Origine delle perturbazioni cosmologiche e campi scalari leggeri

> Filippo Vernizzi Helsinki Institute of Physics

Origine delle perturbazioni cosmologiche e campi scalari leggeri



Filippo Vernizzi Helsinki Institute of Physics

Cosmological perturbations

Primordial cosmological perturbations are the origin of the structure that we observe today



Cosmic microwave background



CMB physics

Large scales: gravitational potential on the *last scattering surface* + time dependence of the gravitational potential Ψ~10⁻⁵.



Adiabaticity

Position of acoustic peak sensitive on the ``equation of state'' of perturbations: **adiabatic perturbations**

$$\frac{\Delta T}{T} = A\cos(kc_s\eta_{dec}), \quad k \sim \ell$$

Limits on ``isocurvature" perturbations





Adiabatic vs isocurvature

Adiabatic:

Perturbation affecting all the cosmological species such that

$$\delta\left(n_{DM} / n_{rad}\right) = 0$$

It is thus associated with a *curvature perturbation*:

$$\psi \approx \frac{\delta \left(\rho_{DM} + \rho_{rad}\right)}{\rho_{DM} + \rho_{rad}}$$



Isocurvature:

Perturbations in the fluid components that does not perturb the geometry

$$\delta(\rho_{DM} + \rho_{rad}) = 0$$

It is thus associated with a *relative entropy perturbation:*

$$\delta\left(n_{_{DM}}\,/\,n_{_{rad}}\right) \neq 0$$

Scale invariance and gravity waves

Scale invariance

Dependence of $\Psi(k)$:

$$|\Psi(k)|^2 k^3 \approx k^{n_s-1} \approx \text{const}$$

n_s=0.99±0.4 (95%)

• Gravity waves

Primordial gravity waves may be present (see inflation). Normalization of temperature anisotropies is sensitive to gravity waves:

 $r = \frac{\text{tensor amplitude}}{\text{scalar amplitude}} < 0.71$ (95%)

Limit on gravity waves. Gravity waves detection: polarization!

Tensor/scalar - spectral index

Polarization

At last scattering, unpolarized *quadrupolar* radiation gets Thomson scattered into **polarized radiation**: *direct snapshot of the conditions at last scattering*.

$$\frac{d\sigma_T}{d\Omega} \propto |\hat{\epsilon} \cdot \hat{\epsilon}'|^2$$

Quadrupolar anisotropies of incoming wave generates linear polarization of the **outgoing wave: ~ velocity field of the fluid**

$$\frac{\Delta T}{T}\Big|_{T} = A\cos(kc_{s}\eta_{dec})$$
$$\frac{\Delta T}{T}\Big|_{P} = A_{P}\sin(kc_{s}\eta_{dec})$$
$$\frac{\Delta T}{T}\Big|_{T}\frac{\Delta T}{T}\Big|_{T}\sum_{r=1}^{N}$$

P

Polarization and gravity waves

- Polarization can be split into a **gradient (***E***-mode)** and a **rotational** part (*B*-mode).
- Scalar (compressional) modes cannot generate circular polarization (at linear order): **they only couple to** *E*

• Lensing (*non-linear effect*) can convert E-mode into B-mode, disturbing the possible observational window for primordial gravity waves.

Gaussianity

• Primordial large scale non-Gaussianity can be constrained:

$$\Psi = \Psi_L + f_{NL} \left(\Psi_L^2 - \left\langle \Psi_L^2 \right\rangle \right)$$

$$\langle \Psi \Psi \Psi \rangle \approx f_{NL} \langle \Psi \Psi \rangle \langle \Psi \Psi \rangle$$
 3-point statistic

• Experimental constraints: *limits on non-Gaussinity*

WMAP: -58 < $f_{_{\rm NL}}$ < 134 (95%)

WMAP (next release): $|f_{NL}| < 20 (95\%)$

> Planck (2007): $|f_{NL}| < 5 (95\%)$

Interpretation: inflation

Adiabaticity, scale invariance, Gaussianity of perturbations are evidence of inflation.

Superluminal expansion	INFLATON
Origin of matter: reheating	INFLATON
Density perturbations	INFLATON

Evidence for inflation

Adiabaticity, scale invariance, Gaussianity of perturbations are evidence of inflation.

Supernova Cosmology Project Knop et al. (2003) 3 No Big Bang 68%, 90%, 95%, 99% 2 self interacting scalar field $\left(\frac{\ddot{a}}{a}\right) = \frac{8\pi}{3m_{\rm Pl}^2} \left[V\left(\phi\right) - \dot{\phi}^2\right]$ $\rho = \frac{1}{2}\dot{\phi}^2 + V(\phi)$ Expands Forever 0 Recollapses Eventually $P = \frac{1}{2}\dot{\phi}^2 - V(\phi)$ Closed φ Open Flat based on speculative and uncertain physics 2

0

3

just the behavior that we observe today!

Inflaton dynamics

- **Overdamped** self interacting scalar field $\ddot{\phi} + 3H\dot{\phi} + V' = 0$
- Drive the homogeneous Hubble expansion

$$H^{2} = \left(\frac{\dot{a}}{a}\right)^{2} = \frac{1}{3m_{Pl}^{2}} \left(\frac{1}{2}\dot{\phi}^{2} + V\right), \quad \frac{\ddot{a}}{a} = \frac{1}{3m_{Pl}^{2}} \left(V - \dot{\phi}^{2}\right),$$

 Inhomogeneous quantum fluctuations, of (comoving) wavenumber k

$$\delta\dot{\phi_k} + 3H\delta\dot{\phi_k} + \left(\frac{k^2}{a^2} + m^2\right)\delta\phi_k = 4\phi\dot{\Psi} + \dots$$

$$c = \frac{m_{_{Pl}}^2}{2} \left(\frac{V'}{V}\right)^2 = -\frac{\dot{H}}{H^2} <<1, \quad \eta = m_{_{Pl}}^2 \frac{V''}{V} = \frac{m^2}{H^2} <<1$$

0

Slow-roll parameters

♦ V(φ)

Physical scales exit the Hubble scale: solution to the cosmological problems...

$$\lambda(t) = \frac{a(t)}{k} \approx \frac{e^{Ht}}{k}, \quad H^{-1}(t) \approx const$$

... and generation of primordial perturbations.

Vacuum fluctuations

Canonical variable *U*, quantization of a scalar field in Minkowski with varying mass

$$u_{k}'' + \left(k^{2} + m^{2}a^{2} + \frac{a''}{a}\right)u_{k} = 0, \qquad u_{k} = a\delta\phi_{k}, \quad '=\frac{\partial}{\partial\eta}, \qquad \frac{a''}{a} < 0$$

[Mukhanov, Brandenberger and Feldman]

Massless limit: if *m* << *H* (inflaton is a light scalar field)

 $\delta \phi_k = \frac{u_k}{a} \approx \frac{e^{i\frac{\kappa}{aH}}}{a\sqrt{2k}} \left(1 + \frac{iaH}{k}\right), \quad \text{inflaton fluctuations$ **normalized**to the*Bunch-Davis*vacuum, i.e. zero point fluctuations in quasi de Sitter spacetime

Power spectrum of inflaton fluctuations

$$P_{\delta\phi}(k) = k^3 \left\langle \delta \phi_k^2 \right\rangle = \frac{k^2}{a^2} \left(1 + \frac{a^2 H^2}{k^2} \right) \approx H^2, \quad k \ll aH \quad \text{Large scales limit}$$

Linear evolution \Rightarrow Gaussian random variables

Fluctuations of any scalar light fields (m < 3H/2), frozen in with H amplitude

• Massive limit: if
$$m >> H$$
 (heavy field)
 $P_{\delta\phi}(k) \approx 0, \quad k << aH$

Adiabatic perturbations

• Inflaton: dominant component in the universe Inflaton fluctuations imprint curvature perturbations in the metric

$$\delta \rho \approx V \delta \phi \qquad \Psi \approx -\frac{3}{2} H \frac{\delta \rho}{\dot{\rho}} \approx \frac{V \delta \phi}{V' \dot{\phi}} \approx \frac{H}{m_{_{Pl}} \sqrt{\varepsilon}}$$

Power spectrum of scalar perturbations

$$P_{\Psi}(k) = \frac{k^3 \left\langle \delta \phi_k^2 \right\rangle}{m_{Pl}^2 \varepsilon} \approx \frac{H^2}{m_{Pl}^2 \varepsilon}, \quad \varepsilon = \frac{m_{Pl}}{2} \left(\frac{V'}{V}\right)^2$$

• Inflationary expansion also generates gravity waves (firm prediction of inflation)

Power spectrum of tensor perturbations

$$P_T(k) = \frac{k^3 \langle h_k^2 \rangle}{m_{Pl}^2} \approx \frac{H^2}{m_{Pl}^2}$$

Tensor/scalar ration depends upon ϵ

$$r = \frac{P_T(k)}{P_{\Psi}(k)} = \varepsilon$$

Weak scale dependence

The flatness of the inflaton potential predicts a small tilt in the scalar and tensor spectra

Scalar spectral index

$$\frac{d\ln\left\langle\delta\phi_{k}^{2}\right\rangle}{d\ln k} = n_{s} - 1$$

 -2ϵ

 $+2\eta$

slow-roll parameters

$$\varepsilon = \frac{m_{Pl}}{2} \left(\frac{V'}{V}\right)^2 = -\frac{\dot{H}}{H^2}, \quad \eta = m_{Pl} \frac{V''}{V} = \frac{m_{\phi}^2}{H^2}$$

- slow changing Hubble rate
- slow evolution outside the horizon
 - finite mass
 - metric backreaction (self gravity) -4ϵ

$$\frac{d \ln \langle h_k^2 \rangle}{d \ln k} = n_T = -2\varepsilon \qquad \qquad r = \frac{P_T(k)}{P_T(k)} = \varepsilon = -\frac{n_T}{2}$$

$$r = \frac{P_T(k)}{P_{\Psi}(k)} = \varepsilon = -\frac{n_T}{2}$$

$$n_s = 1 - 6\varepsilon + 2\eta$$

Non-Gaussianity from inflation

Standard single field inflation predicts small non-Gaussianities related to the scalar spectrum-tilt

Single field non-Gaussianity consistency relation

$$f_{NL} = \frac{n_s - 1}{4} = \frac{-3\varepsilon + \eta}{2} << 2$$

$$\Psi = \Psi_L + f_{NL} \left(\Psi_L^2 - \left\langle \Psi_L^2 \right\rangle \right)$$

[Maldacena, '02, Gruzinov; Criminelli and Zaldarriaga, '04]

Heuristic argument:

$$\delta \rho_{\phi} = V' \delta \phi + \frac{1}{2} V'' \delta \phi^2,$$

need $V''\delta\phi^2/V'\delta\phi$ not too small for non - Gaussianities

$$\frac{V''\delta\phi^2}{V'\delta\phi} = \frac{(V''/V)H}{(V'/V)} \approx \eta\Psi \approx \eta 10^{-5}$$

``easy" to disprove the simplest inflaton scenario

Primordial perturbations: testing inflation

Single scalar field inflation leads to firm predictions

 $P_{\Psi}(k) = \frac{k^3 \langle \delta \phi_k^2 \rangle}{m_{Pl}^2 \varepsilon} \approx \frac{H^2}{m_{Pl}^2 \varepsilon} = \frac{V}{m_{Pl}^4 \varepsilon}$

amplitude

energy scale of inflation

2

$$n_s = 1 - 6\varepsilon + 2\eta$$
 $\varepsilon = -\frac{H}{H^2}, \quad \eta = \frac{m_{\phi}}{H^2}$ slow-roll parameters

gravity waves observational consistency test

$$r = \frac{P_T(k)}{P_{\Psi}(k)} = -\frac{n_T}{2}$$

non-Gaussianity observational consistency test

$$f_{NL} = \frac{n_s - 1}{4} << 1$$

Tensor/Scalar to spectral index

Non-minimal scenarios or even more radical proposal are compatible with data

Change in the future: experimental limits on all these parameters are getting close to the interesting range where distinction between different proposal is possible.

Moduli problem: [Coughlan et al., '83]

Weakly coupled light scalar fields (m < < H) are not diluted during inflation and can dominate the universe and decay during or after nucleosynthesis

Late decay of light fields

Question: WHY???

Why not?

- Scalar fields are abundantly present in **supersymmetry** and **string theory**.
- The **Minimal Supersymmetric Standard Model** contains many flat directions (directions in the field space where V ~ 0): curvaton as flat direction of the MSSM.

[Mazumdar and Enqvist, '03; Enqvist, '04]

- *Light fields* (overdamped during inflation, m<<H) *inherits quantum fluctuations*
 - ⇒ effects on perturbations

Relaxing inflaton constraints

Inflation is very economical but severe constraints on inflaton potential

 $V = (10^{16} \text{GeV})^4$ and $m_{\phi} << H \approx V^{1/2} / m_{Pl} \sim 10^{13} \text{GeV}$

Some inflationary models motivated by particle physics (supersymmetry) require more freedom on energy scale of inflation and mass of the inflaton

[Dimopoulos and Lyth, 2002] [Dvali and Kachru, 2003]

The curvaton can generate perturbations and liberate the inflaton relaxing the constraints on inflaton potential: **division of labour**

Superluminal expansion	INFLATON
Origin of matter: reheating	INFLATON
Density perturbations	CURVATON

Drawback: more difficult to directly test inflation

- is there a motivation for new physics?
- do we really need the curvaton?

Observations in presence of light fields

- energy scale of inflation can be lowered $P_{\Psi}(k) \gg \frac{H^2}{m_{Pl}^2 \varepsilon} = \frac{V}{m_{Pl}^4 \varepsilon}$ energy scale of inflation $V << 10^{16} \text{ GeV}$ • spectral index
 - $n_{\sigma} \ll 1 6\varepsilon + 2\eta \qquad \longrightarrow \qquad m_{\phi} \sim H$
- violation of gravity waves observational consistency test

$$P_T(k) = \frac{V}{m_{Pl}^4} \longrightarrow r << \frac{P_T(k)}{P_{\Psi}(k)} = -\frac{n_T}{2}$$

• **violation** of non-Gaussianity observational consistency test

$$f_{NL} >> \frac{n_s - 1}{4}$$
 strong non-Gaussianities

• possible presence of isocurvature perturbations

Curvaton perturbations

Light field: vacuum quantum fluctuations

$$\delta \phi \approx H$$
, $\delta \sigma \approx H$, $\delta_{\sigma} \approx \frac{\delta \rho_{\sigma}}{\rho_{\sigma}} \approx \frac{m_{\sigma}^2 \sigma \delta \sigma}{m_{\sigma}^2 \sigma^2} \approx \frac{\delta \sigma}{\sigma}$

• Domination of the universe and decay before nucleosynthesis: imprints of curvaton perturbations

$$\Psi = -\frac{1}{2} \frac{\rho_{rad} \delta_{rad} + \rho_{\sigma} \delta_{\sigma}}{\rho} \approx r \delta_{\sigma} \approx r \frac{\delta \sigma}{\sigma} \quad \text{with} \quad \mathbf{r} = -\left(\frac{\rho_{\sigma}}{\rho}\right)_{dec}$$
domination

Curvaton perturbations are generically much larger than inflaton perturbations

$$\delta_{rad} = \delta_{\phi} \approx \frac{\delta \phi}{m_{Pl} \sqrt{\varepsilon}} = \frac{H}{m_{Pl} \sqrt{\varepsilon}} << \delta_{\sigma} = r \frac{\delta \sigma}{\sigma} = r \frac{H}{\sigma} \quad \text{if} \quad \sigma << \sqrt{\varepsilon} m_{Pl} \quad \text{and} \quad r \approx 1$$

New extra parameter: σ expectation value during inflation

Curvaton scale dependence

• Scalar spectral index

$$\frac{d\ln\left\langle\delta\sigma_{k}^{2}\right\rangle}{d\ln k} = n_{\sigma} - 1$$

slow-roll parameters

$$\varepsilon = \frac{m_{Pl}}{2} \left(\frac{V'}{V}\right)^2 = -\frac{\dot{H}}{H^2}, \quad \eta = m_{Pl} \frac{V''}{V} = \frac{m_{\phi}^2}{H^2}$$

- slow changing Hubble rate
- slow evolution outside the horizon
 - finite mass

$$+2\eta_{\sigma}=2\frac{m_{\sigma}^2}{H^2}$$

 -2ε

• metric backreaction (self gravity) -4ε

$$n_{\sigma} = 1 - 2\varepsilon + 2\eta_{\sigma} << n_{\phi}$$

Pure quartic inflation

$$V(\phi) = \lambda \phi^4$$

Quartic inflation with a curvaton

Non-Gaussianities

Inflaton generated non-Gaussianities are constrained to be small due to slow-roll dynamics

 $V'\delta\phi >> \frac{1}{2}V''\delta\phi^2$

If $r = (\rho_{\sigma} / \rho)_{dec}$ small, curvaton perturbations are suppressed, and **non-linear fluctuations** in the curvaton field must become important leading to **stronger non-Gaussianities**.

$$\delta \rho_{\sigma} = m_{\sigma}^{2} \left(\sigma \delta \sigma + \frac{1}{2} \delta \sigma^{2} \right) \quad \text{and} \quad \Psi = r \left(\frac{\delta \sigma}{\sigma} + \frac{1}{2} \frac{\delta \sigma^{2}}{\sigma^{2}} \right)$$

with $\mathbf{r} = \left(\rho_{\sigma} / \rho \right)_{dec}$

Curvaton generated non-Gaussianities

$$f_{NL} = \frac{1}{r}$$
 with $\Psi = \Psi_L + f_{NL} \left(\Psi_L^2 - \left\langle \Psi_L^2 \right\rangle \right)$

[Lyth, Ungarelli and Wands, 03]

may be close to the limit of Planck satellite (2007)

Isocurvature perturbations

• If the curvaton is one of the flat directions of MSSM it may be possible to see it in the laboratory if LHC sees SUSY

• Closer connection to particle physics

Superluminal expansion	INFLATON
Origin of matter: reheating	CURVATON
Density perturbations	CURVATON

 Baryons and leptons may have been generated by the curvaton (Affleck-Dine field)

[Hebecker, March-Russel, Yanagida, '02; Moroi and Murayama, '02;

MacDonald, '03]

(Small) Isocurvature perturbations

Summary

Perturbations:

- Adiabatic
- Gaussian
- Scale-invariant

Observables	Values	
Ρζ	$(2 \times 10^{-5})^2$	
n _s	$\simeq 0$	
r	$\simeq 0$	
f _{NL}	$\simeq 0$	
Isocurvature	$\simeq 0$	

Summary

Perturbations: Fine structure:

- Adiabatic \rightarrow Small isocurvature perts
- Gaussian \rightarrow Small non-Gaussianities
- Scale-invariant → Small deviation from scale-invariance

Observables	Values	INFLATION	CURVATON
Ρζ	$(2 \times 10^{-5})^2$	Y	Y
n _s	≃ 0	Y	Ν
r	$\simeq 0$?	N
f _{NL}	~0	N	Y
Isocurvature	$\simeq 0$	N	Y

A two fields tale

Consider two fields during inflation: one is heavy and the other light: $m >> m_{\gamma}$ $\mathcal{L} = \frac{1}{2}\dot{\phi}^2 + \frac{1}{2}\dot{\chi}^2 - V(\phi,\chi), \quad V(\phi,\chi) = \frac{1}{2}m^2\phi^2 + \frac{1}{2}m_\chi^2\chi^2$ **Heavy field:** Light field: $\ddot{\chi} + 3H\dot{\chi} + m_{\chi}^2\chi = 0$ $\ddot{\phi} + 3H\dot{\phi} + m^2\phi = 0$ - Initially H >> m: ϕ is frozen $H >> m_{\gamma}$: Frozen field. Practically $m_{\gamma} = 0$ φ - Eventually H<<m: ϕ starts oscillating

$$\phi = \Phi e^{-\frac{3}{2}Ht} \cos(mt)$$

Damped oscillations: $\phi \sim a^{-3/2} \sim exp(-3/2Ht)$

From heaviness to lightness

[Langlois, FV, '04]

We add a coupling between the two fields:

$$V(\phi, \chi) = \frac{1}{2}m^2\phi^2 + \frac{1}{2}m_{\chi}^2\chi^2 + \frac{1}{2}g^2\phi^2\chi^2$$
 coupling

Heavy field:

$$\ddot{\phi} + 3H\dot{\phi} + m^2\phi = 0$$

- Initially $H >> m: \phi$ is frozen

φ

- Eventually H<<m: ϕ starts oscillating

$$\phi = \Phi e^{-\frac{3}{2}Ht} \cos(mt)$$

Damped oscillations: $\phi \sim a^{-3/2} \sim exp(-3/2Ht)$

Light field: $m_{\rm eff}^2(t) = g^2 \phi^2(t)$ H << $m_{eff}^2 = g^2 \phi^2 = \text{contant:}$ χ is initially heavy! $\chi(t)$ quickly rolls to zero; $\chi(t) \rightarrow 0$

Field with oscillating mass during inflation:

$$\begin{split} \ddot{\delta\chi}_k + 3\frac{\dot{a}}{a}\dot{\delta\chi}_k + \left(\frac{k^2}{a^2} + g^2\phi^2\right)\delta\chi_k &= 0\\ m_\chi^2 = g^2\Phi^2 e^{-3Ht}\cos^2(mt) \end{split}$$

 χ later **becomes light**: $m_{eff}^2 \rightarrow 0$

From heaviness to lightness

Conclusion

1. More precise data will allow/require to study more detailed models of inflation and cosmological perturbations

- 4. Several mechanism can produce primordial perturbations
- inflaton perturbations during inflation
- *late time decay of a light scalar field (curvaton)*
- 9. Distinct observational tests
- gravitational waves (polarization or advanced LISA)
- (non-)Gaussianities
- residual isocurvature perturbations

