loops):

$$A_{el} \sim (g_s)^{2+2h} \exp\left(-\frac{\alpha' s f(\theta)}{1+h}\right)$$

(from complex saddle trajectory)

All genus resummation (MO, 1990) only justified in an energy window, fails at infinite energy

$$\log(1/g_s^2) < \frac{s}{M_s^2} < \log^3(1/g_s^2)$$

(g_s << 1)

Small, probably too conservative (result given below)

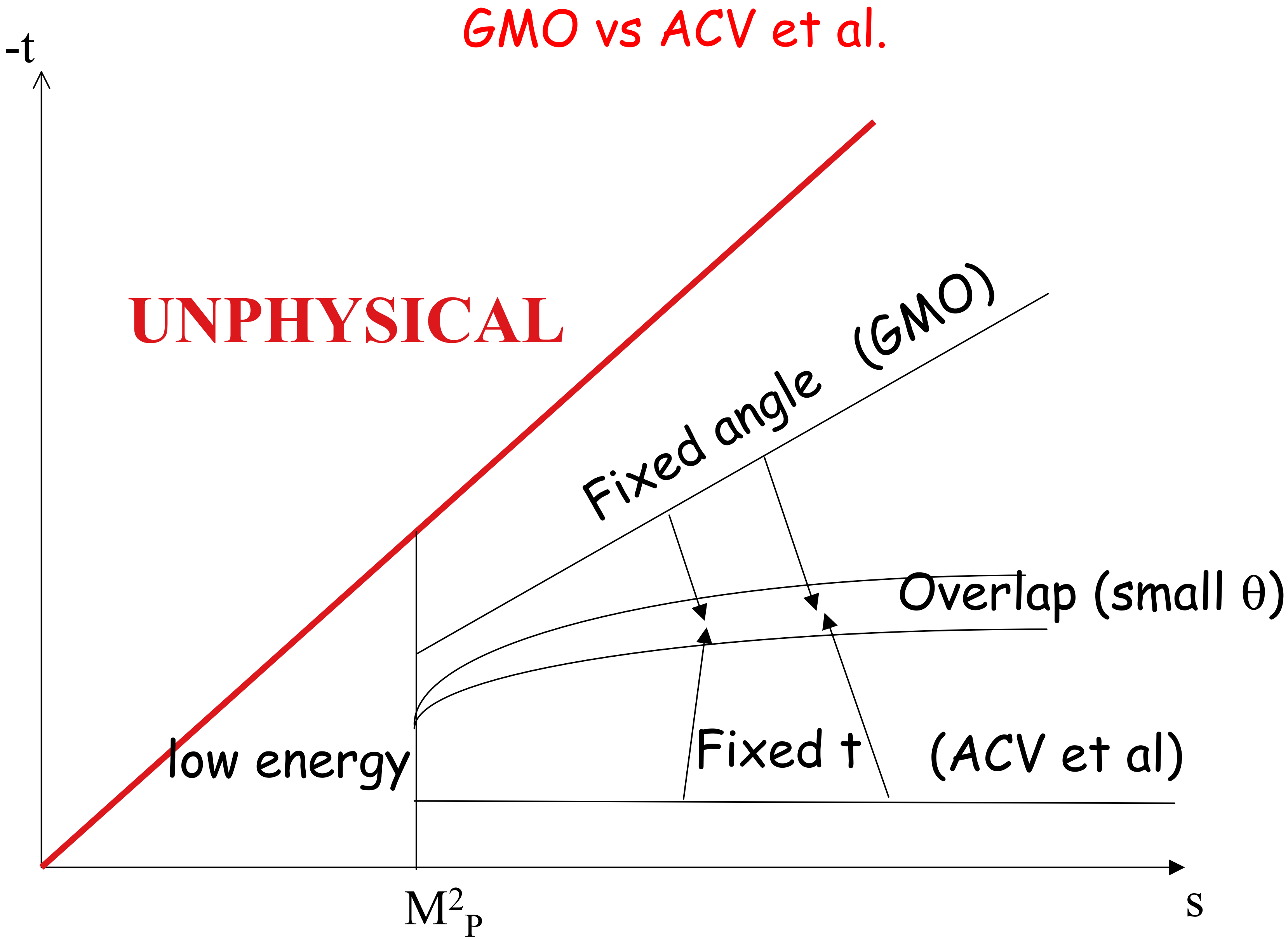
Amati, Ciafaloni, GV (ACV) et al.

- Work in energy-impact parameter space, A(E,b)
- Can go to arbitrarily high energy provided b is also increased accordingly,

$$b > R_S(E) \sim (G_N E)^{\frac{1}{D-3}}$$

- One then goes over to A(E, q~ θ E) by FT and trusts (leading?) contributions coming from the above region of b. In this way one can reach the regime of fixed $\theta << 1$

- In gravity, fixed θ scattering at very high E is dominated by large distance physics (opposite of QCD!). Reason explained below..


GMO vs ACV et al.
-t
UNPHYSICAL
Fixed angle (GMO)
Overlap (small θ)
low energy
Fixed t
(ACV et al)
M²P
s

The comparison (to which we shall come back) is quite striking

$$A_{GMO}(s,\theta) \sim exp\left(-l_s q\sqrt{log(1/\theta^2)log(1/g_s^2)}\right)$$

$$A_{ACV}(s,\theta) \sim exp\left(-l_s q\sqrt{log(\alpha' s)log(1/g_s^2)}\right)$$

Cf. tree level fixed t vs. fixed, small θ

$$(\alpha' s)^{\alpha' t} \; vs. \; exp(\alpha' t\, log(1/\theta^2)$$

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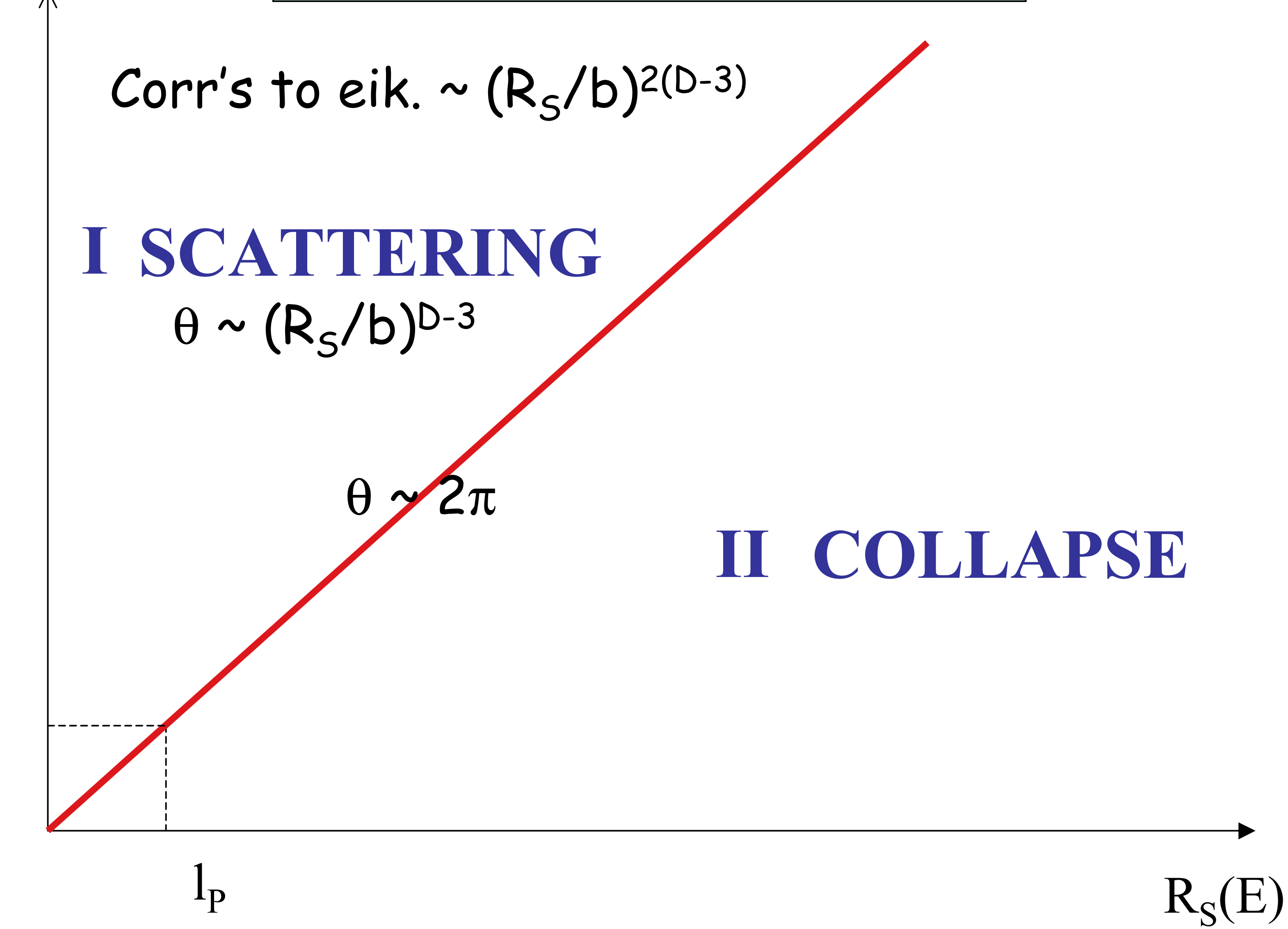
WITHOUT STRING THEORY



CGR arguments for Collapse @ b < R_S

- Penrose 1974 (unpublished)
- CTS arguments:
  1. Eardley and Giddings, gr-qc/0201034,
  2. Giddings and Rychkov, hep-th/0409131
- Finite-size effects:
  1. Yurtsever, 1988
  2. Kohlprath and GV, gr-qc/0203093

$$R_S > b + l_s$$

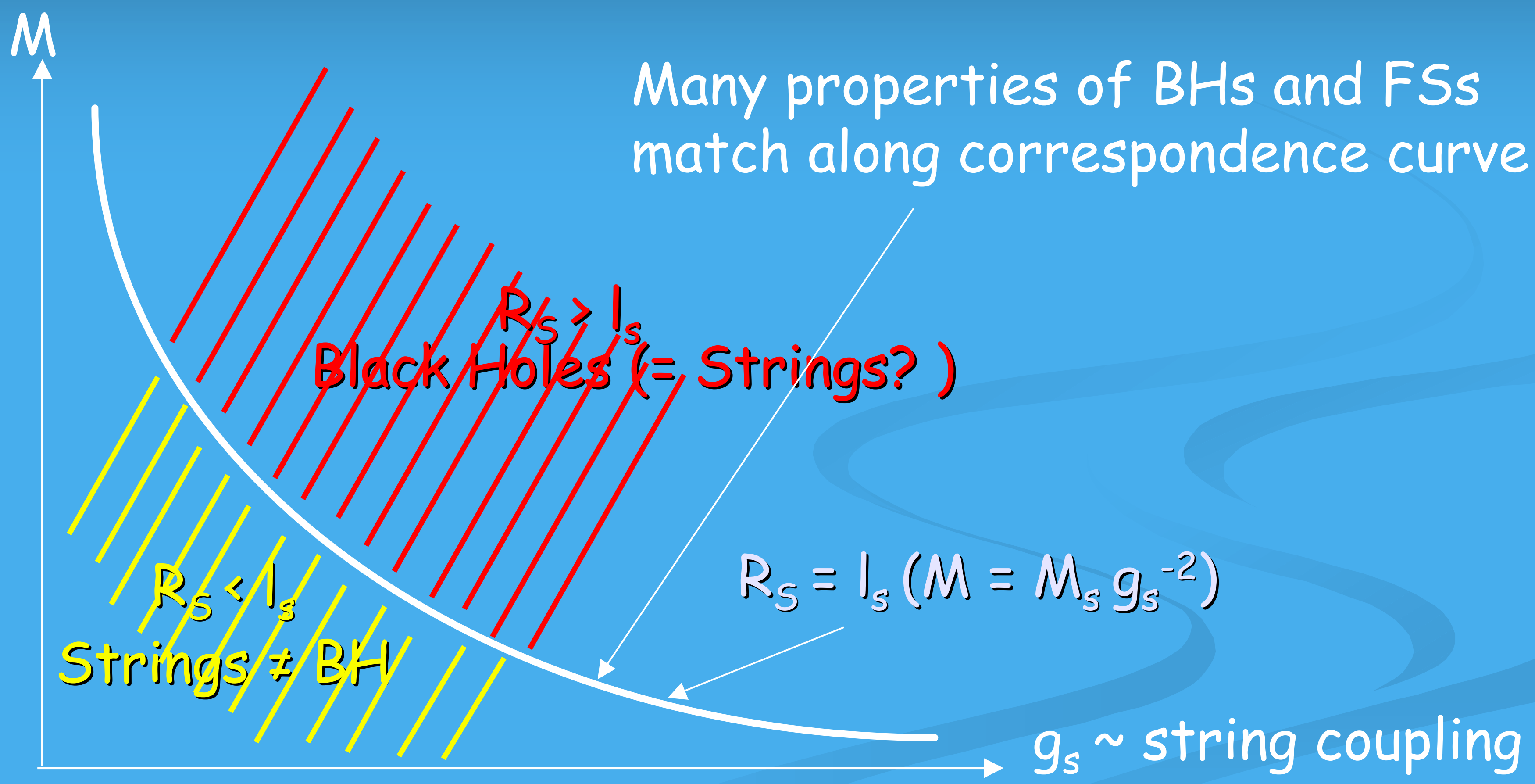
- In string theory the collapse criterion should be amended!

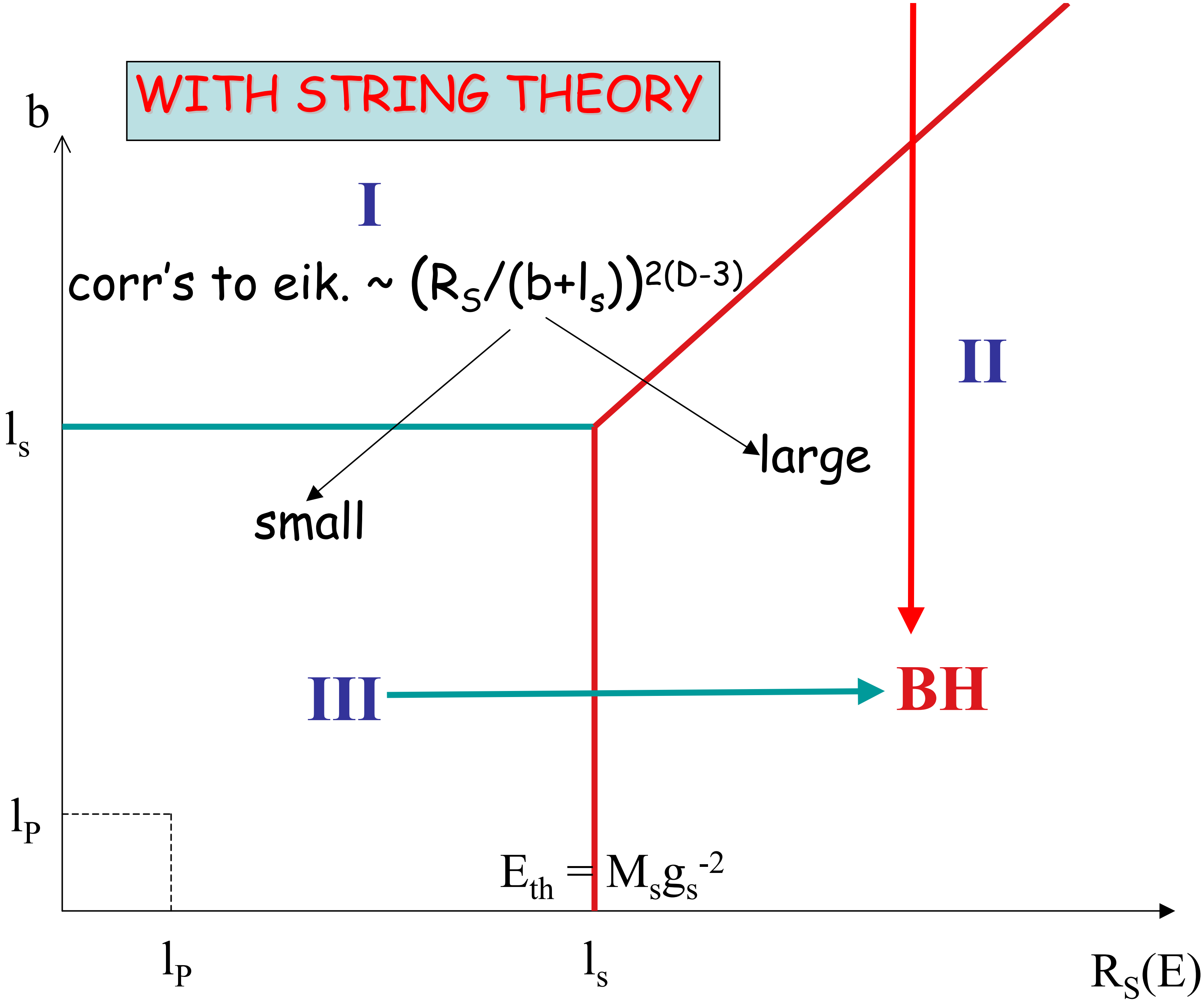
We shall take the string coupling fixed and very small (g_s « 1). Our defs.

$$l_s = \sqrt{2\alpha'} = M_s^{-1}\,,\ 8\pi G_D = l_D^{D-2} = M_D^{2-D}\,, \hbar = 1$$

$$l_s = (g_s)^{-\frac{2}{D-2}} l_D \gg l_D\,,\ i.e.\, M_D = M_s (g_s)^{-\frac{2}{D-2}} \gg M_s$$

String/Black-Hole correspondence





WITH STRING THEORY
b
I
corr's to eik. ~ $(R_S/(b+l_s))^{2(D-3)}$
II
l_s
large
small
III
BH
l_P
$E_{th} = M_s g_s^{-2}$
l_P
l_s
$R_S(E)$

Three regimes in super-Planckian scattering

- I) Small angle scattering (relatively easy)
- II) Large angle and collapse (very hard, all attempts have failed so far)
- III) Stringy (easy again) This is where GMO and ACV can be compared with amazingly good agreement given the completely different approaches (q~ θ E)

$$A_{GMO}(s,\theta) \sim exp\left(-l_s q\sqrt{log(1/\theta^2)log(1/g_s^2)}\right)$$

$$A_{ACV}(s,\theta) \sim exp\left(-l_s q\sqrt{log(\alpha' s)log(1/g_s^2)}\right)$$

Cf. tree level fixed t vs. fixed, small θ

$$(\alpha' s)^{\alpha' t} \text{ vs. } exp(\alpha' t\, log(1/\theta^2))$$

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Approximate (but exactly unitary) S-matrix in regions I and III

Operator formula encoding previous ACV results:

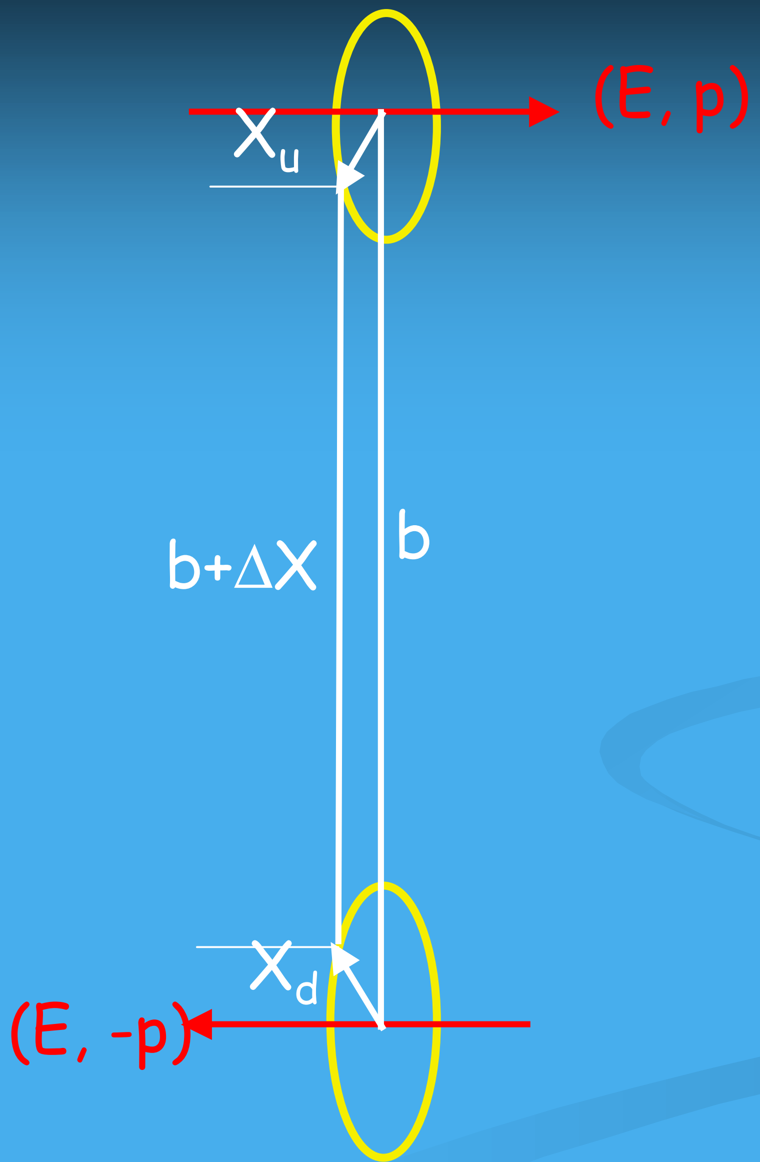
$$S = exp(i\hat{I})\,, \hat{I} = (\hat{\delta} + \hat{\delta}^{\dagger}) + \sqrt{-2i(\hat{\delta} - \hat{\delta}^{\dagger})}(C + C^{\dagger}) = \hat{I}^{\dagger}$$

$$[C, C^{\dagger}] = 1\ ,\ [\hat{\delta}, \hat{\delta}^{\dagger}] = [C, \hat{\delta}] = [C, \hat{\delta}^{\dagger}] = 0$$

$$S = e^{2i\hat{\delta}} e^{i\sqrt{-2i(\hat{\delta}-\hat{\delta}^{\dagger})}\ C^{\dagger}} e^{i\sqrt{-2i(\hat{\delta}-\hat{\delta}^{\dagger})}\ C}$$

$$\delta(E, b) = \frac{1}{(2\pi)^{D-2}} \int d^{D-2}q \frac{A_{tree}(s,t)}{4s}\ e^{-iqb}\ ,\ s = E^2,\ t = -q^2$$

$$\hat{\delta}(E, b) = \frac{1}{4\pi^2} \int_0^{2\pi} \int_0^{2\pi} d\sigma_u d\sigma_d : \delta(E, b + \hat{X}_u(\sigma_u) - \hat{X}_d(\sigma_d)) :$$



(E, p)
X_u
b+ΔX
b
X_d
(E, -p)

Inserting tree-level amplitude (and forgetting ^s!) we get

$$Im\delta \sim \frac{G_D\, s\, l_s^2}{(Y l_s)^{D-2}} e^{-b^2/b_I^2}, \; b_I^2 \equiv l_s^2 Y^2, \; Y = \sqrt{log(\alpha' s)}$$

Therefore, for b >> b_I (Region I), we can forget about C, C^+. In this region we also find:

$$\delta = \left(\frac{b_c}{b}\right)^{D-4} \equiv G_D\, s\, b^{4-D}$$

Inserting this phase shift, and going over to scattering angle θ, we find a saddle point at

$$b_s^{D-3} = \frac{8\pi G_D \sqrt{s}}{\Omega_{D-2}\theta} \qquad \theta = \frac{8\pi G_D \sqrt{s}}{\Omega_{D-2} b^{D-3}}$$

corresponding precisely to the relation between impact parameter and scattering angle in the (AS) metric of a relativistic particle: clearly, fixed θ , large E probe large b

- This also explains why
- Because of eikonal exponentiation, Re δ also gives the average loop-number. Thus the total huge momentum transfer $q = \theta E$ is shared among Re δ gravitons to give:

$$q_{ind.} = \frac{q}{Gsb_s^{4-D}} \sim \frac{\theta}{R_S^{D-3} b_s^{4-D}} \sim 1/b_s$$

meaning that the process is soft at large b_s

- Lesson: while in QCD it is better to get a large transverse momentum via the exchange of as few gluons as possible, in QSG it is better to share it among as many as possible gravitons!

We still have to take into account the operators in δ

Physically, they describe "diffractive excitation" (DE) via graviton exchange. As a result, the elastic amplitude is found to be suppressed:

$$|A_{el}| \sim e^{-2\delta_{DE}} \, , \; \delta_{DE} = \frac{G_D \, s \, l_s^2}{b^{D-2}}$$

This introduces a second critical value of b, b_{DE}, below which diffractive excitation takes over the elastic process:

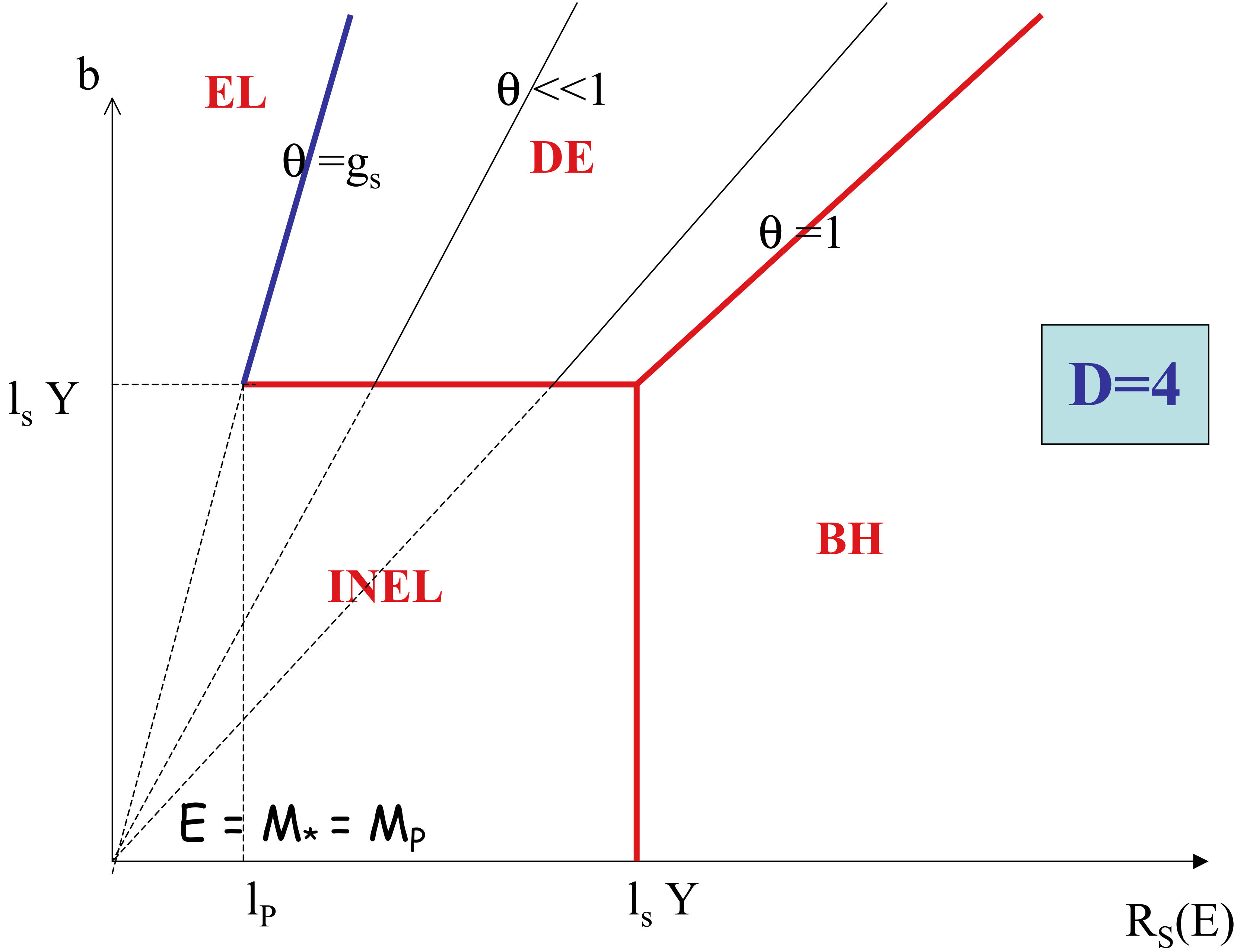
$$b_{DE}^{D-2} = G_D s l_s^2$$

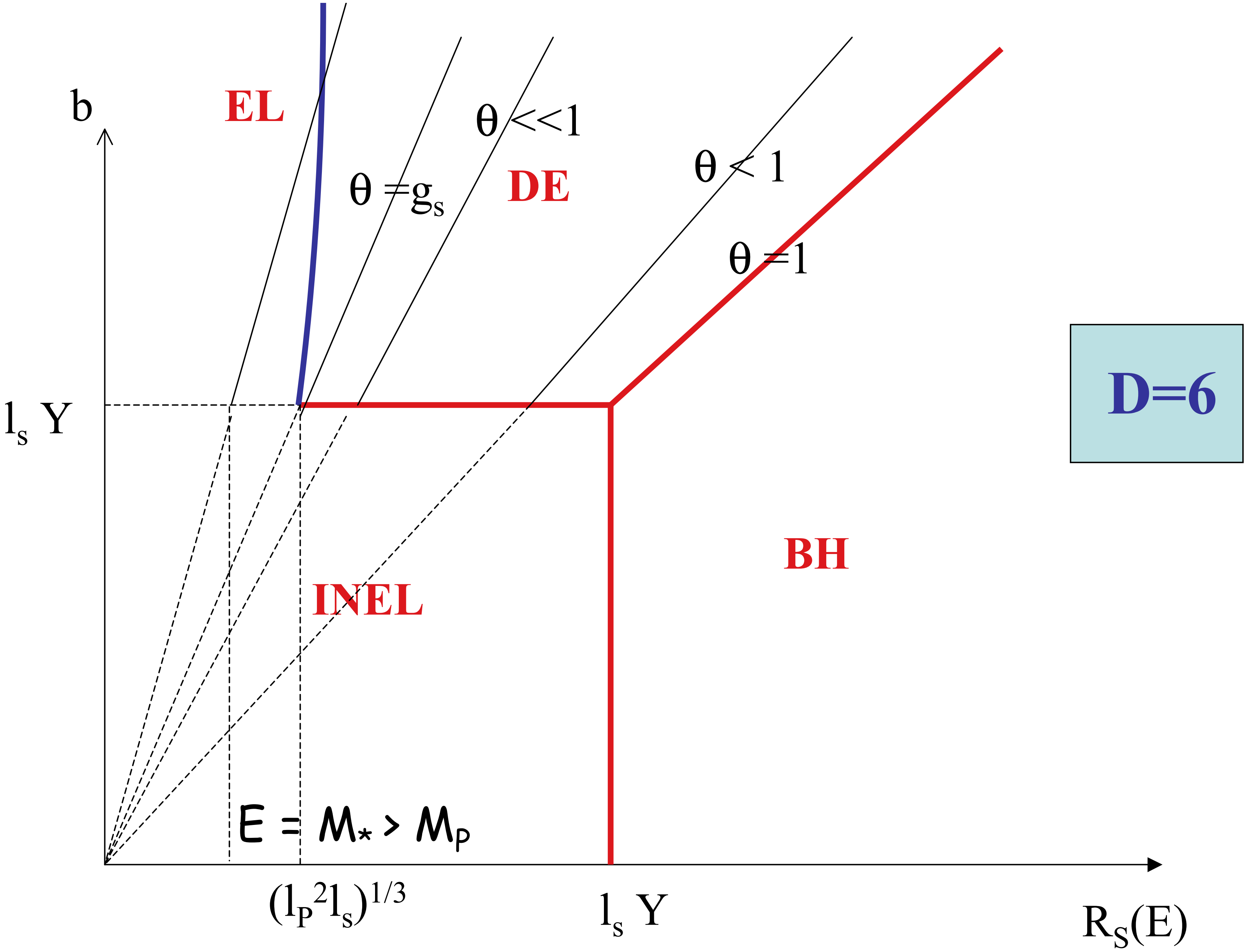
$$b_{DE} = b_I \; at \; s = M_*^2 \equiv M_s^2 / g_s^2$$

NB: $M_*^2 \sim M_s \, M_{th}$

This new scale M_* ($M_* = M_P$ in D=4, $M_* > M_P$ for D>4) will play an important role in the following. Does it have a deep meaning?

For the moment let us look at what it does to our ph. diagram


b
EL
$\theta = g_s$
$\theta << 1$
DE
$\theta = 1$
$l_s Y$
D=4
BH
INEL
$E = M_* = M_P$
l_P
$l_s Y$
$R_S(E)$



b
EL
θ <<1
θ =g_s
DE
θ < 1
θ =1
l_s Y
D=6
BH
INEL
E = M* > M_P
(l_P^2 l_s)^{1/3}
l_s Y
R_S(E)

Region III

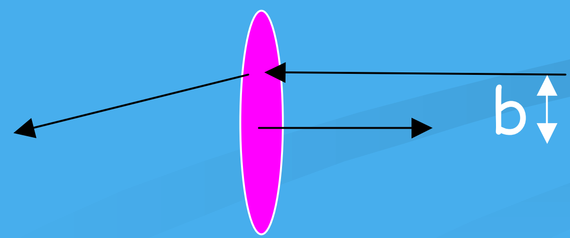
Let us forget for a moment that Im $\delta \neq 0$, C and C^+

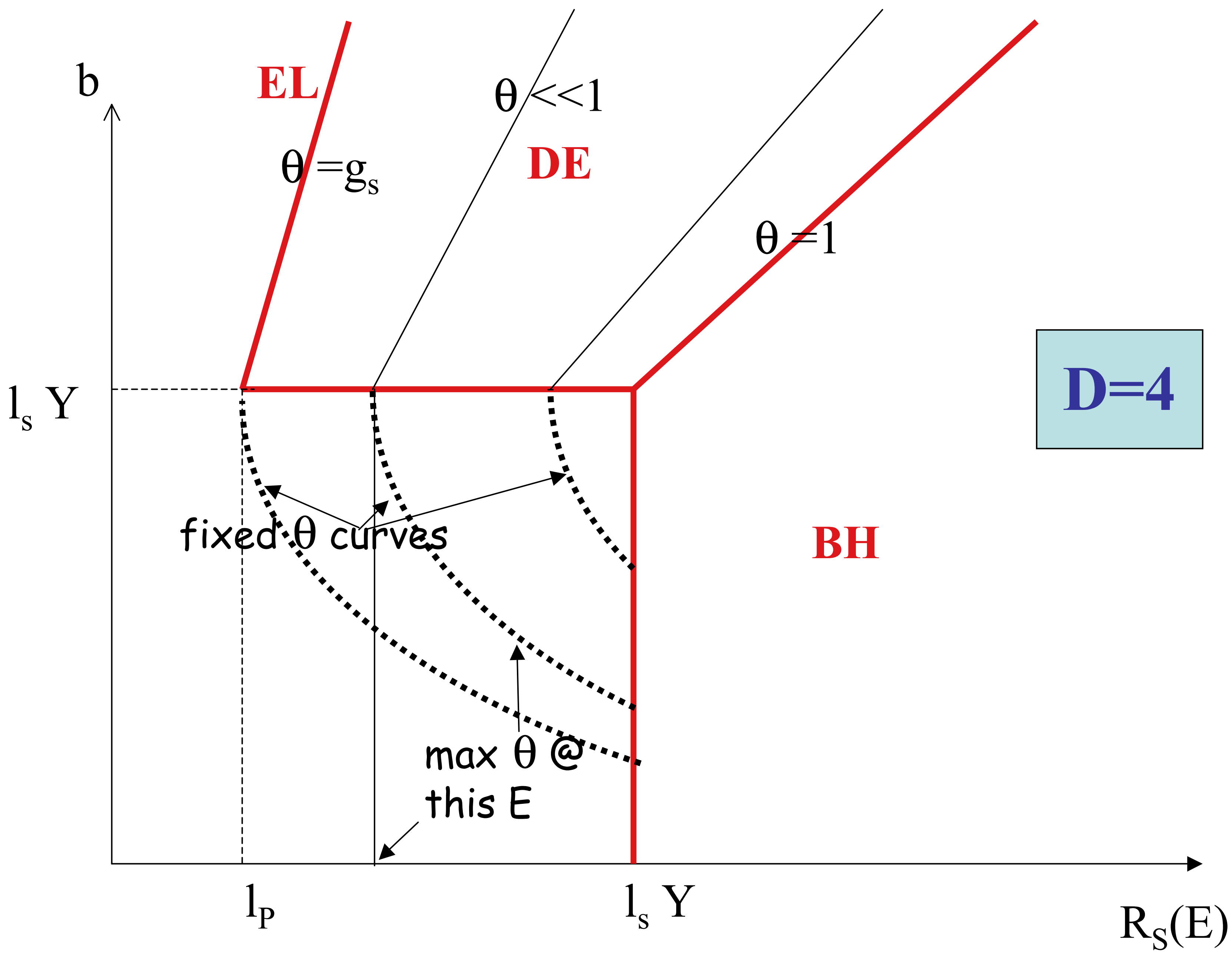
$$Re\delta = -\frac{G_D\, s\, b^2}{(Y l_s)^{D-2}}$$

The saddle point condition now gives the relation:

$$\theta = G_D\, \rho\, b\ ,\ \rho = \frac{E}{(Y l_s)^{D-2}}$$

corresponding to deflection from an homogeneous beam of transverse size $l_s Y$:





b
EL
θ <<1
θ =g_s
DE
θ =1
D=4
l_s Y
fixed θ curves
BH
max θ @
this E
l_P
l_s Y
R_S(E)

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Region III: diffractive excitation

While still neglecting Im $\delta \neq 0$ (C and C^+), let us consider DE
Like in region I one finds:

$$|A_{el}| \sim e^{-2\delta_{DE}}$$

but instead of $\delta_{DE} = \frac{G_D\, s\, l_s^2}{b^{D-2}}$ we now find

$$\delta_{DE} = \frac{G_D\, s\, l_s^2}{(l_s\, Y)^{D-2}}\,,\ b < b_I$$

In other words, diffractive absorption saturates at $b = b_I$

Region III: effects of Im $\delta \neq 0$

The operators C and C^+ are now "activated" , recall:

$$S = e^{2i\hat{\delta}} e^{i\sqrt{-2i(\hat{\delta}-\hat{\delta}^\dagger)}\, C^\dagger} e^{i\sqrt{-2i(\hat{\delta}-\hat{\delta}^\dagger)}\, C}$$

The elastic amplitude, <0|S|0>, is now suppressed as exp(-2 Im δ) and therefore:

$$\sigma_{el} \sim exp(-4Im\delta) = exp\left[-\frac{G_D\, s\, l_s^2}{(Y l_s)^{D-2}}\right]$$

This damping adds to the one already discussed due to DE:

$$|A_{el}| \sim e^{-2\delta_{DE}} \qquad \delta_{DE} = \frac{G_D\, s\, l_s^2}{(l_s Y)^{D-2}}\,,\; b < b_I$$

and combined effect simply doubles the exponent! Note that the exponent is of order $-(s/M_*^2) = g_s^{-2} = S$ @ $E = E_{th}$

Which final states saturate unitarity?

Recall once more:

$$S = e^{2i\hat{\delta}} e^{i\sqrt{-2i(\hat{\delta}-\hat{\delta}^{\dagger})}\, C^{\dagger}} e^{i\sqrt{-2i(\hat{\delta}-\hat{\delta}^{\dagger})}\, C}$$

➔The final state, S|0>, is a coherent state of quanta associated with C, C⁺. What are they? In order to arrive at above expression for S one had to use the AGK rules of Gribov's Reggeon Calculus: C (C⁺) annihilates (creates) a cut gravi-Reggeon*) (CGR) The probability of producing n CGRs obeys a Poisson distribution with an average given by:

$$\langle N_{CGR} \rangle = 4 Im\delta = \frac{G_D\, s\, l_s^2}{(Y l_s)^{D-2}} = O\left(\frac{s}{M_*^2}\right)$$

*) The GR is the stringy graviton; a CGR is whatever is dual to it in the sense of old DHS duality

At this point we can compute the average energy of a final state associated with a single CGR:

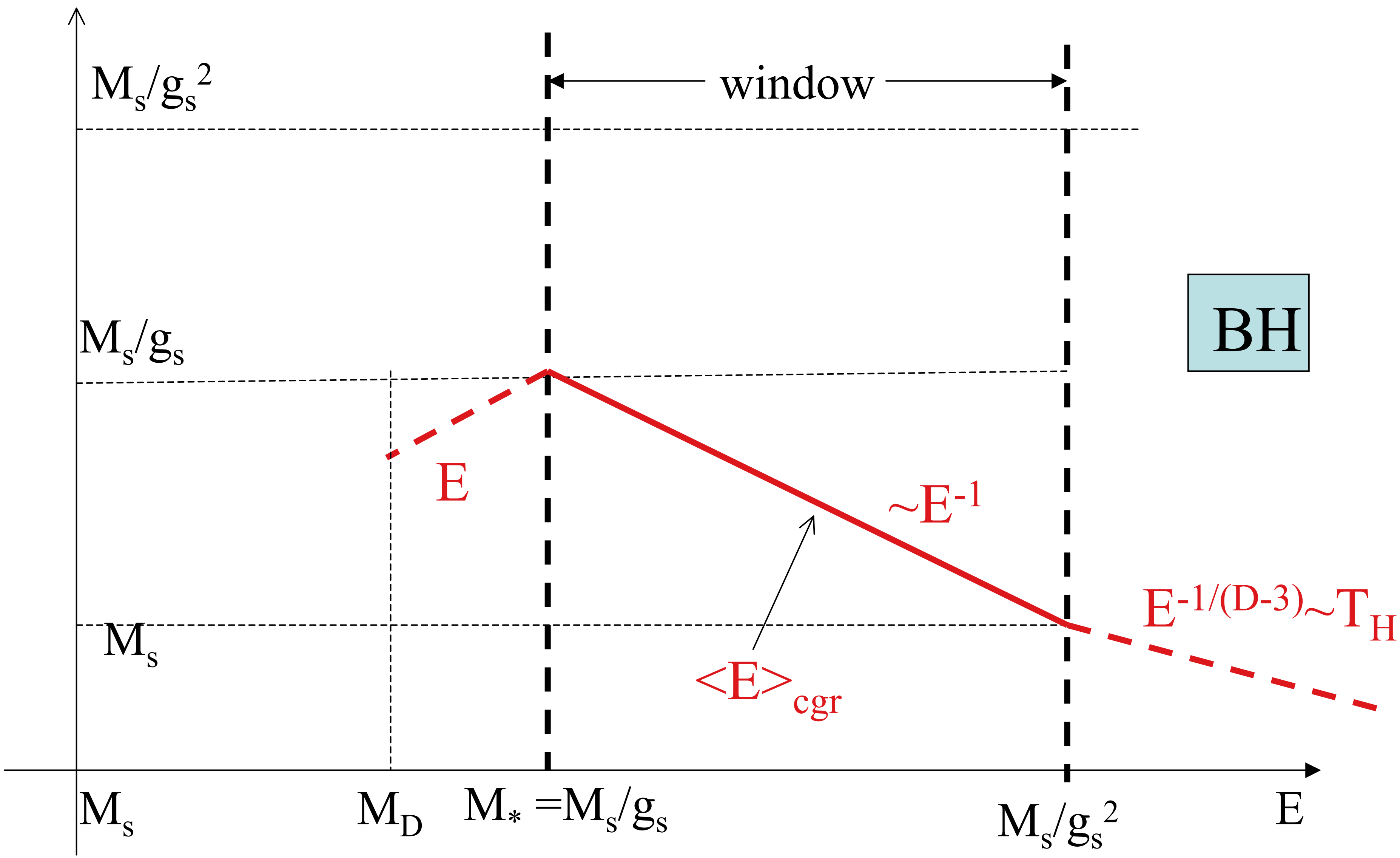
$$\langle E \rangle_{CGR} = \frac{\sqrt{s}}{\langle N_{CGR} \rangle} \sim M_s Y^{D-2} \left(\frac{l_s}{R_S} \right)^{D-3} \sim T_{eff} \equiv \frac{M_*^2}{E} = \frac{M_s^2}{g_s^2 E}$$

We have thus found that the final-state energies obey a sort of «anti-scaling» law

$$\langle E \rangle_{CGR} \sqrt{s} = M_*^2 = M_s^2 g_s^{-2}$$

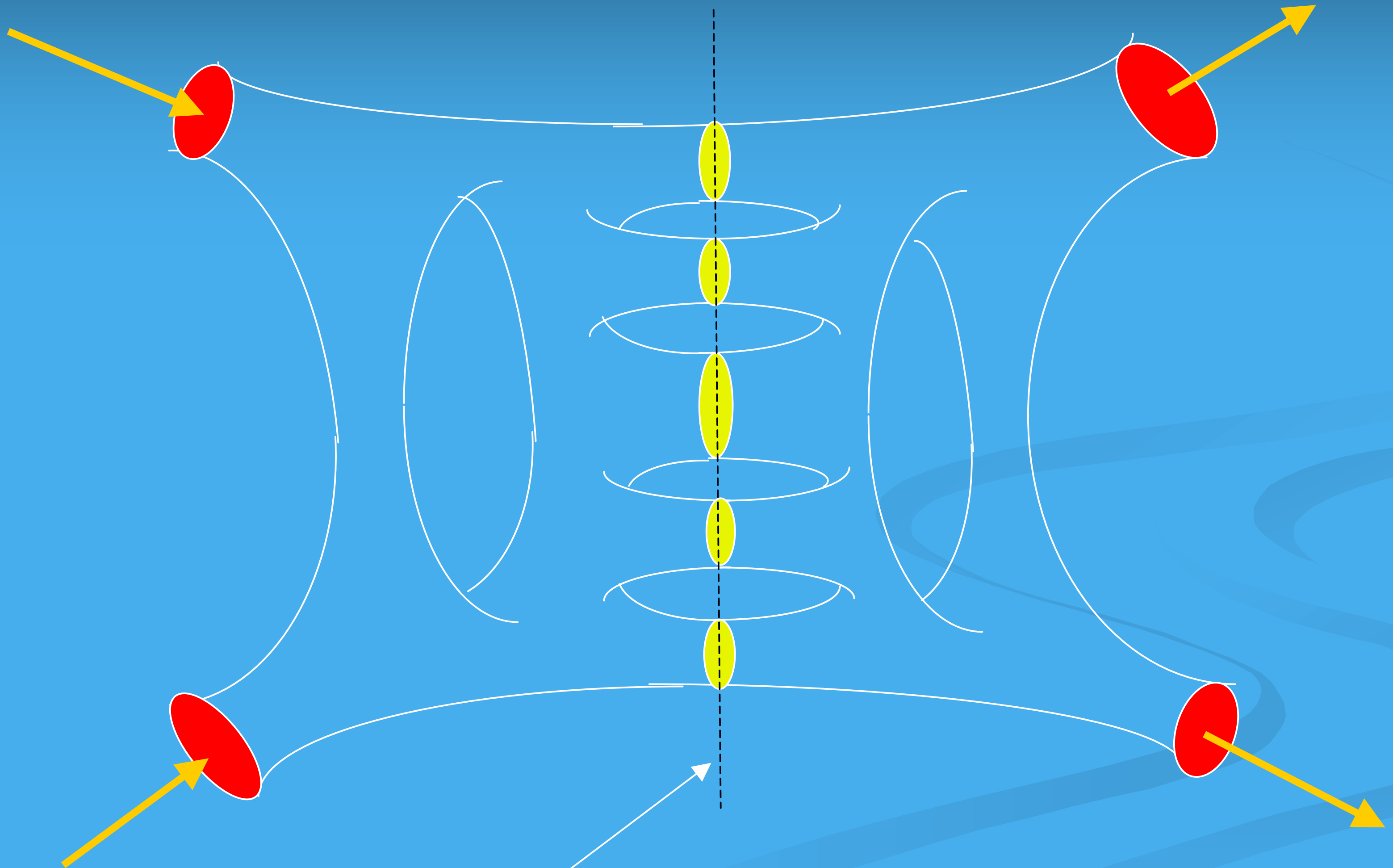
This antiscaling is very unlike what we are familiar with in HEP

It is however similar to what we expect in BH physics!
In particular: For D=4 $T_{eff} \sim T_{Haw}$ even at $E < E_{th}$, while for D>4 $T_{eff} \to T_{Haw}$ for $E \to E_{th}$



window
M_s/g_s^2
M_s/g_s
M_s
BH
E
$\sim E^{-1}$
$E^{-1/(D-3)} \sim T_H$
$<E>_{cgr}$
M_s
M_D
$M_* = M_s/g_s$
M_s/g_s^2
E

Typical final state via the optical theorem



Unitarity cut through 5 CGRs

An interesting question raised by S. Giddings (p.c.)

GMO (and also ACV in the region of overlap) had found:

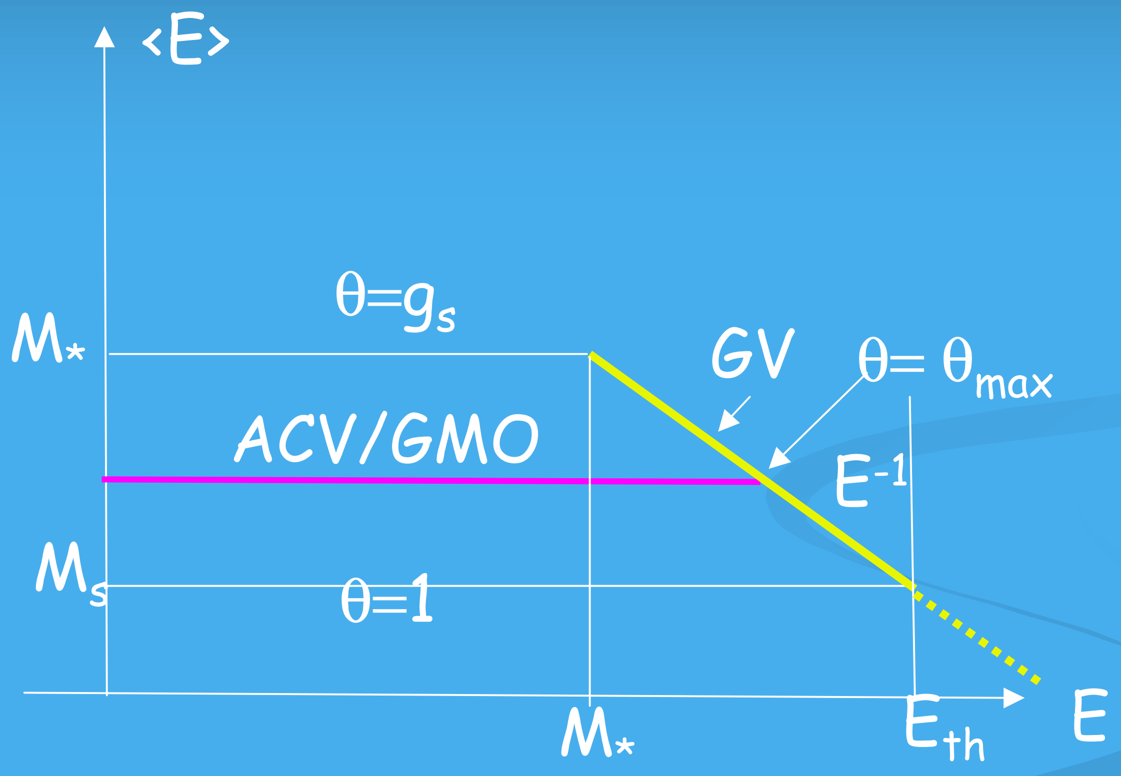
$$\langle 1+h \rangle \sim E$$

And NOT ~ E^2 as I claimed at fixed b < b_I. The answer is simple and instructive. Actually, in the energy window:

$$\langle 1+h \rangle_{GMO/ACV} \sim \frac{\theta}{\theta_{max}} \langle 1+h \rangle_{GV} > \langle 1+h \rangle_{GV}$$

since in region being considered $\theta > \theta_{max}$

On the other hand, $\theta_{max} \sim E$ explains the different E-dependence



<E>
θ=g_s
M_*
GV
θ= θ_max
ACV/GMO
E^-1
M_s
θ=1
M_*
E_th
E

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1. Our results show that, at least (much) below E_{th}, there is no loss of quantum coherence. However the spectra are not thermal
2. When we go above E_{th} we can no-longer neglect “classical” corrections. They correspond to interactions among our CGRs: hopefully, they will turn their Poisson distribution into a (approximately) thermal one for their decay products, but there is no reason to expect a breakdown of unitarity
3. The diffractive states may carry out conserved global quantum numbers (if there are any)

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Summarizing the main points:

- We have been able to recast the main results of ACV in the form of an approximate, but exactly unitary, S-matrix whose range of validity covers a large region of the kinematic energy--angular-momentum plane;

- We have studied the nature of the dominant final states in a window of energy and impact parameter at whose boundary we expect black-hole formation to begin;

- We have found a sort of precocious black-hole behaviour, in particular an ``anti-scaling" dependence of the average energy of the final particles from the initial energy, quite reminiscent of the inverse relation between black-hole mass and temperature;

- This anti-scaling behaviour introduces, through the variable $x = \omega E / M_*^2$, a new energy scale $M_* = M_s / g_s$, whose physical origin we have tried to trace back

- These results may have a twofold application:
  - a conceptual one within the search for an explicit resolution of the information paradox,
  - a more phenomenological one in the context of the string/quantum-gravity signals expected at colliders in models with large extra dimensions.